Wigner-crystal states for the two-dimensional electron gas in a double-quantum-well system

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Using the Hartree-Fock approximation, we calculate the energy of different Wigner-crystal states for the two-dimensional electron gas of a double-quantum-well system in a strong magnetic field. Our calculation takes interlayer hopping, as well as an in-plane magnetic field into consideration. The ground state at small layer separations is a one-component triangular lattice Wigner state. As the layer separation is increased, the ground state first undergoes a transition to two stacked square lattices, and then undergoes another transition at an even larger layer separation to a two-component triangular lattice. The range of the layer separation at which the two-component square lattice occurs as the ground state shrinks, and eventually disappears, as the interlayer hopping is increased. An in-plane magnetic field induces another phase transition from a commensurate to an incommensurate state, similar to that of $\nu = 1$ quantum Hall state observed recently. We calculate the critical value of the in-plane field of the transition and find that the anisotropy of the Wigner state, i.e., the relative orientation of the crystal and the in-plane magnetic field, has a negligible effect on the critical value for low filling fractions. The effect of this anisotropy on the low-lying phonon energy is discussed. An experimental geometry is proposed in which the parallel magnetic field is used to enhance the orientational correlations in the ground state when the crystal is subject to a random potential.

I. INTRODUCTION

An electron gas is expected to condense into a Wigner crystal[1] (WC) below some critical density. This condensation occurs when the Coulomb energy, which tends to localize electrons into individual lattice sites to keep them as far apart as possible from each other, dominates over the kinetic energy, which favors a smooth variation of electron density. In the absence of a magnetic field, the kinetic energy of a two-dimensional electron system (2DES) scales like $K = \hbar^2/m^*a^2$, while the Coulomb energy scales like $V = e^2/\epsilon a$, where a is the mean inter-electron distance and ϵ is the dielectric constant of the host material, and m^* and e are the electron mass and charge, respectively. The relevant parameter is the ratio $r_s = V/K = a/a_B$, where $a_B = \hbar^2\epsilon/m^*e^2$ is the Bohr radius. Monte Carlo simulation[2] predicts that a 2DES crystallizes for $r_s \geq 35$. When a strong magnetic field is applied perpendicular to the 2DES, the situation is changed qualitatively, as the kinetic energy is quenched into discreet Landau levels, and the zero-point fluctuations in the lowest Landau level are confined within a magnetic length $l_0 = (\hbar c/eB_\perp)^{1/2}$, where c is the speed of light and $B_\perp$ is the applied magnetic field. Once l_0 is sufficiently small compared to the typical interparticle distance a, crystallization occurs. The ratio l_0/a can be characterized by the Landau level filling factor $\nu = 2\pi l_0^2 n$, where n is the density of the 2DES. Crystallization will occur for sufficiently small ν for any given density. Theoretical estimates[3] put the critical filling factor of crystallization at about $\nu_c \sim 1/6$. In recent years, especially after the observation of the reentrant insulating phase around the $\nu = 1/5$ quantum Hall state,[4] there has been an increasing interest in the study of the WC states in a 2DES in a strong magnetic field. Many experimental results[4–10] are found to be in some ways consistent with the assumption of a pinned WC as the ground state.

Recent advances in material-growth technology allows the fabrication of high quality double-quantum-well system (DQWS), in which two interacting 2DES are separated by a distance comparable to the mean interparticle distance within the 2DES. This introduces a degree of freedom associated with the third direction. Electron-electron interactions between the layers have been known to lead to quantum Hall states.[11] It also leads to increased stability of the WC state or other charge density wave states.[12] Recent experiments on two layer systems in wide quantum wells below filling factor $\nu = 1/2$ have shown insulating behavior[13] similar to that seen in single layer systems below $\nu = 1/5$. Because Coulomb interactions can lead to mixing of the electronic states of the two wells, more complicated structures of WC states become possible in DQWS. The goal of this paper is to investigate the evolution of the ground state among different possible WC configurations, as the parameters of the DQWS are changed, at a small Landau level filling factor, where a WC is expected to be the ground state. This is accomplished by calculating, in the Hartree-Fock approximation, the ground state energy of the different WC states. Our method of calculation is based on the numerical technique developed in Ref. 14, which is valid in the strong-field limit. We take into consideration interlayer hopping, as well as an in-plane magnetic field. We ignore the finite thickness of the quantum wells and

treat the electron gas as ideally two dimensional. This zero-thickness approximation is an important simplification in the case of a tilted field, since the effect of the in-plane magnetic field can then be included by adding a phase factor to the wave function of the electrons in one of the wells.

At small layer separations, where the electron-electron correlations between the wells are almost as important as the correlations within each individual well, electrons will occupy the symmetric state of a DQWS to minimize Coulomb energy and to take advantage of hopping energy. In this situation, the DQWS behaves essentially like a single layer system. The WC at small filling factor is, therefore, a triangular lattice. As the layer separation increases, the 2DES will seek a state where intralayer correlations are favored over the interlayer correlation. This leads to a transition to a truly two-component system when the intralayer Coulomb energy is more important than the hopping and interlayer interaction energy combined. Under this condition, each individual well forms a WC separately with a lattice constant, which is appropriate for the electron density in its own layer. The two lattices are shifted relative to one another to lower the interlayer static Coulomb energy (Hartree energy). We find that, in the absence of interlayer hopping, the one-component WC is first transformed to a two-component square lattice WC state, and then transformed to a two-component triangular lattice, as the layer separation is increased. The existence of a square lattice WC at intermediate separations can be understood by noticing that the square lattice configurations have lower interlayer interaction energy than that of the triangular lattices. This gain may exceed the difference in intralayer Coulomb energies between a square lattice and a triangular lattice, which is known to be small.[15] The range of the layer separation at which the two-component square lattice exists shrinks, and eventually disappears, as the interlayer hopping is increased, as a result of the expansion of the one-component triangular phase. Our result is consistent with that of Chan and MacDonald,[16] where they treat electrons classically, and with the recent calculation of Narasimhan and Ho.[17]

A DQWS in a tilted magnetic field has been studied recently. A phase transition, driven by the in-plane component of the field, was observed for $\nu = 1$ quantum Hall state,[18] and was explained using an easy-plane itinerant quantum ferromagnetism description.[19] This phase transition happens as a result of the competition between the hopping energy and Coulomb interaction energy. The in-plane magnetic field twists the interlayer phase coherence of the wave functions of electrons in symmetric state. The result is an increase in interlayer Coulomb energy. At small in-plane field, the increase in interlayer Coulomb energy is small compared to the hopping energy, so the electrons will stay in the state dictated by the tunneling part of the Hamiltonian in order to take advantage of the hopping energy. As the in-plane field becomes stronger, the hopping energy is reduced, while the cost in Coulomb energy continues to rise. When the in-plane field is raised beyond a critical value, the DQWS system will undergo a transition to a state in which electrons give up the hopping energy in order to restore the interlayer correlations. The same physics occurs for either a $\nu = 1$ quantum Hall state or a one-component WC state, since for both cases, there is strong interlayer coherence before the application of an in-plane field. Since a WC breaks the rotational symmetry of the system, the phase transition should, in principle, depend quantitatively on the angle between the direction of the in-plane field and the crystal axes. We have calculated the critical value of the in-plane field and found that this dependence is very weak, practically unobservable at small filling factors.

Nevertheless, a parallel magnetic field can have important consequences, particularly for the long-wavelength physics of the system. The broken orientational symmetry of the ground state, due to a parallel magnetic field, implies that there is a restoring force on the crystal if it is misoriented with the field. This effect can be very important if the system is subject to a weak random potential, which destroys long-range orientational order in this system.[20] We propose an experimental geometry, in which the 2DEG is cooled through its freezing transition in the presence of a parallel magnetic field. The force that tends to align the crystal axes perpendicular to the parallel field should enhance the orientational order in the ground state. It has been shown recently[20] that the depinning electric field[5] of a disordered WC is sensitive to the orientational correlations in the system, and tends to be reduced as orientational order increases. Thus, an increase in orientational order can, in principle, be detected experimentally through the transport properties of the system.

This paper is organized as follows. In Sec. II, we describe the Hartree-Fock approximation for the interacting 2DES in a DQWS in a strong magnetic field. In Sec. III, we present and discuss the numerical results. Sec. IV discusses the effect of a parallel magnetic field on some of the crystal properties. A brief summary in Sec. V concludes this paper.

II. HARTREE-FOCK APPROXIMATION

In the absence of an in-plane magnetic field, the Hartree-Fock approximation for a DQWS has been clearly presented in detail in Ref. 14. We need only to extend it to the case of a tilted magnetic field. In a Hartree-Fock approximation, one treats the interacting Hamiltonian as that of free electrons in the mean-field potential determined by a given electronic state and then self-consistently solves for the state. In our present case, the electronic state being sought is a WC. The characteristic of a WC is a periodically modulated charge density. We, therefore, choose the Fourier transformed electron density at corresponding reciprocal-lattice vectors (RLV), as the order parameters of the WC states. Following this idea, we define $\rho_{ij}(\mathbf{q}) = (e^{q^2 l_0^2/4}/g) \int d^2r e^{-i\mathbf{q}\cdot\mathbf{r}} \psi_i^\dagger(\mathbf{r})\psi_j(\mathbf{r})$, where $i, j = 1, 2$, labeling the two layers, $\psi^\dagger$ $(\psi_j(\mathbf{r}))$ is electron creation (annihilation) operator, and the prefactor $e^{-q^2 l_0^2/4} g$ is singled out for the purpose of convenient notation. One obtains, in the lowest Landau level with the Landau gauge,

$$\rho_{ij}(\mathbf{q}) = \frac{1}{g}\sum_{\alpha\beta} e^{-iq_x(\alpha+\beta)/2}\delta_{\beta,\alpha+q_y} C^\dagger_{i\alpha} C_{j\beta}, \quad (1)$$

where α and β are the single-particle states of the lowest Landau level. In the above expression, $g = \Omega/2\pi l_0^2$ is the Landau level degeneracy, where Ω is the area of the DQWS. The order parameters have, by definition, the property that $\langle\rho_{ij}(\mathbf{q})\rangle = \langle\rho_{ji}(-\mathbf{q})\rangle^*$. We restrict ourselves to seek only the WC states, where the charge distribution in one layer is the same as that of the other layer, except that one is rigidly shifted by a displacement of $\mathbf{a}$ relative to the other, i.e., $\langle\rho_{22}(\mathbf{q})\rangle = e^{i\mathbf{q}\cdot\mathbf{a}}\langle\rho_{11}(\mathbf{q})\rangle$. This leaves us with only two independent sets of the order parameters to be obtained. The order parameters are related to Green's function as

$$\langle\rho_{ij}(\mathbf{q})\rangle = G_{ji}(\mathbf{q}, \tau = 0^-), \quad (2)$$

where $G_{ij}(\mathbf{q},\tau) = e^{q^2 l_0^2/4}/g\int d^2 r e^{-i\mathbf{q}\cdot\mathbf{r}} G_{ij}(\mathbf{r},\mathbf{r},\tau)$ and $G_{ij}(\mathbf{r},\mathbf{r}',\tau) = -\langle T_\tau \psi_i(\mathbf{r},\tau)\psi_j^\dagger(\mathbf{r}')\rangle$. In the lowest Landau level, the Green's functions become

$$G_{ij}(\mathbf{q},\tau) = \frac{1}{g}\sum_{\alpha\ \beta} e^{-iq_x(\alpha+\beta)/2}\delta_{\alpha,\beta+q_y} G_{ij}(\alpha,\beta,\tau), \quad (3)$$

with

$$G_{ij}(\alpha,\beta,\tau) = -\langle T_\tau C_{i\alpha}(\tau) C^\dagger_{j\beta}\rangle. \quad (4)$$

The Green's functions are to be obtained by self-consistently solving the equation of motion,

$$\frac{\partial}{\partial\tau} G_{ij}(\alpha,\beta,\tau) + \delta(\tau)\delta_{ij}\delta_{\alpha,\beta} + \langle T_\tau[H, C_{i\alpha}(\tau)] C^\dagger_{j\beta}\rangle = 0. \quad (5)$$

The next step is to approximate the equations of motion in the Hartree-Fock approach. We will treat a tilted magnetic field from the beginning. The result for a perpendicular field is recovered by simply setting the in-plane component of the field $B_\parallel = 0$. Under the zero well-thickness approximation, the only effect of the in-plane magnetic field is to add a phase factor to the electronic wave function of one of the layers. Let $k_B = d/l_\parallel^2$, with d the separation between the wells, and $l_\parallel = (\hbar c/eB_\parallel)^{1/2}$. If one chooses the direction of $B_\parallel$ as the x axis, the single-particle eigenstates become

$$\phi_{1\alpha}(\mathbf{r}) = \frac{1}{\sqrt{l_0\sqrt{\pi}\Omega}} e^{i\alpha y} e^{-(x-\alpha l_0^2)^2/2l_0^2},$$
$$\phi_{2\alpha}(\mathbf{r}) = e^{ik_B y}\phi_{1\alpha}(\mathbf{r}). \quad (6)$$

The Hamiltonian of the DQWS contains kinetic energy (which is a trivial constant for the lowest Landau level), interlayer hopping energy, and the Coulomb interaction energy.

$$H = -\sum_{i\alpha}\mu C^\dagger_{i\alpha} C_{i\alpha} - te^{-k_B^2 l_0^2/4}\sum_\alpha (C^\dagger_{1\alpha} C_{2\alpha-k_B} + C^\dagger_{2\alpha-k_B} C_{1\alpha}) + \frac{1}{2}\sum_{ij}\sum_{\alpha\alpha'\beta\beta'} V_{ij}(\alpha,\alpha',\beta,\beta') C^\dagger_{i\alpha} C^\dagger_{j\beta} C_{j\beta'} C_{i\alpha'}, \quad (7)$$

where μ is the chemical potential to be fixed at the end of calculation for a given electron density, and $V_{ij}(\alpha,\alpha',\beta,\beta')$ is the matrix element of Coulomb potential. The hopping parameter t is suppressed in the presence of an in-plane field by a factor $e^{-k_B^2 l_0^2/4}$, which comes from the matrix element $\langle 1\alpha|t|2\alpha'\rangle = te^{-k_B^2 l_0^2/4}\delta_{\alpha,\alpha'+k_B}$. Physically, this means that the electrons tunnel along the direction of the total magnetic field. Performing the Hartree-Fock pairing $C_1^\dagger C_2^\dagger C_{2'} C_{1'} \approx \langle C_1^\dagger C_{1'}\rangle C_2^\dagger C_{2'} - \langle C_1^\dagger C_{2'}\rangle C_2^\dagger C_{1'}$ on Eq. (7), one obtains

$$H = -g\sum_i \mu\rho_{ii}(0) - gt[\rho_{21}(\mathbf{k}_B) + \rho_{12}(-\mathbf{k}_B)] + g\frac{e^2}{\epsilon l_0}\sum_{\mathbf{q}}\sum_{ij} U_{ij}(\mathbf{q})\rho_{ji}(\mathbf{q}), \quad (8)$$

where $\mathbf{k}_B = k_B\hat{\mathbf{y}}$ and

$$U_{11}(\mathbf{q}) = [V_a(\mathbf{q}) - V_b(\mathbf{q})]\langle\rho_{11}(-\mathbf{q})\rangle + V_c(\mathbf{q})\langle\rho_{22}(-\mathbf{q})\rangle,$$
$$U_{12}(\mathbf{q}) = -V_d(\mathbf{q})\langle\rho_{21}(-\mathbf{q})\rangle, \quad (9)$$

with U_{21} and U_{22} obtained by interchanging the indices 1 and 2. In the above expressions, V_a, V_b, V_c, and V_d are the direct and exchange terms for the interlayer and intralayer Coulomb interactions,

$$V_a(q) = \frac{1}{ql_0}e^{-q^2 l_0^2/2}(1-\delta_{\mathbf{q},0}),$$
$$V_b(q) = \sqrt{\frac{\pi}{2}}e^{-q^2 l_0^2/4} I_0(q^2 l_0^2/4),$$
$$V_c(q) = e^{-qd} V_a(q),$$
$$V_d(q) = \int_0^\infty dx J_0(xql_0) e^{-x^2/2 - xd/l_0}, \quad (10)$$

where J_0 and I_0 are the zeroth order Bessel function and modified Bessel function, respectively. It is worthwhile to notice that since Coulomb scattering is not expected to move electrons from one well to the other, the matrix element $V_{ij}(\alpha,\alpha',\beta,\beta')$ is unchanged by the presence of an in-plane field, i.e., Eq. (9) and Eq. (10) are exactly the same as the expressions for $B_\parallel = 0$. This means that the commutator $[H_c, C_{i\alpha}]$, where H_c is the Coulomb interactions part of the Hartree-Fock Hamiltonian of Eq. (8), is unchanged by the presence of the in-plane field. Letting $A_{ij}(\alpha,\beta) = -\langle T_\tau[H_c, C_{i\alpha}](\tau) C^\dagger_{j\beta}\rangle$, we have

$$A_{ij}(\mathbf{q},\tau) = -\frac{e^2}{\epsilon l_0}\sum_{k\mathbf{p}} U_{ik}(\mathbf{p}-\mathbf{q}) e^{il_0^2\mathbf{q}\wedge\mathbf{p}/2} G_{kj}(\mathbf{p},\tau), \quad (11)$$

where $\mathbf{q}\wedge\mathbf{p} = q_x p_y - q_y p_x$. The effect of the in-plane field is contained explicitly in H_t, the hopping term in

Eq. (8). Denoting $F_{ij}(\alpha,\beta) = -\langle T_\tau[H_t, C_{i\alpha}](\tau)C^\dagger_{j\beta}\rangle$, one obtains

$$F_{11}(\mathbf{q},\tau) = te^{-k_B^2 l_0^2/4}e^{-il_0^2 q_x k_B/2}G_{21}(\mathbf{q}-\mathbf{k}_B,\tau),$$
$$F_{12}(\mathbf{q}+\mathbf{k}_B,\tau) = te^{-k_B^2 l_0^2/4}e^{-il_0^2 q_x k_B/2}G_{22}(\mathbf{q},\tau),$$
$$F_{21}(\mathbf{q}-\mathbf{k}_B,\tau) = te^{-k_B^2 l_0^2/4}e^{il_0^2 q_x k_B/2}G_{11}(\mathbf{q},\tau),$$
$$F_{22}(\mathbf{q},\tau) = te^{-k_B^2 l_0^2/4}e^{il_0^2 q_x k_B/2}G_{12}(\mathbf{q}+\mathbf{k}_B,\tau). \quad (12)$$

One can see, from the above expressions, that in the presence of an in-plane field the hopping Hamiltonian attempts to shift the positions of the nonvanishing interlayer order parameters $\langle\rho_{12}\rangle$ and $\langle\rho_{21}\rangle$ in reciprocal space from $\mathbf{q}=\mathbf{G}$ to $\mathbf{q}=\mathbf{G}\pm\mathbf{k}_B$. This is merely a reflection of the fact that the charge distributions in the two layers will be relatively shifted, because the electrons intend to tunnel along the direction of the total magnetic field. If the electrons of the DQWS are in the symmetric state for $B_\parallel = 0$, where electron distributions in the two layers are directly on top of each other, the inclusion of an in-plane field damages the original interlayer correlations and results in an increase in the interlayer Coulomb energy. In a pseudospin description[19] of the DQWS, the tunneling behaves like a tumbling magnetic field, which twists the interlayer phase coherence of the symmetric states. In an attempt to minimize the total energy, a DQWS is forced to choose between the hopping energy and the Coulomb energy. The electrons can only take advantage of the hopping energy at the cost of increasing the interlayer Coulomb interaction.

Putting Eq. (11) and Eq. (12) into Eq. (5), one obtains the desired equations of motion for G_{11} and G_{21} in Matsubara frequencies,

$$-\frac{e^2}{\epsilon l_0}\sum_{\mathbf{q}'}[e^{il_0^2\mathbf{q}\wedge\mathbf{q}'/2}\widetilde{U}^*_{11}(\mathbf{q}-\mathbf{q}')\widetilde{G}_{11}(\mathbf{q}',i\omega_n) + e^{il_0^2\mathbf{q}\wedge\mathbf{q}'^-/2}\widetilde{U}^*_{21}(\mathbf{q}-\mathbf{q}'^-)\widetilde{G}_{21}(\mathbf{q}'^-,i\omega_n)]$$
$$+(i\hbar\omega_n+\mu)\widetilde{G}_{11}(\mathbf{q},i\omega_n) + te^{-k_B^2 l_0^2/4}e^{-il_0^2 q_x k_B/2}\widetilde{G}_{21}(\mathbf{q}-\mathbf{k}_B,i\omega_n) = \hbar\delta_{\mathbf{q},0},$$
$$-\frac{e^2}{\epsilon l_0}\sum_{\mathbf{q}'}[e^{il_0^2\mathbf{q}^-\wedge\mathbf{q}'/2}\widetilde{U}_{21}(\mathbf{q}'-\mathbf{q}^-)\widetilde{G}_{11}(\mathbf{q}',i\omega_n) + e^{il_0^2\mathbf{q}^-\wedge\mathbf{q}'^-/2}\widetilde{U}_{11}(\mathbf{q}-\mathbf{q}')\widetilde{G}_{21}(\mathbf{q}'^-,i\omega_n)]$$
$$+(i\hbar\omega_n+\mu)\widetilde{G}_{21}(\mathbf{q}^-,i\omega_n) + te^{-k_B^2 l_0^2/4}e^{il_0^2 q_x k_B/2}\widetilde{G}_{11}(\mathbf{q}^-+\mathbf{k}_B,i\omega_n) = 0, \quad (13)$$

where we have adopted the following notations:[14]

$$\widetilde{U}_{ij}(\mathbf{q}) = e^{i\mathbf{q}\cdot\mathbf{a}/2}U_{ij}(\mathbf{q}),$$
$$\widetilde{G}_{ij}(\mathbf{q}) = e^{-i\mathbf{q}\cdot\mathbf{a}/2}G_{ij}(\mathbf{q}),$$
$$\langle\widetilde{\rho}_{ij}(\mathbf{q})\rangle = e^{-i\mathbf{q}\cdot\mathbf{a}/2}\langle\rho_{ij}(\mathbf{q})\rangle,$$
$$\mathbf{q}^\pm = \mathbf{q}\pm\mathbf{k}_B \quad \text{or} \quad \mathbf{q}. \quad (14)$$

Equation (13) and Eq. (14) are intended to be applicable to different possible WC states. We need to set correct conditions for each situation. For the case of a perpendicular magnetic field, one has $\mathbf{k}_B = 0$, $\mathbf{q}^\pm = \mathbf{q}$, and $\mathbf{a}$ as the relative shift of the charge distributions between the layers. As mentioned earlier, there are two possible phases in the presence of an in-plane field, electrons can choose either to take advantage of the hopping energy at the cost of increased Coulomb energy, or to maintain good correlations for reducing the Coulomb energy at the cost of giving up the hopping energy. For the former case, one has $\mathbf{q}^\pm = \mathbf{q}\pm\mathbf{k}_B$, and $\mathbf{a} = k_B l_0^2\hat{\mathbf{x}}$. For the latter case, one has $\mathbf{q}^\pm = \mathbf{q}$ and can effectively set $t = 0$ [see Eq. (8)].

Equation (14) can be rearranged into following compact matrix form, which is convenient for numerical evaluation:

$$[(i\hbar\omega_n+\mu) - D]\widetilde{G} = \hbar B, \quad (15)$$

where

$$B = [1,0,0,0,0,\ldots],$$
$$\widetilde{G} = [\widetilde{G}_{11}(\mathbf{q}_1),\widetilde{G}_{21}(\mathbf{q}_1^-),\widetilde{G}_{11}(\mathbf{q}_2),\widetilde{G}_{21}(\mathbf{q}_2^-),\ldots], \quad (16)$$

with $\{\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\ldots\}$ arranged in the order of increasing magnitude. The nonzero elements of the Hermitian matrix D are

$$D_{2i-1,2j-1} = \frac{e^2}{\epsilon l_0}e^{il_0^2\mathbf{q}_i\wedge\mathbf{q}_j/2}\widetilde{U}^*_{11}(\mathbf{q}_i-\mathbf{q}_j),$$
$$D_{2i-1,2j} = \frac{e^2}{\epsilon l_0}e^{il_0^2\mathbf{q}_i\wedge\mathbf{q}_j^-/2}\widetilde{U}^*_{21}(\mathbf{q}_i-\mathbf{q}_j^-) + \delta_{ij}\widetilde{t},$$
$$D_{2i,2j-1} = \frac{e^2}{\epsilon l_0}e^{il_0^2\mathbf{q}_i^-\wedge\mathbf{q}_j/2}\widetilde{U}_{21}(\mathbf{q}_j-\mathbf{q}_i^-) + \delta_{ij}\widetilde{t}^*,$$
$$D_{2i,2j} = \frac{e^2}{\epsilon l_0}e^{il_0^2\mathbf{q}_i^-\wedge\mathbf{q}_j^-/2}\widetilde{U}_{11}(\mathbf{q}_i-\mathbf{q}_j), \quad (17)$$

where $\widetilde{t} = -te^{-k_B^2 l_0^2/4}e^{-il_0^2 q_x k_B/2}$. Equation (15) can be solved by diagonalizing the matrix D. If V_k and ω_k are, respectively, the kth eigenvector and eigenvalue of D, we obtain[14]

$$\langle\widetilde{\rho}_{11}(\mathbf{q}_i)\rangle = \sum_k^{k_{\max}} V_k(2i-1)V_k(1),$$
$$\langle\widetilde{\rho}_{12}(\mathbf{q}_i^-)\rangle = \sum_k^{k_{\max}} V_k(2i)V_k(1), \quad (18)$$

where $k_{\max}$ is determined by fixing the chemical potential

$$\langle \rho_{11}(0) \rangle = \nu/2. \quad (19)$$

With the order parameters known, we can obtain the ground state energy per electron, which will be shown in the next section, from Eq. (8),

$$\varepsilon = -\frac{1}{\nu} t e^{-k_B^2 l_0^2/4} [\langle \rho_{21}(\mathbf{k}_B) \rangle + \langle \rho_{12}(-\mathbf{k}_B) \rangle] - \frac{e^2}{\nu \epsilon l_0} \sum_{\mathbf{q}} \{ V_d(q^-) |\langle \rho_{12}(\mathbf{q}^-) \rangle|^2 + [V_a(q) - V_b(q) + V_c(q) \cos(\mathbf{q} \cdot \mathbf{a})] |\langle \rho_{11}(\mathbf{q}) \rangle|^2 \}. \quad (20)$$

III. NUMERICAL RESULTS AND DISCUSSIONS

We now discuss our numerical results for the different WC states. For this discussion, we compare the energy of the different WC states and then find the phase diagrams as the sample parameters are changed. The order parameters $\langle \rho_{ij}(\mathbf{q}) \rangle$ are obtained from a numerical analysis on Eq. (15), in which well convergent results are obtained by keeping 16 (24) shells in reciprocal-lattice vectors for triangular (square) WC states. In the following, we will first discuss the situation with a perpendicular field and then the situation with a tilted field.

There are several possible configurations for a WC in a strong perpendicular field for different layer separations and hopping strength. At small layer separations, the electronic states of the different wells mix to form symmetric and antisymmetric states. In the ideal case of $d = 0$, all electrons reside in the symmetric state for any value of hopping. A DQWS under this condition behaves as a single layer system. A WC in the symmetric state has a lattice constant a_0, such that $(\sqrt{3}/2)a_0^2 = 2\pi l_0^2/\nu$. The order parameters at corresponding RLV's are

$$\langle \rho_{ss}(\mathbf{G}) \rangle = \frac{1}{g} \sum_{\alpha\beta} e^{-iG_x(\alpha+\beta)/2} \delta_{\beta,\alpha+G_y} \langle C_{s\alpha}^\dagger C_{s\beta} \rangle \neq 0,$$

$$\langle \rho_{ij}(\mathbf{G}) \rangle = \frac{1}{2} \langle \rho_{ss}(\mathbf{G}) \rangle \quad \text{for} \quad i,j = 1,2, \quad (21)$$

where $C_{s(a)} = (1/\sqrt{2})(C_1 \pm C_2)$. The above expression also shows that the charge distributions for the two layers are directly on top of each other, i.e., $\mathbf{a} = 0$. At finite, but small, layer separations, the WC state of a DQWS is essentially the above one-component triangular (OCT) lattice. For large enough layer separations, the symmetric state is no longer energetically favored, as the system begins to seek a state where electrons within the same layer are more strongly correlated than electrons in different layers. In the large d limit, a DQWS becomes two independent single layers for $t = 0$. Electrons in each well form their own triangular WC. These two-component triangular (TCT) lattices have a lattice constant $(\sqrt{3}/2)a_0^2 = 2\pi l_0^2/(\nu/2)$, larger by a factor of $\sqrt{2}$ than that of the OCT lattice discussed above. To minimize static interlayer Coulomb energy, the two WC's are relatively shifted so that the lattice sites of one WC are at the centers of the triangles of the other WC lattice, i.e., $\mathbf{a} = (1/3)(\mathbf{a}_0 + \mathbf{b}_0)$, where $\mathbf{a}_0$ and $\mathbf{b}_0$ are the primary lattice vectors.[21] A shifted two-component square (TCS) lattice WC state with $\mathbf{a} = (1/2)(\mathbf{a}_0 + \mathbf{b}_0)$, where $\mathbf{a}_0$ and $\mathbf{b}_0$ are the primary lattice constant of the square WC, has lower interlayer Coulomb energy than the above TCT WC state, since the lateral distance between a electron in one layer and its nearest electron in the other layer in a shifted TCS lattice structure is larger than that in a shifted TCT lattice structure. For an intermediate range of layer separations, this TCS lattice structure may become the ground state of a DQWS.

In Fig. 1, we show the energies of the three different WC structures discussed above as functions of layer separation d for different value of hopping t. The lowest energy states are the OCT WC at small d, the TCS WC at intermediate d, and the TCT WC at large d. The range of layer separations at which the TCS WC exists as the ground state shrinks when t is increased, as a result of the expansion of the OCT WC phase. The important conclusion from Fig. 1 is that for weak interlayer hopping, (at least) two structural phase transitions should be expected when the layer separation is increased: first from a OCT WC to a TCS WC, then from a TCS WC

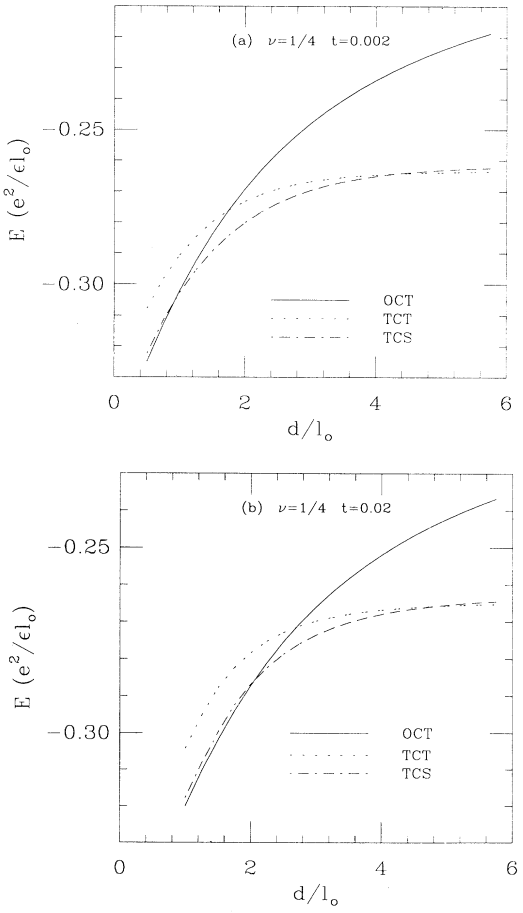

FIG. 1. Energy per particle for a one-component triangular lattice (OCT), a two-component square lattice (TCS), and a two-component triangular lattice (TCT) Wigner crystal, as functions of layer separation d for $\nu = 1/4$ at different values of hopping t. (a) t=0.002. (b) $t = 0.02(e^2/\epsilon l_0)$.

to a TCT WC.

In Fig. 2, $\mathrm{Re}[\langle\rho_{12}(0)\rangle]$, which represents the differences between the electron occupations in the symmetric and in the antisymmetric states, is shown as a function of d. This quantity characterizes the relative strength of interlayer correlations, with respect to intralayer correlations. $\mathrm{Re}[\langle\rho_{12}(0)\rangle]$ is at its maximum value in the one-component regime, where the interlayer and intralayer correlations are treated indiscriminately. It decreases in the two-component regimes as d is increased, due to the fact that the intralayer correlations are more and more favored over the interlayer correlations, but $\mathrm{Re}[\langle\rho_{12}(0)\rangle]$ remains finite for any finite tunneling even in the limit $d \to \infty$. In a pseudospin description, $\mathrm{Re}[\langle\rho_{12}(0)\rangle]$ is the spin magnetization in the direction favored by the tunneling Hamiltonian. The one-component state is the ferromagnetic state with a spontaneously broken symmetry. The finite value of $\mathrm{Re}[\langle\rho_{12}(0)\rangle]$ at large d results from the compromise between the tunneling energy and Coulomb interaction energy. We note that, in general, for $t \neq 0$, $\langle\rho_{12}\rangle \neq 0$, so that physically there is some penetration of electrons in each layer into the interstitial sites of the other layer.[17]

Next, we consider the influence from an in-plane magnetic field. Our discussion will concentrate on the situation where the system was in the symmetric state prior to the application of the in-plane field, since the effects of an in-plane field are the largest in the symmetric state, due to the existence of strong interlayer coherence. As a result of the competition between hopping energy and Coulomb energy, a DQWS in a tilted field can be in either of the two different ground states, depending on the value of the in-plane field. One is the symmetric state WC (SSWC) described by Eq. (21), where $\mathbf{a} = 0$ and the electrons feel no effect of the hopping and $B_\parallel$ has no effect. The other, which is generally relevant at a small in-plane field, is the state in which electrons form linear combinations of the states in the two wells that are displaced by the total magnetic field. We call this state a twisted symmetric state. Defining

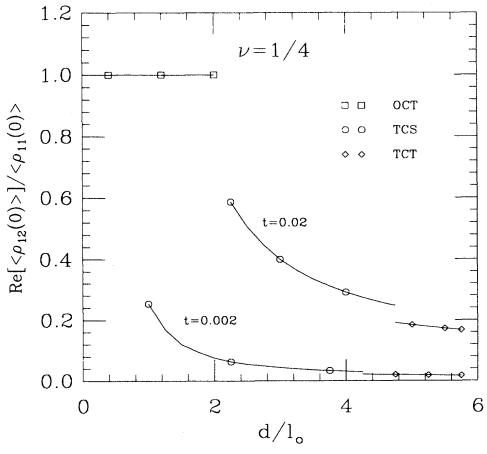

FIG. 2. $\mathrm{Re}[\langle\rho_{12}(0)\rangle]/\langle\rho_{11}(0)\rangle$ versus layer separation d for $t = 0.002$ and $t = 0.02$ $(e^2/\epsilon l_0)$ at $\nu = 1/4$.

$$\mathcal{C}_{s(a)\alpha} = \frac{1}{2}[C_{1\alpha} \pm C_{2\alpha-k_B}], \tag{22}$$

the order parameters for a twisted symmetric state WC (TSSWC) are

$$\langle\varrho_{ss}(\mathbf{G})\rangle = \frac{1}{g}\sum_{\alpha\beta} e^{-iG_x(\alpha+\beta)/2}\delta_{\beta,\alpha+G_y}\langle\mathcal{C}^\dagger_{s\alpha}\mathcal{C}_{s\beta}\rangle \neq 0. \tag{23}$$

The order parameters in layer representation $\langle\rho_{ij}(\mathbf{G})\rangle$ for this TSSWC can be obtained from the above expression by making use of the operator relation of Eq. (22),

$$\begin{aligned}
\langle\rho_{11}(\mathbf{G})\rangle &= \frac{1}{2}\langle\varrho_{ss}(\mathbf{G})\rangle,\\
\langle\rho_{22}(\mathbf{G})\rangle &= \frac{1}{2}e^{iG_xk_Bl_0^2}\langle\varrho_{ss}(\mathbf{G})\rangle,\\
\langle\rho_{12}(\mathbf{G}-\mathbf{k}_B)\rangle &= \frac{1}{2}e^{iG_xk_Bl_0^2}\langle\varrho_{ss}(\mathbf{G})\rangle,\\
\langle\rho_{21}(\mathbf{G}+\mathbf{k}_B)\rangle &= \frac{1}{2}e^{iG_xk_Bl_0^2}\langle\varrho_{ss}(\mathbf{G})\rangle.
\end{aligned} \tag{24}$$

From the above expression, one can see the obvious effects of the in-plane field: it shifts the charge distributions of the two layers relatively by an amount $\mathbf{a} = k_Bl_0^2\hat{\mathbf{x}} = d(B_\parallel/B_\perp)\hat{\mathbf{x}}$. The positions of the nonzero interlayer order parameters $\langle\rho_{12(21)}(\mathbf{q})\rangle$ in the reciprocal vector space are shifted from $\mathbf{G}$ to $\mathbf{G}^\pm = \mathbf{G}\pm\mathbf{k}_B$. The result is an increase in interlayer Coulomb energy. At small values of in-plane field, the increase in the Coulomb energy is small and can be compensated by the hopping energy, so the TSSWC is favored over the SSWC. As the in-plane field increases, the cost in Coulomb energy increases, while the hopping energy decreases. When the gain in the hopping energy can no longer compensate the cost in Coulomb energy for a strong enough in-plane field, the DQWS undergoes a transition from the TSSWC to the SSWC, where the total energy of the system is lowered by giving up the hopping energy to restore the original interlayer coherence. In Fig. 3, we show the energies of the TSSWC and SSWC as functions of the in-plane field for different values of hopping energy. It is clear, from the figure, that the TSSWC is the energetically favored ground state at a small in-plane field, while the SSWC is the energetically favored ground state at large in-plane fields. The critical value of the in-plane field for this phase transition as a function of the hopping energy is shown in Fig. 4, larger critical values of $B_\parallel/B_\perp$ for larger values of hopping t.

Since a WC breaks the rotational symmetry of the system, the properties of a DQWS, in principle, depend on the angle between the in-plane field and the crystal axes. As the WC is pinned by the presence of weak disorder, the angle can be changed by simply sweeping the direction of the in-plane field. This provides a potential opportunity to probe the orientational order of a WC and to find a unambiguous signature of the existence of a WC. In Fig. 3, we show, for $t = 0.02$, $t = 0.08$, and $t = 0.4$, the energies of the TSSWC for both the cases where the in-plane field is perpendicular to (the solid lines), or parallel

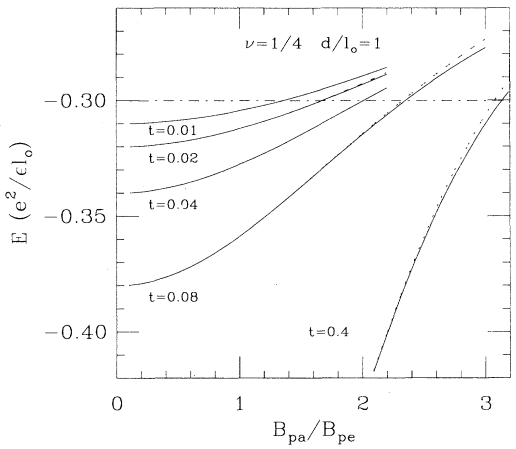

FIG. 3. Energies per particle of a twisted symmetric state Wigner crystal (TSSWC) and a symmetric state Wigner crystal (SSWC) as functions of in-plane field at $\nu = 1/4$ and $d = l_0$ for different value of hopping. The dash-dot line is for the SSWC. The solid lines are for the TSSWC with the in-plane filed perpendicular to one crystal axis. The dot-lines at $t = 0.02$, 0.08, 0.4 $(e^2/\epsilon l_0)$ are for the TSSWC with the in-plane field parallel with one crystal axis.

with (the dot-lines), one of the crystal axes. We can see that the differences are small. The change in the value of critical in-plane field from the different orientations of the field is practically indistinguishable. This is mainly because the phase transition occurs at an in-plane field, where $|\mathbf{a}|$ is small compared to the lattice constant for reasonable sample parameters. However, we should not rule out the possibility that some other quantities may have a measurable dependence on the anisotropy of the WC. For example, the energy difference of the TSSWC for the two different orientations of the in-plane field right before the phase transition is $\Delta\varepsilon \sim 7 \times 10^{-4}(e^2/\epsilon l_0)$ for $t = 0.08(e^2/\epsilon l_0)$, which is on the order of 70 mK for $e^2/\epsilon l_0 \sim 100$ K. *If* the same order of anisotropy exists in the low-lying phonon modes (see the next section),

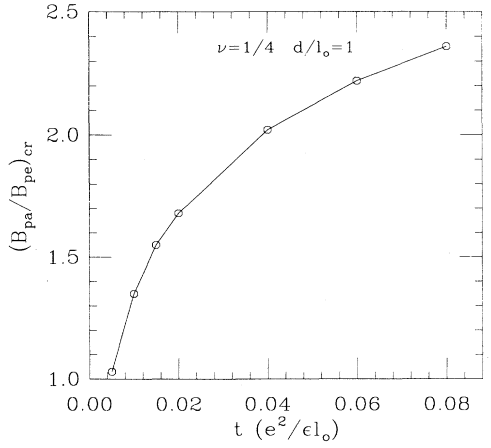

FIG. 4. The critical value of $B_\parallel/B_\perp$, as a function of the hopping for $\nu = 1/4$ and $d = l_0$.

we may have a measurable difference in the anisotropy of the specific heat at low enough temperatures, say, $T < 70$ mK.

Finally, we add a few general comments about the limitations of the model studied in this work. We have ignored both finite thickness corrections, as well as Landau level mixing in our calculations. It is well known that such effects will lead to small corrections in the energies of the states of the system (see, for example, Ref. 3), and obviously will introduce small changes in the precise values of the parameters at which the ground state changes character. Nevertheless, the general picture of the various possible states of system should not be changed by such perturbations. We have also not included disorder to this point. In the presence of disorder, one cannot expect to have true long-range order characteristic of a crystalline ground state. Nevertheless, unless the disorder is extremely large, one expects the ground state to have a finite correlation length, within which the system locally looks like a crystal. The *type* of crystal structure on short length scales should be the same as expected for a pure system. Thus, we expect that as a function of system parameters (i.e., tunneling, layer separation, etc.), one should still see transitions among the different states discussed above within the correlation length.

IV. CRYSTAL PROPERTIES IN A PARALLEL MAGNETIC FIELD

In this section, we discuss some consequences of immersing the double well system in a parallel magnetic field, when the electron system is in the TSSWC—i.e., a lattice of particles in symmetric states of the two wells, with the position of the single-particle orbits in one well displaced in the direction of the *total* applied magnetic field. As was discussed in the previous section, and as shown in Fig. 3, the energy per particle in the electron lattice depends (weakly) on the orientation of the crystal axes relative to the parallel magnetic field. One may think of this effect as an energy cost for having the bond angles of the lattice deviate from some preferred direction. While the energy cost per particle may be small, long-wavelength fluctuations of the lattice—where many bond angles deviate from the preferred direction—will be strongly affected by this energy cost. This will have important effects on the long-wavelength collective modes of the system, as well as the state of the crystal in the presence of a slowly varying random potential (which typically arises in real heterostructure environments).

To model the bond-angle energy, we use a continuum elasticity theory approach. The energy of a two dimensional lattice deformed from a perfect crystal configuration by a displacement field $u(\vec{r})$ may be written as[22]

$$E_0 = \frac{1}{2}\int \frac{d^2r}{a_0^2}\left(2\mu u_{ij}^2 + \lambda u_{kk}^2\right),$$

where μ and λ are Lamé coefficients, a_0 is the lattice constant, $u_{ij} = \partial_i u_j + \partial_j u_i$ is the strain tensor, and repeated indices are summed over. The bond-angle field may be

written in terms of the displacements in the form

$$\theta(\vec{r}) \equiv \frac{1}{2}(\partial_x u_y - \partial_y u_x),$$

so that it is natural to write the energy of a configuration in the presence of parallel magnetic field in the form $E = E_0 + E_\theta$, with

$$E_\theta \equiv \frac{1}{2}\epsilon_0 \int d^2r \theta(\vec{r})^2,$$

where ϵ_0 is a phenomenological parameter describing the energy cost to misalign the crystal with the parallel magnetic field.[23]

This model has been studied in the context of a two-dimensional crystal adsorbed on a periodic substrate,[24] where it was noted that the extra term tends to increase the stability of the crystal, with respect to finite temperature. It is also interesting to note that such a bond-angle term increases the stability of the crystal, with respect to quenched disorder. To see this, consider a weak random potential acting on the WC, which has a long orientational correlation length ξ (or even quasilong-range orientational order[20]). The energy cost to misalign a correlated region with the magnetic field scales as ξ^2, due to E_θ, whereas the energy gained by aligning with the random potential scales only as $\sqrt{N_c} \sim \xi$, where N_c is the average number of electrons in a correlated region. Thus, for weak enough disorder, where ξ is large, one expects the correlated regions to align with the preferred orientational axis.

An interesting possible method to demonstrate this would be to anneal a WC in the double well system in the presence of a parallel magnetic field. It has been shown recently[20] that a WC in the presence of a slowly varying random potential freezes into a state with at best power-law (i.e., quasilong-range) orientational order. By cooling the system in the presence of the parallel field, a preferred orientational axis is picked out for the crystal, leading to the possibility that one could obtain a system with true long-range orientational order. Observing that phenomenon will be possible if the orientational correlation length is large enough at the freezing transition that the E_θ term overcomes thermal fluctuations in the orientation of a correlated region of the lattice; i.e., $\epsilon_0 N_c > T_M$, where T_M is the melting temperature of the crystal. Experimentally, one could probe this effect by measuring the depinning electric field of the lattice, which has been shown to be sensitive to orientational correlations of the crystal;[20] one expects to see a decrease in the pinning field if the orientational correlations are increased. The possibility of creating a WC with long-range order using parallel magnetic fields in a double well system is currently under investigation.[25]

It is also interesting to consider the effect of the bond-bending term on the collective mode spectrum of the WC. It is well known[26] that in a perpendicular magnetic field, the WC supports a phonon mode dispersing as $\omega(q) \propto q^{3/2}$, and there have been attempts to measure this directly using rf absorption.[6] Since the bond-angle term E_θ represents a restoring force on long-wavelength fluctuations, we expect the phonon energy to change in the present condition. For this purpose, we write down the appropriate form for the energy of a crystal deformation in terms of a dynamical matrix that yields the energy $E = E_0 + E_\theta$ in the long-wavelength limit. The low frequency collective mode frequencies are given by λ_α/ω_c, where λ_α are the eigenvalues of the matrix $\sigma_y D$, σ_y is the Pauli spin matrix, and $D = D^0 + D^\theta$ is the total dynamical matrix.[27] D^0, the dynamical matrix in the absence of a parallel magnetic field, has been shown[15] to be

$$D^0_{ij} = b\frac{q_i q_j}{q} + \sum_{\alpha\beta} A_{ij\alpha\beta} q_\alpha q_\beta,$$

where b and $A_{ij\alpha\beta}$ are constants. The dynamical matrix associated with E_θ is easily obtained,

$$D^\theta_{xx} = \frac{\epsilon_0}{4m} q_y^2, \quad D^\theta_{yy} = \frac{\epsilon_0}{4m} q_x^2,$$

$$D^\theta_{xy} = D^\theta_{xy} = -\frac{\epsilon_0}{4m} q_x q_y.$$

By noticing the similarity between D^θ and the second term in D^0, it is easy to show that under the present condition, the low-lying phone mode still disperses like $\omega(q) = Cq^{3/2}$, but the coefficient C is increased, with a contribution from the dynamic matrix D^θ.

V. CONCLUSION

Working in the Hartree-Fock approximation, we calculated the energy of different Wigner states of the two-dimensional electron gas for a double-quantum-well system in a strong magnetic field. We found the phase diagram for the evolution of the WC states in a strong perpendicular magnetic field when layer separation and hopping are changed. In the absence of interlayer hopping, the ground state at small layer separations is a one-component triangular lattice Wigner state, which possesses interlayer coherence. As the layer separation is increased, the ground state first undergoes a transition to a two-component square lattice Wigner state, and then undergoes another transition at an even larger layer separation to a two-component triangular lattice Wigner state. The range of the layer separations at which the two-component square lattice occurs, as the ground state shrinks, and eventually disappears, as the interlayer hopping is increased. We also studied the in-plane magnetic field induced phase transition in the Wigner state, which has so far only been studied experimentally for $\nu = 1$ quantum Hall state. We calculated the critical value of the in-plane field for the transition. We find that the anisotropy of the Wigner state, i.e., the orientation of the crystal with respect to the direction of the in-plane magnetic field, has a negligible effect on the value of the critical in-plane magnetic field for small filling factors. The effect of this anisotropy on the low-lying phonon en-

ergy is discussed. A possible experimental arrangement for observing the in-plane field enhancement of the orientational order in the crystal in the presence of a weak disorder potential is also discussed.

We recently became aware of a study by Narasimhan and Ho (Ref. 17), which uses a Hartree approximation to examine various possible states of this system. While their approach is conceptually simpler than that used here, it has the disadvantage of leaving out exchange effects. However, for small filling factors, such corrections are likely to be small, and indeed where our work overlaps our results are in quantitative agreement. Effects of parallel magnetic fields are not discussed in Ref. 17.

ACKNOWLEDGMENTS

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