Resonant tunneling in the presence of a two-level fluctuator: Low-frequency noise

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We study current noise in a double-barrier resonant-tunneling structure due to dynamic defects that switch states because of their interaction with a thermal bath. The time fluctuations of the resonant level result in low-frequency noise, the characteristics of which depend on the relative strengths of the electron escape rate, the coupling to the defects, and the defect's switching rate. If the number of defects is sufficiently large, the noise is of the $1/f$ type. It is shown that the temperature dependence of the noise intensity becomes more pronounced as the spacer gets thicker.

I. INTRODUCTION

Since dynamic defects exist even in high quality point contacts,[1-3] it is reasonable to expect that they also appear in double-barrier resonant-tunneling structures (DBRTS). A dynamic defect has internal degrees of freedom and can switch between two (or more) metastable configurations, due to its interaction with a thermal bath. Hence, it is often called an *elementary fluctuator* (EF). If the quasibound state in the well is coupled to an EF, the resonant level ε_0 will fluctuate in time following the *switches* of an EF between its states. The fluctuation of ε_0 will induce inelastic tunneling, as well as *low-frequency noise* (LFN) in the conductance.

In our previous work[4] (from now on referred to as I), we have studied the effect of EF on the transmission probability in a DBRTS. It has been shown that the switching of a two-level EF can produce inelastic tunneling with fine structures in the transparency spectrum, whose shape depends on the properties of the EF. In this paper, we will continue to investigate the LFN.

We will consider the physical system as shown in Fig. 1 of I. In the absence of bias, it can be represented by a one-dimensional model Hamiltonian $H = H_e + H_{\text{EF}} + H_{\text{int}} + H_{\text{EF-ph}} + H_{\text{ph}}$. The electronic Hamiltonian $H_e = \sum_{p,\nu}[\varepsilon_{p,\nu}c^\dagger_{p,\nu}c_{p,\nu} + \{V_{p,\nu}c^\dagger_0 c_{p,\nu} + \text{H.c.}\}] + \varepsilon_0 c^\dagger_0 c_0$ describes the electron tunneling in a DBRTS. Due to the finite matrix elements $V_{p,l}$ and $V_{p,r}$, an electron with momentum p tunnels from the emitter (ν=l) to the collector (ν=r) via the resonant level ε_0 in the well. In a structure biased by the voltage V, one has to replace $\varepsilon_{p,l} \to \varepsilon_{p,l} - \mu eV$, $\varepsilon_{p,r} \to \varepsilon_{p,r} + \alpha eV$, where the values of μ and $\alpha = 1 - \mu$ depend on the asymmetry of the barriers. (For convenience, in the following analysis, we set the zero reference energy at ε_0.) Through the interaction $H_{\text{int}} = \frac{1}{2}\sum_{i=1}^N J_i c^\dagger_0 c_0 \sigma_z^{(i)}$, the resonant level is coupled to N two-level EF's embedded in the surrounding environment. The Hamiltonian for these EF's is simply $H_{\text{EF}} = \frac{1}{2}\sum_{i=1}^N E_i \sigma_z^{(i)}$, where $\sigma_z^{(i)}$ is a Pauli matrix, and E_i is the separation between the two levels of the ith EF, which couples to the resonant level ε_0 with strength J_i. Since the system of EF's interacts with a phonon thermal bath, the switching of the EF's is caused by one-phonon transitions $H_{\text{EF-ph}} = \frac{1}{2}\sum_{i,\mathbf{k},\mathbf{q}} M_{\mathbf{q}}^{(i)} \sigma_x^{(i)} b^\dagger_{\mathbf{k}+\mathbf{q}/2} b_{\mathbf{k}-\mathbf{q}/2}$, where $\sigma_x^{(i)}$ is a Pauli matrix.

Let us first outline the physical picture of the LFN. The transmission probability $T(\varepsilon)$ of a tunneling electron from the emitter with energy ε is very sensitive to the energy difference ε-ε_0. This difference determines the total phase change of the wave function of the tunneling electron, which undergoes multiple reflections in the well. If ε-ε_0 is small, the phase change is close to $2\pi k$, where k=$0, \pm1, \pm2, \ldots$. Since ε_0 depends on the states of EF's, when EF's switch states due to their interaction with a thermal bath, $T(\varepsilon)$ fluctuates in time and so causes a current noise. It is important to notice that because the phonon density of states decreases as ω^2 and hence the number of low-frequency phonon modes is very small, the direct electron-phonon coupling cannot produce significant LFN. On the other hand, the relatively high-frequency phonons can activate EF's, which switch states with relatively low characteristic frequency. Thus, the EF's behave as *transformers*, such that the high-frequency phonon fields can generate effectively the LFN.

Our investigation on the LFN requires the knowledge of tunneling transparency $T(\varepsilon)$, which has been studied in details in I. Starting from $T(\varepsilon)$, in Sec. II, we will specify the procedure of calculating the LFN. The characteristic LFN, due to a single EF, depends on the relative strengths of the escape rate of an electron from the well, the coupling between the EF and the resonant level, as well as the switching rate of the EF. These characteristic features will be analyzed in Sec. III. Section IV will consider a large DBRTS system, which contains many EF's. We will solve this problem with the generating-function approach. After introducing the general formulation, we will study in detail the two cases of thick and thin spacers. Both cases exhibit $1/f$-type noise in the parameter region of experimental interest. However, they can be distinguished from the temperature dependence of their noise intensity. Concluding remarks will be followed in the last Sec. V.

II. GENERAL EXPRESSIONS

According to the Büttiker-Landauer formula, the current I through a DBRTS can be expressed in terms of the inelastic tunneling transparency $T(\varepsilon, \varepsilon')$ of an electron with incoming energy ε and outgoing energy ε',

$$I = \frac{2e}{\hbar} \int d\varepsilon \, d\varepsilon' T(\varepsilon, \varepsilon') Q(\varepsilon, \varepsilon') , \tag{1}$$

where

$$Q(\varepsilon, \varepsilon') = n_F(\varepsilon - \mu eV)[1 - n_F(\varepsilon' + \alpha eV)] \\ -n_F(\varepsilon + \alpha eV)[1 - n_F(\varepsilon' - \mu eV)] , \tag{2}$$

n_F is the Fermi function. We restrict ourselves to the noise of the *differential conductance* $G = \partial I/\partial V$ at high bias, $eV \gg k_B T$, that corresponds to a typical experimental situation. In this case, the dimensionless differential conductance $g = (2e^2/\hbar)G$ can be expressed as

$$g = \mu \langle T \rangle_F, \quad \langle A \rangle_F \equiv - \int \frac{\partial n_F(\varepsilon - \mu eV)}{\partial \varepsilon} A(\varepsilon) \, d\varepsilon . \tag{3}$$

Here, $T(\varepsilon) = \int d\varepsilon' \, T(\varepsilon, \varepsilon')$ is the total tunneling transparency. If the temperature is much less than the typical width of the function $T(\varepsilon)$, we have $g(V) = \mu T(\epsilon_V)$, where $\epsilon_V \equiv \epsilon_F + \mu eV$, ϵ_F is the Fermi level.

In this paper, we will set $\hbar = 1$ so that energy and frequency are equivalent. In the absence of EF's, $T(\varepsilon)$ is time independent. Since EF's produce a nonstationary effective field acting on the electrons in a DBRTS, $T(\varepsilon)$ becomes time dependent. This time dependence is just the source of LFN, which we are interested in. Within the *wide band approximation*,[5] the tunneling transparency can be expressed as

$$T(\varepsilon|t) = \gamma_l \gamma_r \int d\eta \, d\tau G(\tau, \eta|t) e^{i\varepsilon(\tau - \eta)}, \tag{4}$$

in terms of the two-particle Green's function $G(\tau, \eta|t)$ defined by Eq. (6) of I.

To calculate the time averaged $\langle T(\varepsilon|t)\rangle_t$, as demonstrated in I, instead of averaging over t, one can average $T(\varepsilon|t)$ over the random processes in EF's and obtain $\langle T(\varepsilon|t)\rangle_f \equiv \langle T(\varepsilon|t)\rangle_t$. According to (3), the pair correlation function of differential conductance

$$s(\tau) \equiv \langle g(\tau) g(0)\rangle_f - \langle g \rangle_f^2$$

can be expressed through the pair correlation function of transparency,

$$F(\varepsilon, \varepsilon'|\tau) \equiv \langle T(\varepsilon|\tau) T(\varepsilon'|0)\rangle - \langle T(\varepsilon|0)\rangle \langle T(\varepsilon'|0)\rangle , \tag{5}$$

where $\langle\,\rangle$ means the average over the random process in EF's. We obtain

$$s(\tau) = \mu^2 \int d\varepsilon d\varepsilon' \frac{\partial n_F}{\partial \varepsilon} \frac{\partial n_F}{\partial \varepsilon'} F(\varepsilon_V, \varepsilon'_V|\tau) . \tag{6}$$

If the temperature is much less than all the energy scales in our problem, then $s(\tau) = \mu^2 F(\varepsilon_V, \varepsilon_V|\tau)$. As a result, the noise in differential conductance is determined by the correlation function F.

According to the definition (4), we have

$$F(\varepsilon, \varepsilon'|\tau) = \gamma_l^2 \gamma_r^2 \int d\zeta_1 d\zeta_2 d\eta_1 d\eta_2 \, e^{i\varphi(\varepsilon, \varepsilon'|\zeta_1 - \zeta_2, \eta_1 - \eta_2)} \\ \times \mathcal{M}(\xi_1, \xi_2, \eta_1, \eta_2|\tau) , \tag{7}$$

where $\varphi(\varepsilon, \varepsilon'|\zeta, \eta) = \varepsilon\zeta + \varepsilon'\eta$, and

$$\mathcal{M}(\xi_1, \xi_2, \eta_1, \eta_2|\tau) = \langle G(\zeta_1, \zeta_2|0) G(\eta_1, \eta_2|\tau)\rangle_f \\ - \langle G(\zeta_1, \zeta_2)\rangle_f \langle G(\eta_1, \eta_2)\rangle_f \tag{8}$$

is the four-time correlation function. We should mention that the above averaged Green's functions are time independent, since we need the Fourier transform $F(\varepsilon, \varepsilon'|\omega)$ of $F(\varepsilon, \varepsilon'|\tau)$, with respect to the variable τ.

We will calculate the correlation function with an *adiabatic approximation*, which takes into account the influence of the fluctuations on the phase of the wave function, but neglects the interlevel transition caused by the switching of EF's between their states (see I and the references therein). According to this approximation, EF's are treated as a source of random fluctuations of the levels' positions, the correlation properties of the fluctuations being determined by the EF-phonon interaction $H_{\text{EF-ph}}$. The same approximation was used in I to calculate the average transparency. The prescription for the adiabatic approximation is first to replace ε_0 by $\varepsilon_0 + \epsilon(t)$, where $\epsilon(t)$ is the random fluctuation of the level's position, and then using (4) to calculate the product $T(\varepsilon|t)T(\varepsilon'|0)$. The so-obtained quantity should be averaged over the random process $\epsilon(t)$, and the procedure is denoted as $\langle\,\rangle_f$.

After specifying the procedure for calculating the correlation functions, we will investigate the LFN under different situations. If the spatial dimensions of a DBRTS is small, there are few EF's in the vicinity of the DBRTS and so the noise is an overlap of few random telegraph processes. Consequently, the noise spectrum is a superposition of few Lorentzian tails. For a relatively large DBRTS surrounded by many EF's, the noise spectrum is governed by the overlap of contributions of different defects, and is strongly dependent on both their spatial distribution and the distribution of their switching rates. As will be shown later, we then arrive at a spectrum of $1/f$ type.

III. LFN DUE TO A SINGLE EF

If the DBRTS is coupled to only one EF, one can put $\epsilon(t) = J\xi(t)$, where $\xi(t)$ represents the random switching process of the EF. The procedure of averaging $\langle\rangle_f$ is simply the average $\langle\rangle_\xi$ over the random process $\xi(t)$ according to the statistics determined by the interaction $H_{\text{EF-ph}}$. In terms of the functions

$$\Xi(\zeta_1,\zeta_2,\eta_1,\eta_2,\tau) \equiv \left\langle e^{iJ\int_0^\infty dt\,\beta(\zeta_1-\zeta_2,\eta_1-\eta_2,\tau,t)\xi(t)}\right\rangle_\xi ,$$
$$\Phi(\zeta_1,\zeta_2) \equiv \left\langle e^{iJ\int_0^\infty dt\,\vartheta(\zeta_1-\zeta_2,t)\xi(t)}\right\rangle_\xi , \quad (9)$$

where

$$\vartheta(x,t) \equiv \Theta(t-x) - \Theta(t) ,$$
$$\beta(x,y,\tau,t) \equiv \vartheta(x,t) + \vartheta(y,t-\tau) ,$$
$$x \equiv \zeta_1 - \zeta_2,\ y \equiv \eta_1 - \eta_2 . \quad (10)$$

The four-time correlation function $\mathcal{M}$ can be expressed as

$$\mathcal{M}(\xi_1,\xi_2,\eta_1,\eta_2|\tau) = e^{-\gamma(\xi_1+\eta_1)}\left[\Xi(\zeta_1,\zeta_2,\eta_1,\eta_2,\tau) - \Phi(\zeta_1,\zeta_2)\Phi(\eta_1,\eta_2)\right] , \quad (11)$$

where $\gamma = (\gamma_l + \gamma_r)/2$. The function Ξ describes the average of the product of the Green's functions over the random processes in EF's, while Φ describes the average of a single Green's function. The appearance of the modulation functions β and ϑ in the expressions for Ξ and Φ is due to the fact that the creation and the annihilation operators $c_0^\dagger$ and c_0 have the time-reversed behavior $c_0 \propto \exp(-i\int\varepsilon(t)dt)$ and $c_0^\dagger \propto \exp(i\int\varepsilon(t)dt)$. Consequently, one should take into account the phase fluctuations for both direct and time-reversed waves. These functions have the form of a set of rectangular pulses, which can overlap with each other. The existence of such phase fluctuations in Ξ and Φ produces in (7) the change of the resonant-tunneling spectra for the averaged product of the Green's functions and for the averaged Green's function, respectively.

There are three quantities with the dimension of frequency: the escape rate 2γ, the shift J of the energy level, and the characteristic EF's switching rate Γ. Γ depends on the interaction strength between the EF and the phonon field, the energy separation of the two EF states, and the temperature. The explicit expression of Γ is given in I. With decreasing temperature, Γ approaches zero. Consequently, if the barriers are thin enough and the temperature is sufficiently low, we have

$$\Gamma \ll \gamma,\ J . \quad (12)$$

This case is referred to as *very slow fluctuator* (VSF). At higher temperatures, we may have either the case of *slow fluctuator* (SF) with

$$\gamma \ll \Gamma \ll J , \quad (13)$$

or the case of *fast fluctuator* (FF) with

$$\gamma \ll J \ll \Gamma . \quad (14)$$

The interplay between the conditions (12), (13), and (14) depends on the temperature, the barrier thickness, and the distance between the EF and the well that determines the interaction strength J. Here, we have used the same classification of limiting cases in I. In the following, we will discuss the VSF case and the general situation that covers both the SF case and the FF case.

In order to calculate the LFN quantitatively, we need to specify the random process $\xi(t)$. We assume that the EF switches randomly in time between the two states with $\xi = \pm 1$, with switching rate Γ_u from the upper state and Γ_d from the lower state. Such random process is often called the *kangaroo* process.[6] For the symmetric case $\Gamma_u = \Gamma_d$, it is called the *random telegraph* process. In the following, we will use the *kangaroo* process for the VSF, and the *random telegraph* process for both the SF and the FF.

A. Very slow fluctuator

If the EF is frozen in one state, the transparency $T(\varepsilon)$ is time independent and the noise vanishes. Since the time $1/\Gamma$ between two switches is much longer than the time $1/\gamma$ required to form the resonant state in the well, each switch of the EF leads to a rigid shift of the *entire* transparency spectrum by an amount of energy J. Consequently, the noise of $T(\varepsilon|t)$ has a *random telegraph* character. This feature is very pronounced if $J \gg \gamma$, because the energy shift of the spectrum is much larger than the width of the spectrum.

The quantitative results for a VSF can be obtained rather simple. Indeed, an EF resides in its states during the time much longer than γ^{-1}, the latter being the typical time to form a resonant-tunneling wave function. As a result, one can assume that the transparency is $T^0(\varepsilon+J)$, during the period when $\xi(t) = +1$, and $T^0(\varepsilon - J)$ when $\xi(t) = -1$. Here,

$$T^0(\varepsilon) = \frac{\gamma_l\gamma_r}{\varepsilon^2+\gamma^2} \quad (15)$$

is the unperturbed transparency. Giving a single-event probability $P_1(\xi)$ and a double-event joint probability $P_2(\xi,\xi_\tau|\tau)$ for the random quantities $\xi \equiv \xi(t)$ and $\xi_\tau \equiv \xi(t+\tau)$, we readily obtain from (5)

$$F(\varepsilon,\varepsilon'|\tau) = \int d\xi\, d\xi_\tau\, \mathcal{K}(\xi,\xi_\tau|\tau) \times T^0(\varepsilon - J\xi)T^0(\varepsilon' - J\xi_\tau) , \quad (16)$$

where

$$\mathcal{K}(\xi,\xi_\tau|\tau) = P_2(\xi,\xi_\tau|\tau) - P_1(\xi)P_1(\xi_\tau) . \quad (17)$$

Thus, the problem of LFN reduces to the calculation of $P_1(\xi)$ and $P_2(\xi,\xi_\tau|\tau)$ for a given random process $\xi(t)$. The procedure outlined above can be justified rigorously from the general analysis basic steps that are outlined later.

To go further, we will assume the *kangaroo* process for the random switching $\xi(t)$. If we define p_ξ as the stationary occupation of the state $\xi(t)$, then $p_{+1} = \Gamma_d/\Gamma$

and $p_{-1} = \Gamma_u/\Gamma$, where $\Gamma = \Gamma_u + \Gamma_d$. In terms of p_ξ, $\mathcal{K}(\xi, \xi_\tau|\tau)$ can be expressed as[7] (see also Appendix A in I)

$$\mathcal{K}(\xi, \xi_\tau|\tau) = [p_\xi \delta(\xi - \xi_\tau) - p_\xi p_{\xi_\tau}]\, e^{-\Gamma|\tau|} . \tag{18}$$

Combining Eqs. (6), (15), (16), and (18), we obtain the noise intensity

$$s(\omega) = \mu^2 \mathcal{L}(\omega|\Gamma) \frac{\Gamma_u \Gamma_d}{\Gamma^2} N(V,T) , \tag{19}$$

where

$$\mathcal{L}(\omega|\Gamma) = \frac{1}{\pi} \frac{\Gamma}{\omega^2 + \Gamma^2} , \tag{20}$$

$$N(V,T) = \left\langle \frac{4\gamma_l \gamma_r \gamma \varepsilon}{(\varepsilon^2 + J^2 + \gamma^2)^2 - 4J^2\varepsilon^2} \right\rangle_F^2 . \tag{21}$$

Consequently, the noise has a Lorentzian spectrum. The envelope function $N(V,T)$ can be expressed in analytical form

$$N(V,T) = \left[\frac{4\gamma_l \gamma_r \gamma \varepsilon_V}{[(\varepsilon_V)^2 + J^2 + \gamma^2]^2 - 4J^2\varepsilon_F^2} \right]^2 , \tag{22}$$

for very low temperatures $k_B T \ll \gamma$, and

$$N(V,T) = \frac{1}{(4k_B T)^2} \frac{\sinh^2(\varepsilon_V/k_B T)\sinh^2(J/k_B T)}{\cosh^4\left[\frac{\varepsilon_V + J}{2k_B T}\right] \cosh^4\left[\frac{\varepsilon_V - J}{2k_B T}\right]} , \tag{23}$$

for high temperature $k_B T \gg \gamma$. One should remind the reader that our zero reference energy is set at the resonant level ε_0. It is then clear that, contrary to the average transparency which has been calculated in I, the noise spectrum remains *symmetric* in ε_F even if the switching rates Γ_u and Γ_d from the upper and the lower state are different. This is illustrated in Fig. 1 where the zero temperature envelope function $N(\varepsilon_V, 0)$ as well as the average transparency spectrum are plotted for both symmetric and asymmetric cases.

B. General case

Next we briefly discuss the general situation that covers the limiting cases of SF and FF. Since it is much more complicated than the VSF case, we will restrict ourselves to the *random telegraph* process when the occupation probabilities of the upper and the lower states are equal. Although this simplified model is valid at high enough temperatures, it nevertheless reproduces the main results with relatively simple formulas.

In general case, one cannot neglect the time dependence of the random quantity $\xi(t)$, while calculating the integrals over time in Eq. (9). However, the functions Ξ and Φ (9) have the form $\Psi(\infty)$, where

$$\Psi(t) = \left\langle e^{iJ\int_0^t w(t')\xi(t')\,dt'} \right\rangle_\xi ,$$

with $w(t') = \beta(\ldots, t')$ or $\theta(\ldots, t')$. The procedure to evaluate such functions has been discussed in detail in I. As in I, we analyze the stochastic differential equation for $\Psi(t)$ equivalent to Eq. (24) from I, with the same boundary conditions. This procedure is also carried out by the transfer matrix method. The straightforward but rather tedious calculations, which we do not present here for brevity, allow us to find the correlation function $F(\varepsilon, \varepsilon'|\tau)$ for an arbitrary relationship between the quantities γ, J, and Γ. If the inequalities (12) hold, one arrives at the results of the previous section. Outside this region, we have analyzed the correlation function $F(\varepsilon, \varepsilon'|\tau)$ at $\tau = 0$ and $|\tau| \to \infty$ (the first quantity is proportional to the integrated over the frequency noise intensity). It is shown that at $\gamma \ll J, \Gamma$,

$$F(\varepsilon, \varepsilon'|\tau) \propto \exp(-|\tau|/\tau_0),$$

where $\tau_0 = \max(\Gamma^{-1}, \Gamma/J^2)$. This behavior is easy to understand, having in mind the analysis of the average transparency in I.

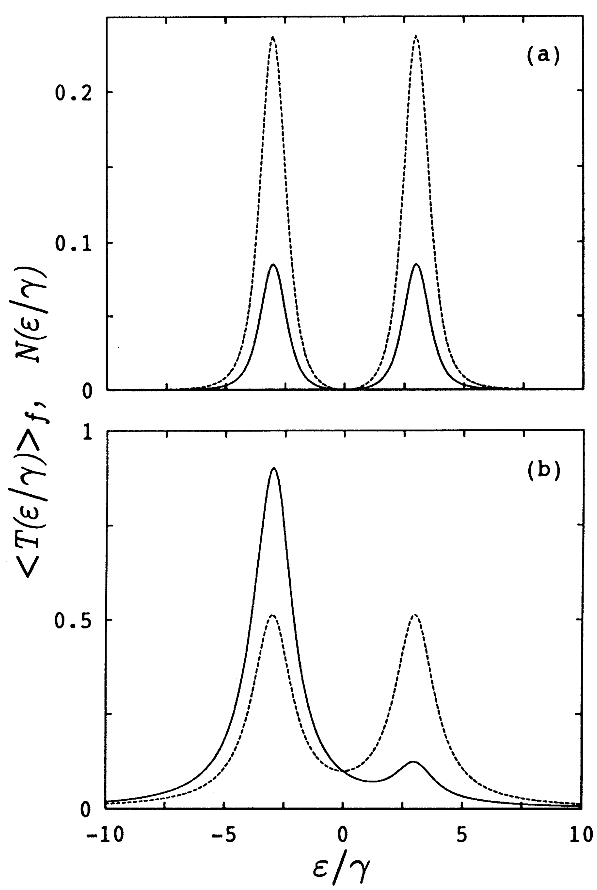

FIG. 1. Average transparency $\langle T(\varepsilon/\gamma)\rangle_f$ (solid curves) and the zero temperature envelope function $N(\varepsilon/\gamma, T = 0)$ (dashed curves), as functions of dimensionless energy ε/γ for $J = 3\gamma$. Panel (a) is for the symmetric case $P_u/\Gamma = P_d/\Gamma = 0.5$, and panel (b) is for the asymmetric case $P_u/\Gamma = 0.9$ and $P_d/\Gamma = 0.1$. The zero reference energy for ε is set at the resonant level ε_0.

To simplify the analysis of the integrated intensity of LFN given by $F(\varepsilon,\varepsilon'|0)$, we assume the condition

$$\gamma \ll \min(\Gamma, J^2/\Gamma) . \tag{24}$$

With the key steps of calculations outlined above, up to the leading term γ/Γ, the final result of the integrated spectrum of noise is obtained as

$$F(\varepsilon,\varepsilon'|0) = \langle T(\varepsilon)\rangle_f \left[\frac{\gamma_r\gamma_l}{(\varepsilon-\varepsilon')^2+4\gamma^2} - \langle T(\varepsilon')\rangle_f \right] . \tag{25}$$

The average transparency $\langle T(\varepsilon)\rangle_f$ is analyzed in detail in I, its width being $\min(\Gamma, J^2/\Gamma)$ under the conditions (24). At very low temperatures, $k_BT \ll \gamma$, the integral noise in differential conductance is determined by $\varepsilon=\varepsilon'=\varepsilon_V$,

$$F(\varepsilon_V,\varepsilon_V|0) = \langle T(\varepsilon)\rangle_f \left[\frac{\gamma_r\gamma_l}{(\gamma_l+\gamma_r)^2} - \langle T(\varepsilon_V)\rangle_f \right] . \tag{26}$$

At $k_BT \gg \gamma$, the LFN noise becomes very small, because the integral of $F(\varepsilon,\varepsilon'|0)$ over ε or ε' vanishes. In this case, the correlation should be analyzed including terms of higher order in γ/Γ.

IV. THE CASE OF MANY FLUCTUATORS: $1/f$ NOISE

For a large enough double-barrier resonant-tunneling structure, there will be many EF's, each of them will couple to the resonant level independently. Therefore, all random processes $\xi^{(i)}(t)$ are independent of each other. In this paper, we are interested in the noise in DBRTS with not very thick barriers. For such DBRTS, we have $\Gamma_i \ll \gamma$, and so we are dealing with the case of VSF's.

To study the correlation functions (7) and (8), we need to calculate the one-event probability and the two-event joint probability similar to those in (16) and (17). To perform such calculation it is convenient to introduce the generating function,

$$K_N(x,y|\tau) = \prod_{i=1}^{N} \left\langle e^{-ix\xi^{(i)}-iy\xi_\tau^{(i)}} \right\rangle_f , \tag{27}$$

for a system of N EF's, where $\xi^{(i)} \equiv \xi^{(i)}(t)$ and $\xi_\tau^{(i)} \equiv \xi^{(i)}(t+\tau)$. The one-event probability $P_1^{(N)}(\epsilon)$ and the two-event joint probability $P_2^{(N)}(\epsilon,\epsilon_\tau|\tau)$ are then derived as the Fourier transforms of $K_N(x,y|\tau)$, with respect to x and to (x,y), respectively. However, for N EF's with random coupling strength $J^{(i)}$, the suitable generating function to work with is

$$Q_N(x,y|\tau) = \prod_{i=1}^{N} \left\langle e^{-ix\epsilon^{(i)}-iy\epsilon_\tau^{(i)}} \right\rangle_f , \tag{28}$$

where $\epsilon^{(i)} = J^{(i)}\xi^{(i)}$ and $\epsilon_\tau^{(i)} = J^{(i)}\xi_\tau^{(i)}$ are the electron energy shifts. We will first analyze the general properties of $Q_N(x,y|\tau)$, and then apply the results to important limiting cases.

A. General formulation

Let us outline the characteristic features of an EF, which depend on the equilibrium probabilities p_{+1} and p_{-1} to find the EF in its upper and lower states, the switching rate $\Gamma=\Gamma_u+\Gamma_d$, and the interaction strength J between the EF and the thermal bath. In general, $p_{\pm 1}$ depend on the energy separation E between the upper and the lower level as

$$p_{\pm 1} = \frac{e^{\mp E/2k_BT}}{2\cosh(E/2k_BT)} .$$

Since both the density of states of the thermal bath and J are energy dependent, the switching rate Γ is also a function of E. Finally, J also depends on the distance r between the tunneling system and the EF.

For $N=1$, one can solve (27) directly to obtain

$$Q_\Gamma(x,y|\tau) = \cos J(x+y) + i[p_{+1}-p_{-1}]\sin J(x+y) + 4p_{+1}p_{-1}\left(1-e^{-\Gamma|\tau|}\right)\sin Jx \sin Jy , \tag{29}$$

and then proceed to derive the results in Sec. III A. For $N>1$, one has to calculate the product of independent generating functions for different EF's. In the limit $N \gg 1$, this product can be approximately calculated with the help of the Holtsmark method,[8] which is commonly used in the theory of optical spectra. This method starts with the identity $\prod_{i=1}^N a_i \equiv \exp(\sum_i \ln a_i)$. Then the sum $\sum_i \ln a_i$ is replaced by $N\langle \ln a\rangle_a$, where $\langle\,\rangle_a$ is the average over the distribution of a_i. In the limit $N \gg 1$, the average $\langle \ln a\rangle_a$ can be well approximated as $\langle a\rangle - 1$. Therefore, for the quantity $Q_N(x,y|\tau)$, we obtain[9]

$$Q_N(x,y|\tau) = \exp\left[-N\langle 1-Q_\Gamma(x,y|\tau)\rangle_{\rm EF}\right] , \tag{30}$$

where $\langle\,\rangle_{\rm EF}$ represents the average over the three parameters E, Γ, and J of the EF's. To go further from (30), one needs to specify the distribution function $\mathcal{P}(E,\Gamma,J)$.

1. *Distribution function*

Since the switching of an EF is caused by the thermal bath, only the EF's with $E \lesssim k_BT$ are active. Consequently, at low enough temperature, the distribution of E should be a smooth function. As discussed in I, this conclusion is valid for the EF's produced by either atomic or electronic disorder. To simplify our calculation, we will use a constant distribution of E with the value P_0, the value of which will be determined later.

However, for a given E, the switching rate $\Gamma(E)$ has a wide range of distribution. To clarify this point, let us consider an EF as a two-level tunneling system, such as a particle (or a group of particles) moving in an effective two-well potential. If in the absence of tunneling the difference between the two levels is Δ and the tunneling coupling between the states is Λ, then the interlevel separation is

$$E = \sqrt{\Delta^2 + \Lambda^2}\,.$$

In the same system, the matrix element for the interwell transition caused by the interaction with thermal bath is proportional to Λ/E. Hence, the transition rate $\Gamma(E)$ is proportional to $(\Lambda/E)^2$. Since the tunneling splitting Λ is an exponential function of the barrier's parameters, the rate $\Gamma(E)$ is distributed in an exponentially wide region, even for a smooth distribution of the barrier's parameters. To model this physical picture, in I we have introduced the energy-dependent rate,

$$\Gamma(E) = \Gamma q(E/k_BT)\,, \quad q(z) = z^\rho \coth\frac{z}{2}\,. \tag{31}$$

The prefactor Γ is a temperature-dependent random quantity distributed between some minimal rate Γ_l and the maximal rate Γ_0. The rate Γ_0 corresponds to *symmetric* EF's, with $\Delta = 0$. Since Γ is distributed in an exponentially wide region, the ratio $\Gamma_0/\Gamma_l \gg 1$. To allow for this exponentially wide distribution, here we assume $\mathcal{P}(E,\Gamma,J) \propto 1/\Gamma$, and hence the integral $\int_{\Gamma_l}^{\Gamma_0} \mathcal{P}(E,\Gamma,J)\,d\Gamma$ is proportional to $L \equiv \ln(\Gamma_0/\Gamma_l) \gg 1$. Both Γ_0 and the exponent ρ in (31) vary with the environment to which the EF's are coupled. For the problem considered in this paper, the EF's are coupled to phonons and so ρ=3 and $\Gamma_0 = (k_BT)^3/E_c^2$, where the characteristic energy E_c is a function of the EF-phonon coupling constant $M_{\mathbf{q}}$ appeared in $H_{\text{EF-ph}}$.[10] It is worthwhile to mention that in systems where the EF-electron coupling dominates, ρ=1 and $\Gamma_0 = \chi k_BT$, with χ being the dimensionless EF-electron coupling constant.

The above estimates are for low temperature, such that the EF's move in the two-well potential by quantum tunneling. At higher temperatures thermal activation becomes increasingly important, and the relaxation rate has a more complicated form.[10] However, the exponential dependence of $\Gamma(E)$ on the barrier's parameters remains valid, and the model (31) leads to qualitatively correct results.

To derive the distribution of J, we assume a uniform spatial distribution of the EF's. At the resonant level separated from an EF by a distance r, the EF produces a dipole elastic or electric field. Since the dipole field also depends on the orientation $\mathbf{n}$ of the dipole moment, the coupling strength can be expressed as $a(\mathbf{n})J$, where $J = A/r^3$ and $\langle|a(\mathbf{n})|\rangle_{\mathbf{n}} = 1$. Consequently, $\mathcal{P}(E,\Gamma,J) \propto 4\pi r^2|\frac{dr}{dJ}| \propto A/J^2$. The minimal value of the coupling strength is $J_l = A/r_{\max}^3 = 4\pi A/3\mathcal{V}$, where $\mathcal{V}$ is the volume of the system. The maximal value J_0 of the coupling strength depends on the geometry of the DBRTS, and will be discussed later.

Summarizing the above analysis, our model distribution function $\mathcal{P}(E,\Gamma,J)$ is then proportional to $P_0\Gamma^{-1}J^{-2}$, where P_0 is an intrinsic property of the system and is temperature independent. Because the value of the constant P_0 is so far undetermined, the proportional constant can be absorbed in P_0, which will be fixed from the normalization condition. We should point out that at a given temperature T, only those EF's with $E \lesssim k_BT$ are dynamically active. Hence the distribution function should be normalized with respect to the temperature-dependent effective number of EF's, $N_{\text{eff}}(T)$, which was defined in I as

$$N_{\text{eff}}(T) \equiv \int_0^\infty dE \int_{J_l}^{J_0} dJ \int_{\Gamma_l}^{\Gamma_0} d\Gamma\, \frac{\mathcal{P}(E,\Gamma,J)}{2\cosh^2(E/2k_BT)} = \left(\frac{3\mathcal{V}}{4\pi A} - \frac{1}{J_0}\right) P_0 L k_B T\,. \tag{32}$$

For the convenience of our future analysis, we will write the model distribution function in the form

$$\mathcal{P}(E,\Gamma,J) = \nu_d \frac{1}{k_BT}\frac{1}{\Gamma}\frac{1}{J^2}\,, \tag{33}$$

where the normalization constant ν_d is determined from Eq. (32). For certain systems to be investigated later, we have $J_0 \to \infty$. In this case, if we define $\bar{r} = [3L\mathcal{V}/4\pi N_{\text{eff}}(T)]^{1/3}$ as the average distance between the EF's, then we have

$$\nu_d = \frac{A}{\bar{r}^3}\,, \tag{34}$$

which measures the coupling strength at the mean distance between the EF's. We emphasize once again that $\nu_d \propto N_{\text{eff}}(T) \propto T$, but the quantity P_0 is temperature independent. It is also important to notice that the results of our calculation should be averaged over the direction of the dipole moment $\langle\,\rangle_{\mathbf{n}}$.

2. Generating function

Now we are ready to analyze the generating function $Q_N(x,y|\tau)$ defined by (30), which has both the real and the imaginary parts. The effect of the imaginary part is to produce a temperature-dependent shift of the resonant level ε_0.[11] Let us define $u_\pm = x \pm y$ to rewrite the real part as

$$-\ln Q_N(x,y|\tau) = (1-\psi_\tau)f(u_+) + \psi_\tau f(u_-)\,, \tag{35}$$

where

$$\psi_\tau = \frac{1}{2L}\int_0^\infty \frac{dz}{\cosh^2\frac{z}{2}} \int_{\Gamma_l}^{\Gamma_0} \frac{d\Gamma}{\Gamma}\left[\frac{1-e^{-\Gamma q(z)|\tau|}}{2}\right] \tag{36}$$

and

$$f(u_\pm) = 2\nu_d \left\langle \int_{J_l}^{J_0} \frac{dJ}{J^2}\{1-\cos[a(\mathbf{n})Ju_\pm]\} \right\rangle_{\mathbf{n}}\,. \tag{37}$$

Using the definition of $q(z)$ given by (31), the integrations in (36) can be performed analytically in the limiting cases to yield

$$\psi_\tau = \begin{cases} (c/L)\Gamma_0|\tau|\,, & |\tau| \ll \Gamma_0^{-1}\,, \\ (1/2L)\ln(\Gamma_0|\tau|)\,, & \Gamma_l^{-1} \gg |\tau| \gg \Gamma_0^{-1}\,, \\ 1/2\,, & |\tau| \gg \Gamma_l^{-1}\,, \end{cases} \tag{38}$$

where $c \approx 12$, if $\rho = 3$ and $c \approx 2.5$ at $\rho = 1$. It is clear that ψ_τ depends on two-dimensionless parameters $\Gamma_0|\tau|$ and $\Gamma_l|\tau|$. In a very wide range

$$\Gamma_0^{-1} \ll |\tau| \ll \Gamma_l^{-1},$$

the τ dependence of ψ_τ is *logarithmic*. Because of such dependence, we will show later that in the frequency interval of experimental interest, the noise spectrum is close to $1/\omega$. This conclusion holds regardless of the sample geometry of the DBRTS discussed below.

The fact that an exponentially wide distribution of relaxation rates produce $1/f$ noise in conductance is well known.[12] The specific feature of the problem under discussion is that the distribution of J is also important—depending on this distribution, different dependences of the noise vs bias can be observed.

To calculate the function $f(u_\pm)$, we need to specify the upper limit J_0 of the integral in (37), which depends on the geometry of the DBRTS. The EF's are located in the doped regions, each of which is separated from a barrier by a spacer. The width of the spacer d is much thicker than the barrier width d_b. Hence, we can set

$$J_0 = A/(d + d_b)^3 \simeq A/d^3 . \tag{39}$$

Another relevant physical system to which our theoretical model also applies is a 2D electron gas, with two sets of split gates. The resulting potential for the tunneling electrons is of the double-barrier type. Between the 2D electron gas and the underlying doped region, there is a layer of undoped region that acts as a spacer. If the thickness of the undoped layer is d, then we simply have $J_0 = A/d^3$. In the following, we will study the DBRTS with either thin spacer or thick spacer.

B. DBRTS with thick spacers

Since J_0 is very small for sufficiently large d, we can expand Eq. (37) in powers of $J_0 u_\pm$. Taking into account that $J_0 \gg J_l$, we obtain

$$f(u_\pm) \simeq \nu_d J_0 \langle a(\mathbf{n})^2\rangle_{\mathbf{n}} u_\pm^2 . \tag{40}$$

Consequently, we have

$$-\ln Q_N(x,y|\tau) = \frac{1}{2}\sigma_+^2(\tau)u_+^2 + \frac{1}{2}\sigma_-^2(\tau)u_-^2 , \tag{41}$$

where

$$\sigma_\pm(\tau) = \sqrt{\nu_d J_0 \langle a^2(\mathbf{n})\rangle_{\mathbf{n}}\,[1 \pm (1 - 2\psi_\tau)]} .$$

We can rewrite the product $\nu_d J_0$ as $J_0^2 N_d$, where $N_d \equiv (d/\bar{r})^3$ measures the effective number of EF's, which broaden the tunneling spectrum. Therefore, the width of the resonant-tunneling spectrum is of the order $J_0\sqrt{N_d}$.

Knowing the generating function $Q_N(x,y|\tau)$, by taking the Fourier transforms with respect to x and to (x,y), we obtain the probabilities (cf. with Ref. 9),

$$P_1^{(N)}(\epsilon) = \mathcal{G}(\epsilon|\sigma_0) , \tag{42}$$

$$P_2^{(N)}(\epsilon,\epsilon_\tau|\tau) = \mathcal{G}(\epsilon_+|\sigma_+(\tau))\mathcal{G}(2\epsilon_-|2\sigma_-(\tau)) , \tag{43}$$

where $\epsilon_\pm \equiv \frac{1}{2}(\epsilon \pm \epsilon_\tau)$, $\sigma_0 = \sigma_+(0)$, $\sigma_\infty \equiv \sigma_\pm(\infty) = \sigma_0/\sqrt{2}$, and

$$\mathcal{G}(\epsilon|\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\epsilon^2/2\sigma^2)$$

is a Gaussian distribution of width σ. The limiting cases can be checked easily by using the asymptotic properties of ψ_τ giving in (38). For large $|\tau|$ such that $\Gamma_l|\tau| \gg 1$, $P_2^{(N)}(\epsilon,\epsilon_\tau) = P_1^{(N)}(\epsilon)P_1^{(N)}(\epsilon_\tau)$ and so the correlation vanishes. At the other limit $\Gamma_0|\tau| \ll 1$, we get $\psi_\tau \to 0$ and $P_2^{(N)}(\epsilon,\epsilon_\tau|0) = \delta(\epsilon - \epsilon_\tau)P_1^{(N)}(\epsilon)$ as expected. Between these two limits, in the wide range $\Gamma_l^{-1} \gg |\tau| \gg \Gamma_0^{-1}$, ψ_τ is much less than 1. Then, the joint probability $P_2^{(N)}(\epsilon,\epsilon_\tau|\tau)$ can be expressed as the product of the one-event probability $P_1^{(N)}(\epsilon)$ and the conditional probability

$$W(\epsilon_\tau,\epsilon|\tau) = \mathcal{G}(2\epsilon_-|2\sigma_-(\tau)) . \tag{44}$$

For $|\tau| \ll 1$, the τ dependence of the above conditional probability has a simple qualitative interpretation. Under this condition, since $\sigma_-^2 \propto \Gamma_0|\tau|$, we arrive at the diffusion law for the level's position variation, with the diffusion coefficient $J_0^2 N_d \Gamma_0$.

Now we will calculate the noise. The correlation function (7) can be expressed in a similar form as (16)

$$F(\varepsilon,\varepsilon'|\tau) = \int d\epsilon\, d\epsilon_\tau\, \mathcal{K}(\epsilon,\epsilon_\tau|\tau) \times T^0(\varepsilon - \epsilon)T^0(\varepsilon' - \epsilon_\tau) , \tag{45}$$

with

$$\mathcal{K}(\epsilon,\epsilon_\tau|\tau) = \mathcal{G}(\epsilon_+|\sigma_+(\tau))\,\mathcal{G}(2\epsilon_-|2\sigma_-(\tau)) - \mathcal{G}(\epsilon|\sigma_0)\mathcal{G}(\epsilon_\tau|\sigma_0) . \tag{46}$$

For sufficiently large N_d, that is, for $d \gg \bar{r}$ such that $\gamma \ll \sigma_-$, the transparency $T^0(\epsilon)$ in the integrand of (45) is a sharp function. Consequently, we have

$$F(\varepsilon,\varepsilon'|\tau) = \mathcal{K}(\varepsilon,\varepsilon'|\tau)T_{\rm tot}^2 , \tag{47}$$

where $T_{\rm tot} \equiv \int d\epsilon T^0(\epsilon)$. At low temperature we can set $\varepsilon = \varepsilon' = \varepsilon_V$, and then the pair correlation function is expressed as

$$F(\varepsilon_V,\varepsilon_V|\tau) \simeq T_{\rm tot}^2\left[\frac{\mathcal{G}(\varepsilon_V|\sigma_+(\tau))}{2\sigma_-(\tau)\sqrt{2\pi}} - \mathcal{G}^2(\varepsilon_V|\sigma_0)\right] . \tag{48}$$

Since $\mathcal{G}(\epsilon_V|\sigma) = \frac{1}{\sqrt{2\pi}\sigma}$ at $\varepsilon_V = \varepsilon_0 = 0$, substituting (48) into (6), we obtain

$$s(\tau) = \frac{T_{\rm tot}^2}{4\pi\sigma_\infty^2} f_G(\tau) , \tag{49}$$

where $f_G(\tau)$ is the dimensionless function,

$$f_G(\tau) = \frac{1}{2\sqrt{\psi_\tau(1-\psi_\tau)}} - 1 . \tag{50}$$

Using the expressions in (38) and taking the Fourier transform $f_G(\omega)$ of $f_G(\tau)$, the noise spectrum for $\varepsilon_F = \varepsilon_0 = 0$ is then calculated. The function $f_G(\omega)$ is shown in Fig. 2 as the solid curve for $\Gamma_0/\Gamma_l = 10^9$. In the region $\Gamma_l \ll \omega \ll \Gamma_0$, $f_G(\omega) \propto \omega^{-0.76}$.

C. DBRTS with thin spacers

When d is small, the product $Ju_\pm$ in (37) can no longer be treated as a small parameter for expansion in the range between J_l and J_0. Instead, the upper limit of the integration J_0 can be well approximated by infinity. Then we obtain the simple result $f(u_\pm) = \pi\nu_d \langle a(\mathbf{n})\rangle_{\mathbf{n}} |u_\pm| = \pi\nu_d|u_\pm|$ and

$$-\ln Q_N(x,y|\tau) = v_+(\tau)|x+y| + v_-(\tau)|x-y|\,, \quad (51)$$

where

$$v_\pm(\tau) = \frac{\pi}{2}\nu_d\,[1 \pm (1-2\psi_\tau)] \quad (52)$$

and $v_\infty \equiv v_\pm(\infty) = \pi\nu_d/2$. By taking proper Fourier transforms of (51), the one-event probability and two-event joint probability are derived as

$$P_1^{(N)}(\epsilon) = \mathcal{L}\left(\epsilon|2v_\infty\right) = \mathcal{L}\left(\epsilon|\pi\nu_d\right)\,, \quad (53)$$

$$P_2^{(N)}(\epsilon,\epsilon_\tau|\tau) = \mathcal{L}(\epsilon_+|v_+(\tau))\mathcal{L}(2\epsilon_-|2v_-(\tau))\,, \quad (54)$$

where $\mathcal{L}(\epsilon|v)$ is a Lorentzian of the variable ϵ and the width v, as defined by (20). Again, in the interval $\Gamma_0^{-1} \ll |\tau| \ll \Gamma_l^{-1}$, we can express the joint probability as the product of $P_1^{(N)}(\epsilon)$ and the conditional probability,

$$W(\epsilon_\tau,\epsilon|\tau) = \mathcal{L}(2\epsilon_-|2v_-(\tau))\,. \quad (55)$$

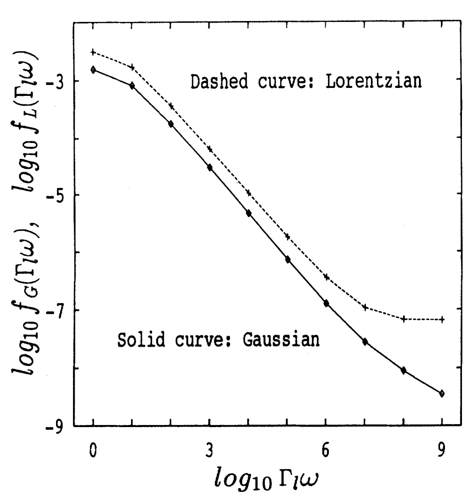

FIG. 2. Dimensionless functions $f_G(\omega)$ (solid line) and $f_L(\omega)$ (dashed line), which are proportional to the noise spectra of DBRTS with thick and thin spacers, respectively. The parameter used for this figure is $\Gamma_0/\Gamma_l = 10^9$, and the frequency is measured in units Γ_l^{-1}.

We should mention that this conditional probability has been analyzed for the case[13] of small $|\tau|$ and for the general case[11] in connection with the spectral diffusion of two-level systems in glasses. Under the condition $|\tau| \ll 1$, we have the relation $v_- \propto \Gamma_0|\tau|$, corresponding to the relation $\sigma_-^2 \propto \Gamma_0|\tau|$ in a DBRTS with thick spacers. Consequently, in a DBRTS with thin spacers, the characteristic value of $|\epsilon_\tau - \epsilon|$ for the Lorentzian diffusion is proportional to $|\tau|$, rather than to $\sqrt{|\tau|}$ as in a DBRTS with thick spacers.

In the limit $\Gamma_l|\tau| \gg 1$, one can show from (53) and (54) that

$$P_2(\epsilon,\epsilon_\tau|\infty) \neq P_1(\epsilon)P_2(\epsilon_\tau)\,.$$

The physical reason of such a correlation at $\Gamma_l|\tau| \gg 1$ has been discussed by Laikhtman[11] in connection to the spectral diffusion in glasses. It has been shown that the correlation diminishes if the positions of EF's are *fixed*. However, for the problem considered here, the spatial distribution of EF's influences both ϵ and ϵ_τ in the similar manner. On the other hand, in conventional experiments for large enough systems, the observed quantity is $\langle F(\epsilon,\epsilon_\tau|\tau)\rangle_{\mathrm{EF}}$. Consequently, the proper expression for the noise should be

$$\mathcal{K}(\epsilon,\epsilon_\tau|\tau) = \mathcal{L}(\epsilon_+|v_+(\tau))\mathcal{L}(2\epsilon_-|2v_-(\tau)) - \mathcal{L}\left(\epsilon_+|v_\infty\right)\mathcal{L}\left(2\epsilon_-|2v_\infty\right)\,. \quad (56)$$

At low temperature $k_BT \ll \gamma \ll \nu_d$, we can set $\epsilon = \epsilon_\tau = \varepsilon_V$ in Eq. (56). From the resulting formula, we derive the pair correlation

$$F(\varepsilon_V,\varepsilon_V|\tau) = \frac{T_{\mathrm{tot}}^2}{\pi}\left[\frac{\mathcal{L}(\varepsilon_V|v_+(\tau))}{v_-(\tau)} - \frac{\mathcal{L}\left(\varepsilon_V|\pi\nu_d/2\right)}{\pi\nu_d/2}\right]\,. \quad (57)$$

By comparing (48) and (57), we see that the pair correlation function depends on $\sigma_-(\tau)$ if the spacers are thick, but on $v_-(\tau)$ if the spacers are thin. In the region $\Gamma_l^{-1} \gg |\tau| \gg \Gamma_0^{-1}$, $v_-(\tau)$ depends on temperature only weakly, while $\sigma_-(\tau)$ has significant temperature dependence. Therefore, when the spacer thickness varies, a change of the temperature dependence of noise intensity will be detected.

Knowing the pair correlation (57), the fluctuation of Ohmic conductance is readily obtained from (6). At $\varepsilon_V = \varepsilon_0 = 0$, we have $\mathcal{L}(\epsilon_V|\sigma) = 1/\pi\sigma$, and then the conductance fluctuation can be expressed as

$$s(\tau) = \frac{T_{\mathrm{tot}}^2}{\pi(\pi\nu_d/2)^2}f_L(\tau)\,, \quad (58)$$

where dimensionless function $f_L(\tau)$ is defined as

$$f_L(\tau) = \frac{v_\infty^2}{v_-(|\tau|)v_+(|\tau|)} - 1 = \frac{(1-2\psi_\tau)^2}{4\psi_\tau(1-\psi_\tau)}\,. \quad (59)$$

Again, by taking the Fourier transform of $f_L(\tau)$, we derive the noise spectrum $f_L(\omega)$ for $\varepsilon_V = \varepsilon_0 = 0$. The result is shown in Fig. 2 as the dashed curve for $\Gamma_0/\Gamma_l = 10^9$. In the region $\Gamma_l \ll \omega \ll \Gamma_0$, $f_L(\omega) \propto \omega^{-0.79}$.

V. DISCUSSION AND CONCLUSION

In the present paper as well as in I, we have demonstrated that the influence of an EF on the resonant tunneling depends strongly on the relative strength between the switching rate Γ, the shift J of the resonant level due to the switching states of an EF, and the electronic escape rate γ from the well. If $\gamma > J$, then the broadening of the resonant level due to the tunneling escape is larger than the separation between the two possible levels $\varepsilon_0 + J$ and $\varepsilon_0 - J$. Hence, the interesting case is $J > \gamma$. In this case, the physical phenomena depend largely on the switching rate Γ. If $\Gamma \ll \gamma < J$, during the time interval that is proportional to p_{+1} (or p_{-1}), the electron tunnels through a static level $\varepsilon_0 + J$ (or $\varepsilon_0 - J$). This will result in two well resolved resonant-tunneling peaks separated by J, and the peak width is of the order γ. On the other hand, if $\Gamma \gg J > \gamma$, during the time interval between two consecutive switching states of an EF, the response of the tunneling system is too slow to form the two resonant levels $\varepsilon_0 \pm J$. Then there will be only one resonant-tunneling peak. The width of this peak is the larger one of J^2/Γ and γ.

It is very difficult to control the switching rate Γ and the coupling strength J. In any disordered material, there exists an exponentially wide distribution of relaxation rates. The value of J depends on the microscopic structure of the defect and its spatial location. On the other hand, the value of γ depends on the barrier width of a DBRTS, and can be well controlled. Therefore, by varying the barrier width of a small-area DBRTS, the effect of an EF on resonant tunneling can be investigated experimentally.

We have shown in our analysis that the correlation time is inversely proportional to Γ if $\Gamma \ll J$, while it is proportional to Γ if $\Gamma \gg J$. Since J is temperature independent, but Γ increases with raising temperature, it will be interesting to detect the crossover temperature at which there is a characteristic change of the behavior of correlation time. However, the exact temperature dependence of Γ varies with the mechanism of switching. For quantum tunneling, $\Gamma \propto T^3$ if the EF interacts with phonons, and $\Gamma \propto T$ if the EF interacts with electrons.[10] For the entirely different situation of activation, $\Gamma \propto \exp(-E_a/k_B T)$. Therefore, in order to achieve this goal, we need a suitable microscopic model to describe the EF, which is not clear at the moment. If we adopt the model in Ref. 10, the estimated crossover temperature is about 10–20 K, and the estimated activation energy is about 100–300 meV.

Besides the temperature crossover, there is also a crossover from the telegraph noise due to a single EF to the $\frac{1}{f}$ noise, due to many EF's. To observe this crossover, it is necessary to know the density of EF's, which depends crucially on the sample preparation procedure.

It is shown that the $\frac{1}{f}$ noise in resonant-tunneling systems is strongly dependent of bias voltage V. The explicit form of this dependence is determined by a spatial arrangement of EF's in the vicinity of the DBRTS. In the presence of thick enough undoped layer (spacer) near a DBRTS, we predict a Gaussian-like V dependence of noise, while a thin spacer leads to a Lorentzian one [compare Eqs. (48) and (57)].

In connection to the experimental observation, one should be aware of the existing low-frequency random fluctuations of the gate voltage in a DBRTS, which can lead to a similar low-frequency noise. To study the intrinsic LFN due the EF, such extrinsic LFN must be deducted. Finally, we should mention another kind of nonequilibrium noise, i.e., the *shot one*,[14] which is important at high enough frequencies. There have been experimental and theoretical studies of shot noise in a DBRTS in the absence of EF's.[15,16] However, it will be very interesting to investigate the shot noise when the tunneling current is influenced by dynamic defects.

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