# Magnetoresistance of multiply connected Al samples

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The magnetoresistance in pure Al samples with interconnected Hall contacts has been investigated. The current in the loop connecting the Hall probes leads to an additional magnetoresistance with both quadratic and linear terms in the magnetic field.

## I. INTRODUCTION

The conventional theory of magnetoresistance in metals[1] predicts that the intrinsic transverse magnetoresistance of a simple metal like, e.g., Al, In, and K, which have a closed Fermi surface and are uncompensated saturates at high magnetic fields. The criterion of a high magnetic field is given by $\omega_c\tau \gg 1$ for the product of the cyclotron frequency $\omega_c = eB/m$ and the mean scattering time $\tau$. In practice, experiment reveals a small linear term in the magnetoresistance.[2] This linear term can be understood as an extrinsic effect for which the importance of thickness variations parallel to the magnetic field has been pointed out experimentally.[3–5] In this work we investigate another geometrical effect on the measured magnetoresistance. For a ring structure short circuiting the Hall contacts of the sample, contributions are observed in the magnetoresistance, which are not only linear in the magnetic field but also quadratic.

The phenomenon of linear magnetoresistance can be explained using Kirchhoff rules of classical electrodynamics for the electrical transport.[3,4] For a stepped sample with different thicknesses parallel to the magnetic field the Hall voltage $V_H = BI/ned$, where $n$ is the charge carrier concentration, $e$ the electronic charge, and $d$ the thickness of the sample, will be different on both sides of the step. From the time-independent Maxwell equation $\vec{\nabla} \times \vec{E} = 0$ follows $\oint \vec{E}\vec{ds} = 0$ for the path integral of the electric field $E$ over a closed loop in the metal. To reconcile the difference in Hall fields on both sides of the step with this condition, the current in the sample will be deviated towards one side of the sample at the step such that an additional voltage drop at that side of the sample compensates the difference in Hall voltages. Such a current diversion in a sample with inhomogeneous thickness explains the observed linear magnetoresistance.

Recently, an alternative explanation for the linear magnetoresistance[6,7] has been given in terms of a quantum-mechanical theory based on the transmission approach for electrical transport.[8] In this model, the carrier transport in a magnetic field is described using the formalism of skipping orbits along the sample sides. The number of quantum channels varies linearly with the sample thickness parallel to the magnetic field. Considering the reflection and transmission of skipping orbit channels at a step in the sample thickness, a multiterminal description for the electrical transport between the contact probes yields the same result as obtained with the classical theory. Both theories have in common that the origin of the linear magnetoresistance is closely related to the Hall effect.

In order to study the edge-state transport in a two-dimensional electron system, the Hall effect has been recently investigated in a Hall bar geometry with a hole in the two-dimensional sample.[9] Such doubly connected samples showed interesting phenomena in the Hall effect, like, for instance, a measurement of the classical Hall effect under null net current injection. These effects have been explained by the topology of the multiply connected sample.

Following the consideration of interconnected current channels in the magnetotransport of a percolating medium by Sarychev, Bergman, and Strelniker,[10] we became interested in studying the influence of a short circuit between the Hall-voltage probes on the magnetoresistance of a pure metallic sample. In Fig. 1 we have schematically drawn a sample with a ring connecting the Hall probes. For an applied current $I$ with the indicated field orientation, the Hall voltage $V_H$ can drive a current

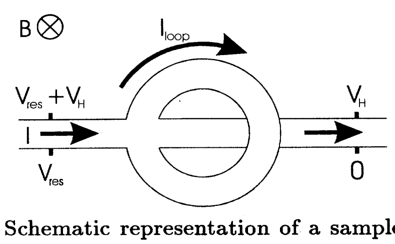


FIG. 1. Schematic representation of a sample with a ring connecting the two Hall-voltage probes. The applied current $I$ causes a Hall voltage $V_H$ which drives a current $I_{\rm loop}$ through the ring. The Hall voltage of $I_{\rm loop}$ yields an additional resistive voltage $V_{\rm res}$ along the sample.

 

$I_{\text{loop}}$ through the loop of the sample. This loop current would cause an additional Hall voltage across the sample. Because the magnetic field acts twice in this simple description, a quadratic term would be expected in the transverse magnetoresistance (measured by the resistive voltage $V_{\text{res}}$ in Fig. 1). The aim of this paper is to investigate such a phenomenon in multiply connected Al samples.

## II. EXPERIMENT

The magnetotransport experiments were carried out on two pure Al samples in the geometry as given in Fig. 2 [sample (a) and sample (b)]. For the indicated magnetic field orientation, the Hall probes have been interconnected with a loop cut out of the same piece of the starting material. The essential difference of the two samples is the fact that the plane of the ring lays for one sample parallel to the magnetic field $B$ [sample (a)], for the other perpendicular to the field [sample (b)].

Sample (a) was cut out of a 99.999% pure Al rod by spark erosion in the dimensions given in Fig. 2(a). With the indicated contact configuration a current $I_{12}$ can be introduced in the sample (contacts 1-2). Not only the resulting transverse magnetoresistance (contacts 3-4) can be measured but also the Hall voltage (4-5) belonging to the applied current and the Hall voltage (6-7) belonging to the current flowing through the loop.

Sample (b) was cut with a scalpel out of a 99.999% pure Al foil of 0.1-mm thickness. Its dimensions are given in Fig. 2 as well. The contact configuration allows us to again measure the magnetotransport properties in various parts of the sample.

Contacts were made by attaching droplets of molten Woods metal in a diluted etching solution. Measurements were done at 2 K in magnetic fields up to 10 T produced by a superconducting solenoid. dc currents up to 200 mA were applied through the samples, resulting in voltages up to roughly 20 $\mu$V. To eliminate the thermal emf's generated in the leads, the voltage averaged for both current polarities was taken. For sample (a) the residual resistance ratio $R(300\text{ K})/R(4.2\text{ K})$ was equal to 4300. For sample (b) the residual resistance ratio was determined to be 930. From these values for the residual resistance ratio one reaches the high-field limit $\omega_c\tau = 1$ at $\simeq$0.17 T for sample (a), and at $\simeq$0.76 T for sample (b).

In some experiments we measured a difference in the voltage measurements of the resistance at magnetic fields of equal magnitude but opposite direction. This behavior can be understood in the frame of the above-mentioned current deviation for samples with thickness variations parallel to the magnetic field. To average over this effect the magnetoresistance $R_{ij}$ of the transport data was determined from the even part of the measured voltages between contacts $i$ and $j$, i.e.,

$$R_{ij} = \frac{V_{ij}(+B) + V_{ij}(-B)}{2I_{12}}. \tag{1}$$

Hall resistances were obtained from the odd part of the measured voltage.

## III. MAGNETORESISTANCE OF SAMPLE (a)

From the measured Hall resistance $R_{45}$ between contacts 4 and 5 of sample (a), the slope $dR_{45}/dB$ = 245 n$\Omega$/T is obtained. In the high-field limit this slope equals $1/ned$ for a metal with charge carrier density $n$ and thickness $d$. In Al, the concentration of charge carriers in the Hall constant $R_H = 1/ne$ changes from 3 electrons per atom at low magnetic fields ($\omega_c\tau \ll 1$) to 1 hole per atom in the limit of high fields ($\omega_c\tau \gg 1$).[11] With $n = 6.10 \times 10^{22}$ cm$^{-3}$ in the high-field limit,[12] and $d$ = 0.42 mm of sample (a), the obtained Hall slope 244 n$\Omega$/T is in excellent agreement with the experiment (the tolerances for the machining are within $\pm 2 \times 10^{-2}$ mm).

For sample (a) we have plotted in Fig. 3 the transverse magnetoresistance $R_{34}$ and the Hall resistance of the loop $R_{67}$. The measured Hall resistance $R_{67}$ proves clearly that a current flows through the ring short circuiting the Hall-voltage probes. Because the measured value $R_{67}$ is about half of the normal Hall resistance $R_{45}$, approximately half of the injected current $I_{12}$ passes through the loop. In Fig. 3 we have also plotted the intrinsic transverse magnetoresistance of the sample with the loop interrupted. This transverse magnetoresistance $R^i_{34}$ for the interrupted loop can be compared with the differ-

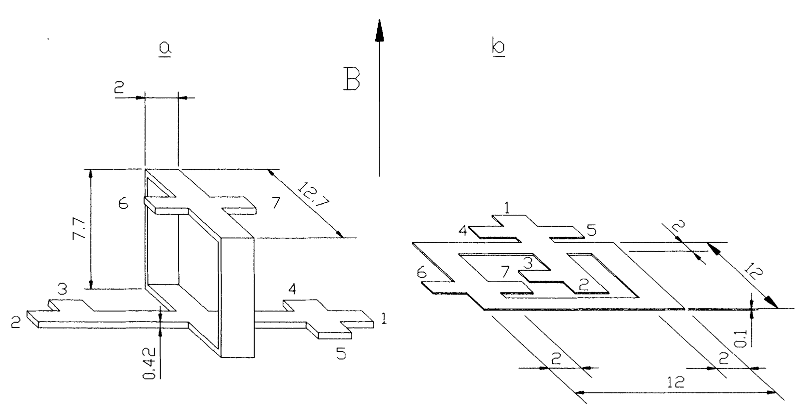


FIG. 2. Geometry of the investigated samples with the dimensions in mm as indicated. For sample (a) the applied magnetic field is parallel to the plane of the ring, and for sample (b) perpendicular.

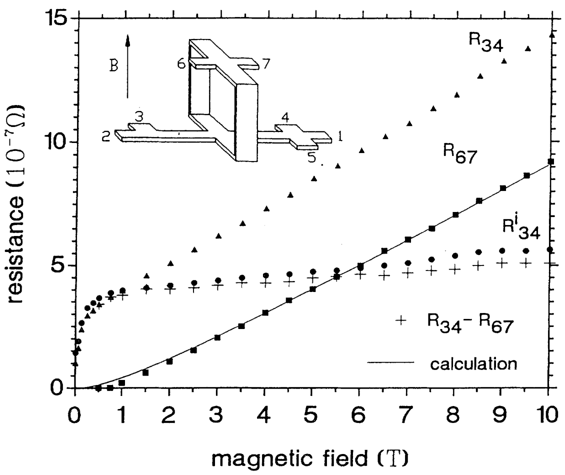


FIG. 3. Magnetoresistance data of sample (a) with the indicated contact configuration. $R_{34} = V_{34}/I_{12}$ is the transverse magnetoresistance, $R_{67} = V_{67}/I_{12}$ the Hall resistance corresponding to the current through the ring, and $R^i_{34} = V_{34}/I_{12}$ the transverse magnetoresistance after interrupting the ring. The crossed data points give the difference $R_{34} - R_{67}$.

ence between the magnetoresistance and Hall resistance for the closed loop represented by crossed data points, i.e., $R^i_{34} \simeq R_{34} - R_{67}$.

Although the Hall field drives a current through the closed loop, the quadratic magnetoresistance mentioned in the Introduction is not observed. To understand the observed linear magnetoresistance we have to consider the electronic transport through a folded sample. For a folded sample in an applied magnetic field a linear magnetoresistance has been observed,[13,14] an effect known as the zig-zag effect. Following the analysis for the observed linear magnetoresistance of a folded sample,[14] we have plotted in Fig. 4(a) the ring in an unfolded representation (conducting strip with different orientations of the magnetic field). In the unfolded representation of sample (a) in Fig. 4(a) we have ignored the parts of the rectangular-shaped ring with the current flow parallel to the magnetic field. In analogy to the explanation of linear magnetoresistance for stepped samples,[3,4,7] the changing sign of the Hall voltage leads to current diversions to one side of the sample resulting in voltage drops along the strip just equal to the difference of two Hall voltages. In this way the integral over the electric field along a closed path [as given by the dashed line in Fig. 4(a)] is zero and Maxwell equations in static fields are obeyed. As a consequence of the current diversion in a magnetic field, the resistance of the ring is linear in the magnetic field. Due to this linear term in the magnetoresistance seen by the current flowing in the ring, the resulting transverse magnetoresistance contains only a linear term instead of the quadratic term mentioned in the Introduction. In the following we will give a quantitative evaluation of the measured transverse magnetoresistance.

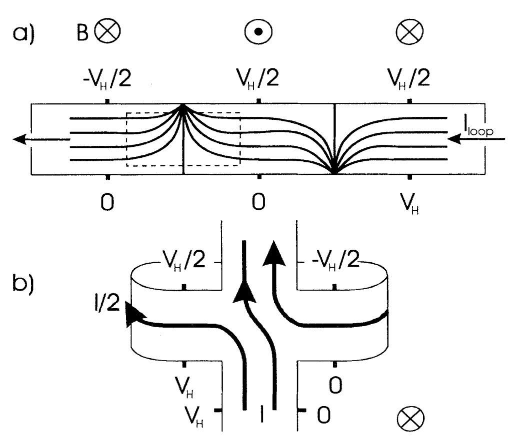


FIG. 4. (a) Current diversion in a ring in an unfolded representation of the ring explaining the linear magnetoresistance for the magnetic field parallel to the plane of the ring. (b) Schematic representation of the current paths near the cross point of the Hall contacts interconnected by means of a ring in a plane parallel to the magnetic field. The corresponding resistive and Hall voltages at the borders of the samples are indicated (ignoring the residual resistivity of the sample).

The Hall voltage $V_{45}$ caused by the current $I_{12}$ and the magnetic field $B$ is driving a current $I_{\text{loop}}$ through the loop generating a second Hall voltage $V_{67}$ measured between the contacts (6-7). The current $I_{\text{loop}}$ depends on the resistance $R_{\text{loop}}$ of the loop through

$$I_{\text{loop}} = V_{45}/R_{\text{loop}}, \tag{2}$$

with the Hall voltage $V_{45} = I_{12}B/ned$. The Hall resistance $R_{67}$ belonging to the current through the loop is then given by

$$\begin{aligned} R_{67} = V_{67}/I_{12} &= (B/ned)I_{\text{loop}}/I_{12} \\ &= (B/ned)^2/R_{\text{loop}}. \end{aligned} \tag{3}$$

The resistance of the loop results from intrinsic as well as geometric effects, i.e.,

$$R_{\text{loop}} = R_{\text{intrinsic}} + R_{\text{geometry}}. \tag{4}$$

The residual intrinsic resistance $R_{\text{intrinsic}}$ is constant at high magnetic fields ($\omega_c\tau \gg 1$) and can be determined from the specific resistivity of Al at room temperature ($\rho = 2.65\ \mu\Omega\,\text{cm}$), the measured residual resistance ratio (RRR equal to 4300), and the geometric dimensions of the ring [see Fig. 2(a)]. For the investigated sample (a) we obtain $R_{\text{intrinsic}} = 870\ \text{n}\Omega$. The geometric resistance $R_{\text{geometry}}$ follows from the above-mentioned zig-zag effect and equals twice the Hall resistance, i.e., $R_{\text{geometry}} = 2B/ned$.

Due to an artifact in the spark-erosion procedure the rectangular ring had two tiny grooves with depth $\simeq 0.05$ mm in the upper and lower parts. These grooves yield additional linear terms in the geometrical part of the magnetoresistance of the ring as studied in detail by Bruls *et al.*[3,4] To take account for these effects we used the thickness of the sample as a free parameter $d_{\text{eff}}$ in the expression for the geometry-related part of the resistance $R_{\text{geometry}}$ of the loop. In Fig. 3 we have plotted the calculated curve of $R_{67}$ for the optimized $d_{\text{eff}} = 0.36$ mm. This value has to be compared with the thickness $d = 0.42$ mm of sample (a). The difference can be explained by the presence of the grooves leading to a correction in the linear magnetoresistance as expected from the zig-zag effect. The deviation between theory and experiment at magnetic fields smaller than 2 T is not surprising since then the condition for high magnetic fields $\omega_c\tau \gg 1$ is not fulfilled.

In Fig. 3, the crossed data points obtained by subtracting the Hall resistance $R_{67}$ of the loop from the magnetoresistance $R_{34}$ for sample (a) show that the additional magnetoresistance $R_{34} - R^i_{34}$ due to the ring structure can be directly compared with the Hall resistance $R_{67}$. In Fig. 4(b) we have drawn the current paths close to the cross point of the attached Hall probes. In the simplified solution of the problem where we ignored the resistive voltages compared to the Hall voltages, half of the applied main current is deviated through the ring. The indicated potentials at the borders show that the additional resistive voltage along the sample equals the Hall voltage belonging to the current $I_{12}/2$ through the ring. The contribution of the loop to the magnetoresistance of the sample is expressed by $R_{34} - R^i_{34} \simeq R_{67}$. The observed small difference between $R_{34} - R_{67}$ and $R^i_{34}$ can be explained by a mechanical deformation in interrupting the ring, which also lowered the RRR of the sample.

## IV. MAGNETORESISTANCE OF SAMPLE (b)

For sample (b), the measured slope $dR_{45}/dB$ of the Hall resistance equals 1090 n$\Omega$/T close to the expected value 1020 n$\Omega$/T for the uniform thickness $d = 100$ $\mu$m of sample (b).

In Fig. 5 the magnetoresistance $R_{34}$ and the Hall resistance $R_{67}$ have been plotted for sample (b) where the magnetic field is now perpendicular to the plane of the ring. For this orientation of the magnetic field with respect to the ring the expected quadratic term in the magnetoresistance is clearly observed. Again the plotted difference $R_{34} - R_{67}$ can be compared with the magnetoresistance $R^i_{34}$ measured after interruption of the loop. In the following we will give a quantitative estimation of the measured quadratic magnetoresistance.

Also in this configuration, the Hall voltage $V_{45}$ caused by the current $I_{12}$ and the magnetic field $B$ is driving a current $I_{\mathrm{loop}}$ through the loop. As in the case of sample (a), in the high-field limit ($\omega_c\tau \gg 1$) the resistance of the loop consists of a constant term $R_{\mathrm{intrinsic}}$ and a geometrical term $R_{\mathrm{geometry}}$. Because the current through the ring flows through a flat foil of homogeneous thickness, the linear magnetoresistance term $R_{\mathrm{geometry}} = 2B/ned_{\mathrm{eff}}$ is now much smaller compared to the intrinsic magnetoresistance of the Al loop. A small contribution to $R_{\mathrm{geometry}}$ could still result from the nonuniform thickness or the nonflatness of the foil. Finally this term should be expressed in an effective thickness decisively larger than the thickness of the foil.

From the geometry of the loop, from the resistivity at room temperature, and from the RRR (=930) we get for $R_{\mathrm{intrinsic}} = 17\,100$ n$\Omega$. Using this value we can fit our data for the Hall resistance $R_{67}$ belonging to the current through the loop in the high-field regime $B \geq 2.5$ T using Eqs. (3) and (4). The dashed curve in Fig. 5 has been obtained for $d_{\mathrm{eff}} = 330$ $\mu$m being more than 3 times bigger as the thickness $d$ of the foil. The Hall effect belonging to this effective thickness is only twice as large as the intrinsic linear magnetoresistance found in the residual value $R^i_{34}$ in the high-field limit. This larger value for the linear magnetoresistance of the loop is in agreement with the longer length of the loop compared to the length of sample (b) (between contacts 3 and 4).

In Fig. 6 we show the current pattern in the limit of very strong fields. The injected current circles first one or more times around the loop before arriving at the drain. In the same figure we have indicated the Hall voltages at the border of the sample. It is readily shown that the additional resistive voltage across the sample due to the ring structure equals the Hall voltage belonging to the

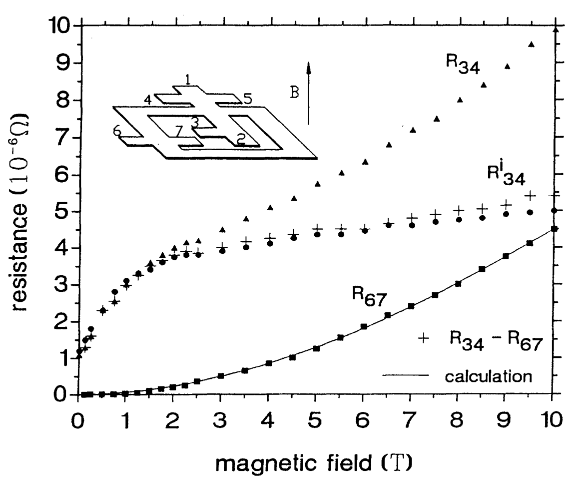


FIG. 5. Magnetoresistance data of sample (b) with the indicated contact configuration. $R_{34} = V_{34}/I_{12}$ is the transverse magnetoresistance, $R_{67} = V_{67}/I_{12}$ the Hall resistance corresponding to the current through the ring, and $R^i_{34} = V_{34}/I_{12}$ the transverse magnetoresistance after interrupting the ring. The crossed data points give the difference $R_{34} - R_{67}$.

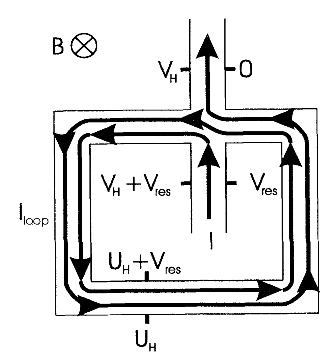


FIG. 6. Schematic representation of the current paths through a sample with a ring perpendicular to the magnetic field. The current $I_{\mathrm{loop}}$, driven by the Hall voltage $V_H$, experiences a resistive voltage drop $U_H$ ($0 < U_H < V_H$) along the loop and generates the Hall voltage $V_{\mathrm{res}}$. This Hall voltage $V_{\mathrm{res}}$ equals the resistive voltage drop between the current contacts.

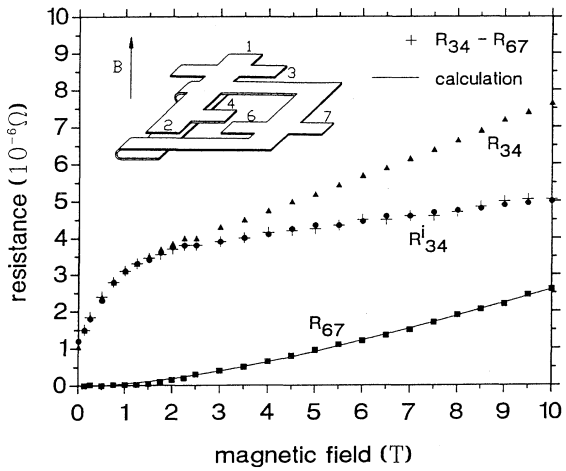


FIG. 7. Magnetoresistance data of sample (b) with a fold in the loop with the indicated contact configuration. $R_{34} = V_{34}/I_{12}$ is the transverse magnetoresistance, $R_{67} = V_{67}/I_{12}$ the Hall resistance corresponding to the current through the ring, and $R^i_{34} = V_{34}/I_{12}$ the transverse magnetoresistance after interrupting the ring. The crossed data points give the difference $R_{34} - R_{67}$.

current through the ring. Therefore we observed in the experiment $R^i_{34} = R_{34} - R_{67}$ [see Fig. 5].

In Fig. 7 we have plotted the experimental magnetoresistance data for the same ring but now with a fold in the ring that short circuits the Hall contacts (see the inset of Fig. 7). The magnetoresistance data show again a quadratic term, but the linear contribution is much stronger. This can be explained by the fold in the ring, which yields a linear contribution to the resistance of the ring. As for sample (a) the current in the ring experiences a resistance linear in the field due to the zig-zag effect. Fitting the data of $R_{67}$ with the expression following from Eqs. (3) and (4) yields an effective thickness $d_{\text{eff}} = 88\ \mu$m. The difference in $R_{\text{geometry}}$ of the folded loop and unfolded loop should reveal the zig-zag effect, such that

$$\left[\frac{2B}{ned_{\text{eff}}}\right]_{\text{folded}} - \left[\frac{2B}{ned_{\text{eff}}}\right]_{\text{unfolded}} = \frac{2B}{ned}. \tag{5}$$

This equality of the last equation is only confirmed within 20%. The discrepancy could probably be explained by a change in the RRR or by a further change in the geometric linear magnetoresistance of the loop in addition to the zig-zag effect due to bending the foil in the folding procedure.

## V. CONCLUSIONS

The magnetoresistance of a multiply connected sample is influenced by the loop interconnecting the Hall-voltage probes. For a loop structure in the plane perpendicular to the applied magnetic field, a quadratic magnetoresistance is observed and can be explained by a current flow in the loop driven by the Hall field. The magnetic field acts twice in a linear way to get the final resistive contribution quadratic in the magnetic field: once through the current through the loop driven by a Hall field, which is linear in the field and depends on the resistance of the loop, and once by the Hall field resulting from the current through the ring. In this geometry the current through the loop can be much larger than the injected current.

For ring structures where the loop current does not only flow in a plane perpendicular to the magnetic field, the zig-zag effect yields a linear term in the resistance seen by the electrons flowing through the loop. This zig-zag effect suppresses the amount of current flowing through the loop in a linear way, yielding only a linear magnetoresistance of the sample with half of injected current flowing through the loop.

In the limit of very high magnetic fields, the linear dependence will always dominate the quadratic one. This can be seen from the expression for the Hall resistance $R_{67}$ [Eqs. (3) and (4)] and the loop-related magnetoresistance in $R_{34}$, once a linear geometric contribution $R_{\text{geometric}}$ exists due to thickness variations.

The magnetoresistance due to the loop structures can be understood in terms of a longer path the electrons have to go through the sample passing one or more times around the loop. Our geometry with the loop in a plane perpendicular to the magnetic field resembles the well-known Corbino geometry (a disk with a central contact and a contact on the periphery). For such a geometry a quadratic magnetoresistance is found corresponding to electrons circling around the disk before reaching the outer contact.

The observed magnetoresistance follows from the boundary conditions imposed by the sample geometry. It may be clear that the conductivity tensor, describing the relation between local fields and currents in the sample in a strong magnetic field, is not influenced by these boundary conditions.

## ACKNOWLEDGMENTS

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