Violation of universality for Ising spin-glass transitions

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Simulation data on Ising spin glasses in dimensions 3, 4, and 5 indicate that the critical exponent η changes with the form of the interaction distribution, which appears to be a pertinent parameter for the spin-glass transition. This result is incompatible with standard second-order transition universality rules which do not seem to hold for Ising spin glasses.

Thanks to renormalization-group theory, the universality rules for the critical exponents at standard second-order phase transitions are extremely well established. These state that the critical exponents depend only on the space dimension d and the number of (spin) components n.[1] No other parameters are pertinent. In spin glasses, which are systems with random and frustrated interactions, it has been widely assumed that the freezing transition can be assimilated to a form of second-order transition to which standard renormalization-group theory can apply, hence that the upper critical dimension is 6 from phenomenological Landau-Ginzburg arguments,[2] and that the appropriate universality rules should be valid, albeit with unconventional values of the exponents. There appears, however, to be no formal proof that universality in the usual sense should continue to hold in such systems; we wish to show here that on the contrary numerical simulation data on Ising spin glasses (ISG's) are consistent with the behavior that one would expect if the form of the interaction distribution were a pertinent parameter for the transition. This leads to a violation of the usual universality rules (identical values of all the exponents for all members of a family of systems with fixed d and n) and even of weak universality[3] (constant ratios β/ν and γ/ν). We will outline some of the consequences of this conclusion.

It is interesting to note first that in real three-dimensional (3D) spin glasses, which are Heisenberg with weak symmetry breaking terms, the exponent η is observed to change from system to system, while ν remains invariant to the accuracy of the experimental estimations (Table I).[4] This empirical evidence suggests that standard universality is not obeyed in spin glasses; the question can be investigated further by numerical techniques.

TABLE I. Experimental critical exponents in three-dimensional spin glasses. The quoted values are derived from the exponents measured directly in the experiments using the standard scaling relations. It can be seen that the ν values cluster around $\frac{4}{3}$, while the η values are very dispersed.

Sample	ν	η	Reference
*Cu*Mn	1.3 ± 0.2	0.39 ± 0.06	5
*Ag*Mn	1.43 ± 0.15	0.46 ± 0.03	6
FeNiP	1.15 ± 0.15	-0.02 ± 0.07	7
FeNiPBA1	1.39 ± 0.2	-0.83 ± 0.1	8
$CdCr_2InS_4$	1.26 ± 0.2	0.18 ± 0.08	9
*Al*Gd	1.53 ± 0.2	-0.15 ± 0.03	10

All the simulations we will discuss have been carried out on ISG's on simple (hyper)cubic lattices with near-neighbor interactions. In dimension 2, where there is full consensus that the freezing temperature T_g is always zero so that there is only a single independent critical exponent,[2] the reported simulation results on ν vary with the form of the interaction distribution.[11,12] Cheung and McMillan[12] pointed out that for a $T=0$ transition, models with continuous interaction distributions and those with discrete distributions should not be in the same universality class. The data[11] in fact show significant differences in ν between the different continuously distributed cases as well as between the discrete and continuous cases. As the mean, variance, and skewness are the same for all the interaction distributions by definition, it is natural to rank them by the value of the next combination of moments, the kurtosis $R=\langle J_{ij}^4\rangle/\langle J_{ij}^2\rangle^2$, where J_{ij} is a near-neighbor interaction strength. R takes the values of 1, $\frac{9}{5}$, 3, and 6 for the $\pm J$, flat, Gaussian, and exponential distributions, respectively. The published data[11] indicate that ν varies regularly with R.

Now let us turn to the results for the exponent η in dimension 3. For the $\pm J$ distribution the most accurate estimates are from dynamic simulations,[13] giving $\eta=-0.22\pm0.05$, and from finite-size scaling, -0.28, if the same ordering temperature is assumed.[14] For the Gaussian distribution, when T_g is taken to be 0.9, then $\eta=-0.45\pm0.05$.[14] Dynamic simulations leading to independent estimates of T_g and of η are consistent with this value and provide additional estimates of η for the exponen-

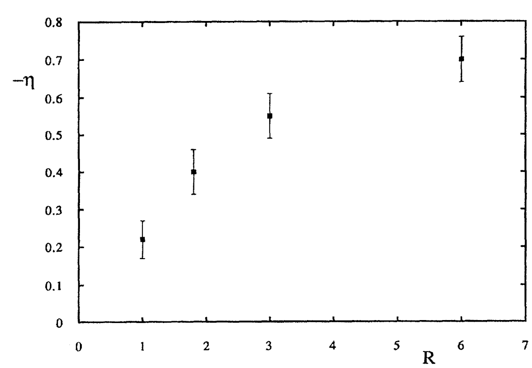

FIG. 1. Values of the critical exponent η for three-dimensional Ising spin glasses plotted against R (see text). Data are for the $\pm J$, flat, Gaussian, and exponential interaction distributions from left to right (Ref. 15).

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tial and flat interaction distribution functions.[15] These values are shown as a function of R in Fig. 1. The results appear to indicate a systematic and continuous variation of η with R; as R is increased, T_g drops and η becomes more negative. Again, the data suggest that in contradiction to the standard universality rules η does not appear to be constant, but to be a function of R.

The results in dimensions 2 and 3 may conceivably be atypical because the systems are, respectively, below and close to the lower critical dimension. We therefore carried out measurements in dimensions 4 and 5, where there is general agreement that there are bona fide spin-glass transitions at nonzero temperatures. In order to estimate critical exponents, it is important to dispose of reliable values of the ordering temperatures. For $\pm J$ distributions, consistent and accurate values for the freezing temperatures T_g have been obtained from series expansion and simulation methods. In dimension 4 two independent series estimates[16,17] and a simulation estimate using the Binder cumulant method[18] [finite-size scaling for the parameter $g_L = 3 - \langle q^4\rangle/\langle q^2\rangle^2)/2$] give $T_g = 2.02 \pm 0.04$, 2.04 ± 0.02, and 2.06 ± 0.02, respectively, i.e., values grouped closely around $T_g = 2.05$. In dimension 6 the series method gives $T_g = 3.027 \pm 0.005$ (Ref. 17) and the Binder cumulant method 3.035 ± 0.01.[19] Again, the agreement is excellent, with the simulation value just slightly higher than the series value. We can expect that in dimension 5 the series values 2.57 ± 0.01 (Ref. 17) will again be very reliable, and for dimensions higher than 6 the series value can be assumed to be very accurate.

For the Gaussian distribution, in dimension 4 the Binder cumulant estimate[14] is 1.75 ± 0.05, but no direct series estimate has been published. However, Singh and Fisher[20] have used a series expansion in $1/(2d-1)$ to provide simple expressions which relate T_g values for different interaction distributions, so that if the value of T_g is known for, say, the $\pm J$ distribution in a given dimension, one can easily estimate the ordering temperature for other interaction distributions. In dimension 4, taking $T_g = 2.04$ for the $\pm J$ distribution, we find $T_g = 1.77$ for the Gaussian distribution by applying the Singh-Fisher rule. The agreement with the numerical estimate[14] is excellent, demonstrating the reliability of this method. (In dimension 3 this rule leads to a prediction of 0.81 for the Gaussian interaction T_g.) In higher dimensions the Singh-Fisher rule should be even more accurate, and so knowing the $\pm J$ ordering temperature we can estimate the Gaussian ordering temperature with confidence.

We took simulation data for ISG's on simple (hyper)cubic lattices with $\pm J$ and Gaussian near-neighbor interactions in dimensions 4, 5, 6, and 7. We have measured the autocorrelation function fluctuations as a function of sample size L for $L = 2-6$ at T near T_g (to $L = 4$ only in dimension 7). We used single-spin update heat-bath dynamics and averaged over from 1500 to 100 samples depending on L and d, taking stringent precautions concerning the sample annealing procedure, following Ref. 14.

In order to check the values of the ordering temperatures, we measured the Binder cumulant g_L for different temperatures near the estimated value of T_g for the 4D Gaussian case and for the 5D $\pm J$ and Gaussian interactions. The scaling rule for g_L, which is dimensionless, is just $g_L = G[L^{1/\nu}(T - T_g)]$, and so if we plot g_L against L for any fixed T near

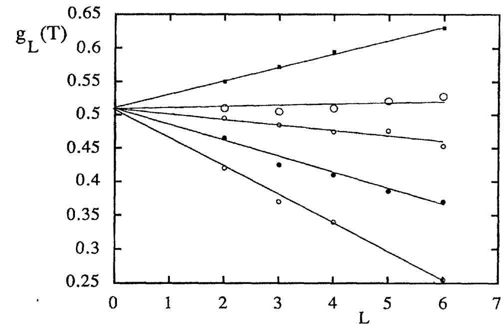

FIG. 2. Binder cumulant as a function of size L at different temperatures for the 4D Gaussian ISG. Temperatures are 1.6, 1.75, 1.8, 1.9, and 2.0 from top to bottom. The data for the temperatures 1.6 and 2.0 were read off the plot given in Ref. 11, Fig. 7, and can be seen to be consistent with the present results taken at intermediate temperatures. The lines extrapolated to $L = 0$ should intersect at $g_L(T_g)$ (see text).

T_g the points should extrapolate to $g_L(T_g)$ for $L = 0$. In dimension 4, ν is near 1,[17] and so we can expect near linear $g_L(L)$ plots at fixed T. Figure 2 shows our data in the form of linear plots together with data read off Fig. 7 of Ref. 11; the results of the two independent simulations are consistent with $T_g = 1.76 \pm 0.01$ and $g_L(T_g) = 0.51 \pm 0.02$. The T_g value confirms that given by Refs. 11,14; the $g_L(T_g)$ is distinctly higher than the value obtained for the 4D $\pm J$ interaction system, where $g_L(T_g) = 0.44 \pm 0.01$.[18] This parameter should be universal if the standard rules are obeyed;[21] the fact that different values are observed for the two different interaction distributions is evidence that the universality rules do not hold.

In dimension 5 similar plots with ν chosen as 0.73 (Ref. 17) are consistent with $T_g = 2.59 \pm 0.02$ and $g_L(T_g) = 0.28 \pm 0.01$ for the $\pm J$ distribution and with $T_g = 2.37 \pm 0.02$ and $g_L(T_g) = 0.31 \pm 0.01$ for the Gaussian distribution. Again, as in 4D, the estimated $\pm J$ value of T_g appears to be marginally higher than the series estimate, the ratio of the $\pm J$ and Gaussian T_g values agrees with the Singh-Fisher rule, and $g_L(T_g)$ appears to be slightly higher for the Gaussian case than for the $\pm J$ case.

Now consider the mean-square fluctuations in the autocorrelation function at equilibrium, $\langle q^2\rangle$. From the scaling rules,[14] at $T = T_g$, $L^{d-2}\langle q^2\rangle$ is proportional to $L^{-\eta}$. In dimension 4, from log-log plots (Fig. 3), we estimate $\eta = -0.26 \pm 0.03$ and -0.59 ± 0.03 for the $\pm J$ ISG and for the Gaussian system if we take $T_g = 2.05$ and 1.75, respectively. For the $\pm J$ case our η estimate is consistent with the values obtained from similar simulations [$\eta = -0.25 \pm 0.01$ (Ref. 18)] and from the less accurate series expansion results [$\eta = -0.1 \pm 0.25$ (Ref. 17)]. For the Gaussian case our data are completely consistent point by point with the results obtained by Bhatt and Young on the same system (Ref. 14, Fig. 15), and our value of η is consistent with independent simulation data of Reger *et al.* ($\eta = -0.6 \pm 0.05$,[22] Fig. 2) and with dynamic scaling results.[15,23] (The less negative value of η quoted in Ref. 14 can be put down to the fact that an overall scaling fit was made to all the data over a wide temperature range and so included points taken at temperatures well away from T_g where a detailed analysis shows that there are deviations from the scaling form valid close to and at

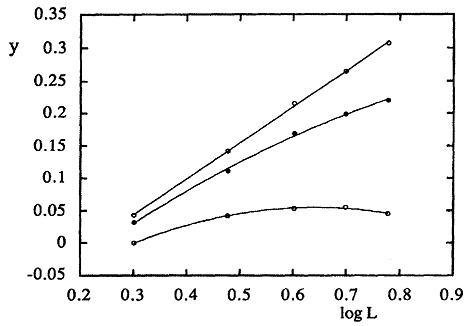

FIG. 3. Autocorrelation fluctuations as a function of temperature and size for the 4D Gaussian ISG. We plot the parameter $y=\log_{10}(L^2\langle q^2\rangle)$ as a function of $\log_{10}(L)$ for different temperatures: $T=1.75$, 1.8, and 1.9 from top to bottom. At the ordering temperature, $L^2\langle q^2\rangle$ is proportional to $L^{-\eta}$, giving a straight line of slope $-\eta$.

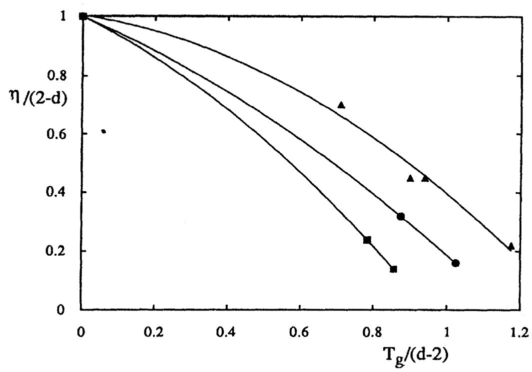

FIG. 4. Data for η in dimensions 3, 4, and 5 for different interaction distributions in terms of a normalized $\eta(2-d)$ against a $T_g/(d-2)$ plot. The curves (upper curve for dimension 3, middle curve for dimension 4, lowest curve for dimension 5) are traced so as to pass through the point $\eta/(2-d)=1$ at $T_g=0$ (see text).

T_g.) The results in Fig. 3 are consistent with T_g being very near 1.75, as the log-log plot is still curved for $T=1.8$.

In dimension 5, the data lead to $\eta=-0.42\pm0.05$ and -0.72 ± 0.05 for the $\pm J$ and Gaussian distributions, respectively, if we assume the series values of the ordering temperatures. If we choose the slightly higher temperatures given by the Binder cumulant data, the η values would be -0.35 ± 0.05 and -0.65 ± 0.05, respectively. The $\pm J$ value is in good agreement with the estimate from the series expansion method [$\eta=-0.38\pm0.07$ (Ref. 17)]; we are aware of no other published estimate for the Gaussian distribution. As the T_g values can be assume to be linked by the Singh-Fisher rule, whichever pair is correct there is a clear difference between the Gaussian and $\pm J$ distribution values of η.

Thus in each of dimensions 3, 4, and 5 consistent simulation data indicate that the exponent η is significantly more negative for the Gaussian distribution than for the $\pm J$ distribution, in contradiction to the universality rule. Even for weak universality,[3] η should remain constant for a given family of systems. It could be objected that the data are taken on "small" samples (even though each $L=6$ sample in dimension 5 contains 7776 spins) so that the asymptotic regime may not have been reached; however, the dynamic data argue against this. For the 3D $\pm J$ system, finite-size scaling on small samples[14] and dynamic measurements on large samples[13] ($L=64$) give the same value of η to within their respective error bars. For dimensions 4 and 5 we can also compare with dynamic data.[15,23] At an ordering temperature the autocorrelation function $q(t)$ tends to λt^{-x} with x related to the other exponents through

$$x=(d-2+\eta)/2z. \qquad (1)$$

Van Hove relaxation dynamics lead to $z=2(2-\eta)$ to a good approximation in ISG's near the upper critical dimension,[24] and so we expect x to be determined uniquely by d and η:

$$x\approx(d-2+\eta)/4(2-\eta). \qquad (2)$$

Thus, if η is more negative, x should be smaller. With much larger samples than those used in the finite-size scaling measurements, we have observed that in each dimension x is significantly smaller for the Gaussian distribution than for the $\pm J$ distribution. In dimension 4, $x=0.195$ and 0.14 for the $\pm J$ and Gaussian distributions, respectively,[23] corresponding to η values of -0.25 and -0.56, in excellent agreement with the finite-size scaling estimates. In dimension 5 the x values are 0.29 and 0.21, leading to equally good agreement with the finite-size scaling values of η. We conclude that even though the L values used in the finite-size scaling work were small because of computational limitations, there is no evidence that there are any significant differences between estimates obtained from results at these sample sizes and from much larger samples.

We can ask ourselves the question of whether it is possible to rationalize the trends observed in the results. Suppose that there exits a control parameter pertinent for a transition; varying this parameter will change both the ordering temperature T_g and exponents continuously. According to the standard scaling laws[14] $\gamma=\nu(2-\eta)$ if $T_g>0$ and $\gamma=d\nu$ if $T_g=0$. At fixed dimension, if the control parameter is changed in such a way as to drive T_g towards zero, η must simultaneously tend to the value $2-d$. Then on an η against T_g plot the set of values for a given family of systems with nonzero T_g in dimension d should lie on a continuous curve culminating at the point $T_g=0$, $\eta=2-d$. When we plot the data for ISG's in dimensions 3, 4, and 5 in this way, the results appear to be fully compatible with such curves (Fig. 4). T_g and η vary together with R in a manner which suggests that R is a pertinent control parameter for the transition. In the same way, according to our data, $g_L(T_g)$ increases as T_g drops with increasing R. This again is logical, because if T_g is being drive towards zero by the pertinent parameter R, $g_L(T_g)$ should be being driven towards 1.

In systems with nonzero T_g, for ν (and hence for α through the scaling relation $\alpha-2=\nu d$) the scaling laws do not impose a rule analogous to that for η and so ν and α can remain independent of R even if η (and hence through scaling rules β, γ, and δ) changes with R.

When the data for η are plotted as a function of d, we find regular behavior from dimension 2 to dimension 5 for each distribution (Fig. 5). If η is to increase to zero at dimension 6 for both distributions as it should do at the upper critical dimension, there must be a rather sudden change of behavior between dimension 5 and dimension 6. The ε expansion curve to order 3 (Ref. 25) is in very poor agreement

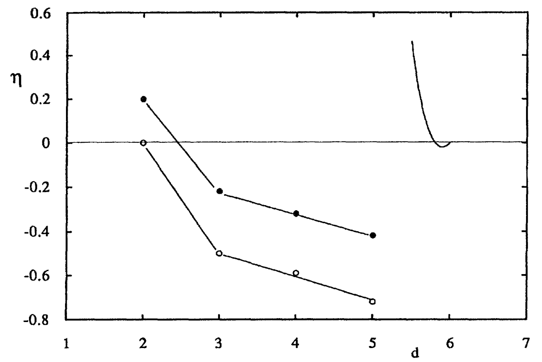

FIG. 5. Data for $\eta(d)$ as a function of dimension d for the $\pm J$ (upper curve) and Gaussian (lower curve) interaction distributions. Dimension 2, Ref. 11; other dimensions, see text. Curve starting at $d=6$, ε expansion to order 3 (Ref. 25).

with the observed behavior of $\eta(d)$ near dimension 6 [in contrast to the excellent prediction for $\eta(d)$ down to $d=2$ for the same order of the expansion in ferromagnetic Ising systems].[26] In 5D our $\pm J$ η estimate is in excellent agreement with the independent series estimate;[17] it appears that the ε expansion method is of doubtful utility in ISG's. To obtain even the right sign for η at $d=5$, the expansion must be arbitrarily truncated after the first term in ε (see Ref. 2 for comments on the ε expansion).

At an upper critical dimension, logarithmic terms appear in finite-size scaling expressions, making estimates of exponents very difficult. This has been shown to be the case at $d=6$ in the $\pm J$ ISG.[19] Our data in 6D are consistent with Ref. 19, but the logarithmic corrections preclude any firm conclusion on the exponent values. Above the upper critical dimension, the notion of correlation volume (and hence of correlation length) should be replaced by correlation number.[27] In consequence, we expect that for any dimension greater than the upper critical dimension the susceptibility of finite-size samples should increase in the same way with the total number of spins N as in the mean-field case. This means that for the ISG's above the upper critical dimension we expect $\langle q^2 \rangle$ to be proportional to $N^{-2/3}$ (as the susceptibility is proportional to $N^{1/3}$ in the mean field). We have checked this in dimension 7 for both the $\pm J$ and Gaussian distributions and have found that this size dependence is followed to within the accuracy of the measurements. This result is consistent with the standard theoretical prediction that the upper critical dimension is indeed 6.[2]

In conclusion, from the discussion of the data on η in dimensions 3, 4, and 5 it is suggested that below the upper critical dimension parameters such as the interaction distribution kurtosis R are pertinent at ISG transitions and can induce continuous changes in exponents so that the usual universality rules are no longer obeyed. It has already been shown that an applied magnetic field also appears to be a pertinent parameter; for the $\pm J$ ISG in dimension 4, η becomes more negative as T_g decreases on the application of a magnetic field[18] in a similar way to the behavior that we have found to occur with increasing R. Other possible pertinent parameters could be the type of lattice, the range of the interactions, a bias in the center of gravity of the interaction distribution, etc.

In the light of the present results, it would appear to be useful to reexamine the theory of the critical behavior of spin glasses. de Almeida[28] has pointed out that the usual linear renormalization-group approach is inappropriate for ISG's as the standard rescaling factor b should be replaced by a set of factors $b^{\alpha\beta}$, and a nonlinear renormalization-group procedure is essential because critical fluctuations are much more important in spin glasses than in conventional magnets. Interaction distribution terms can be expected to appear through the $\rho(K)$ of Eq. (1) of Ref. 28. de Almeida's theoretical conclusions may be relevant to our empirical results, and it would be interesting to firmly establish on theoretical grounds just which parameters can be pertinent for spin-glass transitions and why. It seems possible that there is a much richer critical behavior in ISG's and other spin-glass and spin-glass-like systems than in conventional systems, with no universality classes in the usual sense.

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