PHYSICAL REVIEW B VOLUME 52, NUMBER 18 1 NOVEMBER 1995-II

Low-temperature many-electron hopping conductivity in the Coulomb glass

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(Received 7 February 1995; revised manuscript received 19 June 1995)

A method to study the low-temperature conductivity in the Coulomb glass, i.e., a disordered system with many electrons subject to long-range electron-electron interaction, is introduced. After determining the low-lying many-electron states of the system we calculate the transition matrix between these states, including one-electron hopping as well as many-electron hopping processes. The conductivity is then obtained by an examination of that eigenvector of the transition matrix which describes the stationary state. The results demonstrate that the low-temperature conductivity in interacting systems is strongly influenced by many-electron hopping processes.

I. INTRODUCTION

Theoretical investigations of the low-temperature hopping conductivity in disordered systems of localized electrons with long-range Coulomb interaction such as doped compensated or amorphous semiconductors are still controversially discussed. On the one hand, theories such as the variable-range hopping theory by Mott,[1] have been developed which are based on one-electron hopping processes. For noninteracting systems the variable-range hopping theory predicts an Arrhenius law of the conductivity σ at high temperatures T and a behavior $\sigma(T)\propto\exp[-c(T_0/T)^{1/(d+1)}]$ at low temperatures (the constants c and T_0 depend on the system under consideration and d is the dimension of the system), where the energies of the individual states become more important. Efros and Shkovskii[2] have shown that the Coulomb interaction slightly modifies this behavior so that $\sigma(T)\propto\exp[-c(T_0/T)^{1/2}]$ is independent of the dimensionality of the system. This result is based on single-electron hopping. A special kind of variable-range hopping, which is restricted to hops up to a range R was implemented by Levin *et al.*[3] According to the authors[3] this so-called R model is useful for studying the equilibrium properties of the system, but the restriction strongly influences the determination of the conductivity. For the calculation of the conductivity there exist some Monte Carlo simulations[4,5] which also take into account one-electron hopping only, and which have been evaluated at relatively high temperatures.

On the other hand, it seems unlikely that many-electron processes can be neglected in the theoretical investigation of the conductivity in interacting systems. Especially at low temperatures many-electron hopping is expected to influence the conductivity of the system strongly. The relevance of many-electron effects was pointed out by Pollak[6] as early as 1970 and stressed repeatedly,[7-9] but it remains a matter of controversy.[10] This is at least partly due to the fact that the exact effect of the Coulomb interaction on the density and the nature of the low-lying states was unknown for a long time and therefore an appropriate theoretical treatment of the many-electron conductivity has been impossible until now. Based on assumptions on the many-electron density of states Pollak[7] was able to predict a power law for the temperature dependence of the conductivity.

In recent years different groups have developed and used different algorithms to determine the low-lying states explictly.[11-19] All these approaches, which have been performed after the pioneering but strongly restricted work of Levin *et al.*[3] have demonstrated the huge number of many-electron excitations in the considered energy range.[11-18,20,21] The significance of the many-electron states for the determination of thermodynamic properties was demonstrated subsequently.[12,19,22] Moreover, rate equations describing the relaxation of excited states, were derived and solved by diagonalization[20,23,24] or iteratively.[25] In all these cases many-electron hopping was found to be relevant. This was similarly true in another investigation[26] of relaxation following the most likely relaxation path at each step of the process. That approach was recently[27] extended to the calculation of the low-temperature conductivity, determining a percolation path in the configuration space of the many-electron states. For low temperatures it could be demonstrated in this way that the percolation paths incorporated at least two-electron transitions.

For clarification we point out that even if many-electron excitations would be less important, correlation effects might still be significant for the conductivity: even if one-electron hops dominate the transport, sequential correlations may still be essential. However, as we intend to demonstrate below, many-electron hopping is not to be neglected.

From the experimental point of view, the observed temperature dependence of the conductivity seems to be described in a sufficiently accurate way by one-electron theories. For low temperatures the conductivity can be well described by the theory of Efros and Shklovskii; for higher temperatures, where the Coulomb interaction becomes less important, Mott's law was observed.[28-34] But recently an Arrhenius law of the conductivity at *very low* temperatures has been observed experimentally,[29,33-36] which cannot be understood by the above-mentioned theories. It has been argued, that this strange behavior might be related to the magnetic properties of the systems,[28,35,37] but in some of the experiments a magnetic influence can definitely be excluded.[36]

A Monte Carlo study[38] of only single-electron hopping yielded an activated behavior at temperatures above the Efros-Shklovskii regime. In that study no simultaneous correlations could be included, but sequential correlations between successive one-electron hops were taken into account. On the other hand, the more comprehensive multielectron percolation treatment[27] yielded the Efros-Shklovskii prediction for the temperature dependence of the conductivity, but with a deviating prefactor in the exponent.

Consequently, there is still a lack in the understanding of low-temperature conductivity in disordered systems of localized electrons with long-range Coulomb interaction. With this paper we want to shed some light on this problem. We present a new method to compute the conductivity taking the many-electron transitions into account. The analysis of our results allows us to explain the Arrhenius law at low temperature.

II. TRANSITION RATES BETWEEN MANY-ELECTRON STATES

Our study is based on the Coulomb glass model.[2] The Hamiltonian for $N/2$ electrons on N sites reads

$$H=\sum_{i=1}^{N}(n_i+1/2)\varphi_i+\sum_{\substack{i,j=1\\ i<j}}^{N}\frac{V}{r_{ij}}n_in_j. \tag{1}$$

Here n_i is the occupation number of the ith site which is 1/2 or $-1/2$ for an occupied or unoccupied site, respectively. The disorder in the system is described by the potential φ_i, which is taken from a uniform distribution of width W. The second term of the Hamiltonian characterizes the long-range Coulomb interaction with r_{ij} being the distance of sites i and j. We consider the sites positioned on a regular lattice, i.e., a lattice-gas model. For the results presented below we have chosen $W=V=1$ so that disorder and interaction are equally important.[20,22] Due to the necessary computational effort we restrict ourselves to the examination of systems of size $N=N_x\times N_y=10\times 6$ in two dimensions.

We are aware that this small system size constitutes certainly a severe limitation, because due to the periodic boundary conditions only half the sample length is significant which means that hopping in the x direction is limited to five lattice constants (the extension of the sample in y direction is less significant because we intend to study the current in the x direction, which is the reason for choosing a rectangular instead of a square sample). This length is shorter than the critical correlation length of a current-carrying percolation network, but on the other hand, the notion of such a percolation-based current usually depends on single-electron hops. This means that one should be extremely careful about drawing conclusions from one-electron paths contributing to the conductivity, because on larger samples their share might be reduced. The current-carrying percolation path has also been investigated on the basis of the percolation treatment[27] of the conductivity in the multielectron configuration space. In this case the maximum hopping length of the participating electrons was found to be four for the highest temperatures considered; of course, for lower temperatures this length increases so that the sample size limits the temperature range in which the results are reliable. If, however, many-electron effects turn out to be important for our small samples, it is reasonable to assume that their significance increases for larger samples.

In the first step of our investigation of the low-temperature conductivity we determine the energetically low-lying many-electron states Ψ_k of the systems by means of a Metropolis algorithm as described in Refs. 20 and 22 using periodic boundary conditions. A detailed analysis of these states has shown that a large number of the low-lying states can be reached from the ground state of the system by short-range many-electron hopping rather than by one-electron hopping,[13,20–22,26] which strongly supports the importance of many-electron hopping for the low-temperature conductivity.

Each n-electron tranistion can be constructed from n one-electron hops. In the second step we determine for each pair of states Ψ_k and Ψ_l the initial and final sites $i_m(k)$ and $j_m(l)$ for those hops which yield the shortest possible overall hopping distance

$$R_{kl}=\sum_{m=1}^{n}r_{i_m(k),j_m(l)}. \tag{2}$$

This can be easily performed up to $n\approx 10$ by comparing all $n!$ combinations of initial and final sites. For $n>7$, however, we have used a Metropolis procedure in analogy to the traveling salesman algorithm to optimize the overall hopping distance. Up to $n=12$ the results were explicitly checked by comparison with the systematic analysis of all combinations.

We now examine the system in an electric field $\boldsymbol{\epsilon}=(\epsilon,0,0)$ parallel to the x axis to investigate the conductivity. The transition rate for phonon-induced hopping between Ψ_k and Ψ_l is then given by

$$\gamma_{kl}=\gamma_0e^{-\beta E_{kl}}e^{-2R_{kl}/a} \tag{3}$$

for a process with increasing energy $E_{kl}>0$ at the inverse temperature β. Here the Boltzmann factor $e^{-\beta E_{kl}}$ arises from the phonon density. The second exponential factor is due to the overlap of the electronic wave functions with the localization radius a. For the results presented below we have chosen $a=1/2$ in contrast to previous investigations, because this rather large localization length yields larger transition rates and thus simplifies the computation of the currents.

This phenomenological equation (3) for the many-electron transition rates is only approximate because, e.g., the interaction energy enters only indirectly via the energy difference E_{kl} between the many-particle states. However, the rates should depend explicitly on the interaction, because for vanishing interaction strength correlated hops of two or more electrons become impossible. Moreover, Eq. (3) is approximate, because for the same energy difference E_{kl} and the same overall hopping distance R_{kl} the rate should still depend on the number of displaced electrons, as many-electron hops constitute a higher-order process in comparison to single-electron transitions. Similarly the phonon-assisted hops should become more difficult to correlate if the distance between the hopping electrons (not the length of the hops) increases. On the other hand, two different analytical approaches[8,39] yielded the conclusion that the phenomenological transition probability (3) describes the essential dependence on energy and hopping distance. Due to its simplicity the formula (3) has been widely used to study transport and relaxation properties of the Coulomb glass (see, e.g., Refs. 3,9,22 and 25–27).

For a process with decreasing energy the transition rate (3) simplifies to

$$\gamma_{kl} = \gamma_0 e^{-2R_{kl}/a}. \tag{4}$$

It is worth mentioning that the energy difference E_{kl} is influenced by the electric field and the change of the dipole moment μ_{kl} of the system in the x direction, i.e., the component of the dipole moment in the field direction, associated to the transition via

$$E_{kl} = E_{kl}(\epsilon = 0) + \epsilon \mu_{kl}, \tag{5}$$

where $E_{kl}(\epsilon = 0)$ is the energy difference of the states Ψ_k and Ψ_l.

Considering periodic boundary conditions the dipole moment of a one-electron transition between the sites i and j is given by

$$\mu_{ij} = \begin{cases} x_j - x_i, & |x_j - x_i| \leq N_x/2 \\ x_j - x_i - N_x, & x_j - x_i > N_x/2 \\ x_j - x_i + N_x, & x_i - x_j > N_x/2. \end{cases} \tag{6}$$

In the case of many-electron transitions one has to sum over the n one-electron transitions:

$$\mu_{kl} = \sum_{m=1}^{n} \mu_{i_m(k), j_m(l)}. \tag{7}$$

III. DETERMINATION OF THE CURRENT

To determine the current in the system we consider a master equation, i.e., a rate equation for the occupation probabilities p_l of the states Ψ_l:

$$\frac{d}{dt}\mathbf{p} = \mathbf{\Gamma}\mathbf{p}. \tag{8}$$

Here the vector $\mathbf{p}$ comprises the probabilities p_l and $\mathbf{\Gamma}$ is the matrix of the transition rates γ_{kl} between each two of the states, as given in Eqs. (3) and (4). For the situation without electric field we have previously shown[20,23,24] how the eigenvectors and eigenvalues of the transition matrix $\mathbf{\Gamma}$ can be used to solve the differential equation (8) efficiently for a given initial occupation $\mathbf{p}(t=0)$. In this way the relaxation of an initially excited many-electron state towards the ground state could be followed over many orders of magnitude in time and the importance of many-electron hopping for such relaxation processes was corroborated.[20,23,24]

In the present situation, however, we consider an electric field in Eq. (5). In this case a single electron can be imagined, which hops on a percolation path through the system in the x direction gaining energy within each single hop as a consequence of the electric field. Due to the periodic boundary conditions it is possible that after a few hopping processes the electron reaches its starting point with an energy gain. In other words the electron can move on a closed path through the system gaining energy in each circuit. Therefore the system with periodic boundary conditions is equivalent to a toroidal system embedded in an electrical vortex field. Consequently an electric current exists in the stationary state. This argument needs not to be changed when more than one percolation path or many-electron hopping is included.

Thus the task to determine the electric current can be performed, if the stationary solution of Eq. (8) can be determined, which means that $d\mathbf{p}/dt = 0$ or

$$\mathbf{\Gamma}\mathbf{p} = 0 \tag{9}$$

has to be solved. Consequently, the stationary solution is given by that eigenvector $\mathbf{p}_0 \equiv \mathbf{v}$ of the transition matrix $\mathbf{\Gamma} = (\gamma_{kl})$ which corresponds to the eigenvalue $\lambda = 0$. The knowledge of this eigenvector is the key for a detailed analysis of the conductivity of the system including the effects of many-electron transitions. To determine this eigenvector a homogeneous system (9) of linear equations has to solved.

We note that such an eigenvalue $\lambda = 0$ and thus a nontrivial stationary solution $\mathbf{v}$ always exists. This can be seen in the following way: charge conservation requires that

$$\sum_l p_l = 1. \tag{10}$$

This condition determines the diagonal elements γ_{ll} of the transition matrix according to

$$\sum_k \gamma_{kl} = 0. \tag{11}$$

This means that the rows of the matrix Γ are not linearly independent. Consequently,

$$\det(\mathbf{\Gamma}) = 0. \tag{12}$$

As the determinant of a matrix is equal to the product of all eigenvalues of the matrix, it follows that at least one eigenvalue of $\mathbf{\Gamma}$ has to be zero.

Usually the transition matrix $\mathbf{\Gamma}$ features exactly one eigenvalue $\lambda = 0$. In principle, one could imagine that an accidental degeneracy occurs, taking into account the randomly distributed potentials in the Hamiltonian (1) which yield "random" energy differences E_{kl} and thus "random" matrix elements γ_{kl}. In practice, however, we have never encountered such a degeneracy neither when diagonalizing the entire matrix $\mathbf{\Gamma}$ in our relaxation studies[20,23,24] nor when solving Eq. (9) in the present investigation.

Once the stationary state $\mathbf{v}$ is known it is possible to calculate the related current via

$$I=\sum_l \sum_{k\neq l} \gamma_{kl}\mu_{lk}\nu_l , \tag{13}$$

where ν_l denotes the lth component of the stationary state $\mathbf{v}$. In the linear region of the current-voltage characteristics the conductivity can be obtained as $\sigma=I/\epsilon$. We have checked that the system is in the linear region for $\epsilon\approx kT/100$, but we observe strong deviations from the linear behavior of the current for $\epsilon=kT/10$ in contrast to Refs. 4 and 5.

A sensitive test of the quality of the obtained conductivity is a closer examination of the current path. Between any two sites i and j of the system the current

$$I_{i\to j}=\sum_k \sum_l \gamma_{kl}\sum_{m=1}^{n} (\delta_{i,i_m(k)}\delta_{j,j_m(l)}-\delta_{j,i_m(k)}\delta_{i,j_m(l)})\mu_{ji}\nu_l \tag{14}$$

flows, where the Kronecker δ's ensure that the sites i and j are the initial and final sites (or vice versa) of one of the one-electron hops from which the many-electron transition from state Ψ_k to state Ψ_l is constructed. A typical example of the current is displayed in Fig. 1. It can be seen that different paths through the system contribute. Thus it becomes clear that the above described procedure of determining the stationary solution of the master equation (8) does not single out an optimal percolation path but rather includes all continuous paths through the sample which transfer one or more electrons between periodically related sites without changing the state of the system. Most of these paths, however, are extremely unlikely. This is due to the fact that the stationary solution in the case with a (relatively small) electric field differs only slightly from the thermal distribution without electric field. The occupation probabilities of the states Ψ_k are typically changed by less than 1%. Due to the electric field the transition rates γ_{kl} and γ_{lk} do not fulfill the principle of detailed balance. But nevertheless the total current between two sites, which is determined by many different contributions according to Eq. (14), vanishes in most cases as can be seen in Fig. 1.

The currents between the different sites can be used to check the accuracy of our calculations. In thermal equilibrium, the charge conservation must be granted at each site. Consequently at each site j the equation

$$I_j=\sum_{i\neq j} I_{i\to j}=0 \tag{15}$$

must be fulfilled which is equivalent to $\mathrm{div}(I)=0$. In the case of nonzero I_j the site j would be an electron source or an electron drain. If a site exists where Eq. (15) is violated, an error occurs in the calculation of the current according to Eq. (13). Consequently the value

$$Q=\frac{\Sigma_j|\mathbf{I}_j|}{I}\frac{1}{N} \tag{16}$$

measures the quality of the determined current (13). Q should be much smaller than 1 in any case. We have checked this condition carefully for all our data. In some cases the condition is violated, which can be explained by numerical instabilities in the determination of the stationary state $\mathbf{v}$. Our analysis shows that the exact determination of this eigenvector is the crucial point in the numerical calculation due to the fact that the elements of the transition matrix differ by several orders of magnitude.

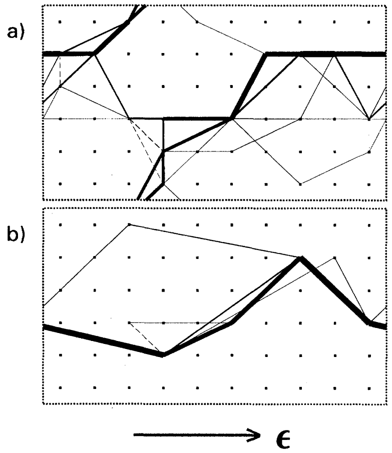

FIG. 1. Current path in a noninteracting system of size $N=10\times6$ with periodic boundary conditions for (a) $\beta=50$ and (b) $\beta=350$. The dots indicate the sites of the system. The line thickness is proportional to the strength of the current contributions according to Eq. (14) before performing the summations in Eq. (14). The largest contribution between two sites therefore covers smaller contributions in the plot so that the current conservation (15) at each site cannot be perceived. Solid and broken lines correspond to a current flow in and opposite to the direction of the electric field ϵ. For clearness the current contributions, which are less than 1% of the maximum current, are neglected.

Another crucial point should be mentioned again, namely the very limited system size. We have taken care in our anlaysis of the current path with respect to variable range hopping that the longest significant hopping length in the x direction is given by half the sample length, i.e., five lattices constants. (Longer hops have to be shortened by shifting one of the participating sites by $\pm N_x$ according to the periodic boundary conditions.) For single-electron hopping one can estimate the hopping length to be of the order of $\sqrt{a/T}$ which yields six lattice constants already for $\beta=72$. This impedes the correct observation of variable-range hopping and we shall mention the consequences for the computed conductivity in the discussion of the results below. But variable-*range* hopping is not our main concern, because variable-*number* hopping will be demonstrated to be significant. In this case the average hopping distance of the participating electrons is of the order of $\sqrt{a/T/n}$, which is smaller than the sample length even for $\beta=200$. In this respect we note that for the investigated sample the number n of displaced electrons was determined[40] to be 3 or 4 in most energetically low-lying excitations.

Another bottleneck of our examination arises from the fact that only a finite number of low-lying states can be included in our investigation. Our results are therefore only valid with sufficient accuracy up to a maximum temperature T. As the density of states increases very strongly with increasing energy[11–17,20,21] it would be necessary to consider many more states to reach slightly higher temperatures. In our studies we concentrated on the lowest 200 or 1000 states, which allows us to perform our calculations on a workstation. For the highest temperatures in our results presented below we have found only small differences depending on the number of states. This is not really surprising, because already the lowest 200 states cover the relevant energy interval for the temperature range considered below. For one-electron hopping one can estimate this energy interval to be of the order of $\sqrt{T/a}$. Although the validity of this formula has to be questioned when multielectron transitions are significant, so that hopping transport becomes feasible already at lower energies, it is nevertheless an upper estimate for the reasonable energy range. For $\beta=50$ this estimate requires the inclusion of about 200 states. This explains that only small differences in the results were obtained upon including the next 800 states. Expanding the set to 1000 states does of course not increase the energy interval very much, because of the drastic increase of the density of many-electron states[11–17,20,21] with increasing energy. But we have explicitly checked that the consideration of another 300 states does not yield any further difference in the results.

As a further test of the power of our method for the calculation of the low-temperature conductivity we have also performed[40] a Monte Carlo simulation of hopping in an electric field, but for a system of very small size (10×1, i.e., one dimensional), for which all 252 many-electron configurations in the half-filled case are exactly known. The one-electron hopping results were comparable with the respective solution of the master equation (8), but the computational effort needed for the Monte Carlo method was 30 times larger than for our new method. Moreover, even then the Monte Carlo simulation featured still strong statistical fluctuations. The reason for the inferiority of the Monte Carlo method in this case can be found in the huge differences of the rates in Eq. (3) for different transitions which to our best knowledge cannot be taken into account by an appropriate importance sampling. This renders the Monte Carlo approch inefficient.

IV. RESULTS

A detailed analysis of all contributions to the current according to Eq. (13) yields some interesting quantities, e.g., the temperature dependence of the mean hopping distance. In the following we summarize the main results of this examination.

(i) In systems without Coulomb interaction, i.e., under neglection of the second term in the Hamiltonian (1), we observe an increase of the hopping distance with decreasing temperature. As an example the current path of a two-dimensional noninteracting system is shown in Fig. 1 for $\beta=50$ and $\beta=350$. The smaller number of corners in the current path for $\beta=350$ in Fig. 1(b) indicates the larger hopping distances. This behavior corresponds to the predictions of the variable-range hopping by Mott.[1] Despite this fact the conductivity does not follow Mott's law in any temperature region. This can be understood as a consequence of the small system size ($N=10\times6$). For the lowest temperatures under consideration there are already some hopping processes which span half the system size [see Fig. 1(b)], which means that the hopping distance cannot increase further with decreasing temperature.

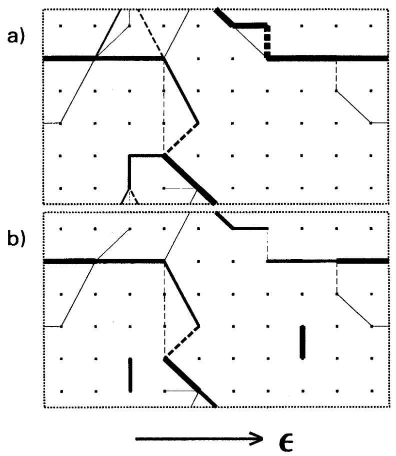

FIG. 2. Current path in an interacting system of size $N=10\times6$ for $\beta=100$. The presentation corresponds to Fig. 1. In (a) all hopping processes are considered whereas in (b) only the contributions of the one-electron hopping processes are included.

(ii) In systems without Coulomb interaction, the neglection of many-electron transitions in the transition matrix, i.e., setting $\gamma_{kl}=0$ for many-electron transitions, yields the same results for the stationary state and the current as the consideration of the full transition matrix. This means that many-electron hopping is not important in noninteracting systems, as commonly expected. We note that even sequential correlations do not exist at all in these noninteracting systems, because the contributing one-electron hops are independent.

(iii) In interacting systems we observe an increase of the mean hopping distance with increasing temperature. This observation is in agreement with the prediction of variable-range hopipng in interacting systems. Again we do not observe the predicted temperature behavior of the conductivity. But as in the noninteracting case, this can be explained by the small system size.

(iv) With decreasing temperature in interacting systems we observe contributions of more and more many-electron hopping processes. The importance of the many-electron hopping processes becomes visible if the current path including all hopping processes is compared to the current path obtained by analyzing the stationary state only with respect to the one-electron transitions. This is demonstrated in Fig. 2. In Fig. 2(b), where the multielectron hopipng processes are neglected, one observes several electron sources and drains in contrast to Fig. 2(a) indicating the importance of many-electron hopping. Especially prominent in this respect is the thick vertical line on the right-hand side of Fig. 2(b). This one-electron hop is completely balanced by the respective contributions of many-electron transitions so that no such line appears in Fig. 2(a). A determination of the stationary state under neglection of all many-electron processes in the transition matrix yields the following result: In some systems there exists still a closed current path through the system but the absolute value of the current decreases in comparison to the consideration of all possible hopping processes; in other systems such a current path does not exist anymore when many-electron hopping processes are neglected, the station-

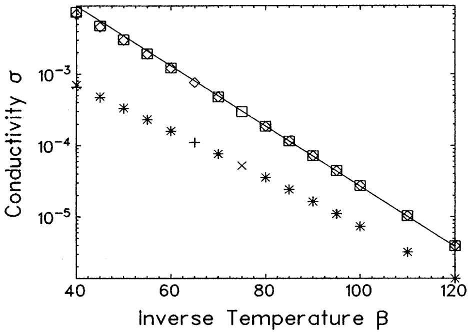

FIG. 3. Temperature dependence of the conductivity (□, ◇) and its many-electron part (+, ×) in the interacting system of size $N = 10 \times 6$. □, + and ◇, × are calculated on the basis of 200 and 1000 low-lying states, respectively. The solid line is a fit of the Arrhenius law in the low-temperature region. The many-electron part of the conductivity is obtained by neglecting all one-electron transitions in Eq. (13), i.e., all those terms in which the sum over m in Eq. (7) or, equivalently, in Eq. (14) contains only one term ($m = n = 1$).

ary state **v** coincides with the thermal distribution without an electric field and the total current in the system vanishes. Considering only contributions of the many-electron transitions to the conductivity an increasing importance of the many-electron processes with decreasing temperature is observed, which is reflected by the comparatively flat slope of the many-electron conductivity in Fig. 3.

(v) In interacting systems the conductivity is smaller than in noninteracting systems. This observation is in accordance with the results of one-electron Monte Carlo simulations.[41] It is not surprising, because the one-electron hopping as well as the many-electron transitions become more difficult as the repulsive interaction energy with the other charge carriers has to be overcome.

(vi) Our main result is that in interacting systems the conductivity follows an Arrhenius law at very low temperatures (cf. Fig. 3). This behavior is in contrast to the result of the percolation treatment[27] of the multielectron conductivity, which yielded the Efros-Shklovskii dependence on temperature.

We do not attribute this difference to the random spatial distribution of the sites chosen in Ref. 27 in contrast to our lattice structure. We do not think that the difference is due to our system size (albeit this is extremely small), as long as the hopping length does not span half the system length of five lattice constants; we have rather used this criterion to limit the temperature range displayed in Fig. 3. We have further checked that the observed Arrhenius law is not a mesoscopic effect specific to the particular random sample: Different samples yield similar behavior. We have also checked as mentioned above that the number of states which were included in our investigation was sufficiently large. At first view it may be surprising that the consideration of 1000 instead of 200 many-electron states seems not to establish further significant hopping paths through the system. Such paths are indeed established but do not contribute to the conductivity at low temperatures, because these paths involve configurations of relatively high energy which are very difficult to reach at low temperatures even under the influence of an electric field in Eq. (5).

We rather believe that the conceptual difference between the percolation treatment[27] and our stationary solution of the master equation (8) is significant: at a given temperature the percolation approach takes into account *one* path through the system while the stationary solution of Eq. (8) comprises all the continuous paths with their appropriate weights [cf. Eq. (14) and the subsequent discussion]. We attribute the contrasting results of Ref. 27 to this different ansatz. We note the same conceptual difference between the relaxation studies[25,26] following the most likely relaxation path at each step of the relaxation process and the more comprehensive solution of the relaxation rate equation [similar to Eq. (8)], which takes all possible relaxation paths into account with their appropriate weights.[20,23,24]

A detailed analysis of the current paths shows that there are always some bottlenecks which have to be passed by the same hopping processes independent of the temperature. It is interesting to note that the same bottlenecks occur for several different paths which make significant contributions to the stationary solution of the master equation (8). It seems that there is no other possibility to pass these regions of the systems. Consequently the transition rates of the same few hopping processes, which are necessary to pass the bottlenecks, determine the conductivity of the systems independent of the temperature. In this case it is obvious that the conductivity follows an Arrhenius law.

V. SUMMARY

To conclude, we have presented a method to determine the conductivity of interacting systems with the full influence of many-electron hopping processes. It turned out that with decreasing temperature the hopping distances as well as the importance of many-electron hopping increase in interacting systems. In some interacting systems no current exists if only one-electron hopping processes are considered. At very low temperatures we observe an Arrhenius law of the conductivity, as recently measured in some experiments.[29,33–36] This temperature dependence seems to be related to bottlenecks in the current path in the interacting systems. In this paper we displayed our results for a two-dimensional system. First computations have been performed in three-dimensional systems of size $N = 5 \times 5 \times 5$ with qualitatively comparable results, in particular yielding the Arrhenius law for the conductivity at very low temperatures, too. Although it is difficult to give quantitative predictions from our examination, as the systems under consideration are very small, it follows nevertheless, that one-electron theories[38] are unsuitable for the description of the transport properties of doped compensated semiconductors at low temperatures.

ACKNOWLEDGMENTS

The authors thank J. Sabczysnki and T. Vojta for helpful discussions. This work was supported by the Deutsche Forschungsgemeinschaft (SFB 262).

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