Electron relaxation in the quantum-Hall-effect geometry: One- and two-phonon processes

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The longitudinal-optical (LO) phonon-assisted electron relaxation has been investigated in a two-dimensional electron system in the quantum-Hall-effect geometry. The phonon emission rates versus inter-Landau-level separations are calculated. One LO- and two LO+LA-phonon emission processes via polar optical and deformation acoustical interactions are considered. To obtain a finite relaxation rate associated with one-phonon emission, the allowance for the Landau-level broadening or for the LO-phonon dispersion is made. Below the LO-phonon energy ω_{LO} within an energy range of the order of $\hbar\sqrt{\omega_B/\tau}$ the one-phonon relaxation rate exceeds 1 ps^{-1}. (Here τ is the relaxation time deduced from the mobility and ω_B is the cyclotron frequency. In $GaAs/Al_xGa_{1-x}As$ heterostructure with the mobility $\mu = 25\ V^{-1}s^{-1}m^2$ this range makes up 0.7 MeV.) The two-phonon emission has a significant contribution to the relaxation above ω_{LO}. At energy separations of the order of ca_B^{-1} (c is the sound velocity and a_B is the magnetic length) the LO+LA-phonon emission provides a mechanism of subpicosecond relaxation while in a wide energy range of the order of $\hbar\omega_B$ the subnanosecond relaxation can be achieved.

I. INTRODUCTION

In recent years there has been much success in using electron-optical phonon interaction to probe the various electronic properties of a two-dimensional electron system (2DES) (see Ref. 1 and references cited therein). The electron-longitudinal-optical (LO) phonon scattering rates in the 2DES have been calculated both for usual bulk LO phonons[2–6] and bulklike confined LO, interface surface optical (SO) modes.[1,7–11] If a magnetic field is applied normal to the electron layer, the electron motion is fully quantized. In this quantum-Hall-effect (QHE) geometry, there has been considerable experimental and theoretical interests in the (resonant) two-dimensional (2D) magnetopolarons, cyclotron resonances,[12–18,6] 2D electron-phonon bound states,[19] cyclotron-phonon resonances,[20] magnetophonon,[21–24] and energy loss[25–27] effects. Phonon emission is the basic mechanism that ensures electron relaxation in various experiments of light radiation[28,29] or Auger[30] processes, etc., where electrons are first created in higher Landau states. However, in such effectively 0D systems with rather thin electron layers and subjected to rather strong magnetic fields, a large separation between Landau levels cannot be covered by an acoustical LA phonon[31] so that the multiphonon 2LA-phonon emission processes become more efficient.[32] An optical-phonon emission requires a precise resonance $\Delta l\omega_B = \omega_{LO}$, $\Delta l = 1, 2, 3, \ldots$, in this regime ($\omega_B$ and ω_{LO} are cyclotron- and LO-phonon frequencies). As far as it moves off from the resonance, the efficiency of this process sharply falls so that out of resonance the LO-phonon emission will be possible by accompaniment of the LA-phonon emission via the two-phonon emission mechanism. Possibly for this reason, except for the work,[33] calculations of the electron-optical-phonon relaxation rates have been restricted thus far to the zero magnetic field case where the LO-phonon emission is the dominant relaxation mechanism with subpicosecond emission time. In Ref. 33, the direct calculation of the electron-phonon scattering rates have been carried out for a quantum-well structure with infinitely high confining wells. It has been obtained, however, that in the case of the polar optical (PO)-phonon scattering, the relaxation rate diverges. This divergence has been physically attributed to the complete quantization of the electron spectrum although it does not make clear why the relaxation rate diverges only for the polar interaction. We suppose that this divergence can be caused by use of the momentum-conservation approximation[4] in the extreme quantum limit, while this approximation fails in this limit.[4] It may be seen later that calculations without the use of the momentum-conservation approximation allow us to avoid this divergence.

Recently, to estimate the intensity of infrared cyclotron radiation from 2D electron gas in a double-barrier quantum-well structure, the LA, 2LA phonon, photon emission, and Auger processes happening in the QHE geometry have been analyzed.[34] The LO+LA-phonon emission processes have not been considered. Although, one may expect that in some experimental situations the electron relaxation from a higher Landau level directly into the lowest one can turn out crucial either due to the LO-phonon emission or, if it is far from the resonance, due to the LO+LA-phonon emission.

The LO-phonon-assisted electron relaxation has been studied in quantum dots.[35,36] The multiphonon relaxation rate calculation in a GaAs quantum dot[36] has indicated the significance of LO+LA processes in an effectively 0D system, which create a window of rapid subnanosecond relaxation. Up to now, however, the study of the LO-phonon-assisted multiphonon processes in the

QHE geometry is missing from the literature.

The aim of the present work is to study theoretically the polar PO-phonon-assisted electron relaxation in the QHE geometry. The calculations incorporate one- (Sec. II) and two-phonon (Sec. III) emission processes for the electron relaxation between bulk Landau states. To obtain a finite relaxation rate in the case of the one-phonon processes, the Landau-level broadening or the dispersion of PO phonons has been taken into account. The electron interaction with bulk PO and interface SO phonons are considered. In a quantizing magnetic field, the emission of a LA phonon via piezoelectric interaction is suppressed in comparison with deformation interaction.[37] Therefore calculations of the LO+LA-phonon emission have been carried out for the deformation potential of DA phonons and for the Frölich coupling of PO phonons. Particular attention is given to the comparison of the electron relaxation in two ways: the relaxation in two emission acts (a PO-phonon emission with a subsequent LA- or 2LA-phonon emission) with the two-phonon relaxation via PO+DA-phonon emission mechanism between the same Landau levels. Especially the emission processes in magnetic fields up to 10 T are considered. On the one hand in such fields the resonance $\Delta l\omega_B = \omega_{\rm PO}$ can be achieved for small values of $\Delta l = 1, 2, 3$. On the other hand, the acoustical-phonon emission at electron transitions between such distant Landau levels is still a rather efficient effect[31,32,38] observable in experiment.[28–30] Numerical results are illustrated by consideration of the PO-phonon-assisted relaxation between Landau levels $l = 3$ and $l = 0$. Section IV contains concluding remarks.

II. ONE-PHONON EMISSION

The 2DES embedded in an elastic medium of a single heterostructure and subjected to a quantizing magnetic field in z direction normal to the electron layer is considered here. The scattering probability at which one phonon of a given mode s and a 3D wave vector $\mathbf{q} = (\mathbf{q}_\perp, q_z)$ is emitted by an electron of the 2DES is given by Fermi's golden rule as

$$W^{s\mathbf{q}}_{nlk\to n'l'k''} = \frac{2\pi}{\hbar}|M^{s\mathbf{q}}_{nlk\to n'l'k'}|^2\delta\left[\varepsilon_{nl} - \varepsilon_{n'l'} - \hbar\omega_s(q)\right]. \tag{1}$$

The electron eigenstate $|nlk\rangle$ is labeled by a subband index n corresponding to the quantization of electron motion in the z direction, by a Landau index l, and an electron momentum x-component k. In the case of the bulk Landau states, the eigenenergy ε_{nl} does not depend on the quantum number k, which counts the degeneracy of a Landau level. The frequency of a phonon in a given mode is $\omega_s(\mathbf{q})$. Because corresponding wave functions $|nlk\rangle$ are factored into a subband function $|n\rangle$ and a Landau oscillator function $|lk\rangle$, the matrix element is represented in the form

$$M_{nlk\to n'l'k'} = B^s(q)Q_{ll'}(q_\perp)I^s_{nn'}(q_s)\delta_{k',k-q_x}, \tag{2}$$

where the modulus squared of the form factor $Q_{ll'}$ is given by[39]

$$Q^2_{ll'}(q_\perp) = \frac{l'!}{l!}e^{-t}t^{l-l'}\{L^{l-l'}_{l'}(t)\}^2, \quad t = \tfrac{1}{2}a_B^2q_\perp^2. \tag{3}$$

Here $L^{l'}_l$ is the Laguerre polynomial and a_B is the magnetic length. One can see that $Q_{ll'}$ does not depend on momenta k and k' apart but depends only on $q_\perp$, which is bound up with the axial symmetry of the magnetic field. In the case of $n = n' = 0$ using the model wave function[40]

$$|n\rangle = \frac{z}{\sqrt{2d^3}}\exp\left(-\frac{z}{2d}\right) \tag{4}$$

for the subband part of the matrix element, one can obtain the following explicit forms:

$$|I^{\rm PO,DA}_{00}(q_z)|^2 = (1+q_z^2d^2)^{-3},$$
$$|I^{\rm SO}_{00}(q_\perp)|^2 = (1+q_\perp d)^{-6}, \tag{5}$$

where d is the length scale of the 2DES in the z direction. It is assumed that d is the smallest parameter of the problem so that the electron relaxation between Landau levels of the lowest subband should be considered.

In Eq. (2), the factors $B^s(q)$ characterize the electron-phonon interaction and are given by

$$B^{\rm DA}(q) = iq^{1/2}\frac{B^{1/2}_{\rm DA}}{L^{3/2}}, \quad \frac{1}{\bar\tau_{\rm DA}} = \frac{mB_{\rm DA}p^3_{\rm PO}}{\pi\hbar^2c} = \frac{\Xi^2p^3_{\rm PO}}{2\pi\hbar\rho c^2} = \frac{1}{4\ \rm ps},$$
$$B^{\rm PO}(q) = \frac{1}{iq}\frac{B^{1/2}_{\rm PO}}{L^{3/2}}, \quad \frac{1}{\bar\tau_{\rm PO}} = \frac{mB_{\rm PO}}{\pi\hbar^3p_{\rm PO}} = 2\alpha_{\rm PO}\omega_{\rm PO} = \frac{1}{0.14\ \rm ps},$$
$$B^{\rm SO}(q_\perp) = \frac{1}{iq_\perp^{1/2}}\frac{B^{1/2}_{\rm SO}}{L}, \quad \frac{1}{\bar\tau_{\rm SO}} = \frac{mB_{\rm SO}}{\pi\hbar^3p_{\rm SO}} = \frac{\alpha_{\rm SO}\omega_{\rm SO}}{\alpha_{\rm PO}\omega_{\rm PO}}\frac{1}{\bar\tau_{\rm PO}} = \begin{cases}(0.50\ \rm ps)^{-1} & \text{for} \quad x = 0.3\\ (0.34\ \rm ps)^{-1} & \text{for} \quad x = 1.\end{cases} \tag{6}$$

Here L is the normalization length, m is the electron effective mass, c is the sound velocity, and $\hbar p_s = \sqrt{2m\hbar\omega_s}$. The constants $B_{\rm DA}$, $B_{\rm PO}$ (Ref. 41), and $B_{\rm SO}$ are bound up, respectively, with the deformation potential constant Ξ, the usual Frölich constant $\alpha_{\rm PO}$, and the electron-SO-phonon coupling $\alpha_{\rm SO}$. The constant $\alpha_{\rm SO}$ for interface modes in a single heterostructure is defined on the analogy of the Frölich constant

$$\alpha_{\rm SO} = \frac{e^2}{\bar{\kappa}_{\rm SO} v_{\rm SO}}, \quad v_s = \frac{\hbar p_s}{m}, \tag{7}$$

where $\bar{\kappa}_{\rm SO}$ has the form[20,1]

$$\bar{\kappa}_{\rm SO} = \frac{\omega_{\rm SO}^2}{2} \sum_{\nu=1}^{2} \kappa_{\infty\nu} \frac{\omega_{{\rm PO}\nu}^2 - \omega_{{\rm TO}\nu}^2}{(\omega_{\rm SO}^2 - \omega_{{\rm TO}\nu}^2)^2}. \tag{8}$$

Here $\omega_{{\rm PO}\nu}$ and $\omega_{{\rm TO}\nu}$ are the frequencies of the longitudinal and transverse phonons, respectively, and the high-frequency dielectric constant is $\kappa_{\infty\nu}$. Index $\nu = 1, 2$ denotes the different media in contact at a heteroface. The frequency of the interface SO phonon is the root of the following equation:[42]

$$\sum_{\nu=1}^{2} \epsilon_\nu(\omega) = 0, \quad \epsilon_\nu(\omega) = \kappa_{\infty\nu} \frac{\omega^2 - \omega_{{\rm LO}\nu}^2}{\omega^2 - \omega_{{\rm TO}\nu}^2}, \tag{9}$$

where ϵ_ν is the dielectric function of the medium ν. Taking for physical parameters of a GaAs/Al_xGa_{1-x}As heterojunction $\omega_{\rm LO1} = 36.62$ MeV, $\omega_{\rm TO1} = 33.3$ MeV, $\kappa_{\infty 1} = 10.9$, and $\omega_{\rm LO2} = 49.8$ MeV, $\omega_{\rm TO2} = 45.1$ MeV, $\kappa_{\infty 2} = 12.0$ if $x = 0.3$ or $\omega_{\rm LO2} = 50.0$ MeV, $\omega_{\rm TO2} = 44.9$ MeV, $\kappa_{\infty 2} = 8.5$ if $x = 1$, one can obtain from Eqs. (7)–(9) $\omega_{\rm SO} = 34.4$ MeV and $\alpha_{\rm SO} = 0.31\alpha_{\rm PO}$ if $x = 0.3$ and $\omega_{\rm SO} = 34.62$ MeV and $\alpha_{\rm SO} = 0.43\alpha_{\rm PO}$ if $x = 1$, which allow us to have the numerical values cited in Eq. (6).

In Eq. (6) instead of the constants B_s the nominal times $\bar{\tau}_s$ (Ref. 41) are defined, which give a visual view of scattering rates. It can be seen that $\bar{\tau}_{\rm SO}$ is 2.5–3.5 times larger than $\bar{\tau}_{\rm PO}$, i.e., generally speaking, the interaction with interface modes should be weaker than with bulk modes.

A. The PO-phonon scattering

The scattering rate of an electron between two Landau levels l and l' interacting with the PO phonon can be represented in the form

$$\begin{aligned}\frac{1}{\tau_{l\to l'}^{\rm PO}} &= \sum_{\mathbf{q},k'} W_{nlk\to n'l'k'} \\ &= \frac{2\pi^2}{L^3} \frac{v_{\rm PO}}{\bar{\tau}_{\rm PO}} \sum_q \frac{1}{q^2} Q_{ll'}^2(q_\perp) \\ &\quad \times |I_{00}(q_z)|^2 \delta[\Delta l\omega_B - \omega_{\rm PO}(q)] \\ &= \frac{\sqrt{\omega_B \omega_{\rm PO}}}{\bar{\tau}_{\rm PO}} D_{ll'}^{\rm PO}(\gamma),\end{aligned} \tag{10}$$

where the overlap integral $D_{ll'}^{\rm PO}(\gamma)$ is given as

$$D_{ll'}^{\rm PO}(\gamma) = \int_0^\infty dx dt \frac{Q_{ll'}^2(t)}{(t+x^2)(1+\gamma^2 x^2)^3} \times \delta[\Delta l\omega_B - \omega_{\rm PO}(\mathbf{q})], \quad \gamma^2 = 2\frac{d^2}{a_B^2}. \tag{11}$$

The PO-phonon emission is governed by the density of final states of a two-particle system: an electron at the level l' and a PO phonon. Both particles have an infinite mass so that this system does not have a continuous spectrum. Therefore, to obtain the finite relaxation rate, the Landau-level broadening or the PO-phonon dispersion is to be taken into account.

1. *Allowance for the Landau-level broadening*

For actual calculations of the scattering rate (10), we have to smear the δ function in Eq. (11). Replacing the δ function by a Lorentzian characterized by the total width $\hbar/\tau$, where τ is the relaxation time deduced from the mobility (e.g., $\mu = 25\ {\rm V^{-1}\,s^{-1}\,m^2} \triangleq \tau = 10$ ps), it is easy to obtain

$$D_{ll'}^{\rm PO}(\gamma) = \frac{\tau}{1 + \tau^2(\omega_{\rm PO} - \Delta l\omega_B)^2} d_{ll'}^{\rm PO}(\gamma), \tag{12}$$

$$d_{ll'}^{\rm PO}(\gamma) = \frac{1}{16} \int_0^\infty dt Q_{ll'}^2(t) \frac{8 + 9\gamma\sqrt{t} + 3\gamma^2 t}{\sqrt{t}(1+\gamma\sqrt{t})^3}, \tag{13}$$

$$d_{l0}^{\rm PO}(0) = \sqrt{\pi} \frac{(2l-1)!!}{2^{l+1} l!}. \tag{14}$$

One may see that $x^2 \simeq t \simeq 1$, i.e., $q_z \simeq q_\perp \simeq a_B^{-1}$, have the main contribution to the integral (11). This isotropic distribution in momenta of emitted phonons is conditioned by the long-range nature of the polar interaction. It forces the scattering time to be governed by the minimum size in the momentum space: $\min\{a_B^{-1}, d^{-1}\}$. Recall that in the case of the DA-phonon scattering, the phonon emission is anisotropic in the momentum space: $q_z \simeq d^{-1} \gg q_\perp \simeq a_B^{-1}$.[38,43,44]

In magnetic fields below 10 T using $d = 3$ nm as a typical value for a GaAs/Al_xGa_{1-x}As heterostructure, we have $\gamma^2 \ll 1$. Therefore in rough estimates the value of $d_{ll'}^{\rm PO}$ at $\gamma = 0$ can be used.

The result of numerical calculations according to Eqs. (10) and (14) at $\gamma \neq 0$ and $\tau = 10$ ps is shown in Fig. 1. The diagram of the scattering rate dependence on the inter-Landau-level separation for $l = 3$ and $l' = 0$ represents a narrow peak at the PO-phonon energy with peak value exceeding 10^2 ps^{-1}. One may see that for detuning 0.2 MeV, the scattering time $\tau_{3\to 0}^{\rm PO}$ increases by an order.

Note that in the case of the relaxation between Landau levels $l = 1$ and $l' = 0$, the scattering-rate dependence on the interlevel spacing is also represented as a Lorentzian with approximately the same width and with the peak value 1.17 times larger than that of the relaxation between levels $l = 3$ and $l' = 0$.

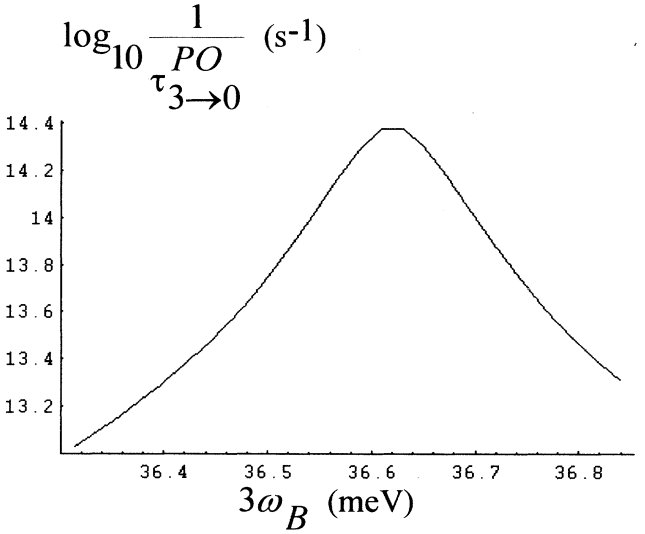

FIG. 1. Calculated Landau-level broadening contribution to the PO-phonon emission rate versus the interlevel spacing $\Delta l\omega_B$ for the electron transitions between $l = 3$ and $l' = 0$ levels in the vicinity of the PO-phonon energy $\omega_{\rm PO}$.

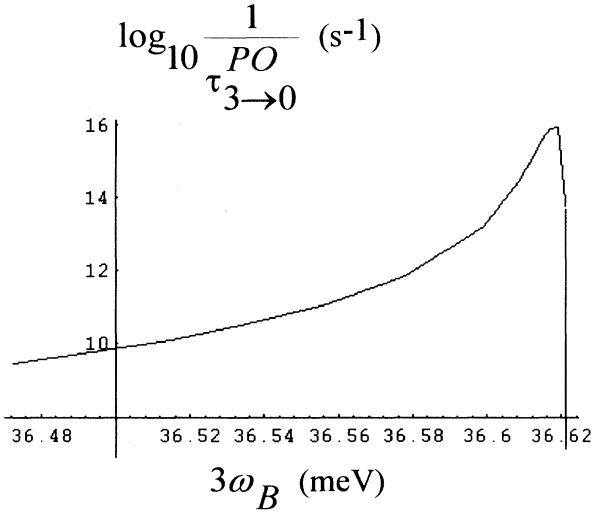

FIG. 2. The PO-phonon dispersion contribution to the PO-phonon emission rate evaluated numerically as a function of the interlevel spacing $\Delta l\omega_B$ for the electron transitions between $l = 3$ and $l' = 0$ levels.

2. *Allowance for the PO-phonon dispersion*

In this case taking $\omega_{\rm PO}(q) = \omega_{\rm PO}(1 - \frac{q^2}{2q_0^2})$ with $\omega_{\rm PO} = 36.62$ MeV and $q_0 = 0.185\ \text{nm}^{-1}$ the overlap integral (11) can be reduced to a one-dimensional integral of the form

$$D_{ll'}^{\rm PO}(\gamma) = \int_0^\infty dxdt \frac{Q_{ll'}^2(t)}{(t+x^2)(1+\gamma^2x^2)^3} \times\delta\left(\Delta l\omega_B - \omega_{\rm PO} + \frac{t+x^2}{2x_0^2}\omega_{\rm PO}\right) = \frac{a}{\omega_{\rm PO} - \Delta l\omega_B}\int_0^1 dx \frac{Q_{ll'}^2(y)}{(1+b^2x^2)^3},$$
$$x_0^2 = q_0^2a_B^2/2,\quad y = a^2(1-x^2),$$
$$a^2 = (1 - \Delta l\omega_B/\omega_{\rm PO})q_0^2a_B^2,\quad b^2 = a^2\gamma^2, \qquad (15)$$

which is evaluated numerically for the electron transitions between $l = 3$ and $l' = 0$ Landau levels (Fig. 2). Because of the energy conservation the electron relaxation via this mechanism is possible only on the low magnetic field side, $3\omega_B < \omega_{\rm PO}$. The scattering rate has a sharp and strongly asymmetric peak immediately below $\omega_{\rm PO}$ with the peak value exceeding 1 fs^{-1}. It can be seen from Eqs. (10) and (15) that the scattering rate decreases exponentially for energies below the peak value while it drops more strongly for energies above the peak value (Fig. 2). As following from the comparison of diagrams of Figs. 1 and 2, the PO-phonon dispersion contribution is greater by an order at the relaxation peak than the Landau-level broadening contribution. The former remains significant even for samples of exceptional quality ($\mu > 100\ \text{V}^{-1}\text{s}^{-1}\text{m}^2$). On the other hand, the allowance for the Landau-level broadening gives rise to a symmetric peak at $\omega_{\rm PO}$ thereby providing the one-phonon relaxation on the upper side of the resonance.

Note that the electron-PO-phonon relaxation rate calculated in a GaAs quantum dot as a function of a dot diameter or, that is the same, of an interlevel spacing has an approximately 3–4 times narrow peak than the peak of the PO-phonon dispersion contribution obtained here in the QHE geometry. Both have approximately the same peak value exceeding 1 fs^{-1}.

B. The SO-phonon scattering

In the case of a single heterostructure, the interface SO phonons do not exhibit dispersion. Therefore for SO phonons, we consider only the Landau-level broadening contribution to the relaxation. To obtain the scattering rate, we have to replace all indexes PO by SO and to take the overlap integral $D_{ll'}^{\rm SO}$ in the form

$$D_{ll'}^{\rm SO}(\gamma) = \frac{\tau}{1+\tau^2(\omega_{\rm SO} - \Delta l\omega_B)^2} d_{ll'}^{\rm SO}(\gamma), \qquad (16)$$

$$d_{ll'}^{\rm SO}(\gamma) = \int_0^\infty dt \frac{Q_{ll'}^2(t)}{\sqrt{t}(1+\gamma\sqrt{t})^6}, \qquad (17)$$

$$d_{l0}^{\rm SO}(0) = \sqrt{\pi}\frac{(2l-1)!!}{2^l l!}. \qquad (18)$$

Note that $t \simeq 1$, i.e., $q_\perp \simeq a_B^{-1}$ have contributed heavily to both integrals in Eqs. (14) and (18). It is clear that the SO-phonon relaxation peak is shifted on the low-energy side by $\omega_{\rm PO} - \omega_{\rm SO} \approx 2$ MeV. Taking $d = 3$ nm and $B = 7$ T, it is easy to obtain $d_{30}^{\rm SO} = 0.023$ and $d_{30}^{\rm PO} = 0.1$. Therefore at the relaxation peak the PO-phonon scattering rate is to SO-phonon scattering rate as

$$\max \frac{\tau_{3\to 0}^{\mathrm{SO}}}{\tau_{3\to 0}^{\mathrm{PO}}} = \frac{\bar{\tau}_{\mathrm{SO}}}{\bar{\tau}_{\mathrm{PO}}}\sqrt{\frac{\omega_{\mathrm{PO}}}{\omega_{\mathrm{SO}}}}\frac{d_{30}^{\mathrm{PO}}}{d_{30}^{\mathrm{SO}}} = \begin{cases} 15.9 & \text{if } x = 0.3 \\ 10.8 & \text{if } x = 1. \end{cases} \tag{19}$$

Thus the SO-phonon relaxation at least by an order is weaker than the relaxation via PO-phonon emission.

III. TWO-PHONON PROCESSES

To calculate the transition probability due to the two-phonon emission, the quasiparticle approach has been exploited. Therefore, the two-phonon contribution to the electron relaxation in the first order of the perturbation theory arising from the interaction Hamiltonian expanded up to second order in the phonon displacement operators can be neglected.[41] The second-order contribution to the probability of an electron transition from a bulk Landau state $|lk\rangle$ into a state $|l'k'\rangle$ of the same lowest subband (for brevity the subband index will be omitted), at which one PO phonon with a 3D wave vector $\mathbf{q}$ and one DA phonon with a 3D wave vector $\mathbf{q}'$ are emitted, is given by

$$W_{lk\to l'k'}^{+\mathbf{q},+\mathbf{q}'} = \frac{2\pi}{\hbar^4}\frac{B_{\mathrm{PO}}B_{\mathrm{DA}}}{L^6}\frac{q'}{q^2}\delta_{k',k-q_x-q_x'}\delta(\Delta l\omega_B - \omega_{\mathrm{PO}} - cq') \times\left|\sum_{\bar{l},\bar{k}}\left\{\frac{\langle l'k'|\exp(-i\mathbf{q}'\mathbf{r})|\bar{l}\bar{k}\rangle\langle\bar{l}\bar{k}|\exp(-i\mathbf{q}\mathbf{r})|lk\rangle}{(l-\bar{l})\omega_B - \omega_{\mathrm{PO}}} + \frac{\langle l'k'|\exp(-i\mathbf{q}\mathbf{r})|\bar{l}\bar{k}\rangle\langle\bar{l}\bar{k}|\exp(-i\mathbf{q}'\mathbf{r})|lk\rangle}{(l-\bar{l})\omega_B - cq'}\right\}\right|^2. \tag{20}$$

The PO+DA-phonon emission rate at electron transitions between Landau levels l and l' can be obtained after summing up over the phonon and the final electron momenta: $\mathbf{q}, \mathbf{q}'$, and k'

$$\frac{1}{\tau_{l\to l'}^{\mathrm{PO+DA}}} = \sum_{k',\mathbf{q},\mathbf{q}'} W_{lk\to l'k'}^{+\mathbf{q},+\mathbf{q}'}. \tag{21}$$

Note that after summation over k' the result cannot depend on gauge noninvariant quantum number k. The explicit calculation of the emission rate can be carried out by considering separately the following two situations: $(\Delta l\omega_B - \omega_{\mathrm{PO}}) \ll \omega_B, \omega_{\mathrm{PO}}$ and $(\Delta l\omega_B - \omega_{\mathrm{PO}}) \lesssim \omega_B, \omega_{\mathrm{PO}}$, where the relaxation is qualitatively different.

(a) $(\Delta l\omega_B - \omega_{\mathrm{PO}}) \ll \omega_B, \omega_{\mathrm{PO}}$. In this energy range the main contribution to the sum over intermediate states $\bar{l}$ in Eq. (20) has the state $\bar{l} = l'$ in the first term and the state $\bar{l} = l$ in the second term. Therefore, the emission rate can be rewritten as

$$\frac{1}{\tau_{l\to l}^{\mathrm{PO+DA}}} = \frac{2\pi}{\hbar^4}\frac{B_{\mathrm{PO}}B_{\mathrm{DA}}}{L^6c^2}\sum_{\mathbf{q},\mathbf{q}'}\delta(\Delta l\omega_B - \omega_{\mathrm{PO}} - cq') \times\frac{Q_{ll'}^2(q_\perp)}{q^2q'}|Q_{l'l'}(q_\perp') - Q_{ll}(q_\perp')|^2 \times|I_{00}(q_z)|^2|I_{00}(q_z')|^2, \tag{22}$$

which again can be reduced to a one-dimensional integral of the form

$$\frac{1}{\tau_{l\to l}^{\mathrm{PO+DA}}} = \frac{1}{\bar{\tau}_{\mathrm{PO+DA}}}\frac{\omega_B}{cp_{\mathrm{PO}}}d_{ll'}^{\mathrm{PO}}(\gamma)\int_0^{\beta^2} dt'\exp(-t') \times\frac{[L_{l'}(t') - L_l(t')]^2}{\sqrt{\beta^2 - t'}[1+\gamma^2(\beta^2-t')]^3}, \qquad \beta = \frac{\Delta l\omega_B - \omega_{\mathrm{PO}}}{\sqrt{2}c}a_B, \tag{23}$$

where a nominal relaxation time is introduced

$$\frac{1}{\bar{\tau}_{\mathrm{PO+DA}}} = \frac{1}{4\bar{\tau}_{\mathrm{PO}}\bar{\tau}_{\mathrm{DA}}cp_{\mathrm{PO}}}, \tag{24}$$

depending only on heterojunction parameters. For a $GaAs/Al_xGa_{1-x}As$ heterojunction with $d = 3$ nm, we have $\bar{\tau}_{\mathrm{PO+DA}} \approx 2.9$ ps.

Actually in this energy range of $\Delta l\omega_B$ the electron transitions take place in the following two ways. (i) Remaining on the level l an electron emits a DA phonon thereby the electron-phonon system is forced to transit into a virtual intermediate state. After the electron emits a second PO phonon, the electron-phonon system turns out in the final state with the real electron on the Landau level l' and with two real DA and PO phonons. (ii) In the second way an electron first emits a PO phonon and simultaneously makes a transition to the level l'. By the emission of a second DA phonon, this virtual intermediate state is forced to transit into the same final state. In both cases, 3D wave vectors of emitted PO and DA phonons are not correlated. The 3D wave vectors of emitted PO phonons have the same isotropic distribution in the momentum space as in the one-phonon emission case. While the momentum distribution of the emitted DA phonons is different in different ranges of $\Delta l\omega_B$. Immediately above the PO-phonon energy $\hbar\omega_B$ so that $\beta \ll 1$ [it corresponds to energies for which $(\Delta l\omega_B - \omega_{\mathrm{PO}}) \ll c/a_B$], electrons emit DA phonons with $q_\perp' \simeq q_z' \simeq a_B^{-1}$. In this case the following asymptotic expression is obtained for the emission rate:

$$\frac{1}{\tau_{l\to l'}^{\mathrm{PO+DA}}} = \frac{16(l-l')^2}{15}\frac{\beta^5}{\bar{\tau}_{\mathrm{PO+DA}}}\frac{\omega_B}{cp_{\mathrm{PO}}}d_{ll'}^{\mathrm{PO}}(\gamma). \tag{25}$$

The essential part of the magnetic field dependence in this range is given by

$$\frac{1}{\tau^{\mathrm{PO+DA}}_{l\to l'}} \propto (B - B_{\Delta l})^5, \tag{26}$$

i.e., the relaxation is enhanced as a fifth power of $B - B_{\Delta l}$ with increase in magnetic field. Here $B_{\Delta l}$ is the magnetic field for which $\Delta l\omega_B = \omega_{\mathrm{PO}}$ takes place.

In the opposite limiting case of $\beta \gg 1$, actually for energies $c/d \lesssim (\Delta l\omega_B - \omega_{\mathrm{PO}}) \ll \omega_B, \omega_{\mathrm{PO}}$, electrons emit DA phonons with $q'_\perp \simeq a_B^{-1} \ll q'_z \simeq d^{-1}$, i.e., the DA-phonon emission is heavily concentrated in a narrow cone around the magnetic field. In this case for the emission rate we obtain

$$\frac{1}{\tau^{\mathrm{PO+DA}}_{l\to l'}} = \frac{1}{\bar{\tau}_{\mathrm{PO+DA}}} \frac{2}{\beta(1+\gamma^2\beta^2)^3} \frac{\omega_B}{cp_{\mathrm{PO}}} d^{\mathrm{PO}}_{ll'}(\gamma). \tag{27}$$

The essential part of the magnetic field dependence is given by

$$\frac{1}{\tau^{\mathrm{PO+DA}}_{l\to l'}} \propto (B - B_{\Delta l})^{-1}, \tag{28}$$

i.e., in this range the relaxation becomes linearly weaker with an increase in the magnetic field.

The results of numerical evaluation of the emission rate as a function of the inter-Landau-level separation $\Delta l\omega_B$ in the whole energy range (a) are illustrated for transitions between Landau levels $l = 3$ and $l' = 0$ (Fig. 3). So far as the two-phonon processes contain the small electron-phonon couplings in the second order, the PO+DA emission gives rise to a peak that is lower than that for the one-phonon emission. The PO+DA relaxation has a sharp onset at low magnetic fields corresponding to energies immediately above $\hbar\omega_{\mathrm{PO}}$, where the peak increases as a fifth power in the magnetic field achieving the peak value exceeding 1 ps^{-1}. At high magnetic fields the peak decreases much slower, linearly in $B - B_{\Delta l}$, so that the PO+DA-phonon emission mechanism gives rise to a rather broad peak than the PO-phonon emission peak.

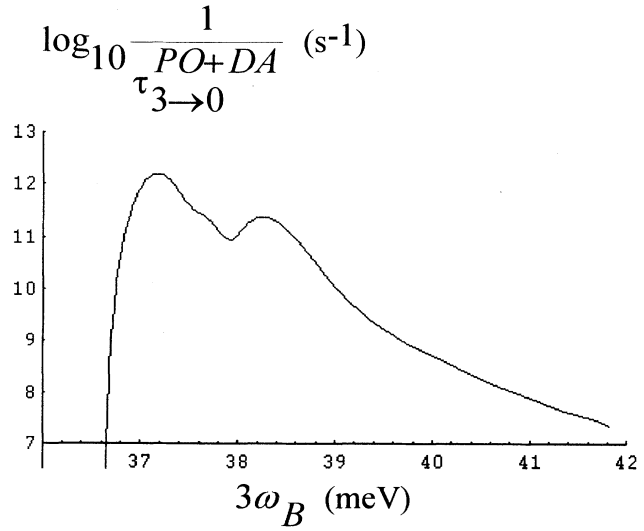

FIG. 3. The PO+DA-phonon emission rate versus the inter-Landau-level separation $\Delta l\omega_B$ for the electron transitions between $l = 3$ and $l' = 0$ levels in the energy range $(3\omega_B - \omega_{\mathrm{PO}}) \ll \omega_B, \omega_{\mathrm{PO}}$.

(b) $(\Delta l\omega_B - \omega_{\mathrm{PO}}) \simeq \omega_B, \omega_{\mathrm{PO}}$. As it follows from the energy conservation the energy of an emitted DA phonon is $cq' \simeq \omega_B$ so that $q' \simeq \omega_B/c \gg a_B^{-1}$. On the other hand, electrons in the states with Landau indexes $l \simeq 1$ have momenta of the order of a_B^{-1}. Therefore at more important electron transitions with $l, l' \simeq 1$ the momentum transmission to the phonon system is also of the same order of a_B^{-1}. Hence in this range electrons should emit phonons with almost oppositely directed momenta of approximately equal absolute values to avoid an additional suppression of the two-phonon emission. The large momentum transferred to each phonon results in the large Landau index $\bar{l}$ for intermediate states in Eq. (20). As far as the quasiclassic description takes place for $\bar{l} \gg 1$, the intermediate state energies are

$$\bar{l}\omega_B \approx \frac{(\Delta\bar{l}\omega_B - \omega_{\mathrm{PO}})^2}{2mc^2} \gg l\omega_B, \omega_{\mathrm{PO}}, cq'. \tag{29}$$

Therefore for the emission rate in the range (b) we obtain

$$\frac{1}{\tau^{\mathrm{PO+DA}}_{l\to l'}} = \frac{3}{2\bar{\tau}_{\mathrm{PO+DA}}} \frac{(2mc^2/\hbar)^2\omega_B}{(\Delta l\omega_B - \omega_{\mathrm{PO}})^3} \frac{1}{p_{\mathrm{PO}}d}. \tag{30}$$

The magnetic field dependence is given by

$$\frac{1}{\tau^{\mathrm{PO+DA}}_{3\to 0}} \propto (B - B_{\Delta l})^{-3}. \tag{31}$$

Because in the range (b) B is of the order of $B_{\Delta l}$ one can replace $B - B_{\Delta l}$ by $B_{\Delta l}$ so that the emission rate dependence on the magnetic field is weak in a rather wide energy range above ω_{PO}.

It is interesting also to compare the PO+DA-phonon emission rate for transitions between Landau levels $l = 3$ and $l' = 0$ with the 2LA-phonon emission rate at transitions between levels $l = 1$ and $l' = 0$. Using the result obtained in Ref. 32, it is easy to obtain

$$\frac{1}{\tau^{\mathrm{PO+DA}}_{3\to 0}} = 8\frac{\tau_{\mathrm{DA}}}{\tau_{\mathrm{PO}}} \frac{\omega^2_{\mathrm{PO}}(2mc^2/\hbar)}{(3\omega_B - \omega_{\mathrm{PO}})^3} \frac{1}{\tau^{\mathrm{2DA}}_{1\to 0}}. \tag{32}$$

Taking $B = 8.4$ T and $d = 3$ nm for a GaAs/$Al_xGa_{1-x}As$ heterojunction, we have $2mc^2 \approx 0.02$ MeV and $\omega_B = 2\omega_{\mathrm{PO}}/5 \approx 14.65$ MeV. So as it follows from Eq. (30) at transitions between Landau levels $l = 3$ and $l' = 0$ one can use $\tau^{\mathrm{PO+DA}}_{3\to 0} = 100$ ns as a characteristic relaxation time in the range (b). Under the same conditions at transitions between levels $l = 1$ and $l' = 0$ for the LA- and 2LA-phonon emission times we have, respectively, $\tau^{\mathrm{DA}}_{1\to 0} = 5.4$ μs (Ref. 31) and $\tau^{\mathrm{2DA}}_{1\to 0} = 15.6\tau^{\mathrm{PO+DA}}_{3\to 0}$. Thus comparison of these times shows the importance of the PO+DA-emission processes in the QHE geometry. In some experimental arrangements the PO+DA-phonon emission mechanism is much more efficient than

the relaxation in two consecutive emission acts: the PO-phonon emission (even under the sharp resonance) + either LA- or 2LA-phonon emission.

IV. SUMMARY

In conclusion, the PO-phonon-assisted electron relaxation is calculated as a function of the inter-Landau-level spacing in the 2DES in the QHE geometry. The PO-, SO-, and PO+DA-phonon emission processes via polar optical and deformation acoustical interactions are considered. The interface SO-phonon relaxation is at least by an order weaker than the relaxation via PO-phonon emission. To obtain a finite relaxation rate associated with one-phonon emission, the allowance for the Landau-level broadening and PO-phonon dispersion is made. Immediately below the phonon energy $\hbar\omega_{\rm PO}$ the PO-phonon dispersion contribution gives rise to a sharp peak with the peak value approximately 0.17 $\rm fs^{-1}$. The Landau-level broadening contribution has a rather broad peak with relatively lower peak value. Below $\omega_{\rm PO}$ within an energy range of the order of $\hbar\sqrt{\omega_B/\tau}$, the one-phonon relaxation rate exceeds 1 $\rm ps^{-1}$ (τ is the relaxation time deduced from the mobility). In a $\rm GaAs/Al_xGa_{1-x}As$ heterostructure with the mobility $\mu = 25\ \rm V^{-1}\,s^{-1}\,m^2$ this range makes up 0.7 MeV.

The two-phonon emission is a controlling relaxation mechanism above $\hbar\omega_{\rm PO}$. For $\Delta l\omega_B$ immediately above $\omega_{\rm PO}$ the PO+DA-phonon relaxation has a sharp onset. The relaxation rate increases as a fifth power in the magnetic field achieving to the peak value exceeding 1 $\rm ps^{-1}$ at energy separations of the order of c/a_B [in GaAs at $B = 7$ T we have $\hbar(c/a_B) \approx 0.4$ MeV]. At higher magnetic fields in the energy range $ca_B^{-1} \lesssim \Delta l\omega_B - \omega_{\rm PO} \lesssim cd^{-1}$ [in GaAs with $d = 3$ nm we have $\hbar(c/d) \approx 1.2$ MeV] the two-phonon peak decreases linearly in the magnetic field. Above $\omega_{\rm PO}$ within the wide energy range (in GaAs this range makes up 5 MeV) the magnetic field dependence of the relaxation rate is rather weak and the subnanosecond relaxation between Landau-levels $l = 3$ and $l' = 0$ can be achieved via two-phonon emission mechanism.

Our analysis demonstrates also that in some experimental situations the PO+DA-phonon emission mechanism is more efficient than the relaxation in two consecutive emission acts: the PO-phonon emission (even under the sharp resonance) with the subsequent emission of either the LA or 2LA phonon.

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