Ginzburg-Landau theory of vortex lattice structure in deformable anisotropic superconductors

P. Miranović and Lj. Dobrosavljević-Grujić
Institute of Physics, P.O. Box 57, 11001 Belgrade, Yugoslavia

V. G. Kogan
Ames Laboratory and Physics Department, Iowa State Universty, Ames, Iowa 50011

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Correlation between the crystal lattice and the vortex lattice in anisotropic (uniaxial) type-II superconductors due to magnetoelastic interactions is studied theoretically. Within the strain-dependent Ginzburg-Landau model, the energy of the magnetoelastic interaction of the vortex lattice is evaluated with the ΔV effect (difference of specific volumes of normal and superconducting phase) as the main source of the elastic strain. For $NbSe_2$ in tilted fields near the upper critical field H_{c2}, the vortex lattice is the same as obtained within the London model in fields well below H_{c2} with magnetoelastic interactions taken into account.

I. INTRODUCTION

It is well established that the Ginzburg-Landau (GL) and London models are valid for describing the mixed state of layered superconductors, provided the scale of material inhomogeneities is much smaller than the coherence length ξ.[1] However, recent studies of the vortex lattice (VL) structure in $NbSe_2$ by neutron diffraction[2] revealed a discrepancy between the theory and experiment. Both London[3] and GL theory[4,5] for uniaxial superconductors predict that when vortices are parallel or perpendicular to the $\hat{c}$ axes, the VL structures, (A) and (B) shown in Fig. 1, belong to a set of structures of the same energy (see Ref. 3 for details) and therefore have the same chance to be observed. When VL is tilted away from these orientations, the degeneracy is removed favoring (A). In fact, only the structure (B) has been seen for all tilt angles[2] at $T/T_c \approx 0.7$. This means that there is a contribution to the vortex interaction which is not included in the London and GL models.

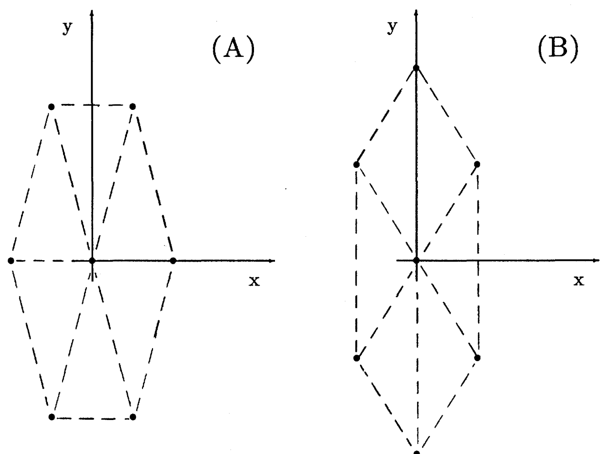

FIG. 1. If vortices are directed along $\hat{c}$ axes, the vortex lattice is hexagonal. When vortices are tilted away from the $\hat{c}$ axis (being rotated about y axis over angle θ), the hexagonal lattice is distorted along the y direction by the factor $1/\gamma$ and along the x direction by γ, where $\gamma^4 = \Gamma^{-2}\sin^2\theta + \cos^2\theta$, and $\Gamma^2 = m_c/m_{ab}$ [the y axis is in the ab plane, the x axis is in the plane $(\hat{c}, \mathbf{B})$]. Two distorted structures, denoted as (A) and (B), are shown.

One such contribution is provided by magnetoelastic (ME) interactions between vortices, which can play a significant role in forming VL.[6] The idea is based on the fact that the specific volumes of the normal and superconducting phases are different (so-called ΔV effect). The "normal" vortex core acts as a source of inhomogeneity, producing local deformations in the surrounding superconducting material. Vortices could interact through the elastic field, and the energy of the ME interaction depends on the VL structure. Incorporating this interaction in the London theory one can explain most of the experimental results for $NbSe_2$.[7] Since the internal stress arises due to the spatial variation of the order parameter, vortex-vortex ME interaction is sensitive to the core structure. In the London domain, the intervortex separation $r \gg \xi$, the cores can be considered as point sources of stress, and the problem is treated by analogy with the thermal expansion of anisotropic bodies subject to point heat sources.[7] Within this model, ME interaction is zero in isotropic materials. Another feature should be mentioned: the ME energy increases with magnetic induction as B^2, the result of the long-range elastic interactions. Since the London energy is only linear in B, one may expect the ME contribution to dominate in high fields approaching H_{c2}. However, in high fields where vortex cores overlap, the London model fails, and one has to take into account the actual order parameter distribution.

We treat the high field case within the GL theory extended to include elastic effects, an approach originally developed for calculation of pinning forces.[8] Within this approach, a finite ME interaction of vortices is predicted for isotropic crystals, unlike the result of the point-source model, where this interaction is absent.[7] This, of course, does not remove the orientational degeneracy of hexagonal VL's. In anisotropic crystals, however, the ME strain contributes differently to energies of different VL structures.

In Sec. II, we present the GL theory of anisotropic deformable superconductors, and discuss the London limit

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valid for $\kappa \gg 1$ in fields well below H_{c2}. In Sec. III, we study the region near H_{c2} and generalize the treatment of Kogan and Clem[4] to deformable superconductors to calculate the free energy for arbitrary field orientation. We discuss briefly our results in the last section.

II. MAGNETOELASTIC ENERGY

Major properties of anisotropic superconductors near the critical temperature T_c are described by the GL theory with the mass tensor.[4,5] A new feature in deformable superconductors is the elastic response of the crystal in the presence of VL. The free energy is[8]

$$\mathscr{F}=\int \left(\alpha|\Psi|^2+\frac{1}{2}\beta|\Psi|^4+\frac{h^2}{8\pi} + \frac{1}{2}M_{ij}^{-1}\Pi_i\Psi\Pi_j^\star\Psi^\star+\frac{1}{2}\lambda_{ijkl}u_{ij}u_{kl}\right) dV. \quad (1)$$

Here $\mathbf{\Pi}=-i\hbar\boldsymbol{\nabla}-(2e/c)\mathbf{A}$, $\hat{M}$ is the tensor of mass, u_{ij} is the strain tensor, λ_{ijkl} are elastic coefficients, and summation over repeated indices is implied. The coefficients α, β, and the tensor $\hat{M}^{-1}$ describe superconducting properties of the material and as such depend on the strain, the idea introduced by Labusch for describing pinning of vortices by strain-producing defects.[8]

It should be noted that our situation differs from that dealt with by Labusch where the strain was not directly related to superconductivity. Indeed, strains u_{ij} in our case are caused by vortices in the mixed state and disappear in the normal phase. Since all terms in the GL energy functional must be proportional to $(1-T/T_c)^2\equiv(1-t)^2$, we obtain from Eq. (1)

$$u_{ij}\propto(1-t)\ . \quad (2)$$

We now show that, unlike the case considered by Labusch, only the coefficient α is affected by strain. To this end, let us turn to the well-established experimental fact: the critical temperature T_c depends on stress.[9,10] For our purpose, it is convenient to present this dependence in terms of strains:

$$T_c=T_{c0}+u_{ik}\left(\frac{\partial T_c}{\partial u_{ik}}\right)_0+\cdots\ . \quad (3)$$

Then the coefficient $\alpha=\hbar^2/2\bar{M}\xi^2(T)$ $[\bar{M}=(M_1M_2M_3)^{1/3}$ and M_i are principal values of the mass tensor] reads

$$\alpha=\frac{\hbar^2}{2\bar{M}\xi_0^2T_c}(T-T_c)\approx\frac{\hbar^2}{2\bar{M}\xi_0^2T_{c0}}\left(T-T_{c0}-u_{ik}\frac{\partial T_c}{\partial u_{ik}}\right) =\alpha_0+\alpha_{ij}u_{ij}+\cdots\ , \quad (4)$$

with

$$\alpha_{ij}=-\frac{\hbar^2}{2\bar{M}\xi_0^2T_{c0}}\left(\frac{\partial T_c}{\partial u_{ij}}\right)_0 \quad (5)$$

[the notation $\xi^2=\xi_0^2/(1-t)$ is used for the GL coherence length]. Clearly, one cannot neglect the u_{ij} dependence of α, since the strain shifts T_c, the point near which the GL energy is expanded. We note that both α and u_{ij} are linear in $(1-t)$, whereas α_{ij} is T independent; thus the renormalized α of Eq. (4) is still linear in $(1-t)$. This means that within the GL approach we should not retain higher order terms in the expansion (4). Also, this implies that other material characteristics, β and M_{ik}, should be taken only to the lowest order in u_{ij}, i.e., they are strain independent.

Note that retaining the higher order terms in $(1-t)$ and taking the strain dependence of the volume element $dV=(1+u_{jj})dV_0$ (Ref. 11) one can show that, in addition to the dominant ΔV effect, spatial variations of the free energy density and a change of the elastic coefficients with respect to those in the normal phase may also contribute to the ME energy.

Substituting α of Eqs. (4) and (5) in $\mathscr{F}$, we obtain the term responsible for the ME coupling:

$$\mathscr{W}=\int \alpha_{ij}u_{ij}|\Psi|^2\ \ dV. \quad (6)$$

Clearly, this term is of the form which could have been guessed on general grounds; we, however, have an explicit expression (5) for α_{ij} in terms of measurable quantities $\partial T_c/\partial u_{ij}$. We further notice that $\int u_{ij}dV$ over the unit cell of VL vanishes (due to periodicity, the displacement u_i vanishes at the cell boundary, and integration by parts yields zero). Therefore, one can write Eq. (6) as

$$\mathscr{W}=\alpha_{ij}\int u_{ij}(|\Psi|^2-\Psi_0^2)dV. \quad (7)$$

It is instructive to obtain the London limit[7] for the ME coupling. In small fields, we model the vortex core by taking $|\Psi|^2$ as zero in the core and $\Psi_0^2=|\alpha_0|/\beta_0$, otherwise. Then one can replace in Eq. (7) $(|\Psi|^2-\Psi_0^2)$ with $-\Psi_0^2S_c\delta(\mathbf{r})$ [$S_c=\pi\xi^2(T)$ is the core area and $\mathbf{r}=(x,y)$ for vortices along z]. We then obtain the ME coupling energy density of VL,

$$w=-S_c\alpha_{ij}\Psi_0^2\ u_{ij}\sum_{\mathbf{r}_v}\ \delta(\mathbf{r}-\mathbf{r}_v)=\eta_{ij}u_{ij}\sum_{\mathbf{r}_v}\ \delta(\mathbf{r}-\mathbf{r}_v); \quad (8)$$

the notation η_{ij} was adopted in Ref. 7. Noting that strain derivatives of T_c are measured by applying uniform stress $p_{ij}=\lambda_{ijlm}u_{lm}$,

$$\frac{\partial T_c}{\partial u_{ij}}=\lambda_{ijlm}\frac{\partial T_c}{\partial p_{lm}}\ , \quad (9)$$

and using $H_{c0}^2/4\pi=\alpha_0^2/\beta_0$, we obtain

$$\eta_{ij}=-S_c\lambda_{ijlm}\left(\frac{\partial T_c}{\partial p_{lm}}\right)\frac{H_{c0}(0)H_{c0}(T)}{4\pi T_c}, \quad (10)$$

where $H_{c0}(T)=H_{c0}(0)(1-t)$. This, in fact, justifies the choice of η_{ij} in Ref. 7 where this quantity has just been guessed on general symmetry grounds.

To make equations dimensionless, we take (as usual) Ψ_0, $\sqrt{2}H_{c0}$, and $H_{c0}^2/4\pi$ as units of the order parameter, magnetic field, and energy density, respectively. The average London penetration depth $\lambda_L=(\bar{M}c^2/16\pi e^2\Psi_0^2)^{1/2}$ is taken as the unit length. In the following, we keep the same notation for dimensionless quantities and indicate the results given in conventional units. In particular, the dimensionless elastic moduli $\hat{\lambda}$ in terms of their conventional counterparts are $4\pi\hat{\lambda}/H_{c0}^2$; α_{ij} of Eq. (5) acquires the dimensionless form

$$\alpha_{ij}=\frac{1}{T-T_{c0}}\frac{\partial T_c}{\partial u_{ij}}\ . \tag{11}$$

The dimensionless free energy then reads

$$\mathscr{F}=\int (F_0+\lambda_{ijlm}u_{ij}u_{lm}/2+\alpha_{ij}u_{ij}|\Psi|^2)dV\ , \tag{12}$$

where

$$F_0=-|\Psi|^2+\tfrac{1}{2}|\Psi|^4+\tfrac{1}{2}\mu_{ij}\Pi_i\Psi\Pi_j^\star\Psi^\star+h^2\ . \tag{13}$$

$\mathbf{\Pi}=(i\kappa)^{-1}\nabla-\mathbf{A}$, $\kappa=2\sqrt{2}eH_{c0}\lambda_L^2/\hbar c$ is the GL parameter, and the inverse dimensionless mass tensor $\hat{\mu}=\bar{M}\hat{M}^{-1}$.

Varying the energy with respect to Ψ^* we obtain the first GL equation, modified in this case by the strain dependence of α:

$$\mu_{ij}\Pi_i\Pi_j\Psi=(1+\alpha_{ij}u_{ij})\Psi-|\Psi|^2\Psi\ . \tag{14}$$

The GL equation for the current density has the standard form.

The system of GL equations should be complemented with the condition of elastic equilibrium $\partial\sigma_{ij}/\partial x_j=0$, where $\sigma_{ij}=\partial F/\partial u_{ij}$ is the stress tensor and F is the free energy density, i.e.,

$$\frac{\partial}{\partial x_j}(\lambda_{ijlm}u_{lm}+\alpha_{ij}|\Psi|^2)=0\ . \tag{15}$$

We now transform the u-dependent part of $\mathscr{F}$ in Eq. (12) using the symmetry of tensors involved [e.g., $\alpha_{ij}u_{ij}=\alpha_{ij}(\partial u_i/\partial x_j)$] and integrating by parts to obtain the ME energy,

$$\mathscr{W}_{\mathrm{ME}}=-\frac{L}{2}\alpha_{ij}\int u_i\frac{\partial|\Psi|^2}{\partial x_j}\ d^2\mathbf{r}. \tag{16}$$

Here the equilibrium Eq. (15) has been used, L is the length of vortices directed along z, $\mathbf{r}=(x,y)$, and j takes on only values 1,2 ($x_1=x$, $x_2=y$).

For periodic VL, all quantities can be expanded in Fourier series. We then have for the average over the VL cell ME energy density

$$\langle w_{\mathrm{ME}}\rangle=-\frac{i}{2}\alpha_{ij}\sum_{\mathbf{Q}} u_i^\star(\mathbf{Q})Q_j\tilde{f}^2(\mathbf{Q})\ , \tag{17}$$

where $\mathbf{Q}$'s form the reciprocal lattice, and $\tilde{f}^2$ is the Fourier transform of $f^2=|\Psi|^2$. The displacement $\mathbf{u}(\mathbf{Q})$ is found from Eq. (15):

$$u_k=iG_{ik}\alpha_{ij}\tilde{f}^2Q_j\,,\qquad G_{ik}=(Q_jQ_l\lambda_{ijkl})^{-1}. \tag{18}$$

We now obtain

$$\langle w_{\mathrm{ME}}\rangle=-\frac{1}{2}\sum_{\mathbf{Q}} S_iG_{ij}S_j^*\,,\quad S_i=\alpha_{ij}Q_j\tilde{f}^2. \tag{19}$$

Note that in all formulas above $Q_z=0$.

The stable VL corresponds to the lowest free energy. In other words, one has to solve three coupled nonlinear differential equations [two GL equations and the equilibrium Eq. (15)]. This difficult task is simplified in high fields, where GL equations can be linearized.

III. VORTEX LATTICE STRUCTURES NEAR H_{c2}

With the help of GL Eq. (14) (modified by the strain), the free energy can be written as

$$\mathscr{F}=\int F\ \ dV=\int (h^2-\tfrac{1}{2}|\Psi|^4)dV+\mathscr{W}_{\mathrm{ME}}\,. \tag{20}$$

In the vicinity of H_{c2} one can evaluate the free energy, as is done in the absence of strain, i.e., with the help of two Abrikosov identities. The *linearized* GL equations read

$$(\mathbf{\Pi}_{c2}\cdot\hat{\mu}\cdot\mathbf{\Pi}_{c2}-1)\Psi_L=0, \tag{21}$$

$$\mathbf{j}_L=\frac{c}{8\pi}\hat{\mu}\cdot(\Psi_L^\star\mathbf{\Pi}_{c2}\Psi_L+\Psi_L\mathbf{\Pi}_{c2}^\star\Psi_L^\star), \tag{22}$$

where $\mathbf{\Pi}_{c2}=(i\kappa)^{-1}\nabla-\mathbf{A}_{c2}$ and Ψ_L, $\mathbf{j}_L$ are solutions of the linear equations. The terms with u_{ij} are omitted here, because the vortex induced strain is zero in the normal phase, and is of the order of $|\Psi|^2$ or higher in the mixed state. Within this approximation, the first Abrikosov identity remains the same as in the strain absence[4]

$$\nabla^2h_x=\epsilon\frac{\partial^2f_L^2}{\partial y^2},$$

$$\nabla^2h_y=-\epsilon\frac{\partial^2f_L^2}{\partial x\partial y},\qquad \epsilon=\frac{\mu_{xz}}{\tilde{\kappa}\mu_{xx}}, \tag{23}$$

$$h_z=H_0-\frac{f_L^2}{2\tilde{\kappa}}\,,$$

where H_0 is a constant, $\tilde{\kappa}=\kappa/\sqrt{\mu_1\mu_{xx}}$, and $f_L^2=|\Psi_L|^2$. The macroscopic magnetic induction is given by $B_z=H_0-\langle f_L^2\rangle/2\tilde{\kappa}$. Slightly below H_{c2} we have to include nonlinear terms. Solutions of the first two GL equations can be taken in the form $\Psi=\Psi_L+\Psi_1$ and $\mathbf{A}=\mathbf{A}_{c2}+\mathbf{A}_1$, where Ψ_1 and $\mathbf{A}_1$ are small corrections. Substituting these in the first GL equation, we obtain for the corrections

$$\begin{aligned}(\mathbf{\Pi}_{c2}\cdot\hat{\mu}\cdot\mathbf{\Pi}_{c2}-1)\Psi_1-\hat{\alpha}\cdot\hat{u}\Psi_L\\ =(\mathbf{\Pi}_{c2}\cdot\hat{\mu}\cdot\mathbf{A}_1+\hat{\mu}\cdot\mathbf{A}_1\cdot\mathbf{\Pi}_{c2})\Psi_L-|\Psi_L|^2\Psi_L\,.\end{aligned} \tag{24}$$

Since the operator $\hat{\mathscr{H}}=\tfrac{1}{2}\mathbf{\Pi}_{c2}\cdot\hat{\mu}\cdot\mathbf{\Pi}_{c2}-1$ is Hermitian,[12] $\int\Psi_L^\star\hat{\mathscr{H}}\Psi_1\ dV=\int\Psi_1^\star\hat{\mathscr{H}}\Psi_L\ dV=0$. Multiplying Eq. (24) by $\Psi_L^\star$ and integrating by parts, we obtain

$$2\langle\mathbf{h_s}\cdot(\nabla\times\mathbf{A}_1)\rangle-\alpha_{ij}\langle f_L^2u_{ij}\rangle+\langle f_L^4\rangle\ =0; \tag{25}$$

here $\mathbf{h}_s$ is the field created by supercurrents $\nabla\times\mathbf{h}_s=(4\pi/c)\mathbf{j}_L$ and $\nabla\times\mathbf{A}_1=\mathbf{H}_0+\mathbf{h}_s-\mathbf{H}_{c2}$. Thus, we have the second Abrikosov identity:

$$\langle h_s^2\rangle+(H_0-\tilde{\kappa})\langle h_{sz}\rangle=-\tfrac{1}{2}\alpha_{ij}\langle f_L^2u_{ij}\rangle+\tfrac{1}{2}\langle f_L^4\rangle. \tag{26}$$

Clearly, this reduces to the result of Ref. 4 in the strain-free case. Using the Abrikosov identities we find

$$\langle f_L^2\rangle=\frac{2\tilde{\kappa}(\tilde{\kappa}-B)}{(2\tilde{\kappa}^2-1)\beta_A-4\tilde{\kappa}^2(\beta_1-\beta_2)+1}, \quad (27)$$

where

$$\beta_A=\frac{\langle f_L^4\rangle}{\langle f_L^2\rangle^2}, \quad \beta_1=\frac{\langle h_{tr}^2\rangle}{\langle f_L^2\rangle^2}, \quad \beta_2=-\frac{\alpha_{ij}}{2}\frac{\langle f_L^2 u_{ij}\rangle}{\langle f_L^2\rangle^2}.$$

Note that $h_{tr}^2=h_x^2+h_y^2$ and that the strain enters via β_2, proportional to $\langle w_{\mathrm{ME}}\rangle$ of Eq. (17). Now, the macroscopic free energy density is

$$\langle F\rangle=B^2-\frac{(2\tilde{\kappa}^2-1)\beta_A-4\tilde{\kappa}^2(\beta_1+\beta_2)+1}{[(2\tilde{\kappa}^2-1)\beta_A-4\tilde{\kappa}^2(\beta_1-\beta_2)+1]^2}(\tilde{\kappa}-B)^2. \quad (28)$$

In the stress-free situation ($\beta_2=0$), the term in the condensation energy $F_c=\langle F\rangle-B^2$ responsible for locking the VL on the crystal is proportional to[4]

$$\beta_1=\left(\frac{\Gamma^{-4/3}}{2\kappa}\frac{(\Gamma^2-1)\sin\theta\cos\theta}{\sqrt{\Gamma^{-2}\sin^2\theta+\cos^2\theta}}\right)^2\sum_{\mathbf{Q}\neq 0}\frac{Q_y^2}{Q^2}\frac{|\tilde{f}_L^2(\mathbf{Q})|^2}{|\tilde{f}_L^2(0)|^2}, \quad (29)$$

where $\Gamma=\sqrt{m_c/m_{ab}}$ is the anisotropy parameter. The sum here is of the order of (β_A-1), so that for high-κ materials $\beta_1\ll\beta_A$. The parameter β_A is the same for both (A) and (B) structures:[13]

$$\beta_A=\sum_{\mathbf{Q}}\frac{|\tilde{f}_L^2(\mathbf{Q})|^2}{|\tilde{f}_L^2(0)|^2}=1.16, \quad (30)$$

i.e., it is the same as in the isotropic case. The Fourier transform $\tilde{f}_L^2(\mathbf{Q})$ is given by[13]

$$\frac{|\tilde{f}_L^2(\mathbf{Q})|^2}{|\tilde{f}_L^2(0)|^2}=\exp\left(-\frac{2\pi}{\sqrt{3}}(p^2+q^2-pq)\right), \quad (31)$$

where

$$Q_x=Q_0 p\sqrt{3}/2\gamma, \qquad Q_y=Q_0(-p/2+q)\gamma \quad (32)$$

for structure (A), and

$$Q_x=Q_0(p-q/2)/\gamma, \qquad Q_y=Q_0 q\sqrt{3}\gamma/2 \quad (33)$$

for (B). Here $p,q=0,\pm1,\pm2,\ldots$, $\gamma^4=\Gamma^{-2}\sin^2\theta+\cos^2\theta$, and $Q_0=2\pi(2B/\sqrt{3}\phi_0)^{1/2}$. Petzinger and Warren[5] have shown that the maximum of β_1, which gives the minimum free energy, corresponds to the (A) structure [this structure minimizes also the London energy in fields $B\ll H_{c2}$ (Refs. 3 and 7)].

In deformable superconductors the free energy (28) is affected by strains via parameter β_2. To calculate β_2, we need an explicit form of α_{ij} and G_{ij} which depends on the crystal symmetry. Hereafter, hexagonal crystal lattice will be considered as an example; generalization to other symmetries is straightforward. In the coordinate system rotated by angle θ around y axis (starting from the situation where the z axis is along $\hat{c}$) the components $\nu_{ij}=(T_{c0}-T)\alpha_{ij}$ that enter the calculation are $\nu_{xx}=\nu_{ab}\cos^2\theta+\nu_c\sin^2\theta$, $\nu_{yy}=\nu_{ab}$, $\nu_{xz}=(\nu_{ab}-\nu_c)\sin\theta\cos\theta$, and $\nu_{xy}=\nu_{yz}=0$. The eigenvalues of $\hat{\nu}$ are

$$\nu_{ab}=(C_{11}+C_{12})\frac{\partial T_c}{\partial p_{ab}}+C_{13}\frac{\partial T_c}{\partial p_c},$$

$$\nu_c=2C_{13}\frac{\partial T_c}{\partial p_{ab}}+C_{33}\frac{\partial T_c}{\partial p_{ab}}, \quad (34)$$

where C_{ij} are the elastic moduli in the crystal frame. Tensors G_{ij} and λ_{ijkl} are given in the Appendix.

We illustrate our results on the example of 2H-$NbSe_2$, for which the elastic moduli are known:[14] $C_{11}=1.47$, $C_{12}=0.38$, $C_{13}=0.11$, $C_{33}=0.53$, and $C_{44}=0.174\times10^{12}$ erg/cm^3. The stress derivatives of T_c are known as well:[9,10] $\partial T_c/\partial p_c\approx5.3\times10^{-10}$ K cm^3/erg, $\partial T_c/\partial p_{ab}=-2.4\times10^{-10}$ K cm^3/erg, where p_c and p_{ab} are "unidirectional pressures" along the c and ab planes, respectively. The tensor ν_{ij} in the crystal frame is given by

$$\nu_{ab}=-3.875\times10^2\ \mathrm{K}, \quad \nu_c=2.28\times10^2\ \mathrm{K};$$

$\Delta C_p/T_c\approx10^4$ erg cm^{-3}/K^2.[15] Effective mass anisotropy is $\Gamma^2\approx10$. These data suffice for the evaluation of the free energy for each particular VL, the task we accomplish numerically.

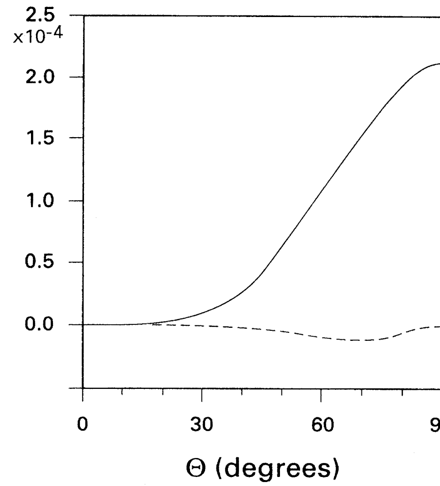

FIG. 2. Relative difference $R=[F_c(A)-F_c(B)]/|F_c(B)|$ of the condensation energies for structures (A) and (B) vs the tilt angle θ without strain (dashed line) and with strain (full line).

In Fig. 2 we plot the ratio $[F_c(A)-F_c(B)]/|F_c(B)|$ as a function of the tilt angle θ. For all angles (except $\theta=0$) structure (B) has a lower energy. For $\theta=0$ the hexagonal 2H-$NbSe_2$ is elastically isotropic in the plane ab; the parameter $\beta_1=0$ and

$$\beta_2=-\frac{(C_{11}+2C_{12})^2}{2C_{11}}\frac{\Delta C_p}{T_c}\left(\frac{\partial T_c}{3\partial p}\right)^2\beta_A \quad (35)$$

is the same for all orientations of the hexagonal VL.

Thus, for 2H-$NbSe_2$ in high fields, the ME interaction causes structure (B) to be preferable, similar to the case of lower fields where the London approach is applicable.[7] The field dependence of the ME interaction responsible for this situation is given by the prefactor $(\tilde{\kappa}-B)^2$ in the free energy; recall that in the London case the ME energy density is $\propto B^2$.

IV. DISCUSSION

We have generalized the GL theory of anisotropic superconductors near the upper critical field[4] to include magnetoelastic effects. We have shown that the ME interaction can change the structure of the VL as compared to the stress-free case. Also, we have formulated approximations under which one can use the London approach for the vortex induced stress in intermediate fields. In so doing we provide justification for the results obtained within the London model.[7] In fact, the form (10) for the tensor η_{ik} has been guessed in Ref. 7 on the basis of symmetry arguments.

Our results are obtained within local anisotropic GL theory.[17,18] Going beyond the local theory, one would like to compare the ME effect with nonlocal corrections to the VL energy. In the London domain, this estimate was possible since in small fields there exists a microscopic nonlocal relation between the supercurrent and the vector potential. For clean $NbSe_2$ (infinite electron scattering time τ), the VL energy due to the nonlocal terms turned out approximately the same as the ME contribution.[7] However, this compound is usually of intermediate purity.[16] The nonlocal corrections, which diminish with decreasing τ, might not be that important except for small tilt angles where the nonlocality might remove the "almost degeneracy" seen in Fig. 2 for $\theta<20°$. Higher order terms in the GL functional such as anharmonicity may play a similar role.[7]

In high fields, Takanaka and Nagashima's nonlocal GL theory[19] for pure superconductors results in replacement of β_A in the free energy expression with $\beta_A+C_{2i}\beta_{2i}$ ($i=1,2,3$). Here β_A and β_{2i} depend only on the lattice geometry. To predict the VL structure, one should know at least the sign of coefficients C_{2i}.[19] However, calculation of various Fermi surface averages entering C_{2i} does not appear feasible, and only rough estimates of C_{2i} can be extracted[21] from the H_{c2} measurements. It is evident from Takanaka and Nagashima's results that C_{2i} decrease with rising temperature as $(1-T/T_c)$ or faster. With decreasing τ, these coefficients are expected to diminish further and eventually to vanish in the dirty limit, where all nonlocal effects disappear.[20] Therefore, the influence of nonlocality versus that of the ME strain on VL structure should diminish in impure samples and with rising temperature (where C_{2i} decrease, whereas β_2 is T independent).

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APPENDIX

The Fourier transform of the elastic Green function G_{ij} is defined as the tensor inverse to $T_{ij}=Q_kQ_l\lambda_{ikjl}$. For example, $G_{xx}=(T_{yy}T_{zz}-T_{yz}^2)/\Delta$ where $\Delta=\det|T_{ij}|$. Further, the moduli λ_{ijkl} with an odd number of y indices are zero, and components of T_{ij} are

$$T_{xx}=\lambda_{xxxx}Q_x^2+\lambda_{xyxy}Q_y^2,\quad T_{xy}=(\lambda_{xxyy}+\lambda_{xyxy})Q_xQ_y,$$

$$T_{xz}=\lambda_{xxzx}Q_x^2+\lambda_{xyzy}Q_y^2,\quad T_{yy}=\lambda_{xyxy}Q_x^2+\lambda_{yyyy}Q_y^2,$$

$$T_{yz}=(\lambda_{xyzy}+\lambda_{yyxz})Q_xQ_y,\quad T_{zz}=\lambda_{xzxz}Q_x^2+\lambda_{yzyz}Q_y^2.$$

The moduli λ_{ijkl} are obtained by transforming from the crystal frame, where they are denoted as C_{ij}:

$$\lambda_{xxxx}=C_{11}\cos^4\theta+C_{33}\sin^4\theta+2(C_{13}+2C_{44})\sin^2\theta\cos^2\theta,$$

$$\lambda_{xyxy}=C_{66}\cos^2\theta+C_{44}\sin^2\theta,$$

$$\lambda_{xxyy}=C_{12}\cos^2\theta+C_{13}\sin^2\theta,$$

$$\lambda_{xxzx}=(C_{11}-C_{13}-2C_{44})\cos^3\theta\sin\theta-(C_{33}-C_{13}-2C_{44})\sin^3\theta\cos\theta,$$

$$\lambda_{xyzy}=(C_{66}-C_{44})\sin\theta\cos\theta,\quad \lambda_{yyyy}=C_{11},$$

$$\lambda_{yyxz}=(C_{12}-C_{13})\sin\theta\cos\theta,$$

$$\lambda_{yzyz}=C_{44}\cos^2\theta+C_{66}\sin^2\theta,$$

$$\lambda_{xzxz}=(C_{11}+C_{33}-2C_{13})\sin^2\theta\cos^2\theta+C_{44}\cos^2 2\theta.$$

For Eq. (19) we need

$$\mathbf{S}\cdot\hat{G}\cdot\mathbf{S}^\star=\frac{\Delta C_p}{T_c}\{(\nu_{xx}^2G_{xx}+2\nu_{xx}\nu_{xz}G_{xz}+\nu_{xz}^2G_{zz})Q_x^2+\nu_{yy}^2G_{yy}Q_y^2+2(\nu_{xx}\nu_{yy}G_{xy}+\nu_{xz}\nu_{yy}G_{yz})Q_xQ_y\}|\tilde{f}^2|^2 \quad (A1)$$

for $\mathbf{Q}\neq0$; we have used here[22] $H_{c0}^2/4\pi=\Delta C_pT_c(1-t)^2$. The $\mathbf{Q}=0$ term in β_2 (i.e., in $-\alpha_{ij}\langle u_{ij}\rangle/2\langle f^2\rangle$) is estimated from Labusch's result for the difference of strain in the superconducting and normal phases:[8] $\langle u_{ij}\rangle/\langle f^2\rangle=\lambda_{ijkl}^{-1}\alpha_{kl}$.

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[12] $\mathbf{\Pi}\cdot\hat{\mu}\cdot\mathbf{\Pi}/2$ is the kinetic energy operator in the magnetic field and, therefore, is Hermitian; see also Appendix in Ref. 4.

[13] We have checked this result which has been obtained by J. Rammer (private communication); also, it is contained implicitly in Ref. 5.

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