Spin-split cyclotron resonance and spatial distribution of interacting electrons

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The interaction coupling of cyclotron transitions with diferent spin orientation was investigated as a function of density for electron inversion layers in GaAs in the magnetic quantum limit. The spatial electron distribution strongly influences the coupling of the electrical dipole transitions, resulting in deviations from the interaction strength for ideally two-dimensional electrons. At the higher densities the coupling is essentially reduced due to the finite thickness of the inversion layer, whereas at sufficiently low densities the coupling reflects the influence of disorder.

 $Kohn¹$ discovered that cyclotron resonance (CR) in a translational-invariant system is independent on the electron-electron interaction. However, if translational invariance is broken by disorder or nonparabolicity of the bands, electron-electron interactions can have a dominant influence on cyclotron resonance.² In GaAs, a reintroduction of electron-electron interactions can be expected due to the energy dependence of the electron effective mass originating from band-coupling phenomena and coupling to longitudinal-optical phonons. A major breakthrough to the understanding of interaction phenomena in cyclotron resonances was achieved recently by Cooper and Chalker³ who demonstrated that for ideally two-dimensional (2D) electron systems in the magnetic quantum limit, electron-electron interactions couple the electrical dipole transitions with different spin orientations. Under such conditions, the frequencies and the intensities for the two spin transitions can no longer be estimated in the framework of the single-particle approximation. With this new concept, previous cyclotronresonance experiments performed on low-density electron inversion layers in GaAs heterostructures could be explained satisfactorily.⁴⁻⁷ We recently demonstrated that electron-electron interactions also couple the electrical dipole transitions between different Landau levels $N \rightarrow N+1$, and that there is essentially no influence of the temperature on the interaction coupling strength.^{8,9}

To explain the striking influence of the electron spin on the $0 \rightarrow 1$ cyclotron transition, Cooper and Chalker compared the experiments to their numerical interaction model for ideally 2D electron systems. Here we attempt to go beyond this ansatz and extend the concept of the interaction-coupled dipole transitions to quasi-twodimensional systems. We would like to demonstrate as well how one can gain information on the spatial distribution of the interacting electrons from cyclotron resonances. We performed a detailed investigation of the influences that rule the interaction coupling of the two spin-resolved electron cyclotron transitions in the magnetic quantum limit in $Al_{1-x}Ga_xAs-GaAs$ single heterojunctions. The density dependence of the coupling strength was studied with gated samples in a density regime 0.5×10^{10} cm⁻² $\leq N_s \leq 5 \times 10^{10}$ cm⁻². The coupling is sensitive to the electron distribution in space, leading to deviations from the $N_S^{3/2}$ dependence of the interaction strength for ideally 2D electrons. At the higher densities, the coupling can be reduced due to the finite extent of the inversion layer in growth direction and screening by the gate, whereas at the lower densities the coupling is considerably enhanced due to the presence of disorder.

The experiment was performed with far-infrared transmission spectroscopy in temperature and magnetic field regimes from 1.5 to 7 K and from 12 to 14 T, respectively. Our samples were two modulation-doped Our samples were two modulation-doped Al_0 ₃Ga₀₇As-GaAs heterostructures grown by molecular-beam epitaxy on semi-insulating (100) substrates with $N_S \approx 8 \times 10^{10}$ cm⁻² and mobilities of order 3.5×10^5 cm²/Vs at liquid-helium temperatures. The layer sequence for sample 1 (2) was 1- (1-) μ m buffer, 100-60-) nm spacer, 40- (80-) nm $Al_{0.3}Ga_{0.7}As$ layer doped with Si to 1×10^{18} cm⁻³, 34- (0-) nm $Al_{0.3}Ga_{0.7}As$, 30-(20-) nm GaAs cap. Four alloyed In contacts and a 5-nm semitransparent NiCr front gate of resistance $Rg \approx 1$ k Ω allowed N_S to be varied continuously via the field effect. The far-infrared transmission in frequency space was obtained with an experimental set up as presented elsewhere.¹⁰ As usual we plot the normalized transmission $T(B)/T(B_0)$, where B and B₀ are the measurement and reference magnetic field strengths, respectively.

Figure ¹ shows the spin-split cyclotron transitions from the ground $N=0$ state to the first excited Landaulevel $N=1$ for different N_S and two temperatures, $T=5$ and 1.5 K, illustrating the inhuence of electron-electron interactions. In a single-particle approximation, the spin splitting is essentially dominated by the bulk Lande gfactor $g_N^* = g_0 + g_1(N + \frac{1}{2})B$, where $g_0 \approx -0.44$ is the g factor at the conduction-band edge and $g_1 \approx 0.01 \text{ T}^{-1}$ is a correction related to the nonparabolicity of the conduccorrection related to the nonparabolicity of the conduction band.¹¹ The transition involving the majority spin

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up occurs at the higher energy, and the expected resonance splitting $\Delta \omega = \omega_1 - \omega_2 = g_1 \mu_B B^2 / \hbar$ is independent of g_0 and proportional to the square of the magnetic field strength. The oscillator strengths of the two transitions should be given by the thermal population and with increasing temperature the amplitudes should approach the same value. In contrast to this expectation, a slight change of the temperature from ⁵ to 1.5 K, results in a reversal of the relative intensities, and with increasing N_S , both transitions collapse in a single line. This behavior has been explained being characteristic for the interaction coupling of electrical dipole transitions in a magnetic field.³

We investigated the N_S dependence of the interaction coupling by analyzing the relative spin intensities in a newly developed analytical model for the high-frequency response of interacting 2D electrons. Our model is equivalent to the one of Cooper and Chalker, but provides a different approach to the interaction coupling of the spin transitions. To describe the intensities and frequencies of the two spin transitions, we start from the

FIG. 1. Gate voltage dependence of the spin-split electron CR for sample 2 at temperatures of (a) 5 and (b) 1.^S K. The bold lines are best fits to the experiment with the interaction model. The fit parameters N_S in 10¹⁰ cm⁻², scattering time τ in ps, and C are given in brackets. Due to a different cool-down history, densities at the same V_g are different for $T=5$ and 1.5 K.

Hamiltonian of a strictly 2D electron layer in a perpendicular magnetic field, which is embedded in a semiconductor with relative dielectric constant c. For a sufficiently dilute electron gas, one can write

$$
H = \sum_{i=1}^{n_1} \hbar \omega_1 (a_i^{\dagger} a_i + \frac{1}{2}) + \sum_{j=1}^{n_1} \hbar \omega_1 (a_{n_1+j} a_{n_1+j}^{\dagger} + \frac{1}{2})
$$

+
$$
\frac{(el)^2}{8\pi \epsilon \epsilon_0} \sum_{\substack{i=1 \ i \neq j}}^{n_1+n_1} \sum_{j=1}^{n_1+n_1} \frac{(a_i^{\dagger} - a_j^{\dagger})(a_i - a_j)}{|\mathbf{R}_i - \mathbf{R}_j|^3},
$$
 (1)

where we used operators $a_i = (\rho_i^x - i \rho_i^y)/\sqrt{2}l$, $a_i^{\dagger} = (\rho_i^x + i\rho_i^y)/\sqrt{2}l, \qquad b_i = (R_i^x + iR_i^y)/\sqrt{2}l, \qquad \text{and}$ $b_i^{\dagger} = (R_i^x - iR_i^y)/\sqrt{2}l$, which relate to the orbital ρ and the guiding-center coordinates $\mathbf{R}_i = \mathbf{r}_i - \boldsymbol{\rho}_i$ of the electrons and follow the commutation relations $[a_i, a_i] = \delta_{ii}$ and $[b_i, b_j^{\dagger}] = \delta_{ij}$. The first two terms describe the spin transitions as uncoupled one-dimensional harmonic oscilators, and n_{\uparrow} and n_{\downarrow} are the populations of the majority and minority spin, respectively. The interaction term was obtained by expanding the Coulomb potential in powers of the orbital coordinates up to second order. This is a good approximation for an electron gas, where there is no overlap from adjacent Landau orbits. Our mean interparticle distance $r_0 = (\pi N_S)^{-1/2}$, is of the order of 50 nm, large compared to the Landau orbit $l = (\hbar/eB)^{1/2} \approx 7$ nm as well as the extent of the inversion layer perpendicular to the layer plane, which is about 10 nm. By rearranging the terms in a_i , a_i^{\dagger} , b_i , and b_i^{\dagger} , one obtains

$$
\frac{1}{|\mathbf{r}_{ij}|} \approx \frac{1}{|\mathbf{R}_{ij}|} + \frac{l^2}{4|\mathbf{R}_{ij}|^3} + \frac{3l^4}{4|\mathbf{R}_{ij}|^5} + \frac{l^2}{2|\mathbf{R}_{ij}|^3} a_{ij}^{\dagger} a_{ij}
$$

$$
- \frac{l^2}{|\mathbf{R}_{ij}|^3} (a_{ij}^{\dagger} b_{ij}^{\dagger} + a_{ij} b_{ij})
$$

$$
+ \frac{l^4}{2|\mathbf{R}_{ij}|^5} (a_{ij}^{\dagger} a_{ij}^{\dagger} b_{ij}^{\dagger} + a_{ij} a_{ij} b_{ij} b_{ij}), \qquad (2)
$$

where double indices indicate separations, e.g., $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$. The fourth term on the right-hand side of Eq. (2) describes the interaction coupling of the cyclotron transitions, whereas the last two terms couple the orbital and guiding-center motions of the electrons. Compared to the fourth, all other terms vary slowly on the time scale of the orbital motion and can be neglected in a first approximation.

We then transform the Hamiltonian Eq. (1) to harmonic coordinates, i.e., $H = \sum \hbar \omega_i (\alpha_i^{\dagger} \alpha_i + \frac{1}{2})$ such that $[\alpha_i, H] = \hbar \omega_i \alpha_i$. It is convenient to define the coordinates $A_1 = (1/\sqrt{n_1}) \sum_{i=1}^{n_1} a_i$ and $A_2 = (1/\sqrt{n_1}) \sum_{i=1}^{n_2} a_{n_1+i}$, which form a subspace of the whole $\{a_i\}$ space, and commute with Eq. (1) in such a manner that the result is a linear combination of A_1 and A_2 . Moreover, the dipole operator for right circular polarized light can be written as a linear combination of A_1^{\dagger} and A_2^{\dagger} and couples only to this subspace. Solving the 2×2 eigenvalue problem

$$
\begin{vmatrix}\n\omega_1 + \omega_{I2} - \omega & -\frac{\omega_{I2}}{\sqrt{\kappa}} \\
-\sqrt{\kappa} \omega_{I1} & \omega_1 + \omega_{I1} - \omega\n\end{vmatrix}\n\begin{bmatrix}\nf_1 \\
f_2\n\end{bmatrix} = 0,
$$
\n(3)

where $\kappa = n_{\perp}/n_{\uparrow}$, $\omega_{I1} = \omega_I/(1+\kappa)$, and $\omega_{I2} = \omega_I \kappa/(1 + \kappa)$, we determine the eigenfrequencies and the two harmonic coordinates $\alpha_i = f_{1i} A_1 + f_{2i} A_2$ $(i=1,2)$. Electron-electron interactions rule Eq. (3) via $f(t-1,2)$. Electron-electron interactions rule Eq. (3) via
 $f(t) = [(el)^2/8\pi\epsilon\epsilon_0] \sum_{i=1}^{n_1+n_1-1} (1/d_i^3)$, reflecting the interaction energy of a single electron at the origin with all others separated by the distances d_i . Without loss of generality we assumed that each electron has the same interaction energy. Because of the large N_S , we replace the sum $(n_1 + n_1 - 1)$ by infinity, and we finally get $\hslash \omega_I = C_{2D} (\pi N_S)^{3/2} [(el)^2 / 8 \pi \epsilon_0]$, where C_{2D} $i(r_0/d_i)^3$ is a dimensionless configuration constant depending on the electron distribution in the layer plane.

Assuming a frequency dependence of Drude type, the high-frequency conductivity of interacting 2D electrons can be written as

$$
\sigma_{+}(\omega) = \frac{N_{S}e\tau}{B} \left[\frac{F_1\omega_1}{1 + i(\omega - \omega_1)\tau} + \frac{F_2\omega_2}{1 + i(\omega - \omega_2)\tau} \right], \quad (4)
$$

where, using abbreviations $\Phi = \omega_I / \Delta \omega$, $\rho = n_\downarrow / (n_\uparrow + n_\downarrow)$,
and $\Theta = [(1 - \Phi)^2 + 4\Phi \rho]^{1/2}$, the transition energies ω_i and intensities F_i of the majority ($i = 1$) and minority $(i=2)$ spin are given by

$$
\omega_i = \omega_\uparrow + \frac{\Phi - 1 - (-1)^i \Theta}{2} \Delta \omega ,
$$

$$
F_i = \frac{\Theta + (-1)^i (\Phi - 1 + 2\rho)}{2\Theta} .
$$
 (5)

The F_i were calculated by projecting the eigenvectors of Eq. (3) to the perturbation Hamiltonian for right circular polarization. Please note that in the parabolic case, $\Delta\omega=0$, this reduces to Kohn's theorem, i.e., $\omega_1=\omega_1+\omega_1$, $\omega_2 = \omega_1 = \omega_1$, $F_1 = 0$, and $F_2 = 1$. In the very weak coupling limit, $\omega_I \ll \Delta \omega$, we determine the transition energies to be $\omega_1 = \omega_1$, $\omega_2 = \omega_1$, and that the oscillator strengths $F_1 = n_{\uparrow} / (n_{\uparrow}+n_{\downarrow})$ and $F_2 = n_{\downarrow} / (n_{\uparrow}+n_{\downarrow})$ are dominated by the thermal population of the spin states.

We determined the configuration constant by fitting
the normalized transmission $T(B)/T(B_0)$ transmission $T(B)/T (B_0)$ $=1-\text{Re}\sigma_{+}(\omega)/\{[1+(\epsilon)^{1/2}]\epsilon_0c+\sigma_{g}\}\text{ with Eqs. (4) and}$ (5) and the thermal population derived from Fermi-Dirac statistics (see Fig. 1). Figure 2 shows the N_S dependence of the experimental configuration for samples ¹ and 2. Cooper and Chalker demonstrated that at low temperatures there is little difference between a solid and liquidlike electron configuration in CR for ideally 2D electron systems. Thus, the configuration constant can be calculated assuming that the guiding-center coordinates are fixed to the positions of a Wigner crystal.³ For a hexagonal Wigner crystal with long-range order, we get $C_{2D} = 6(\sqrt{3}/2\pi)^{3/2} \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} [1/(i^2+j^2-ij)^{3/2}] \approx 1.6$, shown in Fig. 2 by a dashed line. This value is insensitiv

FIG. 2. Experimental configuration constant determined from the spin-split CR for electron inversion layers in GaAs in magnetic field and temperature regimes 12 $T \le B \le 14$ T and 1.5 $K \leq T \leq 7$ K, respectively. The dashed line indicates the configuration constant C_{2D} for an ideally 2D hexagonal Wigner crystal.

to the unit cell of the Wigner crystal, e.g., for a square lattice one obtains nearly the same value.

Most prominently, for both samples the configuration constant increases with decreasing N_S , exceeding the value for ideally 2D electrons by up to an order of magnitude. For sample 2, the configuration constant is enhanced for densities smaller than about 2×10^{10} cm⁻², whereas for sample 1 there is evidence for an enhancement already at higher N_S , indicating a sample dependence. The increase in the configuration constant we attribute to the presence of disorder developing at the lower N_S . Evidence for a metal-insulator transition for electron inversion layers in GaAs at densities of the order of 10^{10} cm⁻² has also been observed previously from inelastic light scattering experiments.¹² In the metalinsulator transition regime, the electrons are no longer distributed evenly in the sample but are believed to be localized in dimples that to some extent might be isolated from each other. Assuming that the electron density in the dimples \tilde{N}_s is larger than the density N_s of the equivalent homogeneous system, we can qualitatively explain the enhancement of the experimental configuration constant in the regime where disorder is important. Since our analysis is based on the assumption of a homogeneous system, we observe a configuration constant, which increases proportional to $(\widetilde{N}_S/N_S)^{3/2}$. Our experiment provides a promising approach to study the metalinsulator transition in semiconductors by cyclotron resonance.

For sample 2, there is a tendency that the configuration constant is reduced compared to C_{2D} at densities larger than about 2×10^{10} cm⁻². A reduction can be expected due to the finite extent of the inversion layer in growth direction and screening by the gate. For a quantitative evaluation of the effect, we calculated the effective

electron-electron interaction potential for quasi-2D electrons in the framework of the Fang-Howard variationalenvelope function¹³

$$
V(r) = \frac{e^2}{4\pi\epsilon\epsilon_0} \int_0^\infty dq \left[F_1 \left(\frac{q}{b} \right) + F_2 \left(\frac{q}{b} \right) \right] J_0(qr) , \qquad (6)
$$

where J_0 is the Bessel function of zeroth order and $F_1(q/b) = [1 + \frac{9}{8}(q/b) + \frac{3}{8}(q^2/b^2)](1+q/b)^{-3}$ and $F_2(q/b) = [1 - \coth(qD)][1 + \coth(qD)]^{-1}[1+(q/\sqrt{1+q})]$ (b) ⁻⁶ are form factors related to the inversion layer thickness and the screening by the gate, respectively. The parameter $b = [12m^*e^{\frac{\lambda}{2}}(N_{\text{depl}} + \frac{11}{32}N_S)/\epsilon \epsilon_0 \hbar^2]^{1/3}$ depends on N_S and the depletion charge N_{depl} in the GaAs buffer and rules the layer thickness. In the definition of $F_2(q/b)$, we neglected the small difference in the dielectric constants of GaAs and $Al_{0.3}Ga_{0.7}As$. In terms of the derivatives of the interaction potential, the configuration constant can be written in the more universal form

$$
C = \frac{4\pi\epsilon\epsilon_0 r_0^3}{e^3} \sum_{i=1}^{\infty} \left[\frac{\partial^2}{\partial r^2} V(r) + \frac{1}{r} \frac{\partial}{\partial r} V(r) \right]_{r=d_i} . \tag{7}
$$

In Fig. 3, the calculated N_S dependence of the configuration constant normalized to C_{2D} is shown for different N_{depl} . In the limit of small N_S , the curvatures and the first derivatives of the interaction potentials for 2D and quasi-2D systems approach the same value if screening by the gate is ignored. Thus, the electrons will always behave as being confined in an infinitely thin electron layer. However, if the gate is included, the configuration constant drops to zero in the limit of vanishing N_S , resulting in a total cancellation of the electron-electron interaction. With increasing N_{depl} , i.e., decreasing layer thickness, the configuration constant approaches the 1imit for ideally 2D electrons. Decreasing N_{depl} is equivalent to approaching the 3D limit with reduced interaction coupling. In our N_S regime from 0.5 to 5×10^{10} cm⁻² the gate gives only a small correction compared to the finite thickness effect, since the distances D between the gate and the inversion layer are large for our samples. The calculated configuration constant is in close agreement with the experimental data of sample
two for $N_{\text{depl}} \approx 5 \times 10^{10} \text{ cm}^{-2}$, a realistic value for our samples.

In principle, the interaction coupling of the dipole transitions should also inhuence the cyclotron resonance of electrons in bulk samples. However, in previous experiments, which were performed at electron concentrations n_e of the order of 10^{14} cm⁻³, the frequencies and intensities of the two spin-transitions were in satisfactory agreement with the predictions in single-particle approximament with the predictions in single-particle approximation.¹¹ An estimation of the interaction coupling strength in the bulk is complicated due to the possibility of the electrons moving freely parallel to the magnetic field direction. However, for sufficiently high magnetic fields and low temperatures we might assume that all electrons have energies close to the band edge of the ground Landau level. Then it is straightforward to show that the interaction energy is given

FIG. 3. Normalized configuration constant C/C_{2D} vs N_S for different N_{depl} . Dotted and solid lines are calculated considering the finite thickness effect with and without screening by the gate, respectively. A gate to inversion layer separation of $D = 120$ nm is assumed.

by $\hbar \omega_I = C_{3D} (4\pi n_e / 3) [(el)^2 / 8\pi \epsilon \epsilon_0]$, where the configuration constant for bulk electrons C_{3D} is defined as in Eq. (7) with $r_0 = (3/4\pi n_e)^{1/3}$. Similarly as in the 2D case, we evaluate C_{3D} assuming that the positions of the guiding-center coordinates are fixed to the positions of a Wig ner crystal. Considering that one has a bodycentered-cubic lattice for bulk electrons¹⁴ and that the difference in the single-particle transition energies is essentially the same for 2D and bulk samples, one obtains a relative coupling strength $3C_{2D}(\pi \hat{N}_S)^{3/2}/C_{3D}4\pi n_e$, which is larger by a factor of about 30 if we assume $N_S \approx 10^{11}$ cm⁻² in the 2D case. A similar interaction coupling strength in the bulk as in the 2D case can be expected only if n_e is increased to about 10¹⁶ cm⁻³. Thus, for the experiment of Ref. 11, the single-particle picture should provide a good approximation.

Finally, we like to comment on the possibility of a collective mode that can exist in a 2D electron system with broken translational invariance. The guiding-center motion is coupled with the orbital motion due to the fifth and sixth terms in Eq. (2), indicating that an inter-Landau-level cyclotron excitation is always associated with an intra-Landau-level excitation of the guidingcenter coordinates. If one neglects the small sixth term, which couples excitations that are active to left and right circular polarization, the motion of the guidingcenter coordinates is solely active to left circular polarization, and the mode dispersion $\hbar\Omega \approx 2\hbar\omega_I = C_{2D} (\pi N_S)^{3/2} [(el)^2 / 4\pi \epsilon \epsilon_0]$ is proportional to B^{-1} . Modes with this B dependence are characteristic for 2D electrons with broken translational invariance. They were predicted to occur in a disordered 2D electron

gas,¹⁵ and are well known for laterally microstructure quantum discs and dots, 16 where these excitations resemble edge magnetoplasmons.

In conclusion, we have studied the density dependence of the spin-split CR of interaction 2D electrons in GaAs in the extreme magnetic quantum limit. The interaction coupling of the electrical dipole transitions deviate from the strength predicted for ideally 2D electrons due to influences of the finite extent of the inversion layer, screening by the gate and disorder. There is evidence for a collective intra-Landau-level mode associated with the

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motion of the guiding-center coordinates. The experimental configuration of this mode is important for the understanding of the properties of interacting twodimensional electron systems in a magnetic field.

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