Hot-electron magnetophonon resonance of quantum wells in tilted magnetic fields

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The nonlinear dc magnetoconductivity of a parabolic quantum well under high tilted magnetic fields is evaluated analytically for the optical-phonon scattering, by using the formalism of the nonlinear-response theory developed previously. The results show two characteristic oscillating behaviors, as a function of the magnetic field, with electric-field-induced resonances occurring in the hot-electron regime when $P\omega_+=\omega_L^*$ and $P\omega_-=\omega_L^*$, where $\omega_\pm$ and ω_L^* are two hybrid eigenfrequencies of the system under the tilted magnetic field and the effective phonon frequencies, respectively, and P is an integer. The resonance peaks appear as either maxima or minima depending on the confinement frequency, the magnetic-field angle, and the electric field.

I. INTRODUCTION

In recent years, there has been considerable interest in understanding the hot-electron magnetophonon resonance (MPR) effects in two-dimensional (2D) or quasi-two-dimensional (Q2D) systems, since it provides useful information on the relaxative transport properties of semiconductors, such as the carrier relaxation mechanism, damping of the oscillations due to the electron-phonon interaction, the intracollisional field effect[1] (ICFE), the phonon frequencies, and the band structure (e.g., the effective mass m^*). Thus many studies[2–5] have been made of the nonlinear transport properties of these systems, associated with the hot-electron (nonlinear) MPR effects. Warmenbol, Peeters, and Devreese[2] studied ordinary and hot-electron MPR in the 2D system (formed in a single heterojunction) theoretically in the framework of the momentum-balance equation. Mori *et al.*[3] also studied the same system using the Kubo formula[6] and the Fang-Howard trial function. For the hot-electron MPR behavior in Q2D quantum-well (superlattice) structures with an infinite square-well potential, Vasilopoulos, Charbonneau, and Van Vliet[4] analyzed hot-electron MPR effects for impurity, and longitudinal optical-phonon scatterings on the basis of the theory developed by Barker,[1] Budd,[7] and Calecki, Palmer, and Chomette.[8] Recently, Suzuki[5] presented a theory of hot-electron MPR for the same model as treated by Vasilopoulos, Charbonneau, and Van Vliet,[4] by utilizing the electric-field-dependent conductivity formula defined in the Ohmic form of nonlinear electric current and the resolvent superoperator method.[9] However, their analysis has mainly focused on the case where the magnetic field is applied in the direction normal to the interface layer of the systems. We see that, in this case, one kind of Landau-level index is formed and the MPR effect arises from the resonant scattering of electrons in Landau levels by phonons. If the magnetic field is tilted with respect to the normal, it serves to add an extra confining potential to the initial confinement, gives rise to two different kinds of Landau-level indices, and causes a dramatic change in the energy spectrum, leading to so-called hybrid magnetoelectric quantization.[10,11] As a result, one may expect different behaviors of the nonlinear dc magnetoconductivity of electrons in such systems. Thus we are motivated to analyze hot-electron MPR effects of a Q2D quantum well in tilted magnetic fields.

In this paper, we present a theory of hot-electron MPR of a Q2D quantum well with parabolic potential well in tilted magnetic fields, by using the field-dependent conductivity formula[12] defined in the Ohmic form of the nonlinear current density. Here we consider a simple model for a Q2D electron gas confined in the quantum-well structures subject to the electric field $\mathbf{E}(\parallel\hat{x})$ and the magnetic field $\mathbf{B}=(B_x,0,B_z)$. For the sake of simplicity, we assume that interaction with optical phonons is the dominant scattering mechanism, in which only bulk modes are treated. Based on this model, we will evaluate the nonlinear dc magnetoconductivity and the field-dependent relaxation rate which are closely related to the hot-electron MPR effects.

The present paper is organized as follows: In Sec. II, we will describe a simple model for the system. In Sec. III, we present the nonlinear dc magnetoconductivity $\sigma_{xx}(E)$ formula related to the field-dependent relaxation rate due to the collision process. In Sec. IV, the relaxation rate for bulk optical-phonon scattering in the Q2D quantum-well structure is calculated on the basis of the nonlinear response theory[12] obtained previously. The hot-electron MPR effect is also discussed for such a system, where special attention is given to the unusual behavior of the hot-electron MPR line shape, such as

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conversion of hot-electron MPR maxima into minima or splitting of the MPR peaks, and shift of the MPR peaks. Concluding remarks are given in Sec. V.

II. MODEL FOR A QUANTUM WELL UNDER HIGH ELECTRIC FIELD

We consider a simple model for the quantum well, in which a one-dimensional electron gas is confined in a heterostructure by a narrow or split gate, and electron motions are free along two directions. We assume that a heterointerface is normal to the z axis, and that the confinement in the z direction is characterized by a parabolic potential of frequency Ω as by Balev and Vasilopoulos.[13] A static magnetic field $\mathbf{B}[=(B_x,0,B_z)]$, and a dc electric field $\mathbf{E}(\|\hat{x})$, respectively, are applied in the transverse tilt direction to the barriers of the potential well (such as realized in the heterointerface) and along the lateral direction of their walls. Then the one-particle Hamiltonian (h_{eE}) is given as

$$h_{eE}=[\mathbf{p}+e\,\mathbf{A}]^2/2m^*+m^*\Omega^2z^2/2+eEx\ , \tag{2.1}$$

where $\mathbf{p}$ and m^*, respectively, are the momentum operator and the effective mass of a conduction electron. By taking into account the Landau gauge $\mathbf{A}=(0,xB_z-zB_x,0)$ and a trial shift of the origin of coordinates, the Hamiltonian can be written as

$$h_{eE}=\frac{p_x^2}{2m^*}+\tfrac{1}{2}m^*\omega_z^2x^2+\frac{p_z^2}{2m^*}+\tfrac{1}{2}m^*\omega_1^2z^2-m^*\omega_x\omega_zxz-eEx_\lambda+\tfrac{1}{2}m^*\tilde{v}_{dz}^2\ , \tag{2.2}$$

where $\omega_x=eB_x/m^*=\omega_c\cos\theta$, $\omega_z=eB_z/m^*=\omega_c\sin\theta$, $\omega_1^2=\omega_x^2+\Omega^2$, and $\tilde{v}_{dz}=\omega_1E(B_z\Omega)^{-1}$. The eigenstates are of the form $\exp(ik_yy)\chi(x-x_\lambda,z-z_\lambda)$, where $x_\lambda=-\hbar k_y(m^*\omega_z)^{-1}-\omega_1^2eE(m^*\omega_z^2\Omega^2)^{-1}$, $z_\lambda=-\omega_xeE(m^*\omega_z\Omega^2)^{-1}$, $\chi(x,z)$ is the wave function for the $x-$ and $z-$ directions, and k_y is the wave vector in the y direction. We can see that Eq. (2.2) represents two coupled harmonic oscillators; hence Eq. (2.2) can easily be diagonalized by an appropriate rotation of coordinates x and z as by Ihm *et al.*[10] and Ryu, Hu, and O'Connell:[11]

$$\begin{bmatrix}x\\z\end{bmatrix}=\frac{1}{\omega_{+-}}\begin{bmatrix}\omega_{+1} & \omega_{-1}\\-\omega_{+z} & -\omega_{-z}\end{bmatrix}\begin{bmatrix}X\\Z\end{bmatrix}. \tag{2.3}$$

Here $\omega_{\pm\mp}^2=\omega_\pm^2-\omega_\mp^2$, $\omega_{\pm1}^2=\omega_\pm^2-\omega_1^2$, and $\omega_{\pm z}^2=\omega_\pm^2-\omega_z^2$, where $2\omega_\pm^2=\omega_c^2+\Omega^2\pm[(\omega_c^2-\Omega^2)^2+4\Omega^2\omega_c^2\cos^2\theta]^{1/2}$. The angle of rotation α is related to the above in that $\sin2\alpha=2\omega_x\omega_z\omega_{-+}^{-2}$. Then the resulting Hamiltonian and the corresponding normalized eigenfunctions and eigenvalues, respectively, are given as

$$h_{eE}=\frac{p_X^2}{2m^*}+\tfrac{1}{2}m^*\omega_+^2X^2+\frac{p_Z^2}{2m^*}+\tfrac{1}{2}m^*\omega_-^2Z^2-eEx_\lambda+\tfrac{1}{2}m^*\tilde{v}_{dz}^2\ , \tag{2.4}$$

$$|n,l,k_y\rangle=(1/L_y)^{1/2}\Phi_n(X-X_\lambda)\times\Phi_l(Z-Z_\lambda)\exp(ik_yy)\ , \tag{2.5}$$

$$E_{nl}(k_y)=(n+1/2)\hbar\omega_++(l+1/2)\hbar\omega_--eEx_\lambda+\tfrac{1}{2}m^*\tilde{v}_{dz}^2\ , \tag{2.6}$$

with $X_\lambda=(\omega_{-z}^2/\omega_{+-}\omega_{+1})x_\lambda+(\omega_{+z}/\omega_{+-})z_\lambda$ and $Z_\lambda=-(\omega_{+z}^2/\omega_{+-}\omega_{-1})x_\lambda-(\omega_{-z}/\omega_{+-})z_\lambda$, where $n(=0,1,2,\dots)$ and $l(=0,1,2,\dots)$, respectively, are the Landau-level indices due to the tilted magnetic field, and $\Phi_n(X)$ and $\Phi_l(Z)$ represent harmonic-oscillator wave functions. The first and second terms in Eq. (2.6) are the quantized kinetic energy in the presence of a tilted magnetic field, whereas the third and fourth terms in Eq. (2.6), respectively, are the potential and kinetic energy occurring from the electronic motion in the presence of an electric field. In the absence of the field, these terms become zero. Therefore, the effect of including the electric field in the electron Hamiltonian (2.1) is to remove the k_y degeneracy of the energy spectrum and to shift the center positions by $\omega_1^2eE(m^*\omega_z^2\Omega^2)^{-1}$ and $\omega_xeE(m^*\omega_z\Omega^2)^{-1}$ in the x and z directions, respectively. As shown in Eqs. (2.5) and (2.6), the electron energy spectrum in the Q2D quantum well is hybrid quantized by the confinements in the z directions and the tilted magnetic field, and the set of quantum numbers is designated by (n,l,k_y). The dimensions of the sample are assumed to be $V=L_xL_yL_z$. In the following, we will utilize Eqs. (2.5) and (2.6) to obtain the transverse magnetoconductivity analtyically. It is interesting to note that the dependence of the single-electron energy spectrum in Eq. (2.6) on the confinement frequency, the electric field, and the magnetic-field angle has an important effect on the nonlinear dc magnetoconductivity and the field-dependent relaxation rates, as well as on the hot-electron MPR effects for a Q2D quantum well. A detailed discussion of these effects will be given explicitly in Secs. III and IV.

III. FIELD-DEPENDENT MAGNETOCONDUCTIVITY

We want to evaluate the nonlinear dc magnetoconductivity $\sigma_{xx}(E)$ for the quasi-two-dimensional electron-gas (Q2DEG) system, subject to the electric field $\mathbf{E}(\|\hat{x})$ and magnetic field $\mathbf{B}[=(B_x,0,B_z)]$, by taking into account the general expression for the nonlinear dc conductivity $\sigma_{i,j}(E)(i,j=x,y,z)$ given in Eq. (4.38) of Ref. 12 and considering the following matrix elements in representation (2.5):

$$|\langle k_y,l,n|j_x|n',l',k_{y'}\rangle|^2=\frac{e^2\omega_+^2\tilde{l}_{B+}^2}{2}[n\delta_{n'n-1}+(n+1)\delta_{n'n+1}]\delta_{ll'}\delta_{k_y,k_{y'}}+\frac{e^2\omega_-^2\omega_{-1}^2\tilde{l}_{B-}^2}{2\omega_{-z}^2}[l\delta_{l'l-1}+(l+1)\delta_{l'l+1}]\delta_{nn'}\delta_{k_y,k_{y'}}\ , \tag{3.1}$$

where $j_x=-(e/m^*)p_x=-(e/m^*)[(\omega_{+1}/\omega_{+-})P_X+(\omega_{-1}/\omega_{+-})P_Z]$ is the x component of a single-electron current operator, $\tilde{l}_{B+}=(\omega_{+1}/\omega_{+-})l_{B+}$, $l_{B+}=(\hbar/m^*\omega_+)^{1/2}$, $\tilde{l}_{B-}=(\omega_{-z}/\omega_{+-})l_{B-}$, $l_{B-}=(\hbar/m^*\omega_-)^{1/2}$, and the Kronecker symbols $(\delta_{n'n\pm1},\delta_{l'l\pm1},\delta_{n'n},\delta_{l'l},\delta_{k'_y,k_y})$

denote the selection rules which arise during the integration of the matrix elements for each direction. It should be noted that the matrix element of the current operator in Eq. (3.1) is directly proportional to the nonlinear dc magnetoconductivity, which contains two types of contribution as follows: one corresponding to the first term of the right-hand side in Eq. (3.1) is related to the current carried by the electron hopping motion between the localized cyclotron orbits, i.e, n and $n\pm1$; and the other corresponding to the second term is caused by the current carried by electron hopping motion between the localized cyclotron orbitals, i.e., l and $l\pm1$.

For the calculation of the nonlinear dc magnetoconductivity $\sigma_{xx}(E)$ for the Q2D quantum well, we apply the general expression for the electric-field-dependent dc conductivity $\sigma_{ij}(E)(i,j=x,y,z)$ given in Eq. (4.38) of Ref. 12 to the quantum well modeled in Sec. II, by using the selection rules of Eq. (3.1) and replacing the λ_1 and λ_2 states with the representation (2.5). Then $\sigma_{xx}(E)$ can be obtained easily by the sum of the two hopping parts: $\sigma_{xx}^{nh}(E)$, due to the Landau-level index n; and $\sigma_{xx}^{lh}(E)$, due to the Landau-level index l. These are

$$\sigma_{xx}^{nh}(E)\approx\frac{e^2\tilde{l}_{B+}^2}{V\hbar^2\omega_+}\sum_{n,l,k_y}(n+1)[f(E_{nl}^0(k_y))-f(E_{n+1l}^0(k_y))]\tilde{\Gamma}(n+1,l,k_y;n,l,k_y) \tag{3.2}$$

$$\approx\frac{m^*e^2\omega_z\tilde{l}_{B+}^2}{2\pi\hbar^3\omega_+L_z}\frac{\exp(\beta_eE_F)}{\sinh(\beta_e\hbar\omega_+/2)\sinh(\beta_e\hbar\omega_-/2)}\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)\ , \tag{3.3}$$

$$\sigma_{xx}^{lh}(E)\approx\frac{e^2\omega_{-1}^2\tilde{l}_{B-}^2}{V\hbar^2\omega_-\omega_{-z}^2}\sum_{n,l,k_y}(l+1)[f(E_{nl}^0(k_y))-f(E_{nl+1}^0(k_y))]\tilde{\Gamma}(n,l+1,k_y;n,l,k_y) \tag{3.4}$$

$$\approx\frac{m^*e^2\omega_z\omega_{-1}^2\tilde{l}_{B-}^2}{2\pi\hbar^3\omega_-L_z\omega_{-z}^2}\frac{\exp(\beta_eE_F)}{\sinh(\beta_e\hbar\omega_+/2)\sinh(\beta_e\hbar\omega_-/2)}\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)\ , \tag{3.5}$$

where $f(E_{nl}^0(k_y))$ is the Fermi-Dirac distribution function with $E_{nl}^0(k_y)=(n+1/2)\hbar\omega_++(l+1/2)\hbar\omega_-$, and the quantities $\tilde{\Gamma}$, which appear in terms of the collision broadening due to the electron-phonon interaction, play the role of the width in the spectral line shape. We assumed that $\tilde{\Gamma}\ll\hbar\tilde{\omega}_\pm[=E_{n+1l}(k_y)-E_{nl}(k_y), E_{nl+1}(k_y)-E_{nl}(k_y)]$, and that the shift becomes zero, to observe the oscillatory behavior of the hot-electron MPR as some other authors did.[4,5] To obtain the nonlinear dc magnetoconductivity of Eqs. (3.3) and (3.5), we assumed that the f's in Eqs. (3.2) and (3.4) are replaced by the Boltzmann distribution function for nondegenerate semiconductors, i.e, $f(E_{nl}^0(k_y))\approx\exp[\beta_e(E_F-E_{n,l}^0(k_y))]$, where $\beta_e=1/k_BT_e$, with k_B being Boltzmann's constant, T_e the hot-electron temperature, and E_F the Fermi energy. We further performed the sum over n (if n is large) by writing $\sum n\exp(-\alpha n)=-(\partial/\partial\alpha)\sum\exp(-\alpha n)$, summing the geometric series, and carrying out the one summation with respect to k_y in $\sum_{n,l,k_y}$ in terms of the following relation:[5]

$$\sum_{k_y}(\cdots)\rightarrow(L_y/2\pi)\times\int_{-m^*\omega_zL_x/2\hbar-eE\omega_1^2/\hbar\omega_z\Omega^2}^{m^*\omega_zL_x/2\hbar-eE\omega_1^2/\hbar\omega_z\Omega^2}dk_y(\cdots)\ , \tag{3.6}$$

since the upper and the lower limits are obtained from the fact that the electrons should be within the crystal dimensions in the x direction, i.e., $-L_x/2\le x\le L_x/2$. It should be noted that $\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$ [or $\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)$] is referred to as the field-dependent relaxation rate associated with the states $n+1$ (or $l+1$) and n (or l) since the field-dependent relaxation (or collision) time $\tau(E)$ can be estimated from $\tau(E)\approx\hbar/\tilde{\Gamma}$, and also that it depends on the confinement frequency, the electric field, and the magnetic-field angle since these parameters are included in the eigenstate energy $E_{nl}(k_y)$. In particular, the dependence of $\tilde{\Gamma}$ on the applied electric field leads to the field-induced electronic relaxation process known as ICFE, i.e., the accelerating effect of the electric field.[1,9,12] As seen from Eqs. (3.3) and (3.5), the nonlinear dc magnetoconductivity $\sigma_{xx}(E)$ is closely related to the field-dependent relaxation rate $\tilde{\Gamma}$. Thus the electronic transport properties (e.g., electronic relaxation processes, ICFE, ordinary and hot-electron magnetophonon resonances, etc.) in the Q2D quantum-well structures can be studied by examining the behavior of $\tilde{\Gamma}$ as a function of relevant physical parameters introduced in the theory. The general form of the field-dependent relaxation rate $\tilde{\Gamma}$ is given in Ref. 12, which was obtained for both the weak- and strong-coupling cases with respect to the electron-background (phonon and/or impurity) interaction. In this paper, we will use the general form of the weak-coupling case since that for the strong-coupling case is so complicated that we cannot evaluate the field-dependent relaxation rate analytically. However, the δ-function singularities in $\tilde{\Gamma}$, appearing when the general relaxation rate $\tilde{\Gamma}$ for the weak-coupling case is taken into account, can be removed by introducing an appropriate parameter, as will be clarified later. The detailed description will be given in Sec. IV.

IV. ELECTRIC-FIELD-INDUCED MAGNETOPHONON RESONANCES IN TILTED MAGNETIC FIELDS

For the calculation of the field-dependent relaxation rates $\tilde{\Gamma}$ in Eqs. (3.3) and (3.5), we choose the interaction potential $C(q)$ introduced by Fröhlich:[4,5] $|C(q)|^2=Df(q)/V$, with $f(q)=1/q^2$ for polar-LO-phonon scattering and $f(q)=1$ for nonpolar-LO-phonon scattering, where D and α are the constant of the polar interaction and the dimensionless polaron coupling constant, respectively, where we assume that the phonons are dispersionless (i.e,. $\hbar\omega_{\mathbf{q}}\approx\hbar\omega_L\approx$const, where ω_L is the optical-phonon frequency) and bulk (i.e., three-dimensional). In other words, we have neglected any changes in the electron-phonon interaction brought about by the Q2D confinement of the electrons and the surface roughness effect. Furthermore, we need the following matrix elements in the representation (2.5):

$$|\langle k_y,l,n|\exp(\pm i\mathbf{q}\cdot\mathbf{r})|n',l',k_y'\rangle|^2 =|J_{nn'}(u_1)|^2|J_{ll'}(u_2)|^2\delta_{k_y,k_y'\pm q_y}\ , \quad (4.1)$$

$$|J_{nn'}(u_1)|^2=\frac{n_n!}{n_m!}e^{-u_1}u_1^{n_m-n_n}[L_{n_n}^{n_m-n_n}(u_1)]^2\ , \quad (4.2)$$

where $n_n=\min\{n,n'\}$, $n_m=\max\{n,n'\}$, $u_1=\tilde{l}_{B+}^2(q_x^2+b_1^2q_y^2)/2$, $b_1=\omega_+\omega_{-z}^2/\omega_{+1}^2\omega_z$, $u_2=\tilde{l}_{B-}^2(q_z^2+b_2^2q_y^2)/2$, and $b_2=\omega_-\omega_{+z}^2/\omega_z^2\omega_x$, and where $L_n^m(u)$ is the associated Laguerre polynomial.[14] The detailed derivation of the relaxation rate and its general expression in the lowest-order approximation for the weak-coupling case of an electron-phonon system can be seen in Eq. (4.39) of Ref. 12. Transforming the sum over $\mathbf{q}$ in Eq. (4.39) of Ref. 12 into an integral form in the usual way, and considering Eq. (3.6) and the interaction potential for optical-phonon scattering, the Q2D version of this quantity associated with the electronic transition between the states $|n,l,k_y\rangle$ and $|n_1,l_1,k_{1y}\rangle$ can be evaluated as

$$\tilde{\Gamma}(n_1,l_1,k_{1y};n,l,k_y) =\frac{m^*\omega_z AD}{2(2\pi)^3\hbar}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dq_x dq_y dq_z f(q) \times\Bigg\{\sum_{(n',l')\neq(n_1,l_1)}|J_{n_1n'}(u_1)|^2|J_{l_1l'}(_2)|^2 \times(N_0+\tfrac{1}{2}\pm\tfrac{1}{2})\delta[(n-n')\hbar\omega_+ +(l-l')\hbar\omega_- \mp S(q_y)\mp\hbar\omega_L] + \sum_{(n',l')\neq(n,l)}|J_{n'n}(u_1)|^2|J_{l'l}(u_2)|^2 \times(N_0+\tfrac{1}{2}\pm\tfrac{1}{2})\delta[(n'-n_1)\hbar\omega_+ +(l'-l_1)\hbar\omega_- \pm S(q_y)\pm\hbar\omega_L]\Bigg\}\ , \quad (4.3)$$

where $A=L_xL_y$, $S(q_y)=\hbar Eq_y/B_z$, n' and l' indicate the intermediate localized Landau level indices, and N_0 is the optical-phonon distribution function given by $N_{\mathbf{q}}=[\exp(\beta\hbar\omega_{\mathbf{q}})-1]^{-1}$ with $\omega_{\mathbf{q}}=\omega_L$. Here $\beta=1/k_BT$, T being the (lattice) temperature. It should be noted that the Landau-level indices n_1 and l_1 given in Eq. (4.3), respectively, are replaced by $n+1,l$ and $n,l+1$ in Eqs. (3.3) and (3.5). The energy-conserving δ functions in Eq. (4.3) imply that when the electron undergoes a collision by absorbing energy from the field, its energy can only change by an amount equal to the energy of a phonon involved in the transitions. This in fact leads to electric-field-induced magnetophonon resonance (EFIMPR) effects due to the Landau levels.[5] The remarkable thing is that two kinds of EFIMPR effect arise from two different Landau-level indices n and l. We can see these effects from the conditions $(n',l')\neq(n,l)$ and $(n',l')\neq(n_1,l_1)$ in the summations of Eq. (4.3), which give rise to two cases: (1) $\sum_{n'\neq n_1}\sum_{l'}$ and $\sum_{n'\neq n}\sum_{l'}$ leading to the EFIMPR effect due to the Landau level n, whereby $\hbar\omega_+\gg\tilde{\Gamma}$ is satisfied; and (2) $\sum_{n'}\sum_{l'\neq l_1}$ and $\sum_{n'}\sum_{l'\neq l}$, leading to the EFIMPR effect due to the Landau level l, whereby $\hbar\omega_-\gg\tilde{\Gamma}$ is satisfied since the EFIMPR in the Q2D quantum-well structure is due essentially to the electric-field-induced inter-Landau-level (inelastic resonant phonon) scattering.

First let us now calculate the field-dependent relaxation rate $\tilde{\Gamma}$ associated with the EFIMPR effect due to the Landau-level index n. As shown in Eq. (4.3), the field-dependent relaxation rate $\tilde{\Gamma}$ involves integrations with respect to q_x, q_y, and q_z in Cartesian coordinates. The integral over q_x, q_y, and q_z is very difficult to evaluate analytically since it must be done separately for each n and n'. So, to simplify the calculations, we replace $S(q_y)$ in the argument of the δ function by the potential-energy difference $eE\Delta\bar{x}$ across the spatial extent $\Delta\bar{x}$ of a Landau state as some authors did.[4,5] To obtain the field-dependent relaxation rate given in a simple form, we further assume[4,5] that n and n' are very large. We can then make an approximation that $n'\pm1\approx n'$. Setting $n'-n=-P$ in the emission term and $n'-n=P$ in the absorption term, and noting that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dq_x dq_y dq_z f(q)|J_{n\pm Pn}(u_1)|^2|J_{l'l}(u_2)|^2 \equiv F_{ll'}(P) \quad (P=1,2,3,\ldots) ,$$

we obtain, for the electron hopping motion due to the Landau-level index n,

$$\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$$

$$\approx \Lambda_1 \sum_P \left\{ (2N_0+1)F_{ll}(P)\delta[P-\omega_L^*/\omega_+] + \sum_{l'=1}\sum_{l'\neq l} (N_0+\tfrac{1}{2}\pm\tfrac{1}{2})F_{ll'}(P)\delta[P-(\omega_L^*\pm\Delta l\omega_-)/\omega_+] \right\} , \quad (4.4)$$

$$\approx \Lambda_1(2N_0+1)\mathrm{Re}\left\{F_{ll}\left[\frac{i\gamma+\hbar\omega_L^*}{\hbar\omega_+}\right]\right\}\Psi\left[\frac{\gamma}{\hbar\omega_+},\frac{\omega_L^*}{\omega_+}\right] + \Lambda_1\sum_{l'\neq l}(N_0+\tfrac{1}{2}\pm\tfrac{1}{2})\mathrm{Re}\left\{F_{ll'}\left[\frac{i\gamma+\hbar(\omega_L^*\pm\Delta l\omega_-)}{\hbar\omega_+}\right]\right\}$$
$$\times\Psi\left[\frac{\gamma}{\hbar\omega_+},\frac{\omega_L^*\pm\Delta l\omega_-}{\omega_+}\right] , \quad (4.5)$$

where $\Lambda_1 = m^*\omega_z AD/((2\pi)^3\hbar^2\omega_+)$, $\Delta l = l'-l$, $\omega_L^* = \omega_L + eE\Delta\bar{x} \approx \omega_L + eE\sqrt{\hbar/m^*\omega_L}$,[5] and Re{ $\cdots$ } means "the real part of." To obtain Eq. (4.5), we have replaced the δ functions in Eq. (4.4) by Lorentzians with a width parameter γ,[4,5] applied Poisson's summation formula[15] for $\sum_p$ in Eq. (4.4), and taken into account the property[4,5]

$$\Psi(a,b) = 1 + 2\sum_{s=1}^{\infty} e^{-2\pi sa}\cos(2\pi sb) = \frac{\sinh(2\pi a)}{\cosh(2\pi a)-\cos(2\pi b)} \quad (a>0) . \quad (4.6)$$

We see from Eq. (4.4) that the nonlinear dc magnetoconductivity (3.3) associated with the field-dependent relaxation rates shows the resonant behaviors EFIMPR at $P\omega_+ = \omega_L^*$, and $\omega_L^* \pm \Delta l\omega_-$ (P is an integer) for $\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$. It is very interesting to point out that additional EFIMPR peaks (subsidiary peaks) appear whenever the nonresonant Landau-level transition ($l\rightarrow l'$) can take place for the relevant separation between the Landau levels. Similarly, the field-dependent relaxation rate associated with the EFIMPR effect due to the electron-hopping motion for the Landau-level index l is given by

$$\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$$

$$\approx \Lambda_2 \sum_P \left\{ (2N_0+1)F_{nn}(P)\delta[P-\omega_L^*/\omega_-] + \sum_{n'\neq n} (N_0+\tfrac{1}{2}\pm\tfrac{1}{2})F_{nn'}(P)\delta[P-(\omega_L^*\pm\Delta n\omega_+)/\omega_-] \right\} , \quad (4.7)$$

$$\approx \Lambda_2(2N_0+1)\mathrm{Re}\left\{F_{nn}\left[\frac{i\gamma'+\hbar\omega_L^*}{\hbar\omega_-}\right]\right\}\Psi\left[\frac{\gamma'}{\hbar\omega_-},\frac{\omega_L^*}{\omega_-}\right]$$
$$+\Lambda_2\sum_{n'\neq n}(N_0+\tfrac{1}{2}\pm\tfrac{1}{2})\mathrm{Re}\left\{F_{nn'}\left[\frac{i\gamma'+\hbar(\omega_L^*\pm\Delta n\omega_+)}{\hbar\omega_-}\right]\right\}\Psi\left[\frac{\gamma'}{\hbar\omega_-},\frac{\omega_L^*\pm\Delta n\omega_+}{\omega_-}\right] , \quad (4.8)$$

where

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dq_x dq_y dq_z f(q)|J_{n'n}(u_1)|^2|J_{l\pm Pl}(u_2)|^2 \equiv F_{nn'}(P)$$

with $P=l'-l$, $\Lambda_2 = m^*\omega_z AD/((2\pi)^3\hbar^2\omega_-)$, and $\Delta n = n'-n$. We assumed[5] that $\gamma_i=\gamma$ ($i=1$, 2, and 3) for the collision-damping terms in Eq. (4.5) and $\gamma_i=\gamma'$ ($i=4$, 5, and 6) for that in Eq. (4.8). In this case, the nonlinear dc magnetoconductivity, Eq. (3.3), associated with the field-dependent relaxation rates shows the resonant behaviors EFIMPR at $P\omega_- = \omega_L^*, \omega_L^* \pm \Delta n\omega_+$ (P is an integer) for $\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$. If the weak-field limit ($E\rightarrow 0$) is taken in Eqs. (4.4), (4.5), (4.7), and (4.8) (i.e., $\omega_L^* = \omega_L$), these equations give the resonant magnetic fields for the ordinary MPR effect. Equations (4.5) and (4.8) give the general description of the EFIMPR oscillations due to the Landau-level indices n and l in the Q2D quantum-well structure. For the EFIMPR effect due to the Landau-level index n, the first and second terms of Eq. (4.5), respectively, show that the period of the oscillation is given under the condition of $P=\omega_L^*/\omega_+$ and exhibits additional complexity of oscillations with the subsidiary (EFIMPR) peaks appearing in terms of $P\omega_+ = \omega_L^* \pm \Delta l\omega_-$. The field-dependent relaxation rates of Eq. (4.8) associated with the EFIMPR effect due to the Landau-level index l have another oscillatory period $P=\omega_L^*/\omega_-$ and the subsidiary (EFIMPR) peaks appear at $P\omega_- = \omega_l^* \pm \Delta n\omega_+$. The peak positions of these oscillations also depend on the confinement frequency, the magnetic-field angle, and the electric field, as well as the relevant energy separation between the Landau-level indices. It should be noted that the field-dependent relaxation rate $\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)$ gives the same results [Eqs. (4.5) and (4.8)] if the intermediate Landau-level indices n' and l' are large enough.

To visualize the series of resonance positions, in Figs.

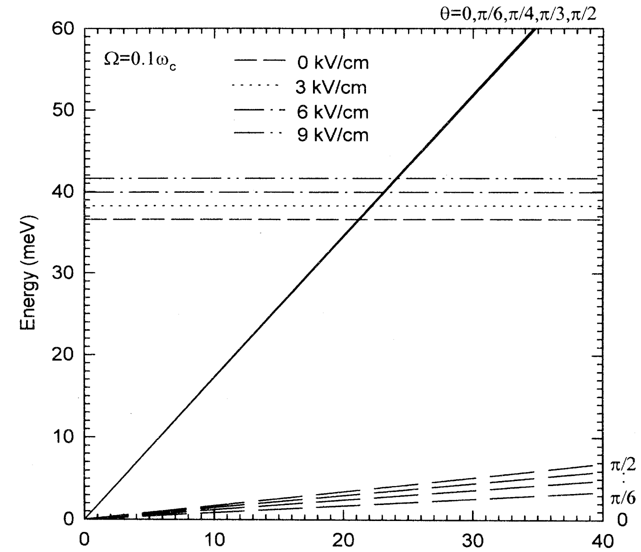

FIG. 1. Energy diagram for $\Omega=0.1\omega_c$. The magnetic-field angle is indicated for the long-dashed and solid lines, which are for $\hbar\omega_-$ and $\hbar\omega_+$, respectively. The dash-double-dotted, dash-dotted, dotted, and medium dashed lines are for $\hbar\omega_L^*$ depending on the strength of the electric field.

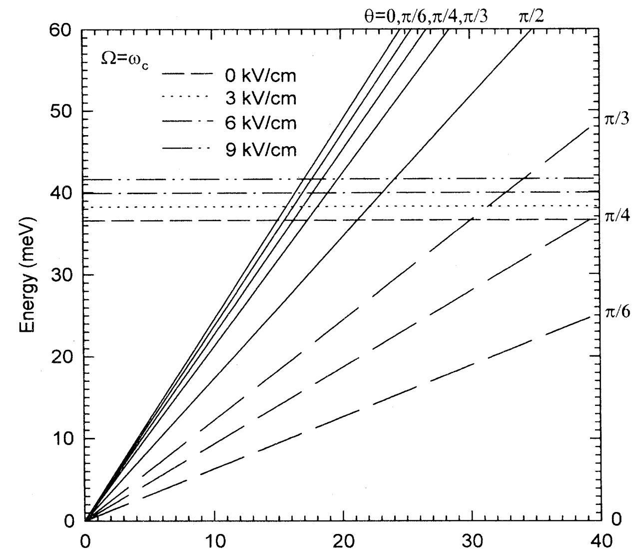

FIG. 3. Energy diagram for $\Omega=\omega_c$. The magnetic-field angle is indicated for the long-dashed and solid lines, which are for $\hbar\omega_-$ and $\hbar\omega_+$, respectively. The dash-double-dotted, dash-dotted, dotted, and medium dashed lines are for $\hbar\omega_L^*$ depending on the strength of the electric field.

1, 2, 3, and 4 we plotted the energies of the initial and final states given by $P\hbar\omega_\pm=\hbar\omega_L^*$, where, for simplicity, we will consider the case of $P=1$. It is shown that the crossing points give the resonance fields, which depend on the confinement frequency, the magnetic-field angle, and the electric field. For the numerical results presented in this paper, material constants are taken for GaAs, i.e., $m^*/m_e=0.067$ and $\hbar\omega_L=36.6$ meV. As shown in Figs. 1 and 4, the resonance peak positions at $\hbar\omega_+=\hbar\omega_L^*$ for weak ($\Omega=0.1\omega_c$ and below) and strong ($\Omega=5\omega_c$ and above) confinement frequencies are nearly unaffected by the magnetic-field angle, whereas those at $\hbar\omega_-=\hbar\omega_L^*$ depend on the magnetic-field angle. These peak positions are shifted to the higher-magnetic-field side from the or-

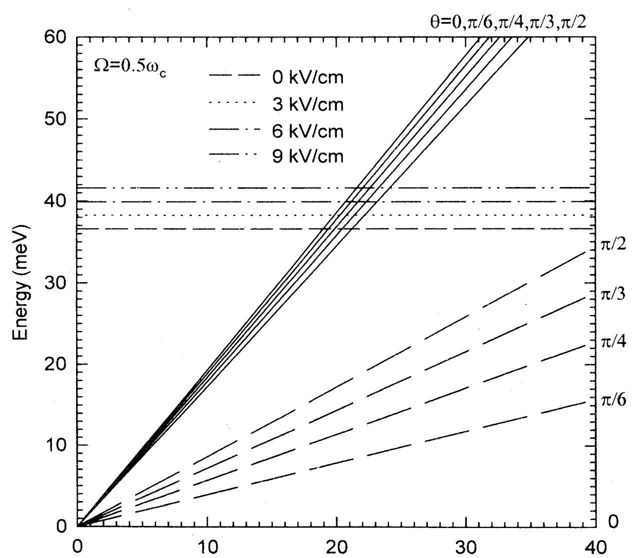

FIG. 2. Energy diagram for $\Omega=0.5\omega_c$. The magnetic-field angle is indicated for the long-dashed and solid lines, which are for $\hbar\omega_-$ and $\hbar\omega_+$, respectively. The dash-double-dotted, dash-dotted, dotted, and medium dashed lines are for $\hbar\omega_L^*$ depending on the strength of the electric field.

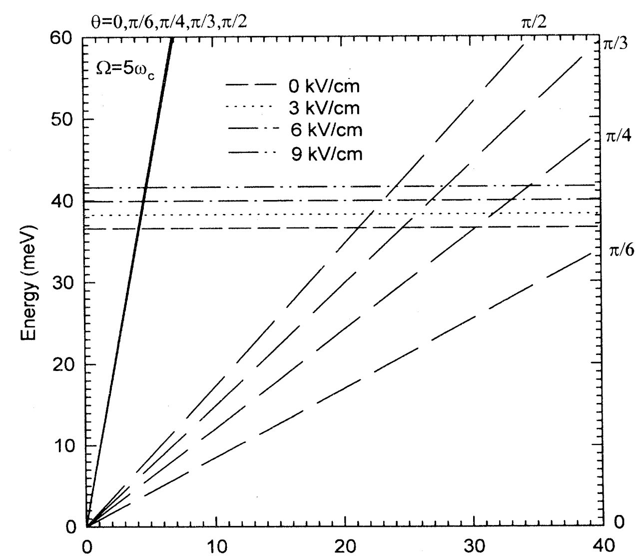

FIG. 4. Energy diagram for $\Omega=5\omega_c$. The magnetic-field angle is indicated for the long-dashed and solid lines, which are for $\hbar\omega_-$ and $\hbar\omega_+$, respectively. The dash-double-dotted, dash-dotted, dotted, and medium dashed lines are for $\hbar\omega_L^*$ depending on the strength of the electric field.

dinary MPR peaks at $\omega_{\pm}=\omega_L$ as the strength of electric fields increases. For strong confinement frequency $\Omega \geq \omega_c$ the resonance peaks associated with $\hbar\omega_-$ can appear in the case of $P=1$ or above, while for weak confinement frequency these peaks can occur at higher values than $P=1$. Unlike the case of Figs. 1 and 4, the peak positions at $\hbar\omega_+=\hbar\omega_L^*$ in Figs. 2 and 3 depend on the magnetic-field angle. Unfortunately, we are not aware of any relevant experimental and theoretical work to compare our theory with. Therefore, to test the validity of this prediction, additional experiments and theories are needed.

V. CONCLUDING REMARKS

So far, we have presented a theory of hot-electron MPR and investigated the physical characteristics of the EFIMPR effect in the Q2D quantum-well structure, where a Q2DEG confined by the parabolic potential well in the z direction is subjected to electric ($\mathbf{E}\|\hat{x}$) and magnetic fields $\mathbf{B}=(B_x,0,B_z)$. The origin of this formalism[12] dates back to the discovery of the theory of nonlinear static conductivity. On the basis of this formalism, the field-dependent relaxation rate for the weak-coupling case has been utilized with respect to the electron–LO-phonon interaction, and its behavior (relaxative transport process) has been discussed in connection with the EFIMPR effect.

It is shown from Eqs. (3.4) and (3.5) that the nonlinear dc magnetoconductivity σ_{xx} appears in the form of two types of contributions associated with the selection rulers of the current-density operator. The nonlinear dc magnetoconductivities σ_{xx}^{nh} and σ_{xx}^{lh}, respectively, are directly proportional to the field-dependent relaxation rates $\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$ and $\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)$ for the electron-hopping motion due to the Landau-level indices n and l. It should be noted that the field-dependent relaxation rates for the electron-hopping motion are closely related to the directionality of the magnetic field, since one of the two relaxation rates for the electron-hopping motion disappears if the magnetic field is applied in a specific (x or z) direction. For the EFIMPR effect due to the Landau-level index n, the field-dependent relaxation rates have oscillatory period $P=\omega_L^*/\omega_+$, and subsidiary EFIMPR peaks, which exhibit additional complexity of oscillations, appearing at $P\omega_+=\omega_L^*\pm\Delta l\omega_-$. On the other hand, the field-dependent relaxation rates associated with the EFIMPR effect due to the Landau-level index l have another oscillatory period $P=\omega_L^*/\omega_-$ and subsidiary EFIMPR peaks appearing at $P\omega_-=\omega_L^*\pm\Delta n\omega_+$, where P is an integer. Note that if the magnetic field is applied in a specific direction ($\theta=0$ or 90°), one of the two different EFIMPR peaks disappears, since, in this case, the eigenvalues in Eq. (2.6) contain the Landau-level index n or l alone. As seen from Figs. 1, 2, 3, and 4, the EFIMPR peak positions are closely related to the confinement frequency, the magnetic-field angle, and the strength of the electric field. The resonance peak positions at $\hbar\omega_+=\hbar\omega_L^*$ for weak ($\Omega=0.1\omega_c$ below) and strong ($\Omega=5\omega_c$ above) confinement frequencies are nearly unaffected by the magnetic-field angle, whereas those at $\hbar\omega_-=\hbar\omega_L^*$ depend on the magnetic-field angle. These peak positions are shifted to the higher-magnetic-field side from the ordinary MPR peaks at $\omega_{\pm}=\omega_L$ as the strength of electric fields increases. For strong confinement frequency $\Omega\geq\omega_c$ the resonance peaks associated with $\hbar\omega_-$ can appear in the case of $P=1$ or above, while for weak confinement frequency these peaks can occur at higher values than $P=1$. It is also shown from Figs. 2 and 3 that the peak positions at $\hbar\omega_+=\hbar\omega_L^*$ depend on the magnetic-field angle. It is noted that our results for the field-dependent relaxation rate and the nonlinear dc magnetoconductivity are based on an approximation in which the $S(q_y)$ terms of Eq. (4.3) have been replaced by the potential-energy difference $eE\Delta\bar{x} \approx eE\sqrt{\hbar/m^*\omega_L}$ as in Ref. 5, and that we have not taken into account any modification of the electron-phonon interaction brought about by the confinement of phonons (we used the interaction for bulk phonons). Furthermore, any analytical expression for the integration over $\mathbf{q}$ of Eqs. (4.5) and (4.8) has not been made for $F_{ll'}(P)$ and $F_{nn'}(P)$ since we are interested in the resonant behaviors alone. In conclusion, we can expect that our results can help understand qualitatively the physical characteristics on the EFIMPR effect of the Q2D quantum-well structure in tilted magnetic fields.

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