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Luminescence and efficiency of an ideal photovoltaic cell with charge carrier multiplication

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We discuss the potential of an ideal photovoltaic cell with charge carrier multiplication by impact ionization. Thermodynamics requires that carrier multiplication modifies luminescence as well. Such a cell may be described as a hot-carrier cell where interband equilibrium is inhibited and where thermalization is restricted to the band edges. We show that in the limit of low band-gap energy this cell is equivalent to a thermal absorber coupled to a Carnot engine. In conjunction with a spectral filter, a finite band-gap cell with carrier multiplication can even surpass the work obtainable from a selective thermal absorber. In principle, charge carrier multiplication can be tailored such that a single-gap cell matches a fully selective conversion device. A high-efficiency, thin-film cell with carrier multiplication may be thought of as a single-band-gap cell on the basis of a low-band-gap material, with efficiency close to the value of a fully selective device. Low absorptivity for photons below the threshold for carrier multiplication might be realized with an indirect transition. When the solar irradiance exceeds a certain threshold all photons above gap energy may be used.

I. INTRODUCTION

Shockley and Queisser[1] defined an ideal photovoltaic cell without carrier multiplication by the requirements that 1. each photon with energy higher than the gap energy creates exactly one pair of charge carriers, 2. the charge carriers recombine only radiatively or in the external load circuit, 3. the charge carriers rapidly thermalize within the conduction and valence band. Under these requirements, a maximum efficiency of 30% for diluted blackbody solar radiation is predicted. This is significantly lower than the maximum efficiency obtainable from a selective thermal absorber in conjunction with a Carnot engine of 52.28%, a fully selective conversion device[2] of 66.68%, or complete and reversible conversion[3] of 72.6%. In this contribution, we always refer to diluted blackbody radiation with a net solar irradiance of $E^{\mathrm{sol}} = 1000\,\mathrm{W m^{-2}}$, a temperature of the sun's photosphere of $T_S = 5780\,\mathrm{K}$, and an ambient temperature of $T_a = 300\,\mathrm{K}$. We also assume that the cell is at ambient temperature, $T_0 = T_a$.

Many concepts have been considered to overcome the stringent efficiency limits of the classical one-gap cell. Two concepts are particularly aimed to match the spectral response of the device to the broad solar radiation: In multigap cells, the incoming radiation is split in two or more bands each being absorbed by a cell of dedicated band gap.[2] In solar thermophotovoltaics,[4] an intermediate thermal absorber/emitter increases the number of photons above gap energy.

The hot-carrier cell discussed in Ref. 5 is in effect equivalent to a hybrid cell at high temperature, where a negative voltage is used to decrease reradiation losses. This increases the work obtainable from the heat output by overcompensating the invested electrical energy. We recalculated the results for diluted blackbody irradiance as defined here and found that this hybrid cell has a maximum efficiency of 66.46%, higher than the selective absorber which has 52.28%, and very close to the maximum efficiency of a fully selective device of 66.68%.

Recently, it has been shown that the restriction to unit quantum efficiency may be overcome by high-energy carriers that generate additional charge carriers by impact ionization before thermalizing. This possibility has been briefly discussed in a footnote in Ref. 1 and taken up recently by various authors.[6,7] Ultimate efficiencies in the absence of luminescence of about 60% are predicted.[6] Including luminescent recombination maximum efficiencies of 43% are calculated.[8] Quantum efficiencies larger than unity have been measured in Si solar cells.[7–9] The authors propose a combination of an indirect fundamental transition with a direct transition at higher energies in a suitable band structure. In this paper, we explore the potential of carrier multiplication for the conversion of solar radiation with a single-band-gap photovoltaic cell.

II. THEORY

The luminescence of an ideal photovoltaic cell for photon energies e exceeding gap energy is given by[10–12]

$$N_e^{\mathrm{lum}}(e, T_0, \Delta\mu) = \frac{g(e)}{\exp\left(\frac{e-\Delta\mu}{k_B T_0}\right) - 1}, \tag{1}$$

where $\Delta\mu$ is the potential difference of electrons and holes, which in equilibrium is equal to the chemical potential of the photons at the cell temperature T_0. As a special case, zero chemical potential, $\Delta\mu = 0$, yields the spectral photon flux density $N_e^{\mathrm{th}}(e,T) = N_e^{\mathrm{lum}}(e,T,0)$ of a thermal emitter as given by Planck's equation. N_e represents the number of photons per area and per photon energy interval for hemispheric radiation with both polarizations. In this case, the mode density $g(e)$ is

$$g(e) = \frac{2\pi e^2}{h^3 c^2}, \tag{2}$$

where h is Planck's constant, k_B is Boltzmann's constant, and c is the speed of light.

In the following quantitative evaluation, we model the incoming radiation N_e^s as blackbody solar radiation diluted by ambient blackbody radiation.

$$N_e^s(e) = f N_e^{\mathrm{th}}(e, T_S) + (1-f) N_e^{\mathrm{th}}(e, T_a). \tag{3}$$

Here, f is the dilution factor and T_a, the ambient temperature. The solar irradiance is defined as the net input:

$$E^{\mathrm{sol}} = \int_0^\infty e\left[N_e^s(e) - N_e^{\mathrm{th}}(e, T_a)\right] de = f\,\sigma\,(T_S^4 - T_a^4), \tag{4}$$

where σ is the Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$. The efficiency is defined as the work output power P related to the solar rather than the total incoming irradiance:

$$\eta = \frac{P}{E^{\mathrm{sol}}}. \tag{5}$$

The luminescence of a cell with charge carrier multiplication is deduced with an argument generalized from Refs. 4 and 2. Consider a radiation source emitting radiation of temperature T in equilibrium with the cell in open circuit with voltage $U = \Delta\mu/q_0$, as depicted in Fig. 1, where q_0 denotes the elementary charge. A narrow bandpass filter reflects all radiation, except photons with energy close to e. Assume that each photon transmitted by the filter generates κ pairs of charge carriers, i.e.,

$$\phi_e \rightleftharpoons \kappa e^- + \kappa h^+, \tag{6}$$

where ϕ_e denotes a photon of energy e, e^- an electron, and h^+ a hole. Charge carrier multiplication is expressed by a value of κ exceeding unity; the κ carriers are assumed to finally thermalize at the temperature of the cell.

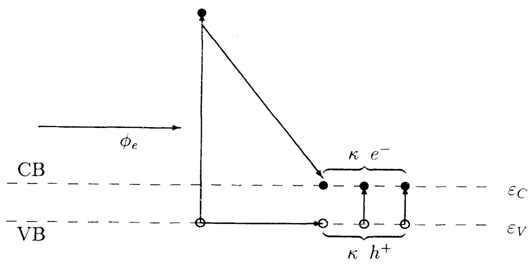

FIG. 1. Generation of κ pairs of charge carriers by impact ionization. In the example, the kinetic energy of the electron, which has been brought from the valence band (VB) to the conduction band (CB) in the primary absorption of the photon ϕ_e, is used to produce another two pairs of carriers.

We consider the conversion of a single photon by which the photon flux of the irradiation exceeds the flux of the luminescence. This extra photon delivers the charge κq_0 at the cell, thus, the electrical energy delivered is $\kappa\Delta\mu$. On the other hand, in reversible operation the electrical energy equals the photon energy times the Carnot efficiency, thus

$$e\left(1 - \frac{T_0}{T}\right) = \kappa\Delta\mu. \tag{7}$$

In other words, the chemical potential of the photons equals the sum of the chemical potential of the electrons and of the holes, as required for the reaction Eq. (6). Note that the chemical potential of the hole is minus the value for the electron, hence $\Delta\mu$ is the difference of the chemical potential of electrons in the conduction band to the value in the valence band.

From this equation, we can replace the radiance temperature T by the chemical potential $\Delta\mu$:

$$\frac{e}{k_B T} = \frac{e - \kappa\Delta\mu}{k_B T_0}. \tag{8}$$

Just like the luminescence in conventional photovoltaic devices is the time-reversed process of the corresponding absorption, in the case of charge carrier multiplication, luminescence may arise from the recombination of a high-energy electron produced after one or more Auger recombinations. Thus κ electrons contribute to produce high-energy photons. This can be visualized from Fig. 1, by inverting all arrows.

In equilibrium, the luminescence photon flux $N_e^{\mathrm{lum}}(e, T_0, \Delta\mu, \kappa)$ equals the thermal photon flux $N_e^{\mathrm{th}}(e, T)$ and the luminescence for a cell with cell temperature T_0, chemical potential $\Delta\mu$ (voltage $\Delta\mu/q_0$), temperature T_0 and multiplication factor κ is

$$\begin{aligned} N_e^{\mathrm{lum}}(e, T_0, \Delta\mu, \kappa) &= \frac{g(e)}{\exp\left(\frac{e - \kappa\Delta\mu}{k_B T_0}\right) - 1} \\ &= N_e^{\mathrm{lum}}(e, T_0, \kappa\Delta\mu). \end{aligned} \tag{9}$$

Hence, the power-voltage characteristic of the ideal cell with radiative recombination only is

$$\begin{aligned} P_{\mathrm{el}}(\Delta\mu) = \Delta\mu \int_{e_G}^\infty & \kappa(e)[N_e^s(e) \\ & - N_e^{\mathrm{lum}}(e, T_0, \kappa(e)\Delta\mu)]de. \end{aligned} \tag{10}$$

The emitted photons of energy e have chemical potential $\kappa(e)\Delta\mu$. This form of the luminescence, with κ according to Eq. (23), has been proposed in Ref. 13.

Different processes with different values of κ may take place in parallel, each with a probability $p_\kappa(e)$, $\sum_\kappa p_\kappa(e) = 1$. The power-voltage characteristic with such a distribution is

$$P_{\mathrm{el}}(\Delta\mu) = \sum_{\kappa} \Delta\mu \int_{e_G}^{\infty} p_{\kappa}(e)\kappa(e)[N_e^s(e) - N_e^{\mathrm{lum}}(e, T_0, \kappa(e)\Delta\mu)]de. \quad (11)$$

The distribution $\{p_\kappa\}$ with the maximum power is one for a certain value of κ and zero otherwise. Hence, in the case of a free cell design, the sum over κ may be omitted. In the case of experimental data, we generally do not know the actual distribution, but only the quantum efficiency $\eta_{\mathrm{QE}}(e)$,

$$\eta_{\mathrm{QE}}(e) = \sum_{\kappa} p_{\kappa}(e)\kappa, \quad (12)$$

and hence we also omit the sum when using experimental data.

In order to describe the electronic distribution after primary photon excitation, we assume that in each energy interval $[\varepsilon, \varepsilon + d\varepsilon]$, all electrons are in equilibrium and may thus be described with temperature T_0 and a quasi Fermi level $\mu(\varepsilon)$, written as $\mu_C(\varepsilon)$ and $\mu_V(\varepsilon)$ in the conduction and valence band, respectively. The difference between the quasi levels of the conduction edge, $\mu_C(\varepsilon_C)$, and of the valence edge, $\mu_V(\varepsilon_V)$, is the work $\Delta\mu = q_0 U$ obtainable in an external circuit,

$$\Delta\mu = \mu_C(\varepsilon_C) - \mu_V(\varepsilon_V); \quad (13)$$

the gap energy e_G is $\varepsilon_C - \varepsilon_V$.

Alternatively, we could consider hot electrons with a temperature $T(\varepsilon)$, with $\mu(\varepsilon)$ related to $\mu_C(\varepsilon_C)$ in the conduction and as $\mu_V(\varepsilon_V)$ in the valence band. The two descriptions are related by

$$\mu_C(\varepsilon) - \mu_C(\varepsilon_C) = (\varepsilon - \varepsilon_C)\left(1 - \frac{T_0}{T_C(\varepsilon)}\right), \quad (14)$$

$$\mu_V(\varepsilon_V) - \mu_V(\varepsilon) = (\varepsilon_V - \varepsilon)\left(1 - \frac{T_0}{T_V(\varepsilon)}\right). \quad (15)$$

Equilibrium of the cascade impact ionization reaction

$$e^-(\varepsilon_2) + h^+(\varepsilon_1) \rightleftharpoons \kappa(\varepsilon_1, \varepsilon_2)\left[e^-(\varepsilon_C) + h^+(\varepsilon_V)\right] \quad (16)$$

requires

$$\mu_C(\varepsilon_2) - \mu_V(\varepsilon_1) = \kappa(\varepsilon_1, \varepsilon_2)\Delta\mu. \quad (17)$$

If different pairs $(\varepsilon_1, \varepsilon_2)$ with the same difference $e = \varepsilon_2 - \varepsilon_1$ have different κ, this adds up with probabilities p_κ, as described in Eq. (11). When $\kappa(\varepsilon_1, \varepsilon_2)$ is a function of $\varepsilon_2 - \varepsilon_1$ only, this function uniquely determines all quasi levels:

$$\mu_C(\varepsilon) - \mu_C(\varepsilon_C) = [\kappa(\varepsilon - \varepsilon_V) - 1]\Delta\mu, \quad (18)$$

$$\mu_V(\varepsilon_V) - \mu_V(\varepsilon) = [\kappa(\varepsilon_C - \varepsilon) - 1]\Delta\mu. \quad (19)$$

High values of $\kappa(e)$ result in carriers with large $\mu_C(\varepsilon_V + e) - \mu_C(\varepsilon_C)$, i.e., at high temperature. Note that it is essential for an efficient cell that the temperature at the band edges equals T_0; hot electrons with a uniform temperature throughout each band have high luminescent losses.

It is interesting to note that $\kappa < 1$, e.g., $\kappa = 1/2$, might be interpreted as the inverse of the diode quality factor (Ref. 14, Chap. 3) e.g., by describing two-photon emission through states in the middle of the gap. This procedure would formally continue $\mu(\varepsilon)$ into the band gap. A two-photon process has been considered in Ref. 15.

Note that the case of arbitrary $\kappa(e)$ gives a new degree of freedom similar to the situation for a fully selective device. In a fully selective photovoltaic device each photon energy is converted at a dedicated potential $\Delta\mu(e)$, optimized for that particular spectral density. With the definition of a voltage depending on photon energy, $\Delta\mu'(e) = \kappa(e)\Delta\mu$, Eq. (10) describes a fully selective device, formulated in the photovoltaic representation:

$$P_{\mathrm{el}} = \int_{e_G}^{\infty} \Delta\mu'(e)[N_e^s(e) - N_e^{\mathrm{lum}}(e, T_0, \Delta\mu'(e))]de. \quad (20)$$

We might equally well use the representation of a thermal absorber, by defining an absorber temperature $T_A(e)$, with $\kappa(e)\Delta\mu/e = 1 - T_0/T_A(e)$. Any fully selective conversion device has the same optimum efficiency. Note that any value of $0 < \Delta\mu < e_G$ may be used.

The optimum charge carrier multiplication factor $\kappa^{\mathrm{opt}}(e)$ of an equivalent single-gap photovoltaic cell with fully selective behavior is given by $\kappa^{\mathrm{opt}}\Delta\mu = \Delta\mu^{\mathrm{opt}}(e)$, where $\Delta\mu$ is the chemical potential of the single-gap cell and $\Delta\mu^{\mathrm{opt}}(e)$ maximizes the power in Eq. (20),

$$\Delta\mu^{\mathrm{opt}}(e) = \underset{\Delta\mu}{\mathrm{argmax}} \left\{\Delta\mu\left[N_e^s(e) - N_e^{\mathrm{lum}}(e, T_0, \Delta\mu)\right]\right\}. \quad (21)$$

Figure 2 shows the optimum chemical potential $\Delta\mu^{\mathrm{opt}}(e)$

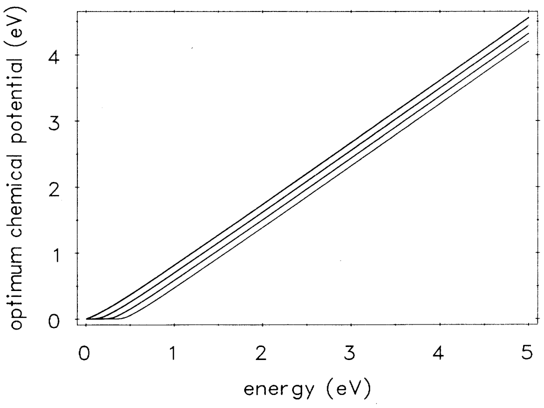

FIG. 2. Optimum chemical potential $\Delta\mu^{\mathrm{opt}}(e)$ of a hypothetical fully selective photovoltaic device illuminated by diluted blackbody radiation as a function of photon energy e, for different values of the solar irradiance E^{sol} equal to 10 W m^{-2} (lowest curve), 10^3 W m^{-2}, 10^5 W m^{-2}, and 10^7 W m^{-2} (top curve). The optimum charge carrier multiplication factor $\kappa^{\mathrm{opt}}(e)$ of a single-gap photovoltaic cell with fully selective behavior is given by $\kappa^{\mathrm{opt}}\Delta\mu = \Delta\mu^{\mathrm{opt}}(e)$, where $\Delta\mu/q_0$ is the external voltage of the single-gap cell.

of a hypothetical fully selective photovoltaic device, as a function of photon energy e when illuminated by diluted blackbody radiation, for different values of the solar irradiance $E^{\rm sol}$, i.e., for different dilution factors f. For all photon energies, except for very low values, the optimum potential is a linear function of e; the slope is $1 - T_a/T_S$, and the curves for different f are shifted by $k_B T_a \ln(f)$. The same behavior is found for the open circuit chemical potential.[16]

From Eq. (14) and Eq. (18) follows that, for $\kappa(e)\Delta\mu = \Delta\mu^{\rm opt}$, the electron temperature is

$$T_C(\varepsilon_V + e) = \frac{T_0}{1 - \dfrac{\Delta\mu^{\rm opt}(e) - \Delta\mu}{e - e_G}}. \qquad (22)$$

This relation requires that to exclude photons with $\Delta\mu^{\rm opt}(e) < \Delta\mu$.

III. INTEGER MULTIPLICATION FUNCTION

In Ref. 8, the following idealizing form has been proposed for $\kappa(e)$:

$$\kappa^{\rm int}(e) = {\rm Int}\left(\frac{e}{e_G}\right). \qquad (23)$$

Figure 3 shows the luminescence of an ideal photovoltaic cell with charge carrier multiplication, according to Eq. (23) as a function of photon energy for different values of the chemical potential. When the gap energy is small compared to the energy of the incoming photons, $e_G \ll k_B T_S$, the Int function in Eq. (23) may be omitted for all photons. In this case, the electrical power from the cell is:

$$P \underset{e_G\to 0}{\longrightarrow} \frac{\Delta\mu}{e_G}\int_{e_G}^{\infty} e \left(N_e^s(e) - \frac{g(e)}{\exp\left(\dfrac{e(1-\frac{\Delta\mu}{e_G})}{k_B T_0}\right) - 1}\right) de. \qquad (24)$$

We replace $T_0/(1 - \Delta\mu/e_G)$ by a different temperature T_A. Then, the power in Eq. (24) is the work obtainable from a black thermal absorber at temperature T_A, in conjunction with a Carnot engine converting heat at temperature T_A with ambient temperature T_0.[4]

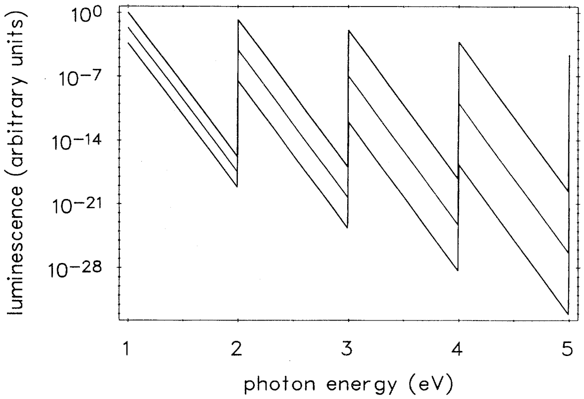

FIG. 3. Luminescence (spectral energy flux density) of an ideal photovoltaic cell with charge carrier multiplication $\kappa^{\rm int}$ as a function of photon energy e/e_G referred to the gap energy, for different values of the chemical potential, $0.7e_G$ (bottom), $0.8e_G$, and $0.9e_G$ (top). The gap energy is 1 eV, the cell temperature 300 K.

IV. LINEAR MULTIPLICATION FUNCTION

For Ge the quantum yield, which in this case is the factor κ used in this work, is reported in Ref. 17. We approximate it as

$$\kappa(e) = 1 + \beta_s \left[\frac{e - e_G}{e_G} - \beta_t\right]^+, \qquad (25)$$

with values given by $\beta_s = 0.3$ and $\beta_t = 2.2\,{\rm eV}/0.7\,{\rm eV} - 1 = 2.14$. The $[\,]^+$ indicates that negative values are treated as zero. The threshold photon energy for subsequent impact ionization is $(\beta_t + 1)e_G$, the impact efficiency is β_s. When all excess energy is transferred to the electron in the conduction band, the threshold for a process $e^- \to 2e^- + h^+$ is $1.5e_G$ when the masses m_e of the electron and m_h of the hole produced by impact ionization are equal, and 1 for $m_h \ll m_e$. The values $\beta_t = 0$ and $\beta_s = 1$ are equivalent to an ideal selective thermal absorber.

With Eq. (25), $\kappa(e)$ is piecewise a linear function. In any interval $[e_1, e_2]$, where κ is a linear function of the photon energy e, $\kappa(e) = \gamma_0 + \gamma_1 e$, the power $P_{\rm el}(e_1, e_2)$ obtained from the radiation in this photon interval is

$$P_{\rm el} = \Delta\mu \int_{e_1}^{e_2} (\gamma_0 + \gamma_1 e) \left(N_e^s(e) - \frac{g(e)}{\exp\left(\dfrac{e - (\gamma_0 + \gamma_1 e)\Delta\mu}{k_B T_0}\right) - 1}\right) de \qquad (26)$$

$$= \Delta\mu \int_{e_1}^{e_2} \left\{ \gamma_0 \left[N_e^s(e) - N_e^{\rm lum}\left(e, \frac{T_0}{1 - \gamma_1\Delta\mu}, \frac{\gamma_0\Delta\mu}{1 - \gamma_1\Delta\mu}\right)\right] + \gamma_1 \left[e\,N_e^s(e) - e\,N_e^{\rm lum}\left(e, \frac{T_0}{1 - \gamma_1\Delta\mu}, \frac{\gamma_0\Delta\mu}{1 - \gamma_1\Delta\mu}\right)\right]\right\} de. \qquad (27)$$

The term with γ_0 corresponds to a net flux of number of photons, the term with γ_1 corresponds to a net flux of their energy. Thus, the power in each interval may be calculated without numerical integration using standard integrals of the spectral photon and energy flux of the source and the luminescence of a black cell with no charge carrier multiplication. For the form in Eq. (25), we use the two intervals $[e_G, (\beta_t+1)e_G]$, with $\gamma_0=0$, $\gamma_1=1$ and $[(\beta_t+1)e_G, \infty]$ with $\gamma_0=1-\beta_s(\beta_t+1)$, $\gamma_1=\beta_s$.

Figure 4 and Fig. 5 show the efficiency of an ideal cell with carrier multiplication as a function of photon energy given by Eq. (25), with impact efficiency $\beta_s=0$, 1/3, 2/3, and 1, together with the efficiency of the selective thermal absorber. The impact threshold was assumed to be $\beta_t=1$ in Fig. 4 and $\beta_t=2$ in Fig. 5. No filter is used, i.e., the absorptivity for all photons with energy above e_G is one. The efficiency shows a maximum which is somewhat higher than without carrier multiplication and slightly shifted to a lower gap energy. This has also been found in Ref. 8 with Eq. (23). Most solar photons have not enough energy for significant carrier multiplication.

For most curves, there is also a local maximum at low gap energies that is completely absent when there is no carrier multiplication. At low gap energy, the current greatly exceeds the short circuit cell of a one-gap cell without carrier multiplication; however, luminescence in the energy range $[e_G, (1+\beta_t)e_G]$ inhibits open circuit voltages large enough to surpass the efficiency at high gap energies. All values for $\beta_s > 0$ and of the thermal absorber converge for $e_G \to 0$ to the value of the black absorber. Even for $\beta_s=1$ $(\beta_t=1)$, the efficiency of 36% is significantly lower than for the selective thermal absorber, 52.28%. With $\beta_t=2$, there is nearly no improvement compared to the cell without multiplication. For this configuration, a low value of the threshold β_t is needed for high efficiency.

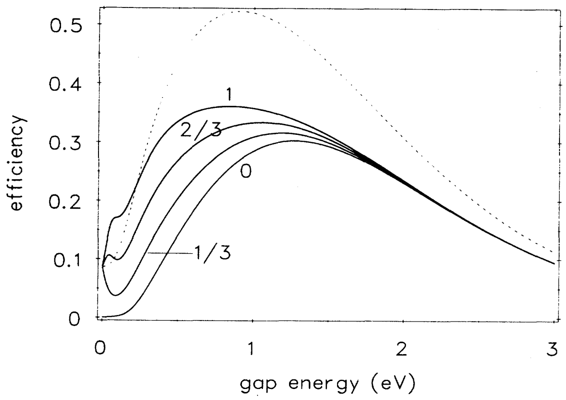

FIG. 4. Efficiency of an ideal cell with charge carrier multiplication as a function of photon energy, according to Eq. (25) with impact efficiency $\beta_s=0$, 1/3, 2/3, and 1, and impact threshold $\beta_t=1$. For $\beta_s=0$, there is no impact ionization. The broken line corresponds to the ideal selective thermal absorber.

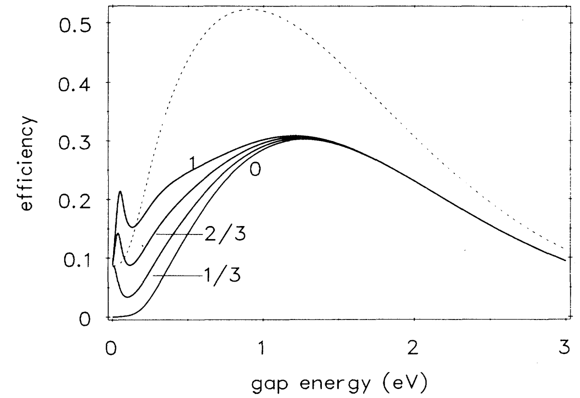

FIG. 5. Same as Fig. 4, but with impact threshold $\beta_t=2$.

A. High irradiance

Figure 6 shows the efficiency of an ideal photovoltaic cell with carrier multiplication in maximum power operation for diluted blackbody irradiance with varying solar irradiance, for different values of the impact efficiency $\beta_s=0$, 0.2, 0.4 0.6, 0.8, and 1, and impact threshold $\beta_t=2$. For each irradiance and each value of β_s, the gap energy of maximum efficiency is taken.

Without carrier multiplication, the efficiency weakly increases with the irradiance, due to the logarithmic increase of the open circuit voltage with irradiance. With carrier multiplication, the efficiency increases much more

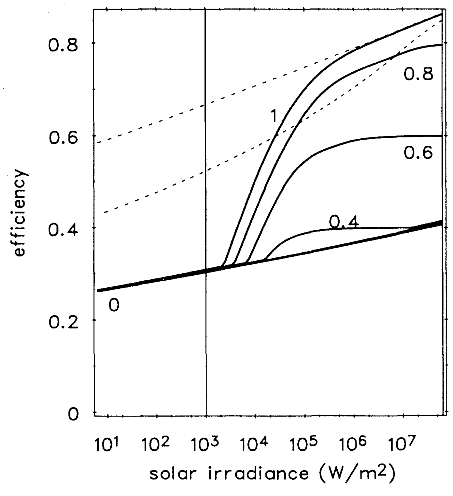

FIG. 6. Efficiency of an ideal photovoltaic cell with carrier multiplication $\kappa(e)$, according to Eq. (25) in the maximum power operation for diluted blackbody irradiance, ambient temperature 300 K; for different values of the impact efficiency $\beta_s=0$, 0.2, 0.4 0.6, 0.8, and 1, and impact threshold $\beta_t=2$. For $\beta_s=0$, there is no impact ionization. The gap energy is optimized at each irradiance. The broken lines correspond to the selective thermal absorber (lower curve) and to the fully selective device (upper curve). The vertical lines indicate $E^{\mathrm{sol}}=1000$ W m^{-2} and $E^{\mathrm{sol}}=6.33\times10^7$ W m^{-2} $(f=1)$.

rapidly with irradiance; for high values the efficiency exceeds the value of the ideal selective thermal absorber and comes close to the value of the fully selective device. There is a remarkable change in the behavior of the curve that occurs for $\beta_s = 0.6$ at a value of the solar irradiance of about $E^{\mathrm{sol}} = 7000\,\mathrm{W\,m^{-2}}$. Two different regions, with low and with high gap energy e_G, compete for the global maximum. This behavior can also be seen in Fig. 4 and Fig. 5, where for low gap energy a local maximum occurs.

For low E^{sol}, the optimum gap energy in the high-energy regime is advantageous, since luminescence at low energies is suppressed, but the range of photon energies $[e_G, (1+\beta_t)e_G]$, where the multiplication factor κ is one, is not used efficiently. On the contrary, with the optimum gap energy in the low-energy regime, the multiplication factor κ can match the optimum chemical potential depicted in Fig. 2, and net losses in the range of photon energies $[e_G, (1+\beta_t)e_G]$ are not significant for high E^{sol}. For lower values of E^{sol}, the high-band-gap maximum is the global and the low-band-gap maximum the local maximum; for higher values of E^{sol}, the reverse is true. For $\beta_s = 0.6$ and $E^{\mathrm{sol}} = 7000\,\mathrm{W\,m^{-2}}$, the two maxima with equal efficiency are $e_G = 0.035\,\mathrm{eV}$ and $e_G = 1.2\,\mathrm{eV}$; the ratio of the currents at these maxima is about 30.

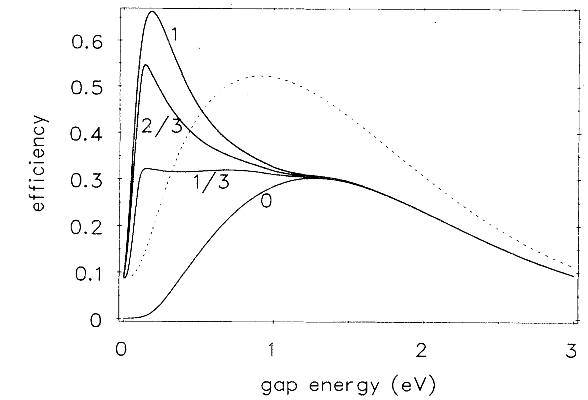

FIG. 7. Same as Fig. 5, but with absorptivity zero for photons with energy less than a certain threshold, which is chosen such as to optimize efficiency.

B. Filter

It is remarkable that the linear dependence with a threshold of the experimentally observed impact ionization, as modeled by Eq. (25), is very close the optimum multiplication factor, which matches the fully selective device as depicted in Fig. 2. The experimentally observed slope of $\beta_s = 0.3$ in Ge, however, is lower than the optimum slope of $1 - T_a/T_S = 0.95$. For matching $\kappa(e)\Delta\mu$ to the chemical potential of the radiation, $\Delta\mu$ should be $(1 - T_a/T_S)/\beta_s e_G$. Hence $\Delta\mu$ exceeds e_G, and formally, the condition for laser action is fulfilled. In theory a perfect filter could be deployed in order to suppress luminescence in the region where $\Delta\mu$ exceeds e_G, but it seems problematic in practice. However, even for $\Delta\mu < e_G$, such a filter is useful.

In Fig. 7, we show the efficiency for the same conditions as in Fig. 5, but with the assumption that the absorptivity is zero for all photons with energies below a certain threshold $e_t \in [e_G, (1+\beta_t)e_G]$, where net losses occur. The combination of carrier multiplication, low band gap and selectivity, can make a cell more efficient than the ideal selective absorber, even for a threshold β_t as high as 2. For $\beta_s \to 1$, the efficiency is close to the value of the fully selective device. Such a filter might be realized with a thin-film cell on the basis of an indirect semiconductor, where the threshold energy for impact ionization coincides with the first direct transition. Below threshold, absorption of the indirect transition is low.

V. CONCLUSIONS

Carrier multiplication by impact ionization processes offers an additional degree of freedom for improving the conversion efficiency of single-gap photovoltaic cells. The spectral behavior of the multiplication factor may be adjusted to optimize the cell to the incoming radiation. Such a cell can convert radiation to work as efficiently as a fully selective device.

Luminescence of an ideal quantum device with charge carrier multiplication is that of an ideal single-gap cell, where the chemical potential of the photons is multiplied by the number of charge carrier pairs that contribute to the emission of one photon. The possibility of carrier multiplication is always present. Indeed Si and Ge are used for particle detectors, where the number of carriers produced is taken as a measure of the energy of high-energy particles.

For photovoltaic conversion, this effect is not significant due to the high gap energy in relation to typical photon energies from solar radiation. A low-band-gap cell with carrier multiplication, in conjunction with low absorptivity immediately above gap energy or with a radiation concentration of about 100, has the potential to improve the efficiency of photovoltaic cells.

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