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Sound transmission phenomena and phase-separation kinetics at the superfluid-normal interface of liquid ^{3}He-^{4}He mixtures

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The transmission and reflection of sound at the interface between ^{3}He-dilute and ^{3}He-concentrated phases of a liquid ^{3}He-^{4}He mixture are studied. The effect of phase separation on sound phenomena is involved in terms of the effective growth coefficient calculated under assumptions of the existence of phase equilibrium at the interface. At low temperatures the phase-separation kinetics is governed by the processes associated with the excitation and propagation of second sound modes to compensate the difference in ^{3}He concentration of adjacent phases. The growth coefficient is found to vanish at the maximum concentration point on the separation line of the ^{3}He-dilute phase. Accordingly, the behavior of the sound transmission coefficient as a function of separation pressure displays a small smooth maximum at the pressure of the maximum concentration point and the reflection of the second sound wave from the normal ^{3}He-concentrated phase reaches its maximum value at the same pressure.

I. INTRODUCTION

For more than a decade a great deal of experimental and theoretical interest has been attracted to the macroscopic quantum tunneling phenomena accompanying the decay of a metastable system described by a macroscopic parameter. A noticeable portion of attention has been paid to the low temperature nucleation phenomena in a metastable condensed medium. As the most typical systems of interest we can mention overpressurized liquid helium,[1] supercooled *A* phase of superfluid ^{3}He,[2] cavitation of bubbles in liquid ^{4}He at negative pressure,[3] and nucleation of vortices in superfluid ^{4}He.[4]

The first systematic studies of the low temperature separation kinetics[5] have been recently done down to 0.4 mK in the two-component metastable system as a supersaturated liquid ^{3}He-^{4}He mixture, which is apparently the most prominent and unique system for observing the bulk quantum nucleation in a condensed medium. The decay of the metastable homogeneous state of a supersaturated ^{3}He-^{4}He mixture is supposed to be associated with the formation and growth of a droplet of an energetically favorable phase enriched by ^{3}He. Essentially the growth of a droplet represents the surmounting of a certain potential barrier originating from the positive interfacial tension until the droplet will reach the critical size and will be capable of converting the system on the whole into the stable state. At high temperatures, when the nucleation rate is governed by thermal fluctuations, the key parameter determining the nucleation rate is the height of a potential barrier.

For sufficiently low temperatures, when the thermal activation should be succeeded by quantum effects related to the tunneling through a potential barrier, the kinetic or growth properties of the interfacial boundary begin to play a principal significance in the determination of the nucleation rate. Phenomenologically, the growth properties of the interface and their influence on the quantum nucleation rate can be described by the incorporation of two kinetic terms in addition to the potential energy in the equation of droplet growth. One of them is related to the kinetic energy of a droplet and characterized by the effective mass of a droplet. The other term, described by a friction coefficient, represents a drag force hindering the growth of a droplet and is connected with the energy dissipation resulting from the irreversibility of relaxation processes associated with disturbing the medium from the equilibrium state. Essentially the terms represent the imaginary or reactive part and the real or resistive part in the reciprocal of the complex response function or generalized growth coefficient relating the interface growth rate to the imbalance between states of phase equilibrium. A knowledge of the effective mass and friction coefficient is of principal importance because these quantities, in addition to barrier height, determine the thermal-quantum crossover temperature, the magnitude, and the temperature dependence of nucleation rate in the quantum regime.

However, the theoretical calculation of the effective mass and the friction coefficient for a droplet in superfluid ^{3}He-^{4}He mixture comes across definite difficulties associated with the boundary conditions at the interface between ^{3}He-dilute and ^{3}He-concentrated phases. The point is that, in order to determine the spatial distributions of fluid velocities, ^{3}He concentration, temperature and pressure in both the ^{3}He-dilute and ^{3}He-concentrated phases and then the friction coefficient and the effective mass of a droplet, it is not sufficient to use only the continuity conditions for the flows of total mass, ^{3}He mass, energy, and momentum density across the interface. As a result, some supplementary assumptions on the boundary conditions are necessary. In particular, one can suppose the lack of dissipation in the mixture[6] or the sticking of the normal component of superfluid ^{3}He-dilute phase to the surface of ^{3}He-concentrated phase.[7] Accordingly, we will arrive at the different friction coefficients and effective masses of a droplet, entailing different nucleation rates and thermal-quantum crossover temperatures.

Unfortunately, the accuracy attained in the low tempera-

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ture nucleation experiments, the drastic exponential dependence of nucleation rate on the supersaturation of mixture, and the lack of microscopic description for nucleation of small-sized droplets do not allow us to make an undeniable distinction in favor of any assumption on the boundary conditions at the interface. In such a situation, especially when realistic microscopic calculations of quantum nucleation in ^{3}He-^{4}He mixture are absent, any experiments which can determine boundary conditions and elucidate the nature of kinetic growth processes at the interfacial region are desirable.

In order to examine the kinetic growth properties of the interface, we should first disturb it from the state of phase equilibrium by means of varying temperature, pressure, and concentration from the equilibrium values separately or in combination and next observe the response of the interface. In particular, the deviation from phase equilibrium can be provoked by an incident acoustic wave which induces modulations of pressure at the interface. Similar experiments were proposed and performed in the course of studying the kinetic properties and phase conversion phenomena for the liquid-solid[8] and vapor-liquid[9] interfaces of helium. The pressure modulations should cause the processes of phase separation at the interface between two adjacent phases. The transmission of a sound wave depends strongly on the interface mobility.[8] Provided the response of the interface on the pressure modulations is slow, i.e., the interface growth rate is extremely small and phase conversion from one phase to another is not so efficient, the sound transmission and reflection will be the same as for the case of two immiscible liquids. On the contrary, if the interface has a high mobility and phase conversion between phases occurs at an infinite rate, the system is able to retain the phase equilibrium state and to smooth the magnitude of pressure modulations down to zero. This means that a sound wave incident onto the interface from one medium should not cause any pressure modulations at the interface and in the other medium. Hence in this ideal case the sound transmission into the other phase vanishes. Thus the coefficients of sound transmission and reflection may give certain information on the rate of conversion processes between two phases.

In the present paper we consider the transmission and reflection of sound waves at the interface between normal ^{3}He-concentrated and superfluid ^{3}He-dilute phases of liquid ^{3}He-^{4}He mixture. In addition, we shall discuss the connection of the above phenomena with the phase separation kinetics in liquid ^{3}He-^{4}He mixture and compare their features with the single-component system such as the liquid-solid boundary in pure helium.

II. TRANSMISSION OF SOUND ACROSS THE SUPERFLUID-NORMAL INTERFACE

So, let us consider the normal incidence of a monochromatic acoustic sound wave from the normal ^{3}He-concentrated phase onto an infinite flat plane which separates the normal phase from the superfluid ^{3}He-dilute phase of a liquid ^{3}He-^{4}He mixture. In addition to the displacement of the interface the modulations of pressure can induce the modulations of temperature and concentration at the interface. Hence, in addition to the reflected and transmitted acoustic sound waves, a second sound wave will arise propagating into the bulk of the superfluid phase. In the opposite case of a sound wave incident from the superfluid phase of the mixture, an excited second sound wave traveling back will appear in addition to the reflected and transmitted acoustic sound waves. Naturally, this results in a certain asymmetry for the coefficients of sound transmission through the superfluid-normal interface. Of course, the asymmetry should not be large since the excitation of second sound by means of an acoustic pressure wave is not so effective.

As usual, in the treatment of acoustic problems we assume all the quantities, e.g., as pressure P, concentration c, and temperature T, differ slightly from their constant equilibrium values in order to deal with the linearized equations. For simplicity, we also neglect the dissipative contribution connected with the coefficients of viscosity. Solving the equations of motion for the bulk of phases, the plane harmonic wave solutions should be found. Next, these solutions are matched at the interface.

To gain further simplifications, we neglect all heat effects associated with the thermal expansion and with the release of latent heat during phase conversion. Strictly speaking, the first condition implies vanishing entropies of both phases and can be valid only in the limit of temperatures much smaller than the degeneration temperature of mixture $T \ll T_F$, where the second sound wave is represented mainly by the oscillations of a ^{3}He component and the velocity of second sound becomes independent of temperature. Thus the amplitudes of waves can generally be written as

$$\begin{aligned} \delta P(xt) &= \delta P_\tau e^{ik_1 x - i\omega t} + \beta(\omega)\,\delta c\, e^{ik_2 x - i\omega t}, \quad k_1 = \frac{\omega}{u_1}, \\ \delta c(xt) &= \delta c\, e^{ik_2 x - i\omega t}, \quad k_2 = \frac{\omega}{\sqrt{u_2^2 - i\omega D}}, \end{aligned} \tag{1}$$

in the superfluid $x>0$ phase and

$$\begin{aligned} \delta P'(xt) &= \delta P_0 e^{ik' x - i\omega t} + \delta P_r e^{-ik' x - i\omega t} \\ &\quad + \beta'(\omega)\,\delta c' e^{\lambda' x - i\omega t}, \quad k' = \frac{\omega}{u'}, \\ \delta c'(xt) &= \delta c' e^{\lambda' x - i\omega t}, \quad \lambda' = \sqrt{\frac{-i\omega}{D'}}, \end{aligned} \tag{2}$$

in the normal $x<0$ phase, the positive direction of the x axis being chosen from the normal into the superfluid phase. Here δP_0, δP_r, and δP_τ are the amplitudes of incident, reflected, and transmitted acoustic waves. In general, the oscillations of pressure can give rise to the concentration oscillations δc and $\delta c'$ which will propagate in the form of a second sound wave into the bulk of the superfluid phase and decay at the diffusion length $\sqrt{\omega/D'}$ into the normal phase. The coefficients $\beta(\omega)$ and $\beta'(\omega)$ determining the connection between oscillations of pressure and concentration in the second sound wave and in the diffusive mode can be approximated as

$$\beta(\omega) = \frac{u_2^2 - i\omega D}{1 - (u_2^2 - i\omega D)/u_1^2}\,\frac{\partial\rho}{\partial c} \approx (u_2^2 - i\omega D)\,\frac{\partial\rho}{\partial c}, \tag{3}$$

$$\beta'(\omega)=-\frac{i\omega D'}{1+i\omega D'/u'^2}\frac{\partial\rho'}{\partial c'}\approx -i\omega D'\frac{\partial\rho'}{\partial c'}. \tag{4}$$

The quantities D and D' denote the diffusion coefficients for ^{3}He atoms in the superfluid and normal phases, u_1 and u' are velocities of acoustic sound waves, and u_2 is the second sound velocity neglecting the entropy term[10]

$$u_2^2=\frac{c^2\rho_s}{\rho_n}\frac{\partial Z}{\partial c}. \tag{5}$$

Here c is the mass concentration of ^{3}He atoms and the thermodynamic potential Z conjugated to variable c is defined according to

$$Z=\frac{\mu_3}{m_3}-\frac{\mu_4}{m_4}, \tag{6}$$

where μ_3 and μ_4 are the chemical potentials of ^{3}He and ^{4}He atoms in mixture and m_3 and m_4 are the masses of particles. For our purposes, the weak coupling between the concentration and pressure oscillations in acoustic waves can be neglected and the corresponding terms proportional to $\exp(ik_1x-i\omega t)$ and $\exp(\pm ik'x-i\omega t)$ are omitted in the expressions for $\delta c(xt)$ and $\delta c'(xt)$. Note also that in the expressions for the first and second sound velocities and in what follows we neglect terms proportional to the square of the derivative of density with respect to concentration $(\rho^{-1}\partial\rho/\partial c)^2$ compared with unity, but we retain first-order terms.

With the neglect of viscosity and latent heat effects one can formulate the following boundary conditions at the interface $x=0$:

$$J=\rho'(V'-\dot\zeta)=j-\rho\dot\zeta,\quad j=\rho_nV_n+\rho_sV_s, \tag{7}$$

$$J_3=\rho'c'(V'-\dot\zeta)+g'=\rho c(V_n-\dot\zeta)+g, \tag{8}$$

$$P'=P \text{ or } \delta P'=\delta P, \tag{9}$$

where $\dot\zeta$ is the velocity of the interface and V', V_n, and V_s are the fluid velocity of normal phase, normal, and superfluid velocities of the ^{3}He-dilute phase. The quantities g and g' represent the diffusion flows in which the barodiffusion terms are involved as well,

$$\begin{gathered} g=-\rho D\nabla c-\rho D_P\nabla P,\quad D_P=-\frac{D}{\rho^2}\frac{\partial\rho/\partial c}{\partial Z/\partial c}, \\ g'=-\rho' D'\nabla c'-\rho' D_P'\nabla P',\quad D_P'=-\frac{D'}{\rho'^2}\frac{\partial\rho'/\partial c'}{\partial Z'/\partial c'}. \end{gathered} \tag{10}$$

It is obvious that the conditions Eqs. (7)–(9) incorporate conservation laws of total mass, ^{3}He mass, and equality of pressures resulting from the conservation of the momentum density flow.

In order to determine unknown amplitudes δP_r, δP_τ, δc, and $\delta c'$, the above boundary conditions are insufficient and should be augmented. It is evident that the incidence of sound waves onto the interface induces the phase conversion processes disturbing both the interface and the immediate bulk adjacent to the interface. In this bulk region the relaxation toward equilibrium occurs owing to collisions between particles of the medium. In the low frequency $\omega\tau\ll 1$ sound limit, τ being the collision time, we can assume that the thickness of the interface is much smaller compared with the sound wavelength and particles of a medium have many collisions at the distance of wavelength. Under these conditions one can believe that the relaxation of the interface is much faster than that in the bulk where the motion toward a state of phase equilibrium retards due to frequent collisions. In other words, we may assume that the phase boundary between ^{3}He-dilute and ^{3}He-concentrated phases can be treated as being in a state of local phase equilibrium in spite of its disturbance by the pressure modulations of incident sound waves. This assumption leads to the boundary conditions of equality of chemical potentials for both ^{3}He and ^{4}He components of mixture. In turn, this is completely identical to the assertion that the oscillations of concentration in each phase beside the interface will follow the pressure modulations in exact accordance with the phase-separation lines $c=c(P)$ and $c'=c'(P)$ on which the conditions of phase equilibrium are fulfilled precisely,

$$\begin{gathered} \delta c=\frac{dc(P)}{dP}\delta P,\quad x=+0, \\ \delta c'=\frac{dc'(P)}{dP}\delta P',\quad x=-0. \end{gathered} \tag{11}$$

As a final remark, it is to be added that the same hydrodynamic boundary conditions of equalities of chemical potentials for ^{3}He and ^{4}He atoms were employed before[11] for studying processes of the low frequency two-phase sound propagation in the direction parallel to the superfluid-normal interface under the acoustic waveguide geometry.

We are now in the position to calculate the unknown amplitudes δP_r, δP_τ, δc, and $\delta c'$ using the boundary conditions Eqs. (7)–(9) and (11). Omitting the algebraic calculations, let us note only the usefulness of employing the Clapeyron-Clausius relations valid on the phase equilibrium surface of binary mixture

$$\begin{gathered} v'-v-(c'-c)\frac{\partial v}{\partial c}=(c'-c)\frac{\partial Z}{\partial c}\frac{dc}{dP}, \\ v'-v-(c'-c)\frac{\partial v'}{\partial c'}=(c'-c)\frac{\partial Z'}{\partial c'}\frac{dc'}{dP}, \end{gathered} \tag{12}$$

which can be derived by differentiating two equalities for chemical potentials of each component with respect to pressure at fixed temperature.[7] Here $v=1/\rho$ and $v'=1/\rho'$ are the specific volumes of the phases. Defining acoustic coefficients of reflection and transmission as ratios of the pressure amplitudes of reflected and transmitted waves to the amplitude of an incident wave, one has for the case of incidence from normal phase into superfluid phase within an accuracy $u_2^2/u_1^2\ll 1$,

$$r=\frac{\delta P_r}{\delta P_0}=\frac{Y-Y'-YY'\xi_\omega}{Y+Y'+YY'\xi_\omega}, \tag{13}$$

$$\tau=\frac{\delta P_{\tau}}{\delta P_0}=\frac{2Y(1-\beta(\omega)dc/dP)}{Y+Y'+YY'\xi_{\omega}}, \tag{14}$$

where the effective kinetic growth coefficient ξ_{ω} is given by the relation

$$\xi_{\omega}=\sqrt{u_2^2-i\omega D}\rho\frac{\partial Z}{\partial c}\left(\frac{dc}{dP}\right)^2+\sqrt{-i\omega D'}\rho'\frac{\partial Z'}{\partial c'}\left(\frac{dc'}{dP}\right)^2. \tag{15}$$

Here for the acoustic impedances of superfluid and normal phases we introduced notations $Y=\rho u_1$ and $Y'=\rho' u'$, where u_1 and u' are the sound velocities of the corresponding phases. The effective growth coefficient expresses finally[8] the relation $J\sim\xi_{\omega}\delta P$ between the net mass flow J, Eq. (7), through the interface and the modulations of pressure δP inducing the imbalance between the two phases.

As is seen, in the low frequency $\omega\tau\ll 1$ sound limit the effective low temperature growth coefficient of the superfluid-normal interface in a mixture is determined by a sum of two terms. The first term, predominant as $\omega\rightarrow 0$, is associated with the emission of the second sound mode into the superfluid phase, and the second term is due to the diffusive concentration mode in the normal phase. The smaller the second sound velocity and diffusion coefficients, the smaller the mobility of the interface. For the normal-normal interface, the second sound velocity should be put equal to zero. Since at low temperatures the ^{3}He-concentrated phase in ^{3}He-^{4}He mixture is practically pure ^{3}He and the pressure slope $dc'/dP\propto[1-c'(P,T)]$, the second term in Eq. (15) vanishes so that the low temperature dynamics of the flat interface is governed by the ^{3}He propagation processes in the ^{3}He-dilute phase alone, i.e.,

$$\xi_{\omega}=\rho\left(\frac{v'-v-(c'-c)\partial v/\partial c}{c'-c}\right)^2\frac{\sqrt{u_2^2-i\omega D}}{\partial Z/\partial c}$$
$$=\sqrt{u_2^2-i\omega D}\rho\frac{\partial Z}{\partial c}\left(\frac{dc}{dP}\right)^2. \tag{16}$$

It is to be noted that the effect of separation processes at the interfacial boundary on the sound transmission should not be large since the second sound velocity is small compared with the first sound velocity. As a function of pressure, the effective growth coefficient vanishes at the maximum concentration point $P=P_m$ of the phase separation line where $dc/dP=0$. Hence the sound transmission reaches its maximum equal to the ordinary acoustic transmission. Accordingly, the reflection is also given by the ordinary acoustic-mismatch value.

Such a pressure dependence of acoustic coefficients can easily be understood. In fact, if the phase conversion occurs and the interface moves at a certain rate with respect to fluid velocities, the deficiency or excess of ^{3}He concentration proportional to the difference $(c'-c)$ which inevitably appears at the interface during phase conversion should be compensated. However, at the maximum concentration point, according to Eq. (11), the pressure modulations are not sufficient to induce any variations in ^{3}He concentration at the interface, entailing the lack of any process as the second sound mode or diffusion capable of changing the concentration beside the interface. As a result, no phase conversion can occur at the interface and the flows of total mass J and ^{3}He mass J_3 across the interface vanish. Hence the interface velocity $\dot{\zeta}$ equals the fluid velocities V', V_n, V_s and the sound transmission and reflection are determined by the ordinary formulas of acoustic-mismatch theory.

It is interesting to emphasize that such behavior of interface mobility at the maximum concentration point $dc/dP=0$ in a ^{3}He-^{4}He mixture is in contrast to the behavior of the liquid-solid ^{3}He interface at the point of the minimum solidification pressure $dP/dT=0$ where the liquid-solid ^{3}He mobility reaches the maximum value.[12] Unlike multicomponent systems, in the single-component system the minimum pressure point $dP/dT=0$ related to zero value of latent heat corresponds to vanishing difference in the entropies of the two phases so that in the thermal sense the phases become indiscernable and the interface has a higher mobility. In mixtures a similar situation could be realized at the point of equal concentrations $c'=c$ where any necessity in the processes which change the concentration and therefore lead to energy dissipation is absent. As regards the excitation of second sound, the pressure amplitude in the second sound mode, obviously, is not large compared with the pressure amplitude of transmitted sound. Introducing the excitation coefficient as a ratio of the pressure amplitude in the second sound mode to the pressure amplitude of the incident wave, one obtains

$$\gamma=\frac{\delta P_{\mathrm{ex}}}{\delta P_0}=\frac{\beta(\omega)\delta c}{\delta P_0}=(u_2^2-i\omega D)\frac{\partial\rho}{\partial c}\frac{dc}{dP}\frac{2Y}{Y+Y'+YY'\xi_{\omega}}. \tag{17}$$

It is natural that the transmission coefficient of the acoustic wave in Eq. (14) is reduced by the excitation coefficient of the second sound mode. In accordance with the above speculations the excitation of second sound by means of the interface oscillation vanishes at the maximum concentration point.

To gain a further insight, it is worthwhile to give an expression connecting the flow of total mass across the interface with the pressure modulations. With the aid of Eqs. (1), (2), and (7) one can find at interface $x=0$,

$$J=\frac{V'-j/\rho}{v'-v}=\frac{\sqrt{u_2^2-i\omega D}}{c'-c}\rho\frac{dc}{dP}\delta P. \tag{18}$$

As expected, the total mass flow vanishes and changes its direction with respect to pressure deviations from equilibrium value at the maximum concentration point. The distinction between the factor connecting the mass flow J and pressure modulation δP in Eq. (18) and the effective growth coefficient ξ_{ω} involved in sound coefficients Eqs. (13) and (14) is an additional feature of the binary mixture compared with a single-component medium.[8] This fact is completely associated with the necessity to take into account the dependence of the specific volume of the mixture on concentration, i.e., the term $\partial v/\partial c\neq 0$ cannot be neglected. The difference between the fluid velocity of normal phase and the normal component velocity of superfluid phase at the interface $x=0$ is given by

$$V' - V_n = \left(\frac{\rho - \rho'}{c' - c} \frac{\sqrt{u_2^2 - i\omega D}}{\rho'} - \frac{\rho_s}{\rho_n} \frac{c\,\partial Z/\partial c}{\sqrt{u_2^2 - i\omega D}} \right) \frac{dc}{dP} \delta P. \tag{19}$$

In principle, any two conditions like the above Eqs. (18) and (19) relating the jumps of mass flows and fluid velocities to the pressure modulation are sufficient to describe sound phenomena at the interface.

III. REFLECTION OF A SECOND SOUND WAVE

In a similar way the reflection of a second sound wave from the superfluid-normal interface of a ^{3}He-^{4}He mixture can be analyzed. Let us consider the second sound wave incident from the superfluid ^{3}He-dilute phase onto the boundary with the normal ^{3}He-concentrated phase. As before, we assume the low temperature limit $T \ll T_F$ where the second sound mode can be treated as modulations of ^{3}He concentration alone. In addition, we assume the normal phase consisting of pure ^{3}He, i.e., concentration $c' = 1$. Thus in the normal phase we can find only the first sound mode

$$\delta P'(xt) = \delta P' e^{-ik'x - i\omega t}, \quad k' = \omega/u', \quad x < 0. \tag{20}$$

In the superfluid phase we have the incident and reflected modes of second sound and, in addition, the excited mode of first sound propagating away from the interface into the ($x > 0$) bulk of the ^{3}He-dilute phase. The amplitudes of concentration and pressure modulations can be represented as

$$\begin{aligned} \delta c(xt) &= \delta c_0 e^{-ik_2 x - i\omega t} + \delta c_r e^{ik_2 x - i\omega t} + \alpha(\omega)\,\delta P_1 e^{ik_1 x - i\omega t}, \\ k_2 &= \frac{\omega}{\sqrt{u_2^2 - i\omega D}}, \\ \delta P(xt) &= \beta(\omega)\,\delta c_0 e^{-ik_2 x - i\omega t} + \beta(\omega)\,\delta c_r e^{ik_2 x - i\omega t} \\ &\quad + \delta P_1 e^{ik_1 x - i\omega t}, \quad k_1 = \frac{\omega}{u_1}. \end{aligned} \tag{21}$$

The quantity $\alpha(\omega)$ which relates the pressure modulations to the concentration modulations in the first sound mode,

$$\alpha(\omega) \approx \frac{\partial v/\partial c}{\partial Z/\partial c} \frac{u_2^2 - i\omega D}{u_1^2},$$

can be neglected in the final result within the accuracy of ratio of sound velocities $u_2^2/u_1^2 \ll 1$. The analogous quantity $\beta(\omega)$ for the second sound mode is defined by expression Eq. (3).

Using the same boundary conditions of the continuity of flows Eqs. (7)–(9) and assumption Eq. (11) on the phase equilibrium at the interface, we arrive at the coefficient r_2 of second sound reflection from the normal phase

$$r_2 = \frac{\delta c_r}{\delta c_0} = -\frac{Y + Y' - YY'\xi_\omega}{Y + Y' + YY'\xi_\omega}. \tag{22}$$

The reflection coefficient r_2 is defined by the ratio of amplitudes of reflected to incident waves. The effective kinetic growth coefficient ξ_ω is determined by the same expression Eq. (16) as for acoustic sound phenomena. The relative energy loss in the second sound wave during reflection is given by

$$\epsilon = 1 - |r_2|^2 = \frac{4(Y + Y')YY'\xi_1}{(Y + Y' + YY'\xi_1)^2 + (YY'\xi_2)^2};$$

$$\xi_\omega = \xi_1 + i\xi_2. \tag{23}$$

Since the effective growth coefficient ξ_ω is not large compared with the inverse acoustic impedance Y^{-1}, the reflection of second sound from normal phase is close to unity. Concerning the behavior of the reflection as a function of phase-separation pressure, one should note that the maximum value of the reflection reaches at the maximum concentration point $P = P_m$ where $\xi_\omega = 0$ and the reflection reduces slightly for pressures departing from $P = P_m$. The pressure amplitude $\delta P'$ of an acoustic sound wave propagating into the bulk of the normal ^{3}He-concentrated phase can be expressed in terms of concentration modulations δc_0 in the incident wave of second sound,

$$\delta P' = \frac{2YY'}{Y + Y' + YY'\xi_\omega} \sqrt{u_2^2 - i\omega D}\,\rho \frac{\partial Z}{\partial c} \frac{dc}{dP} \delta c_0. \tag{24}$$

IV. SUMMARY

To conclude, we have examined the effect of phase separation in a liquid ^{3}He-^{4}He mixture on the transmission and reflection of low frequency $\omega\tau \ll 1$ sound modes across the superfluid-normal interface between ^{3}He-dilute and ^{3}He-concentrated phases in the low temperature $T \ll T_F$ limit where the heat effects can be neglected. Provided the sound wavelength is much larger compared with the interface width and mean free path of excitations in the medium, we may assume that the interface mobility is limited mainly by the energy dissipation processes due to collisions in the bulk beside the interface and that the intrinsic mobility of the interface is large enough to suppose the existence of local phase equilibrium at the interface. Under these conditions the separation kinetics is governed mostly by the processes of second sound propagation in the ^{3}He-dilute phase, which take care of delivering or removing the concentration difference $(c' - c)$ released during the phase separation of the mixture. As sound frequency ω increases, an additional contribution into the effective interface mobility from diffusion processes enhances. For the normal-normal interface, the effective growth coefficient is associated with the diffusion alone and is proportional to $\sqrt{\omega D}$, D being the diffusion coefficient. Such an ω dependence is also inherent in phase conversion processes driven by heat conduction.

Certainly, all the above speculations cease to be valid for transmission and reflection of high frequency $\omega\tau \gg 1$ sound when particles of a medium have no collisions at the distances compared with the wavelength. In this case the energy dissipation near the interface can be neglected. The coefficients of sound reflection Eq. (13) and transmission Eq. (14) conserve their previous form but, of course, the acoustic impedances should be replaced by the impedances for high frequency sound with the corresponding velocities instead of acoustic sound velocities. The new kinetic growth coefficient will depend on growth properties of the interface alone. In the absence of a clear picture for the interaction between the interface and ^{3}He atoms coupled with a ^{4}He cloud in the dilute phase the estimates of the interface growth coefficient and its pressure dependence are not completely clear. We, however, believe that the experimental study of the subject described here will give a more clarified picture of the interfacial kinetics in ^{3}He-^{4}He mixtures.

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