Composite-fermion analysis of the double-layer fractional quantum Hall system

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Effect of interlayer tunneling in the double-layer fractional quantum Hall system at the total Landau-level filling of $\nu = 1/m$ (m: odd integer) is analyzed with the composite-fermion approach in which the flux attachment is directly applied to the electron-electron interaction. A comparison with a numerical result indicates that the vertically coupled Laughlin liquids may be regarded as a system of composite fermions with *reduced* interparticle interactions and *unchanged* interlayer tunneling, which makes the quantum-Hall regime, identified by a gap in the pseudospin-wave excitation mode, wider as ν becomes $1/3, 1/5, \ldots$.

The fractional quantum Hall (FQH) state is a strongly correlated quantum liquid realized in a two-dimensional electron system in strong magnetic fields.[1] Recently, much attention is focused on what happens when two such liquids are coupled face to face, as realized in quantum wells,[2,3] and the layer degrees of freedom in such double-layer systems are often described by a pseudospin.

Although a (spin-polarized) double-layer system may at first seem analogous to a single-layer system of spin 1/2 electrons from the analogy between pseudospin and real spin, an essential difference does indeed exist, due to two factors: One is the interlayer tunneling, which makes single-particle wave functions split into symmetric and antisymmetric (SAS) ones about the center of the double-layer structure, with an energy separation, Δ_{SAS}. The second is the controllability of the intralayer versus interlayer electron-electron interaction strengths by the layer separation, which makes the symmetry degrade from SU(2) to U(1).

In particular, the double-layer FQH state for the total Landau level filling of $\nu = 1/m$ (m=odd integer) is pseudospin polarized in analogy with an easy-plane ferromagnet.[4] The two factors above are exactly responsible for the pseudospin polarization: (i) The tunneling gap acts as a magnetic field along the x axis for the pseudospin, thereby pushing the electrons into the symmetric band. (ii) The intralayer and interlayer repulsive electron correlations make the ground state represented by Halperin's Ψ_{mmn},[5] an extension of Laughlin's state to two fermion species, where the same power for intralayer and interlayer correlations (i.e., $m = n$), realized unless d is too large, represents a pseudospin-polarized state.[6]

Thus the layer separation, d, normalized by the magnetic length, $\ell \equiv \sqrt{c\hbar/eB}$, and the strength of tunneling, Δ_{SAS}, normalized by the Coulomb-interaction energy, $e^2/\epsilon\ell$ (ϵ=dielectric constant), are the relevant dimensionless parameters, and we can think of a phase diagram on the parameter plane. For an *integer* $\nu = 1$, the pseudospin-polarized quantum Hall state evolves continuously from the correlation-dominated (or "two-component") character (Ψ_{111}) to the tunneling-dominated (or "one-component") character. In the single-mode approximation (SMA),[7,8] this crossover, with no intervening non-QHE (quantum Hall effect) region, is described by a continuous rotation of the pseudomagnon vacuum in a Bogoliubov transformation. The continuity is consistent with an experimental result by Murphy *et al.*[2]

There, the boundary of the QHE region is identified from a finite gap in the charge excitations. Since the ground state is pseudospin polarized, the low-lying excitation is a Goldstone (pseudospin-wave) mode, where the densities of two layers fluctuate in a correlated manner.[7–9] MacDonald *et al.* have, in fact, obtained the QHE region for $\nu = 1$ from the gap in the pseudospin-wave mode with the SMA.[7] The excitation spectrum for $\nu = 1$ has also been examined with the random-phase approximation (RPA) or Hartree-Fock method.[9]

A natural question then is how we can extend the physics of the double-layer system with the interlayer tunneling to the *fractional* $\nu = 1/3, 1/5, \ldots$. The SMA calculations of the excitation spectrum have only been done in the absence of interlayer tunneling for Ψ_{mmn}, with $\nu = 2/(m + n)$.[10] An intriguing way that we propose here is to apply a recent way of looking at the ordinary (single-layer) FQH system, in terms of the composite-fermion picture[11] or the Chern-Simons gauge field theory,[12] to the fractional double-layer system.

The composite-fermion (CF) picture asserts that a quantum Hall liquid of electrons in an external magnetic field B corresponding to $\nu = p/(2mp + 1)$ (m,p integers) is equivalent, in a mean-field sense, to a liquid of composite fermions each carrying $2m$ flux quanta in a magnetic field $B_{\text{eff}} \equiv B - B_{\nu=1/2m}$ corresponding to an integer $\nu = |p|$. There is a mounting body of numerical[13] and experimental[14] evidence supporting this picture.

It is a highly nontrivial problem whether the CF picture can correctly describe the excitations that is dominated by both interaction and tunneling. As for the excitation spectra, a Chern-Simons (CS) approach, in which the fluctuation around the mean CS gauge field is treated with the RPA, has been developed,[15] but there the effective mass of the particle remains a quantity difficult to fix. In fact the Chern-Simons RPA calculations for the

double-layer system[16,17] are still some way from an accurate description of the intra-Landau-level excitations. In this context it may be desirable to manipulate the interaction, so that we adopt here a particular CF picture due to the present authors[18] that directly plugs the flux-attachment transformation into the electron-electron interaction (Haldane's pseudopotential), which enables us to obtain parameter-free results for the pseudospin-wave excitation.

We can then identify the QHE region from the charge (pseudospin-wave) excitation spectrum, calculated from such a composite-fermion picture, in the $\nu = 1/m$ (m=odd integer) double-layer FQH system of spin-polarized electrons. A comparison with the exact result for a finite system shows that the picture indeed gives quantitatively accurate long-distance physics. This provides an analytic calculation of the excitation spectrum for a fractionally filled system with interlayer tunneling.

Let us now mention two issues inherent in the double-layer FQH system. One is whether there is an essential difference between a lateral tunneling of composite particles (in a side-by-side geometry)[19] and the vertical tunneling (in a face-to-face geometry). We shall show that the $\nu = 1/m$ double-layer FQH system can be regarded as a system of composite fermions with the *reduced* interparticle interaction, while the tunneling, when vertical, is *unchanged*. The increased relative importance of the tunneling makes the QHE region for $\nu = 1/m$ in the phase diagram, obtained here for $m = 3, 5$, wider for larger m.

Second, in the absence of interlayer tunneling, the pseudospin-polarized state has a broken $U(1)$ symmetry and may have a spontaneous interlayer phase coherence.[4] The Goldstone (pseudospin-wave) mode restoring this broken symmetry is, in fact, gapless and k linear in analogy with a magnon in an XXZ ferromagnet. An introduction of the interlayer tunneling enforces the $U(1)$ symmetry to break, thereby introducing a gap in the pseudospin-wave mode. The issue is whether the gapless Goldstone mode in the absence of interlayer tunneling will signify a Josephson-like effect.[16,20] We shall also touch upon this problem. Hereafter, we ignore the real spin degrees of freedom or the finite thickness of each layer for simplicity.

The CF picture for the pseudospin wave is as follows. When we attach $(m - 1)$ flux quanta from the external field to each electron for an odd $m = \nu^{-1}$, the relative angular momentum n between electrons translates into the relative angular momentum $n - (m-1)$ between composite fermions.[21] Since $B_{\text{eff}} = B/m$ is thereby reduced from the bare field by a factor of $1/m$, the magnetic length ℓ changes into $\tilde{\ell} = \sqrt{m}\ell$ in a mean-field sense, while the number of single-particle states per unit area, $1/2\pi\ell^2$, is also reduced by a factor $1/m$.

For the motion within a layer, we can work with the spherical geometry to make the relevant quantum number the angular momentum. As one maps stereographically a flat system onto a spherical one, the translational symmetry is translated into the rotational symmetry, where the wave number k relates to the total angular momentum L as $k = L/R$ with R being the radius of the sphere. When the total magnetic flux going out of the

sphere is $2S$ (an integer due to Dirac's condition) times the flux quantum, the relation to ν is $2S = \nu^{-1}N - \delta$ with N being the number of electrons and δ an integer.

The transformation into the CF picture is then given by

$$2\tilde{S} = 2S/m = N - 1,$$

$$\tilde{V}^{\sigma\sigma'}_{2\tilde{S}-[n-(m-1)]} \bigg/ \frac{e^2}{\epsilon\tilde{\ell}} = V^{\sigma\sigma'}_{2S-n} \bigg/ \frac{e^2}{\epsilon\ell}, \tag{1}$$

where σ and σ' $(= 1, 2)$ are layer indices, while $V^{11}_{2S-n} = V^{22}_{2S-n}$ and $V^{12}_{2S-n} = V^{21}_{2S-n}$ are the intralayer and interlayer pseudopotentials for the relative angular momentum $n = 2S - J$, respectively.[22]

We can now plug this transformation into the $\nu = 1$ SMA formula for the pseudospin-wave mode, which is expressed in terms of $6j$ symbols, $\left\{{SSL \atop SSJ}\right\}$, in the spherical geometry.[8] Then we arrive at the desired expression for the pseudospin-wave spectrum, ω_L, for $\nu = 1/m$ as

$$\omega_L = \sqrt{e_L(e_L + 2\lambda_L)},$$

$$e_L = \Delta_{\text{SAS}} + \sum_{J=0}^{2\tilde{S}}(2J + 1)(-1)^{2\tilde{S}-J}\tilde{V}^{12}_J\left[\frac{1}{2\tilde{S}+1}\right.$$

$$\left. -(-1)^{2\tilde{S}-J}\left\{{\tilde{S} \ \tilde{S} \ L \atop \tilde{S} \ \tilde{S} \ J}\right\}\right],$$

$$\lambda_L = \sum_{J}^{2\tilde{S}-J:\text{odd}}(2J + 1)(\tilde{V}^{11}_J - \tilde{V}^{12}_J)(-1)^{2\tilde{S}-J}$$

$$\times\left\{{\tilde{S} \ \tilde{S} \ L \atop \tilde{S} \ \tilde{S} \ J}\right\}, \tag{2}$$

where the range of the total angular momentum now reduces to $0 \leq L \leq 2\tilde{S}$. Since the pseudopotentials are shifted to a higher side of the relative angular momentum (where the potential is softer), this approach can be called the pseudopotential-shifted single-mode approximation.[23] We can confirm that, in the absence of tunneling ($\Delta_{\text{SAS}} = 0$), the above formula reduces to a gapless Goldstone mode with $\left\{{SS0 \atop SSJ}\right\} = (-1)^{2S-J}/(2S + 1)$. As for the tunneling, we can regard that the flux attachment (or the singular gauge transformation) does not affect the vertical interlayer tunneling.

We now compare the composite-fermion SMA result with numerical ones for finite systems. Figure 1 shows the low-lying excitation spectrum at $\nu = 1/3$ for $d/\ell = 1.0$ and $\Delta_{\text{SAS}}/(e^2/\epsilon\ell) = 0.01$ (a) or 0.05 (b). For the numerical diagonalization we take a five-electron system, the largest size tractable in the presence of tunneling.[24] The charging energy is naturally included in the diagonalization.

We can see that the pseudospin-wave mode in the finite system does indeed appear in the truncated range $0 \leq L \leq 2\tilde{S} = N - 1$, while naively there is no reason why the states should not extend for $0 \leq L \leq 2S = m(N-1)$. As for the dispersion curve itself, the CF prediction, Eq. (2), exhibits a good agreement with the exact result up to the wave number $k \sim \ell^{-1}$. This is the case with both Fig. 1(a), where a precursor of the softening of the pseudospin-wave mode is visible, and Fig. 1(b) with one-component character (see below). Thus, the mes-
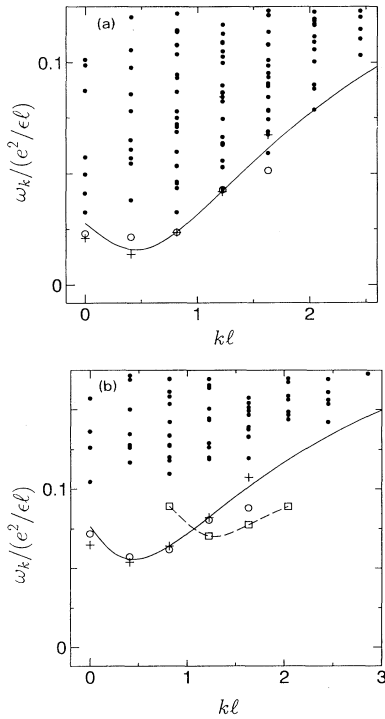
FIG. 1. The excitation spectrum from the ground state (origin of the figure) for a double-layer five-electron FQH system at $\nu = 1/3$, which comprises pseudospin-wave excitations (open circles) and other excitations (solid circles), is shown for $d/\ell = 1.0$, and $\Delta_{SAS}/(e^2/\epsilon\ell) = 0.01$ (a) or 0.05 (b). The results for the pseudospin-wave excitation in the composite-fermion SMA for the same number of electrons (crosses) and for a 51-electron system (solid line) are also shown. In (b), the roton excitations within the symmetric band are also displayed by open squares. The roton and pseudospin-wave excitations are, respectively, connected by curves as a guide to the eye.
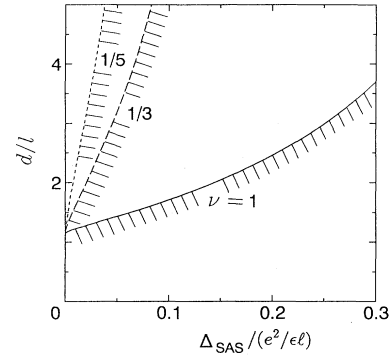

FIG. 2. The composite-fermion SMA result for the phase diagram for the $\nu = 1/m$ pseudospin-polarized quantum Hall state [the region below solid line ($\nu = 1$), broken line ($\nu = 1/3$), or dotted line ($\nu = 1/5$)].

sage here is that the $\nu = 1/m$ double-layer FQH system may be mimicked by a system of composite fermions at $\nu = 1$ with effectively *reduced* interparticle interactions for pseudospin waves even in the presence of tunneling. It may be interesting if these collective modes are experimentally observed, e.g., with grated samples.[25]

If we have a closer look at Fig. 1(b), we can see that $\Delta_{SAS}/(e^2/\epsilon\ell) = 0.05$ is large enough to make the ground state almost a $\nu = 1/3$ Laughlin liquid within the symmetric band with little mixing of the antisymmetric band. This is signaled by the fact that the lowest-lying excitation is a magnetoroton mode within the symmetric band (identified by its pseudospin), where the intraband roton costs less energy than the interband pseudospin wave. The increased importance of Δ_{SAS} relative to the interparticle interaction is responsible for the situation.

The increased importance of Δ_{SAS} also appears in the phase diagram on the $\Delta_{SAS} - d$ plane for the $\nu = 1/m$ double-layer FQH state. The result (Fig. 2), obtained by identifying the softening of the pseudospin-wave mode as the disappearance of the QHE gap, shows that the QHE region widens as ν becomes $1/3, 1/5, \ldots$. This is

again because the electron-electron interaction is effectively weakened as we attach two, four, ... fluxes, which in turn reduces the mixing of the antisymmetric state in the ground state to push the system toward the one-component FQH state. Thus, the CF picture gives a natural explanation of the persistence of the one-component character of the $\nu = 1/3$ FQH state observed in a wide single quantum well,[3] where Δ_{SAS} is intrinsically large, while the reduction of the interaction, due to the expansion of the wave function in wide wells, evoked in Ref. 3, will be quantitatively a secondary contribution.

Now we comment on the possibility of the "Josephson-like" interlayer current in the $\nu = 1/m$ double-layer system, as discussed by several authors.[16,20] Since this effect should be related to the transition between Halperin's $\Psi^{N_1}_{mmm}$ states $[N_1(=0, 1, \ldots, N)$: the number of electrons in layer 1], which are the $k = 0$ states, attention should be paid on the gap at $k = 0$. When the interlayer transfer of electrons occurs over the whole two-dimensional area, a finite gap emerges at $k = 0$ [explicitly, $\Delta_0 \equiv \sqrt{\Delta_{SAS}(\Delta_{SAS} + 2\lambda_0)}$ from Eq. (2)], and this will, as pointed out by MacDonald and Zhang,[10] refute the dc Josephson-like effect. The suppression of the "Josephson effect" is also discussed in the side-by-side geometry by Feng et al.,[19] who pointed out that a composite particle leaves behind phase fluctuations of the CS field when it tunnels laterally. By contrast, in a vertical tunneling in a double-layer geometry, a composite particle can readily tunnel since it does not have to shake off the flux quanta as seen in the present result. Therefore, the "Josephson effect" may still be possible in a geometry where the two layers have only a weak (e.g., spatially localized) vertical tunneling that may be regarded as a perturbation, while the interlayer electron correlation over the whole area continues to "lock" the CS phase.

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[23] In the conventional SMA (Ref. 10), the correlation effect is plugged into the radial distribution function of the ground state, which is assumed to be Halperin's Ψ_{mmn}, while in the present CF approach we can continue to use the analytic formula for $\nu = 1$.

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