Inelastic light scattering from a modulated two-dimensional electron gas in magnetic fields

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(Received 27 July 1995)

We study inelastic light scattering from a two-dimensional electron gas subject to a weak in-plane one-dimensional periodic potential modulation and a perpendicular magnetic field. Electronic Raman scattering is considered for (i) inter-Landau-level transitions, which result in a Raman shift of $\sim 2\omega_c$, and (ii) transitions involving collective excitations at the hybridized magnetoplasmon frequency and a modulation-induced, low-frequency intra-Landau-level plasmon. Raman cross sections and their dependencies on the magnetic field strength and the modulation potential/density are examined.

I. INTRODUCTION

Magnetoresistance oscillations due to the commensurability of the electron magnetic orbit and the periodicity of a weak in-plane one-dimensional (1D) or two-dimensional (2D) potential modulation of a two-dimensional electron gas (2DEG), known as Weiss oscillations,[1–5] have been given a classical[6] and a quantum-mechanical[2–5,7] interpretation. The latter considers the modulation-induced formation of minibands centered about the originally sharp, equally spaced Landau levels. The widths of these Landau bands are periodic functions of the magnetic field. This leads to a density of states periodic in inverse magnetic field and to the Weiss oscillations in the magnetoresistance. We have predicted commensurability oscillations in the magnetoplasmon spectrum of the modulated system[8] and, related to it, oscillations in the electron energy-loss spectrum of the same system.[9] In addition to possible experimental verifications of such predictions with infrared transmission spectroscopy or fast electron energy-loss spectroscopy, it is also likely that the predicted plasma spectrum can be revealed in inelastic light-scattering experiments. This paper deals with the theoretical aspects of the inelastic light scattering from such a modulated 2DEG. We consider both the single-particle contribution to the Raman scattering cross section, and the contribution due to collective excitations of the many-electron system. Both aspects of the electron dynamics have distinctive features due to the simultaneous presence of the quantizing magnetic field and the modulation potential. Specifically, the scattered light intensity due to the transitions between the Landau minibands is directly related to the widths of the latter, while the scattered intensity due to collective excitation is related to both the potential strength and the period of the modulation. Thus a careful analysis of inelastic light scattering data from the modulated 2DEG in a quantizing magnetic field should provide much useful information on the electronic system, its dynamics, and its interactions.

This paper is organized as follows: In Sec. II we briefly describe the formulation of the scattering cross section for inelastic light scattering from a modulated 2DEG in magnetic fields and discuss Raman scattering by single Landau electrons of the modulated 2DEG. In Sec. III we examine Raman scattering by collective excitations of the Landau electrons in the modulated 2DEG. Section IV contains a summary and concluding remarks.

II. RAMAN SCATTERING BY SINGLE LANDAU ELECTRONS

The problem of light scattering from electrons in semiconductor crystals has been studied by many authors over the years.[10–14] The Hamiltonian describing a band electron (of charge e, effective mass m) in the presence of an external dc magnetic field, and irradiated by an electromagnetic field is given by

$$H = (1/2m)(\vec{\pi} - e\vec{A})^2 + V(\vec{x}), \tag{1}$$

where $\vec{\pi} = \vec{p} - e\vec{A}_0$, $\vec{p}$ is the electron momentum, $\vec{A}_0$ is the vector potential associated with the dc magnetic field $\vec{B}$, $\vec{A}$ is the vector potential of the electromagnetic field of the laser light, and $V(\vec{x})$ is the periodic potential of the crystal. The Hamiltonian (1) is usually written as $H = H_0 + H_1 + H_2$, with

$$H_0 = (\pi^2/2m) + V(\vec{x}) \tag{2}$$

taken as the unperturbed part, and

$$H_1 = -(e/m)\vec{A}\cdot\vec{\pi}, \tag{3}$$

and

$$H_2 = (e^2/2m)A^2, \tag{4}$$

as perturbations describing the interaction of the electron with the electromagnetic field. For single-particle excitations within the dipole approximation ($q\ell \ll 1$, $q = 2\pi/\lambda$, λ is the wavelength of the laser light, and

$\ell = \sqrt{\hbar/eB}$ is the magnetic length), H_2 does not contribute, and H_1 contributes to second order. However, this contribution of H_1 to light scattering vanishes for free electrons because of the equal spacing of the Landau levels. In bulk semiconductors such as InSb, the Landau-level separations become unequal due to the band nonparabolicity, resulting in nonvanishing Raman scattering cross sections. In such cases the selection rule for the allowed transitions is

$$\Delta n = 0, \pm 2, \tag{5}$$

where n is the Landau-level index. Obviously, $\Delta n = 0$ is for elastic scattering, and $\Delta n = \pm 2$ corresponds to the Stokes and anti-Stokes processes. At very low temperatures, only the Stokes process is possible. In a modulated 2DEG, the Landau levels are broadened into minibands by the modulation, leading to finite scattering cross sections for inelastic light-scattering processes. In Fig. 1 we show possible transitions between the Landau bands (only initial and final states are indicated, intermediate states are not shown). Figure 1(a) depicts the situation where the Fermi level lies between the nth and $(n+1)$th Landau bands. In this case the net transition is $n \to n+2$. Figure 1(b) shows the situation in which the Fermi level lies within the nth Landau band. In this case both the $n \to n+2$ transition and the $n-1 \to n+1$ can occur. We assume here that the lattice temperature is low enough so that all electronic states below the Fermi level are occupied, and those above the Fermi level are initially empty.

The differential scattering cross section (per solid angle, per frequency shift) for single Landau electron transitions is given by[13]

$$\frac{d^2\sigma}{d\Omega d\omega} = \left(\frac{e^2}{m}\right)^2 \frac{\omega_f}{\omega_i} |M_{fi}|^2 \delta(\hbar\omega_i + E_i - \hbar\omega_f - E_f), \tag{6}$$

with

$$M_{fi} = \frac{1}{m}\sum_r \left[\frac{\langle f|\hat{\epsilon}_f \cdot \vec{\pi}|r\rangle\langle r|\hat{\epsilon}_i \cdot \vec{\pi}|i\rangle}{E_i - E_r + \hbar\omega_i} + \frac{\langle f|\hat{\epsilon}_i \cdot \vec{\pi}|r\rangle\langle r|\hat{\epsilon}_f \cdot \vec{\pi}|i\rangle}{E_i - E_r - \hbar\omega_f}\right]; \tag{7}$$

here $\hat{\epsilon}_i$ and ω_i are the polarization and frequency of the incident photon, and $\hat{\epsilon}_f$ and ω_f are the polarization and frequency of the scattered photon. $|i\rangle$ and $|f\rangle$ are the initial and final electronic states of eigenenergies E_i and E_f, respectively, while $|r\rangle$ presents a complete set of intermediate states with eigenenergies E_r. The frequency shift is given by $\omega = \omega_i - \omega_f$.

For a weak 1D potential modulation of the form

$$V(x) = V_0 \cos(2\pi x/a), \tag{8}$$

applied to the 2DEG subject to a constant uniform magnetic field perpendicular to the plane of the 2DEG, $\vec{B} = B\hat{z}$, the eigenstates of $\pi^2/2m$ and approximately

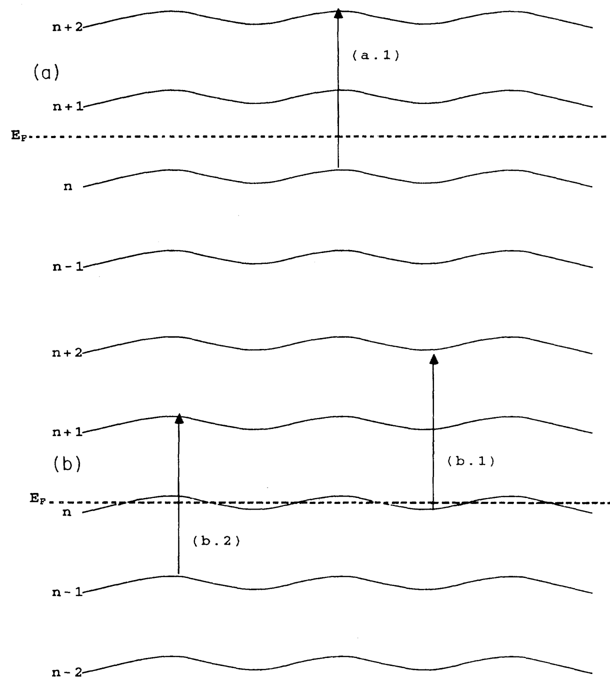

FIG. 1. Allowed transitions of inter-Landau-level Raman scattering for a modulated 2DEG. Only initial and final states are shown for clarity. (a) Fermi level is between the nth and the $(n+1)$th Landau levels. (b) Fermi level lies within the nth Landau band.

those of H_0 are

$$|nk_y\rangle = L^{-1/2}\exp(ik_y y)u_n(x, x_0), \tag{9}$$

where L is a normalization length in the y direction along which the motion is free-electron-like, and $u_n(x, x_0)$ is the normalized harmonic oscillator eigenfunction centered at $x_0 = -\hbar k_y/m\omega_c$. The eigenvalues of H_0, obtained by first-order perturbation theory for a weak modulation, are

$$E_{nk_y} = (n + 1/2)\hbar\omega_c + V_n \cos(2\pi x_0/a), \tag{10}$$

where $V_n = V_0 \exp(-X/2)L_n(X)$, with $X = (2\pi/a)^2/2m\omega_c$, and $L_n(X)$ is a Laguerre polynomial.

In this case the differential cross sections can be evaluated analytically, resulting in the following closed-form expressions corresponding to the transitions depicted in Fig. 1. Note that for simplicity we assume that both the incident and scattered light beams are polarized in the x direction, and that all transitions are vertical ($k_y = k'_y$).

A. Fermi level between the nth and $(n+1)$th Landau bands

The $n \to n+2$ transition is written as

$$d^2\sigma/d\Omega d\omega = (e^2/m)^2(\omega_f/\omega_i)(m\omega_c^3/8\pi^2)(n+1)(n+2)\left[\theta\left(1 - \left|\frac{\hbar\Delta\omega}{V_{n+2}-V_n}\right|\right)\Big/\sqrt{(V_{n+2}-V_n)^2 - (\hbar\Delta\omega)^2}\right]$$
$$\times\left[1\Big/\left(\Delta\omega\frac{V_n - V_{n+1}}{V_{n+2}-V_n} + \delta\omega_i\right) + 1\Big/\left(\Delta\omega\frac{V_{n+2}-V_{n+1}}{V_{n+2}-V_n} - \delta\omega_i\right)\right]^2, \tag{11}$$

where $\theta(x)$ is the Heaviside unit step function, $\delta\omega_i = \omega_i - \omega_c$ and $\Delta\omega = \omega_i - \omega_f - 2\omega_c$.

B. Fermi level within the nth Landau band

The $n \to n+2$ transition is the same as in Sec. II A, but restricted for

$$E_F > E_n - \hbar\Delta\omega V_n/(V_{n+2} - V_n), \tag{12}$$

where E_F is the Fermi energy, and $E_n = (n + 1/2)\hbar\omega_c$.

The $n-1 \to n+1$ transition is

$$d^2\sigma/d\Omega d\omega = (e^2/m)^2(\omega_f/\omega_i)(m\omega_c^3/8\pi^2)(n+1)n\left[\theta\left(1 - \left|\frac{\hbar\Delta\omega}{V_{n+1} - V_{n-1}}\right|\right)\Big/\sqrt{(V_{n+1} - V_{n-1})^2 - (\hbar\Delta\omega)^2}\right]$$
$$\times\left[1\Big/\left(\Delta\omega\frac{V_{n-1} - V_n}{V_{n+1} - V_{n-1}} + \delta\omega_i\right) + 1\Big/\left(\Delta\omega\frac{V_{n+1} - V_n}{V_{n+1} - V_{n-1}} - \delta\omega_i\right)\right]^2. \tag{13}$$

The differential scattering cross sections given above for single Landau electron transitions are evaluated numerically for typical values of the various quantities involved, and are exhibited in Fig. 2. All three types of transitions lead to essentially the same line shape, but with relative magnitudes differing by several orders of magnitude. The $n-1 \to n+1$ transition has the highest scattering cross section, which is shown in Fig. 2. The $n \to n+2$ transitions have scattering cross sections that are at least three orders of magnitude smaller. The line shape reflects the fact that the differential cross section vanishes at exactly the frequency shift $\omega_i - \omega_f = 2\omega_c$. It also clearly shows the density-of-states singularities at the edges of the Landau bands.

III. RAMAN SCATTERING BY COLLECTIVE EXCITATIONS OF A MODULATED 2DEG IN A MAGNETIC FIELD

Transitions involving collective excitations are described by the perturbation $H_2 = e^2A^2/2m$. In this case the differential scattering cross section is given by

$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{\mathcal{A}}{\pi}\left(\frac{e^2}{m}\right)^2\frac{|\hat{\epsilon}_i\cdot\hat{\epsilon}_f|^2}{e^{-\omega/k_BT} - 1}\Pi_2^{2D}(\vec{q},\omega)$$
$$\times\int dz dz' e^{iq_z(z-z')}|\zeta(z)|^2|\zeta(z')|^2. \tag{14}$$

Here $\mathcal{A}$ is the area of the 2DEG, k_B is the Boltzmann constant, T is the lattice temperature, and $\zeta(z)$ is the envelope function arising from the confinement of the electron to the quantum well. Π_2^{2D} is the imaginary part of the density-density correlation function of the modulated 2DEG, which is related to the inverse dielectric function (imaginary part) as $\Pi_2^{2D}(\vec{q},\omega) = K_2^{2D}(\vec{q},\omega)/v_q$, with $v_q = 2\pi e^2/\kappa q$ (κ is the background dielectric constant.)

We have obtained the inverse dielectric function for the modulated 2DEG within the random-phase approximation,[8] and also in the classical hydrodynamic approximation.[15] To simplify the evaluation of the differential scattering cross section we shall make the plasmapole approximation to the inverse dielectric function. This yields

$$K_2^{2D}(\vec{q},\omega) = \pi[(\omega^2 - \omega_c^2)/(\omega_+^2 - \omega_-^2)]$$
$$\times\left[\delta\left(1 - \frac{\omega_-^2}{\omega^2}\right) - \delta\left(1 - \frac{\omega_+^2}{\omega^2}\right)\right]\mathrm{sgn}(\omega), \tag{15}$$

where the two plasma mode frequencies of the modulated 2DEG are given by[15]

$$\omega_+^2 = \omega_{p,2D}^2 + \omega_c^2, \tag{16}$$

with $\omega_{p,2D}^2 = 2\pi e^2 n_0 q/\kappa m$, and

$$\omega_-^2 = \frac{V_0^2}{2\kappa m^2\omega_c^2}\left(\frac{2\pi}{a}\right)^3\frac{q_y^2}{q^2}\left[J_0\left(\frac{2\pi}{a}R_c\right)\right]^2, \tag{17}$$

where $R_c = k_F\ell^2$ is the cyclotron radius at the Fermi energy.

Numerically calculated scattered light intensities due to the plasma excitations are shown in Fig. 3, as a function of the frequency shift. Here we took $T = 0$. The highest peak in the spectrum is due to excitations of the principal magnetoplasmon at the frequency ω_+. The smaller peak is due to the excitation of the new plasma mode of the modulated 2DEG at ω_-. In this calculation we have assumed a constant collision broadening of $\Gamma = 0.5$ meV, and have replaced the δ functions by Lorentzians in the inverse dielectric function (15).

The contribution to the Raman cross section due to the commensurability plasmon of the modulated 2DEG can be further examined by integrating the differential cross section over the frequency range about ω_-. Such an integrated intensity is shown as a function of the inverse magnetic field strength in Fig. 4. The much discussed commensurability oscillations are clearly present in the

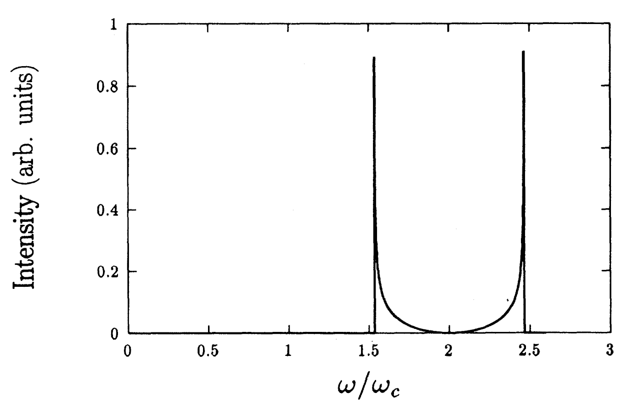

FIG. 2. Calculated scattered light intensity in arbitrary units for single-Landau-electron transitions of a modulated 2DEG. Here $B = 0.5$ T, $n_0 = 3.16 \times 10^{11}$ cm^{-2}, $a = 1000$ Å, $\omega_i = 100$ meV, and $V_0 = 1$ meV. All materials parameters not specified here are taken to be typical values of a GaAs heterostructure.

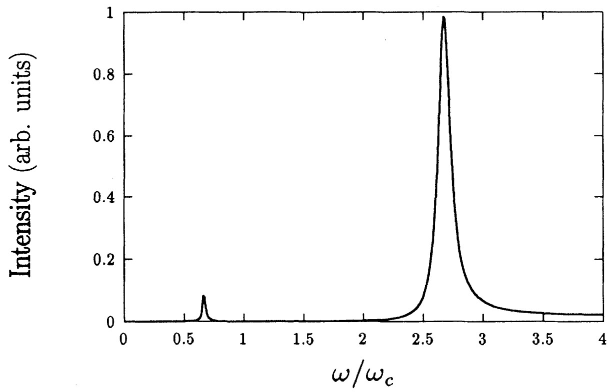

FIG. 3. Calculated scattered light intensity in arbitrary unit for collective Landau electron excitations of a density modulated 2DEG. Here $B = 0.4$ T, $n_0 = 3.16 \times 10^{11}$ cm^{-2}, $a = 1000$ Å, $q = 0.1k_F$, $\Gamma = 0.5$ meV, and $V_0 = 1$ meV.

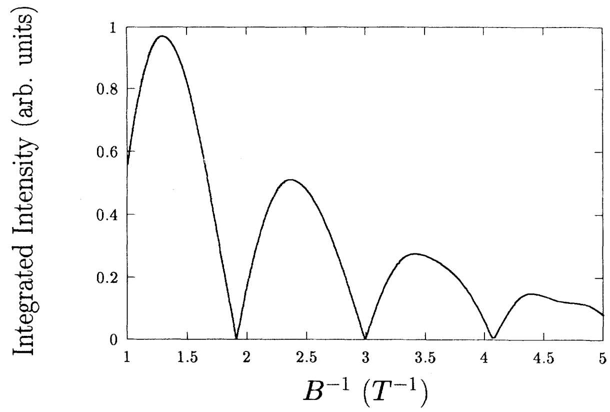

FIG. 4. Integrated scattered light intensity of the lower peak (ω_-) in Fig. 3, as a function of inverse magnetic field. Numerical values used are same as in Fig. 3.

integrated intensity, with increasing prominence for lower magnetic fields. It seems that an inelastic light-scattering experiment should be able to unveil the Weiss oscillations in modulated systems, in addition to the conventional transport experiments.

IV. SUMMARY

The inelastic light-scattering study carried out here is intended to open a new avenue of investigation of the Weiss oscillations, one that is spectroscopic in nature. To this end we have shown that it is indeed possible, by examining the Raman cross sections of either single-Landau-electron processes or collective plasma excitations, to obtain information germane to the modulation and its interplay with the applied magnetic field.

The single-Landau-electron Raman cross section bears out both the Landau miniband widths and the van Hove singularities at its edges. It vanishes characteristically at precisely the frequency shift $2\omega_c$. On the other hand, the Raman scattered spectrum for collective excitations exhibits both the high-frequency, principal magnetoplasmon mode and the modulation-induced, low-frequency commensurability magnetoplasmon. The latter has a total scattering cross section (integrated over all frequency range) that is periodic in inverse magnetic field, with period given by the commensurability condition.

Which of the two types of Raman scattering processes studied here is actually observed experimentally depends largely on the scattering configuration and polarization of the incident light[16–18] (although their contributions to scattering power are comparable in magnitude). We have only considered the simplest possible scattering configuration and polarization here. However, the general features of inelastic light scattering from a modulated 2DEG in a magnetic field are not expected to be affected by the polarization of light and the scattering configurations.

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