Exchange and correlation in the quasi-one-dimensional electron gas: The local-field correction

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The local-field correction for the quasi-one-dimensional electron gas in cylindrical quantum wires with wire radius R_0 is calculated within the sum-rule approach of the self-consistent theory of Singwi, Tosi, Land, and Sjölander. The local-field correction is expressed by a generalized Hubbard form with two coefficients, which are determined self-consistently. Numerical results for the exchange energy and the correlation energy are presented for $0 < r_s < 20$, where r_s is the random-phase-approximation parameter. We find that the exchange energy in the low-density regime is strongly enhanced compared to two and three dimensions: $\varepsilon_{ex}(r_s \to \infty) \propto -\ln(r_s)/r_s$. For high density we find $\varepsilon_{ex}(r_s \to 0) \propto -a^*/R_0$, where a^* is the Bohr radius. For the correlation energy we get $\varepsilon_{cor}(r_s \to 0) \propto -r_s^2/R_0^2$. The local-field correction strongly reduces the correlation energy for small carrier density if compared with the random-phase approximation. We study the pair-correlation function, the plasmon dispersion, and the compressibility, and we describe the effects of exchange and correlation on these quantities. The parameter for weak coupling in wire systems is described by $R_s = 4r_s a^*/\pi R_0 < 1$ and strong coupling corresponds to $R_s > 1$.

I. INTRODUCTION

The random-phase approximation (RPA) is a very efficient theory to describe the dielectric properties of the interacting electron gas in the high-density limit.¹ Plasmons and electron-hole excitations are described by the RPA. The local-field correction (LFC) takes into account corrections to the RPA due to the effects of exchange and correlation and becomes important at low carrier density.² A self-consistent approach for the LFC and the static-structure factor (SSF) was proposed by Singwi, Tosi, Land, and Sjölander (STLS) (Ref. 3) and was used to calculate the LFC for the three-dimensional,³ the two-dimensional⁴ and the one-dimensional electron gas.^{5,6} For a review on the STLS approach, see Ref. 7.

The LFC determines the dielectric function, the plasmon dispersion, the ground-state energy, the compressibility, and the chemical potential of the interacting electron gas. Monte Carlo computations have been used to study many-body effects in the three-dimensional⁸ and in the two-dimensional electron gas.⁹ The ground-state energies have been calculated with high precision. Good agreement has been found for the ground-state energy calculated within the STLS approach and Monte Carlo calculations for $r_s < 20$. r_s is the RPA parameter. For one-dimensional electron gases, such calculations have been performed only recently for an oscillator confinement, however, in an external periodic potential.¹⁰

A sum-rule version of the STLS approach was recently used to calculate the LFC of a Bose condensate¹¹ and an electron gas¹² at temperature zero in two and three dimensions. In this approach, the LFC is described by an analytical form with two coefficients (two-sum-rule approach), which are calculated using the STLS approach for wave numbers $q \rightarrow 0$ and ∞ . The analytical form of the LFC represents a generalization of the Hubbard form¹³ of the LFC and takes into account exchange and correlation. In Hubbard's original expression for a three-dimensional electron gas, only exchange effects are taken into account. When the two coefficients of the sum-rule approach are known, the LFC is given in an analytical form and can be used in more complex calculations, for instance, the calculation of the plasmon dispersion. In this paper, we describe the LFC for the quasione-dimensional electron gas within the sum-rule approach. The Hubbard form of the LFC for wire systems was proposed originally in Ref. 14, where cylindrical wires have been studied and an analytical result for the electron-electron interaction potential has been derived.

It was recently demonstrated in experiments^{15,16} that the collective modes in semiconductor quantum wires can be described by the RPA. In these experiments, the electron density was sufficiently large in order to neglect the effects of a finite LFC. With experiments as performed in Refs. 15 and 16, but at lower density, one could study the LFC as calculated in this paper. On the other hand, we think that a study of many-body effects via the LFC is an important issue by itself in order to understand better many-body effects in systems with reduced dimension.

The exchange energy in wire systems depends on the wire radius and was calculated already in Ref. 17. It is a characteristic property of quasi-one-dimensional systems that a geometric parameter (the wire radius R_0) enters the interaction effects and the exchange energy and the correlation energy depend on the RPA parameter and the confinement parameter R_0 : $\varepsilon_{\rm ex}(r_s, R_0)$ and $\varepsilon_{\rm cor}(r_s, R_0)$. In this paper, we present numerical and analytical results for the correlation energy and the exchange energy.

The paper is organized as follows. The model and the theory are described in Sec. II. In Sec. III, we present the results for the local-field correction and the paircorrelation function. Numerical and analytical results for the ground-state energy are given in Sec. IV. We apply the theory in Sec. V and calculate the plasmon dispersion, the compressibility, and the chemical potential. In Sec. VI, we discuss our results and we conclude in Sec. VII. Some additional results are given in the Appendix: the Hartree energy in cylindrical wires and simplified confinement models are briefly discussed.

II. MODEL AND THEORY

A. Model

We consider cylindrical quantum wires¹⁴ with the wire axis in the z direction. With radius R_0 , the confining potential U_c in the xy plane is given by $U_c(r \le R_0) = 0$ and $U_c(r > R_0) \rightarrow \infty$. The electron gas in the wire is characterized by the one-dimensional carrier density N. The RPA parameter for one-dimensional systems is described as $r_s = 1/2Na^*$. $a^* = \varepsilon_L / m^* e^2$ is the effective Bohr radius defined with the effective electron mass m^* , the background dielectric constant ε_L , and the electron charge e. For Planck's constant, we use $h = 2\pi$. The energy is expressed in units of the effective Rydberg Ry* $Ry^* = m^* e^4 / 2\varepsilon_L^2$. For given by GaAs with $m^*=0.067m_e$ and $\varepsilon_L=13$, the relevant parameters are $a^* = 100$ Å and Ry* = 5.4 meV. m_e is the electron mass in vacuum. The electron density N defines the Fermi wave number k_F via $N = 2g_v k_F / \pi$. g_v is the valley degeneracy and in this paper, we use $g_v = 1$. In the following, we measure wave numbers q in units of $1/R_0$ as $x = qR_0$. An important inverse length scale in the Fourier space is the wave number $q_0 = 2/r_s^{1/2}a^*$. In fact, q_0 is the screening scale in the Bose condensate, where only correlation effects are present.¹¹ The Fermi energy ε_F is given by $\varepsilon_F/Ry^* = (\pi/4g_v r_s)^2$ and the Fermi wave number by $k_F a^* = \pi / 4g_v r_s$.

The Coulomb interaction potential V(q) in the Fourier space is expressed as¹⁴

$$V(q) = \frac{e^2}{2\varepsilon_L} f(qR_0) , \qquad (1a)$$

with

$$f(x) = \frac{144}{x^2} \left[\frac{1}{10} - \frac{2}{3x^2} + \frac{32}{3x^4} - 64 \frac{I_3(|x|)K_3(|x|)}{x^4} \right].$$
(1b)

 $I_3(x)$ and $K_3(x)$ are Bessel functions. We mention that $f(x \ll 1) = 4 \ln(2/x)$ and $f(x \gg 1) = 14.4/x^2$ and the interaction potential in quasi-one-dimensional systems depends explicitly on the confinement parameter R_0 : the limit $R_0 = 0$ cannot be studied.

B. The local-field correction

In the STLS approach³ for one-dimensional systems, the LFC G(q) is given by a one-dimensional k integral over the static-structure factor (SSF) S(k), which is determined by the LFC. The coupled equations for the self-consistent STLS approach must be solved numerically. In one dimension, one finds⁵

$$G_{\rm STLS}(q) = \frac{1}{2\pi N} \int_{-\infty}^{\infty} dk \frac{q(q-k)V(q-k)}{q^2 V(q)} [1 - S(k)] .$$
⁽²⁾

One can show that $G_{\text{STLS}}(q \to 0) \propto 1/V(q \to 0)$ and $G_{\text{STLS}}(q \to \infty) \propto \text{const} - O(1/q^2)$. Note, that $G_{\text{STLS}}(q) \propto 1/N \propto r_s$. The Hartree-Fock approximation (HFA) is obtained by replacing S(k) in (2) by $S_0(k)$, the SSF of the free electron gas, which means that only exchange effects are taken into account and correlation effects are neglected.

We suggest the generalized approximating (GA) expression for the LFC in one dimension as

$$G_{\rm GA}(q) = r_s \frac{a^*}{\pi R_0 C_{21}(r_s, R_0)} \times \frac{f\{[q^2 R_0^2 + q_0^2 R_0^2 / C_{11}(r_s, R_0)^2]^{1/2}\}}{f(qR_0)} .$$
(3)

For the two- and three-dimensional electron gas, it is known⁷ that within the Hubbard approximation¹³ the LFC has the form

$$G(q) \propto V([q^2 + k_F^2]^{1/2}) / V(q) = q^{d-1} / (q^2 + k_F^2)^{(d-1)/2}$$

The expression for $G_{GA}(q)$ can be interpreted as a Hubbard-type expression¹³ for the LFC, where the values for $q \rightarrow 0$ and ∞ are determined by the two parameters $C_{11}(r_s, R_0)$ and $C_{21}(r_s, R_0)$. Exchange and correlation effects are taken into account within a self-consistent calculation of $C_{11}(r_s, R_0)$ and $C_{21}(r_s, R_0)$. The limiting behaviors of the LFC are given by

$$G_{\rm GA}(q \to 0) = r_s a^* f(q_0 R_0 / C_{11}) / \pi R_0 C_{21} f(q R_0 \to 0)$$

and

$$G_{\rm GA}(q \rightarrow \infty) = r_s a^* / \pi R_0 C_{21}$$
.

Supposing that $G_{\text{GA}}(q \to \infty) < 1$, we conclude that $C_{21} > r_s a^* / \pi R_0$.

In the Hubbard (H) approximation only the exchange effects are taken into account in an approximating way in order to obtain an analytical expression for the LFC. The following expression was used in Ref. 14:

$$G_{H}(q) = \frac{1}{2g_{v}} \frac{f([q^{2} + k_{F}^{2}]^{1/2}R_{0})}{f(qR_{0})} .$$
(4)

With (3), we conclude that

$$C_{11,H}(r_s, R_0) = 8g_v r_s^{1/2} / \pi$$
(5a)

and

$$C_{21,H}(r_s, R_0) = 2g_v r_s a^* / R_0 \pi .$$
(5b)

Note, that $C_{21,H}$ increases with decreasing density and decreasing wire radius. $C_{11,H}$ does not depend on the wire radius. Within the HFA, we get a slightly different behavior for the coefficient $C_{11,HFA}$; compare Eq. (5a)

with Eq. (11).

Using the analytical expression for the LFC, we find the following two equations to determine the two coefficients $C_{i1}(r_s, R_0)$ within the two-sum-rule approach:

$$\frac{f(q_0R_0/C_{11}(r_s,R_0))}{C_{21}(r_s,R_0)} = 2\int_0^\infty dx [f(x) + x df(x)/dx] [1 - S(x)] \quad (6a)$$

and

$$\frac{1}{C_{21}(r_s, R_0)} = 2 \int_0^\infty dx \left[1 - S(x) \right] \,. \tag{6b}$$

We mention that x in Eq. (6) corresponds to qR_0 . Similar equations have been derived before for the Bose condensate¹¹ and the electron gas¹² in two and three dimensions. In the following, we use $C_{11} = C_{11}(r_s, R_0)$ and $C_{21} = C_{21}(r_s, R_0)$.

For $R_0 = a^*$ and $r_s > 3$, the pair-correlation function g(z=0), determined by C_{21} , becomes slightly negative. In these cases, we proceed as follows: we determine C_{21} by the condition $G_{GA}(q \rightarrow \infty) = 1$ and calculate C_{11} by the one-sum-rule approach via Eq. (6a).

C. The static-structure factor

From the interacting Bose gas, ¹¹ we know that the SSF contains the LFC G(q). For the interacting electron gas, the SSF must include exchange effects. ¹² In the following, we use an analytical form of the SSF and we suggested the following generalized approximating (GA) expression:¹²

$$S_{\rm GA}(q) = \frac{1}{\left[1/S_0(q)^2 + 1/S_p(q)^2\right]^{1/2}} \ . \tag{7}$$

The term $S_0(q)$ in (7) represents the particle-hole spectrum and the second term represents the plasmon excitations. The SSF of the free-electron gas in one dimension is given by

$$S_0(|q| < 2k_F) = |q| / 2k_F \tag{8a}$$

and

$$S_0(|q| \ge 2k_F) = 1$$
 . (8b)

The SSF $S_{p}(q)$ for plasmons is defined as¹¹

$$S_p(q) = \left[\frac{q^2}{4mNV(q)[1 - G_{\rm GA}(q)]}\right]^{1/2}.$$
 (9)

The SSF, as given in Eq. (7), corresponds to a generalized Feynman-Bijl form, where, however, one-particle excitations *and* collective excitations are taken into account.

Our analytical expression for the SSF represents a very powerful formulation of the relevant physics in manyparticle systems. For $e^2=0$, we get $S_{GA}(q)=S_0(q)$ and this approximation describes the SSF in the HFA. For G(q)=0, we obtain with $S_p(q)=[q^2/4mNV(q)]^{1/2}$ the mean spherical approximation (MSA) (Ref. 9) for the SSF: $S_{GA}(q)=S_{MSA}(q)$. For small wave numbers, we find $S_{GA}(q \rightarrow 0) \propto [q^2/\ln(2/qR_0)]^{1/2}$. For large wave numbers, we get $S_{GA}(q \rightarrow \infty) = S_0(q > 2k_F) = 1$.

In the following, we solve the equation (6) for the coefficients C_{11} and C_{21} , with Eq. (7) for the SSF. We describe three approximations: (i) the Hartree-Fock approximation with $S_{GA}(q) = S_0(q)$ ($C_{11,HFA}$ and $C_{21,HFA}$), (ii) the MSA for the LFC with $S_{GA}(q) = S_{MSA}(q)$ ($C_{11,MSA}$ and $C_{21,MSA}$), and (iii) the generalized approximation with $S_{GA}(q)$ (C_{11} and C_{21}). The equations (6), (7) are much easier to solve than the full STLS equations. Moreover and more important, with C_{11} and C_{21} , we get an analytical expression for the LFC, which can be used in more complex calculations. We calculated C_{11} and C_{21} with a personal computer (Power Macintosh 8100).

III. RESULTS

A. The two-sum-rule approach

In the HFA, the coefficients are given by

$$\frac{f(q_0R_0/C_{11,\text{HFA}})}{C_{21,\text{HFA}}} = 2\int_0^{2k_FR_0} dx [f(x) + x df(x)/dx] [1 - S_0(x)]$$
(10a)

and

$$\frac{1}{C_{21,HFA}} = 2 \int_{0}^{2k_F R_0} dx \left[1 - S_0(x) \right] .$$
 (10b)

With Eq. (8), we find

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$$C_{11,\text{HFA}}(r_s \to 0) = 2g_v (5r_s)^{1/2} [14.4 \ln(\pi R_0 / 2g_v r_s a^*) -9.7094]^{1/2} / 3\pi , \quad (11a)$$

$$C_{11,\text{HFA}}(r_s \to \infty) = 4g_v \exp[\frac{1}{2}]r_s^{1/2}/\pi$$
, (11b)

and

$$C_{21,\rm HFA} = 2g_v r_s a^* / R_0 \pi \,. \tag{12}$$

Numerical results for the coefficients $C_{11,\text{HFA}}$ and $C_{21,\text{HFA}}$ are given in Table I for $R_0 = a^*$ and different values of r_s . These numerical results are in agreement with the analytical results as described by Eqs. (11), (12). The factor -9.7094 in Eq. (11a) is determined by

$$\lim_{\beta \to \infty} \left\{ \int_0^\beta dx \; x f(x) - 14.4 \ln(\beta) \right\} = -9.7094 \; . \tag{13}$$

The detailed form of the confinement, which determines the integral given in Eq. (13), appears in the second order term of $C_{11,\text{HFA}}(r_s \rightarrow 0)$, see Eq. (11a).

For $R_0 = a^*$, our results for the coefficients $C_{11,MSA}$, $C_{21,MSA}$, C_{11} , and C_{21} are given in Table I. We find that C_{11} and C_{21} increases strongly with increasing r_s and the r_s dependence of C_{21} is stronger than of C_{11} . For $r_s < 0.4$, our numerical results are very near to the results in the HFA: $C_{i1,HFA} = C_{i1} \approx C_{i1,MSA}$. The difference be-

r _s	$C_{11,\mathrm{HFA}}(a^*)$	$C_{21,\mathrm{HFA}}(a^*)$	$C_{11,MSA}(a^*)$	$C_{21,MSA}(a^*)$	$C_{11}(a^*)$	$C_{21}(a^{*})$
0.01	0.3774	0.006 37	0.3774	0.006 37	0.3774	0.006 37
0.04	0.6316	0.025 46	0.6314	0.025 45	0.6315	0.025 45
0.1	0.8695	0.063 66	0.8666	0.063 31	0.8676	0.063 39
0.4	1.4551	0.2546	1.3104	0.2229	1.3605	0.2323
0.6	1.7261	0.3820	1.3697	0.2921	1.4899	0.3189
0.8	1.9599	0.5093	1.3597	0.3406	1.5541	0.3900
1	2.1690	0.6366	1.3271	0.3770	1.5902	0.4516
2	3.0078	1.2732	1.1694	0.4901	1.7150	0.7121
3	3.6626	1.9099	1.0758	0.5643	1.8887	0.9666
4	4.2185	2.5465	1.0146	0.6233	2.1059	1.2352
6	5.1553	3.8197	0.9362	0.7181	2.6048	1.8096
8	5.9474	5.0930	0.8857	0.7949	3.1341	2.4128
0	6.6463	6.3662	0.8492	0.8608	3.6663	3.0301

TABLE I. Parameters for the local-field correction: $C_{11,HFA}(R_0)$, $C_{21,HFA}(R_0)$, $C_{11,MSA}(R_0)$, $C_{21,MSA}(R_0)$, $C_{11,MSA}(R_0)$

tween $C_{i1,\text{HFA}}$ and C_{i1} is due to correlation effects. These effects are small for $r_s \ll 1$, but they are large for $r_s > 1$. In general, we find $C_{21,\text{HFA}} > C_{21} > C_{21,\text{MSA}}$, which leads for large wave numbers to $G_{\text{MSA}}(q) > G_{\text{GA}}(q) > G_{\text{HFA}}(q)$. It is clear from $G_{\text{MSA}}(q) > G_{\text{GA}}(q)$ that the MSA overestimates correlation effects.

In Fig. 1, the LFC for $r_s = 1$ and $R_0 = a^*$ is shown in the HFA $[G_{\text{HFA}}(q)]$ and in the sum-rule approach $[G_{\text{GA}}(q)]$. The difference between $G_{\text{GA}}(q)$ and $G_{\text{HFA}}(q)$ is due to correlations. The LFC calculated in the HFA of the STLS approach [Eq. (2) with S(k) replaced by $S_0(k)$] is also shown $[G_{\text{STLS},\text{HFA}}(q)]$. Note that $G_{\text{HFA}}(q)$ is in good agreement with $G_{\text{STLS},\text{HFA}}(q)$. We mention that the LFC is large for small wave numbers, due to the fact that $G(q \rightarrow 0) \propto 1/V(q) \propto 1/\ln(2/qR_0)$. In d dimensions the behavior for small wave number is $G(q \rightarrow 0) \propto q^{d-1}$ for d > 1.

With decreasing density the LFC increases as shown in



FIG. 1. Local-field correction G(q) versus wave number q for RPA parameter $r_s = 1$ and wire radius $R_0 = a^*$. The solid line represents the two-sum-rule approach. The dotted line represents the HFA of the two-sum-rule approach and the dashed line the HFA of the STLS approach.

Fig. 2 for $r_s = 1$, 4, and 10 for $R_0 = a^*$. This increase reflects the increasing importance of correlation effects: within the HFA, one finds $G_{\text{HFA}}(q \to \infty) = 1/2g_v$.

Our results for the SSF S(q) versus q are shown in Fig. 3 for $r_s = 1$, 4, and 10 for $R_0 = a^*$. With increasing r_s the SSF increases due to the decrease of k_F (electron-hole excitations) and the increasing importance of the plasmon dynamic (correlation). From Fig. 3, we conclude that 1-S(q) decreases with increasing r_s and, with Eq. (6b); this implies that C_{21} increases with increasing r_s . This is confirmed by the numerical results given in Table I.

The coefficients C_{11} and C_{21} depend on the confinement described by R_0 . In Table II, we present representative results versus r_s for $R_0 < a^*$ ($R_0=0.2a^*$ and $0.5a^*$) and in Table III for $R_0 > a^*$ ($R_0=2a^*$ and $4a^*$). Our results given in Tables I–III can be used to estimate the coefficients for $0.1 < R_0/a^* < 5$.

B. The pair-correlation function

The SSF determines the pair-correlation function g(z). One gets



FIG. 2. Local-field correction G(q) versus wave number q for wire radius $R_0 = a^*$ and different values of the RPA parameter: $r_s = 1$, 4, and 10. The solid lines represent the two-sum-rule approach. The dashed lines represent the one-sum-rule approach for $r_s = 4$ and 10.



FIG. 3. Static-structure factor S(q) in the two-sum-rule approach versus wave number q for wire radius $R_0 = a^*$ and different values of the RPA parameter: $r_s = 1$, 4, and 10.

$$g(z) = 1 - r_s \frac{2a^*}{\pi R_0} \int_0^\infty dx \cos(xz/R_0) [1 - S(x)], \qquad (14)$$

with $x = qR_0$. With $S(q) = S_{GA}(q)$, we derive the analytical result¹¹

$$g_{\rm GA}(0) = 1 - r_s a^* / \pi R_0 C_{21}(r_s, R_0)$$
 (15a)

In the MSA with $S(q) = S_{MSA}(q)$, we get

$$g_{\text{MSA}}(0) = 1 - r_s a^* / \pi R_0 C_{21,\text{MSA}}(r_s, R_0)$$
 (15b)

and in HFA with $S(q)=S_0(q)$, one finds $g_{\rm HFA}(0)=\frac{1}{2}$. $g_{\rm GA}(0)$ is given by $C_{21}(r_s, R_0)$ and can be calculated with the numerical results given in Tables I–III. Therefore, we have not given the results for this quantity in a table.

The pair-correlation function g(0) versus r_s is shown in Fig. 4 for $R_0 = a^*$ in different approximations. Within the MSA for the SSF, we find a negative pair-correlation function for $r_s > 1.3$. This result corresponds to the large negative values found for g(0) within the RPA for threedimensional systems:^{1,2} the MSA and the RPA for the SSF strongly overestimate correlation effects. In the generalized approximation, $g_{GA}(0)$ approaches smoothly a

TABLE II. Parameters for the local-field correction: $C_{11}(R_0)$ and $C_{21}(R_0)$ for $R_0 = a^*/5$ and $a^*/2$ and for various values of r_s .

r _s	$C_{11}(a^*/5)$	$C_{21}(a^*/5)$	$C_{11}(a^*/2)$	$C_{21}(a^*/2)$
0.01	0.3056	0.0318	0.3473	0.0127
0.1	0.6962	0.3075	0.7815	0.1259
0.4	1.0059	0.9615	1.1986	0.4305
0.6	1.0658	1.2789	1.2801	0.5759
0.8	1.1165	1.5690	1.3236	0.6992
1	1.1687	1.8495	1.3581	0.8119
2	1.4687	3.2570	1.5579	1.3434
3	1.7971	4.7220	1.8143	1.8941
4	2.1289	6.2279	2.0971	2.4711
6	2.7778	9.3033	2.6846	3.6727
8	3.3992	12.420	3.2685	4.9059
10	3.9962	15.557	3.8383	6.1537



FIG. 4. Pair-correlation function g(z=0) versus RPA parameter r_s for wire radius $R_0 = a^*$. The solid line represents the two-sum-rule approach. The dashed and dotted lines represent g(z=0) calculated, respectively, with the SSF $S_{MSA}(q)$ and $S_0(q)$.

very small negative value for large r_s .

 $g_{\rm GA}(0)$ versus r_s is shown in Fig. 5 for $R_0 = 0.2a^*$, $0.5a^*$, a^* , $2a^*$, and $4a^*$. With decreasing wire radius, the Coulomb interaction effects increase and one expects that the correlation effects increase, too. With decreasing R_0 the pair-correlation function decreases, due to the increasing importance of correlation effects. Most important is the fact that even for large r_s the pair-correlation function is only slightly negative $[g_{GA}(0) > -0.056$ for $R_0 = a^*$], see Fig. 5. For small r_s , we find $g_{\text{GA}}(0) \approx g_{\text{HFA}}(0) = 0.5$. From general arguments, it is clear that g(0) must be positive. The negative value for the pair-correlation function for large r_s is a known defect of the STLS approach.⁷ However, in the two-sumrule version of the STLS approach, $g_{GA}(0)$ is only slightly negative and the negative value is very small even for very large r_s , see Fig. 5.

The pair-correlation function g(z) versus z is shown in Fig. 6 for $r_s = 2$ and $R_0 = a^*$. The dotted line represents the HFA with $S(q) = S_0(q)$ and is given by

TABLE III. Parameters for the local-field correction: $C_{11}(R_0)$ and $C_{21}(R_0)$ for $R_0=2a^*$ and $4a^*$ and for various values of r_s .

r _s	$C_{11}(2a^*)$	$C_{21}(2a^*)$	$C_{11}(4a^*)$	$C_{21}(4a^*)$
0.01	0.4059	0.003 18	0.4322	0.001 59
0.1	0.9656	0.0318	1.0660	0.0159
0.4	1.5318	0.1222	1.7208	0.0627
0.6	1.7163	0.1739	1.9516	0.0917
0.8	1.8276	0.2179	2.1135	0.1183
1	1.8934	0.2554	2.2276	0.1420
2	2.0021	0.3958	2.4375	0.2288
3	2.0879	0.5146	2.4779	0.2906
4	2.2165	0.6350	2.5239	0.3465
6	2.5702	0.8955	2.6994	0.4599
8	2.9982	1.1795	2.9679	0.5837
10	3.4594	1.4779	3.2999	0.7186



FIG. 5. Pair-correlation function g(z=0) in the two-sumrule approach versus RPA parameter r_s for wire radius $R_0 = 4a^*, 2a^*, a^*, a^*/2$, and $a^*/5$.

$$g_{\rm HFA}(z) = 1 - \frac{1}{2g_v} \frac{\sin^2(k_F z)}{(k_F z)^2}$$
, (16)

where $g_{\text{HFA}}(0)=1-1/2g_v$ and $g_{\text{HFA}}(z^*)=1$ for $k_F z^*=n\pi$ $(z^*/a^*=4ng_v r_s)$ and *n* is an integer. The solid line in Fig. 6 represents the calculation with the analytical SSF, including the local-field correction within the sum-rule approximation. The approximations where $G(q)=G_{\text{HFA}}(q)$ and G(q)=0 are used in SSF are also shown in Fig. 6. Within these approximations g(0) is negative at this density.

C. The one-sum-rule approach

Within the two-sum-rule approach, the effective potential $V(q \rightarrow \infty) [1-G(q \rightarrow \infty)]$ becomes slightly negative for $r_s > 3.3$ and $R_0 = a^*$. In order to obtain the large qcharacteristics of the system without this incorrect behavior, we can use the one-sum-rule approach for C_{11} , where $C_{21} = r_s a^* / \pi R_0$ is fixed by the condition that



FIG. 6. Pair-correlation function g(z) versus distance z for $r_s=2$ and $R_0=a^*$. The solid line represents the two-sum-rule approach. The dotted, dashed, and dashed-dotted lines represent the HFA [Eq. (16)], and the approximations with G(q)=0 and $G(q)=G_{\rm HFA}(q)$ used for the SSF, respectively.

TABLE IV. Parameter $C_{11}(r_s, R_0)$ for the local-field correction, calculated within the one-sum-rule approach, for various values of r_s and R_0 , where $G_{GA}(q \to \infty) = 1$.

r _s	$C_{11}(a^*/5)$	$C_{11}(a^*)$	$C_{11}(4a^*)$
6	3.4336	3.0229	2.8065
8	4.4606	3.838	3.2973
10	5.4810	4.6659	3.8155

 $G_{\rm GA}(q \rightarrow \infty) = 1$. The one-sum-rule coefficient C_{11} has been given in Table IV.

By comparing C_{11} , calculated within the two-sum-rule approach (Tables I–III and r_s large), with C_{11} , calculated within the one-sum-rule approach (Table IV), we notice a difference of about 30% for $r_s = 10$. This difference is due to the fact that C_{11} enters in the LFC via $f(q_0R_0/C_{11})$, which is given by a logarithmic behavior for small q_0R_0/C_{11} . On the other hand, C_{21} enters the LFC as a prefactor: C_{21} calculated for large r_s within the twosum-rule approach is very near to the value calculated with $C_{21}=r_sa^*/\pi R_0$ within the one-sum-rule approach by fixing $G_{GA}(q \to \infty)=1$. We think that the two-sumrule approach gives better estimates for the correlation energy (and the compressibility) than the one-sum-rule approach.

In Fig. 2, we have shown the results for the LFC within the one-sum-rule approach for $r_s = 4$ and 10. Note that the difference between the one-sum-rule approach and the two-sum-rule approach is very small.

IV. GROUND-STATE ENERGY

The ground-state energy ε_0 per particle can be expressed as²

$$\varepsilon_0(r_s) = \varepsilon_{\rm kin}(r_s) + \varepsilon_C(r_s) + \varepsilon_{\rm int}(r_s) . \qquad (17)$$

The kinetic energy per particle of a one-dimensional electron gas is given as

$$\varepsilon_{\rm kin}(r_s)/{\rm Ry}^* = \varepsilon_F/3{\rm Ry}^* = \pi^2/48g_v^2 r_s^2 . \tag{18}$$

 $\varepsilon_C(r_s)$ represents the direct Coulomb (C) term¹⁸ related to the fact that the subband energies change, due to band-bending effects and many-body effects, when the subbands are populated with carriers. The charge density, due to the dopant atoms, also shifts the subband energies: in low-dimensional systems, only global neutrality is realized, while local neutrality is violated. The direct Coulomb term depends largely on the distribution of dopants¹⁹ and is not the issue of this paper: we use in the following $\varepsilon_C(r_s)=0$. However, we give some estimates for this term in the Appendix A by taking into account the band-bending effects in lowest order perturbation theory.

The interaction energy per particle is written as¹

$$\varepsilon_{\rm int}(r_s)/\mathbf{R}\mathbf{y}^* = -\frac{a^*}{2\pi R_0} \int_0^1 d\lambda \int_0^\infty dx \ f(x)[1-S(x,\lambda)] ,$$
(19)

with λ as the coupling constant and $x = qR_0$. We note that the SSF in (19) is a function of λ and in our theory given by

$$S_{\rm GA}(q,\lambda) = \frac{1}{[1/S_0(q)^2 + \lambda/S_p(q,\lambda)^2]^{1/2}} , \qquad (20a)$$

with

$$S_p(q,\lambda) = \left[\frac{q^2}{4mNV(q)[1-G_{GA}(q,\lambda)]}\right]^{1/2}$$
. (20b)

 $G_{GA}(q,\lambda)$ is given by Eq. (3) with $C_{11}(r_s,R_0)$ and $C_{21}(r_s,R_0)$ replaced by $C_{11}(r_s,R_0,\lambda)$ and $C_{21}(r_s,R_0,\lambda)$, the coefficients obtained by substituting S(q) by $S(q,\lambda)$ in Eq. (6). We mention that $C_{i1}(r_s,R_0,\lambda=0) = C_{i1,\text{HFA}}(r_s,R_0)$ and $C_{i1}(r_s,R_0,\lambda=1) = C_{i1}(r_s,R_0)$ for i=1,2.

A. Exchange energy

The exchange energy per particle is obtained by using $1/S_p(x,\lambda)^2=0$ in Eq. (20a). Integrating over λ gives

$$\varepsilon_{\rm ex}(r_s)/{\rm Ry}^* = -\frac{a^*}{2\pi R_0} \int_0^{2k_F R_0} dx \, f(x) [1 - S_0(x)] \,. \quad (21)$$

Using the self-energy, the exchange energy of quasi-onedimensional systems was already calculated in Ref. 17. The exchange energy can be expressed as an integral over the self-energy. In that case, a double integral has to be performed.¹⁷ In our present formulation, one of the integrations corresponds to the calculation of the SSF in the HFA. It was found¹⁷ that the absolute value of the exchange energy decreases with increasing r_s and increasing R_0 and that in quasi-one-dimensional systems the exchange energy depends on r_s and R_0 and cannot be expressed by a simple analytical form. In three and two di-



FIG. 7. Exchange energy per particle ε_{ex} versus RPA parameter r_s for $R_0=0.2a^*$, $0.5a^*$, a^* , $2a^*$, and $4a^*$. The dashed and dotted lines represent the asymptotical results for $\varepsilon_{ex}(R_s \rightarrow 0)$ [according to Eq. (22a)] and for $\varepsilon_{ex}(R_s \rightarrow \infty)$ [according to Eq. (22b)], respectively, for $R_0=0.2a^*$, a^* , and $4a^*$. The arrows indicate $R_s=1$, see Eq. (23).

mensions the exchange energy can be expressed in an analytical form:^{2,18} $\varepsilon_{ex}(r_s) \propto -Ry^*/r_s$.

In Fig. 7, we show the exchange energy ε_{ex} versus r_s for different wire radii. With decreasing wire radius, the absolute value of the exchange energy increases and the absolute value of the exchange energy decreases with decreasing density. Some numerical results for the exchange energy are given in Table V for $R_0=0.2a^*$, $0.5a^*$, a^* , $2a^*$, and $4a^*$.

With analytical expansions for f(x) (Ref. 14) one can derive, with Eq. (21), analytical expressions for the exchange energy for large and small carrier density. We find for small R_s ,

TABLE V. Exchange energies per particle $\varepsilon_{ex}(R_0)$ (in units of the effective Rydberg Ry^{*}) for $R_0 = a^*/5$, $a^*/2$, a^* , $2a^*$, and $4a^*$ and for various values of r_s .

r _s	$-\varepsilon_{\rm ex}(a^*/5)$	$-\varepsilon_{\rm ex}(a^*/2)$	$-\epsilon_{\rm ex}(a^*)$	$-\varepsilon_{\rm ex}(2a^*)$	$-\varepsilon_{\rm ex}(4a^*)$
0.001	11.819	4.776	2.397	1.201	0.601
0.01	10.664	4.543	2.330	1.182	0.596
0.1	6.362	3.414	1.958	1.066	0.561
0.2	4.624	2.770	1.707	0.979	0.533
0.4	3.117	2.068	1.385	0.853	0.490
0.6	2.405	1.679	1.179	0.763	0.455
0.8	1.980	1.426	1.034	0.692	0.427
1	1.694	1.247	0.925	0.636	0.402
2	1.019	0.792	0.623	0.462	0.318
3	0.747	0.595	0.481	0.370	0.267
4	0.596	0.482	0.396	0.312	0.231
6	0.431	0.355	0.297	0.240	0.185
8	0.341	0.284	0.241	0.198	0.156
10	0.284	0.238	0.204	0.169	0.135
15	0.203	0.172	0.149	0.126	0.103
20	0.1595	0.1365	0.1192	0.1019	0.0847
30	0.1131	0.0978	0.0862	0.0747	0.0632
40	0.0884	0.0769	0.0683	0.0596	0.0510

$$\varepsilon_{ex}(R_s \to 0)/Ry^* = -\frac{a^*}{R_0} \left[2.4082 + \frac{18}{5\pi} R_s [\ln(R_s/2) - 1/2] \right]$$

(22a)

and for large R_s ,

$$\varepsilon_{\rm ex}(R_s \to \infty)/{\rm Ry}^* = -\frac{2a^*}{\pi R_0} \frac{1}{R_s} [\ln(R_s) + {\rm C} + 3/2 - 73/120 + [\ln(R_s) + 1949/1680 - {\rm C} + 7/12]/12R_s^2].$$
(22b)

The parameter R_s is defined as

$$R_s = 4g_v a^* r_s / \pi R_0 \tag{23}$$

and C=0.577 is Euler's constant. The leading term $-\ln(r_s)/2g_v r_s$ in Eq. (22b) is in agreement with the result given in Ref. 17 for $g_v = 1$. We find that our analytical results in Eq. (22) have a quite large validity range and can be used by experimenters. In fact, we find that $\varepsilon_{\rm ex}(R_s \rightarrow 0)$ is a very good approximation to our numerical results for $R_s < 1$ and $\varepsilon_{\rm ex}(R_s \rightarrow \infty)$ is a good approximation for $R_s > 0.8$, see Fig. 7.

We mention that a universal scaling exists for the exchange energy,

$$\varepsilon_{\rm ex}(R_s)R_0/a^* = F(R_s) \tag{24}$$

and $F(R_s)$ is a function of R_s , but not of R_0 and g_v . A similar scaling was already described in Ref. 17 for the self-energy. Note, that in Table V the parameter R_s changes by a factor 10^6 : $R_s = 255$ for $r_s = 40$ and $R_0 = 0.2a^*$ and $R_s = 0.003$ for $r_s = 0.001$ and $R_0 = 4a^*$.

B. Correlation energy

The correlation energy per particle is defined as $\varepsilon_{cor}(r_s) = \varepsilon_{int}(r_s) - \varepsilon_{ex}(r_s)$. With Eqs. (19), (21), we find

$$\varepsilon_{\rm cor}(r_s)/{\rm Ry}^* = -\frac{a^*}{2\pi R_0} \int_0^1 d\lambda \int_0^\infty dx \ f(x) [S_0(x) - S(x,\lambda)] \ . \tag{25}$$

In Fig. 8, we show the correlation energy ε_{cor} versus r_s (0.1 < $r_s < 20$) for different wire radii. With decreasing wire radius, the absolute value of the correlation energy increases, and we find a minimum at intermediate density r_s^* . r_s^* increases with increasing R_0/a^* . The general behavior of the correlation energy versus r_s (with a minimum at r_s^*) is similar to the correlation energy of two-dimensional systems with a finite width as realized in quantum wells.^{4,20} Note that the RPA-like MSA (the dotted line in Fig. 8) overestimates the correlation energy. The dashed line in Fig. 8 was calculated with the HFA for the LFC and is in very good agreement with the sum-rule approach for $R_s < 1$. Numerical results for the



FIG. 8. Correlation energy per particle ε_{cor} versus RPA parameter r_s for $R_0 = 0.2a^*$, $0.5a^*$, a^* , $2a^*$, and $4a^*$. The solid lines represent the two-sum-rule approach. The dashed lines represent the Hubbard-like results calculated with $G_{HFA}(q)$ in the SSF. The dotted lines represent the RPA-like results calculated with $S_{MSA}(q)$.

correlation energy are given in Table VI for $R_0 = 0.2a^*$, $0.5a^*$, a^* , $2a^*$, and $R_0 = 4a^*$ and $0.1 < r_s < 20$. These results can be compared with Monte Carlo calculations when available in the literature.

In two dimensions and three dimensions, it was found that the correlation energy at low density behaves as $\varepsilon_{cor}(r_s \rightarrow \infty, d=2) \approx \varepsilon_{cor}(r_s \rightarrow \infty, d=3) \propto 1/r_s$.^{8,9} From Fig. 8, we conclude that for quasi-one-dimensional systems, the correlation energy decreases for small density $(r_s > r_s^*)$, however, it appears that the confinement effects are still very strong for $r_s \leq 20$, see Table VII. Consequently, we argue that for $r_s \leq 20$, the correlation energy is not only determined by r_s , but also by R_0 .

From Fig. 8, we conclude that in the high-density limit, the correlation energy decreases to zero. It is well known that in three dimensions, the correlation energy behaves as $\varepsilon_{cor}/Ry^*=0.0622 \ln(r_s)+O(r_s^0)$ (Ref. 1) and diverges for small r_s . In two dimensions the correlation energy becomes constant for large densities: $\varepsilon_{cor}/Ry^*=-0.38+O(r_s \ln r_s)$.²¹

With $G_{GA}(q)=0$ in Eq. (20), we obtain $S_{MSA}(q)$ and the correlation energy within the RPA-like MSA. In the high-density regime, the correlation energy within the MSA can be calculated analytically and we find the analytical result²²

$$\varepsilon_{\rm cor}(R_s << 1)/{\rm Ry}^* = -B_{\rm cor} \frac{g_v^3}{\pi^4} \frac{a^{*2}}{R_0^2} r_s^2$$
, (26a)

with $B_{\rm cor}$ given by

$$B_{\rm cor} = \int_0^\infty dx \ x f(x)^2 = 36.616 \ . \tag{26b}$$

The factor B_{cor} depends on the detailed form of the confinement via f(x), see the Appendix B for a different model of the confinement. Note that the correlation en-

r _s	$-\varepsilon_{\rm cor}(a^*/5)$	$-\varepsilon_{\rm cor}(a^*/2)$	$-\varepsilon_{\rm cor}(a^*)$	$-\varepsilon_{\rm cor}(2a^*)$	$-\varepsilon_{\rm cor}(4a^*)$
0.1	0.057 24	0.011 80	0.003 24	0.000 85	0.000 216
0.2	0.1499	0.037 32	0.011 11	0.003 05	0.000 799
0.4	0.3009	0.098 09	0.033 70	0.01007	0.002 766
0.6	0.3908	0.1519	0.058 88	0.01907	0.005 480
0.8	0.4403	0.1932	0.082 60	0.028 86	0.008 675
1	0.4658	0.2234	0.1033	0.038 71	0.012 17
2	0.4622	0.2800	0.1634	0.078 53	0.030 28
3	0.4129	0.2770	0.1813	0.1006	0.045 08
4	0.3671	0.2605	0.1825	0.1112	0.055 55
6	0.2984	0.2248	0.1696	0.1156	0.066 94
8	0.2519	0.1959	0.1536	0.1115	0.070 85
10	0.2187	0.1735	0.1393	0.1051	0.071 07
12	0.1938	0.1559	0.1272	0.098 45	0.069 55
14	0.1744	0.1418	0.1171	0.092 28	0.067 27
16	0.1589	0.1302	0.1085	0.08673	0.064 72
20	0.1354	0.1124	0.0949	0.077 37	0.059 64

TABLE VI. Correlation energies per particle $\varepsilon_{cor}(R_0)$ (in units of the effective Rydberg Ry^{*}) for $R_0 = a^*/5$, $a^*/2$, a^* , $2a^*$, and $4a^*$ and for various values of r_s .

ergy increases strongly with increasing valley degeneracy $\varepsilon_{cor}(R_s \ll 1) \propto g_v^3$. Equation (26) can be written as $\varepsilon_{cor}(R_s \ll 1) \propto -g_v R_s^2$, which supports our claim that the relevant parameter for interaction effects is R_s .

In Fig. 9, we show numerical results for the correlation energy (divided by r_s^2) versus r_s for $0.005 < r_s < 0.2$ and $R_0 = a^*$. For $r_s < 0.01$, the numerical results are well fitted by Eq. (26), the dotted line in Fig. 9. For $r_s = 0.01$ the exact result is $\epsilon_{cor}\!=\!-0.372\!\times\!10^{-4}$ Ry* and the analytical result according to Eq. (26) is $\epsilon_{cor} = -0.376 \times 10^{-4}$ Ry^{*}. The effect of a finite LFC increases with decreasing density; compare in Fig. 9 the solid line with the dashed line, which describes the MSA. The difference between the correlation energies calculated with $G_{\rm HFA}(q)$ and with $G_{\rm GA}(q)$ is negligible for small r_s (less than 2% for $R_0 = a^*$ and $r_s < 0.5$, see Table I). This difference becomes, however, important for large r_s , see Fig. 8.

C. Comparison: d=1, 2, 3

In two-dimensional and three-dimensional systems the exchange energy is given by $\varepsilon_{ex}(r_s) \propto -1/r_s$. We conclude that for the *d*-dimensional electron gas, $|\varepsilon_{ex}(r_s, d=1)| \gg |\varepsilon_{ex}(r_s, d=2)| > |\varepsilon_{ex}(r_s, d=3)|$ for $r_s \rightarrow \infty$, while $|\varepsilon_{ex}(r_s, d=1)| < |\varepsilon_{ex}(r_s, d=3)| < |\varepsilon_{ex}(r_s, d=2)|$ for $r_s \rightarrow 0$. This means that exchange effects are very important in the low-density range of the interacting one-dimensional electron gas: $\varepsilon_{ex}(r_s \rightarrow \infty, d=1)/\varepsilon_{ex}(r_s \rightarrow 0, d=3) \propto \ln(r_s)$. For large density, we get $\varepsilon_{ex}(r_s \rightarrow 0, d=1)/\varepsilon_{ex}(r_s \rightarrow 0, d=3) \propto 1/\varepsilon_{ex}(r_s \rightarrow 0, d=3) \propto a^*r_s/R_0$ and the exchange energy is strongly reduced if compared with the three-dimensional electron gas.

In Fig. 10, we show the interaction energy in units of the exchange energy versus r_s for $R_0 = a^*/5$, $a^*/2$, a^* , $2a^*$, and $4a^*$. Deviations of $\varepsilon_{int}/\varepsilon_{ex}$ from 1 indicate the importance of correlation effects. We conclude from Fig. 10 that correlation effects are stronger compared to exchange effects for thinner wires and that $\varepsilon_{int}/\varepsilon_{ex}$ in-

creases with decreasing density. Similar results have been found for three and two dimensions.^{8,9} For d=3and 2, the ratio at $r_s = 10$ was $\varepsilon_{int}/\varepsilon_{ex} = 1.403$ and $\varepsilon_{int}/\varepsilon_{ex} = 1.505$, respectively. For one-dimensional systems with $R_0 = a^*$, we find for $r_s = 10$ the ratio $\varepsilon_{int}/\varepsilon_{ex} = 1.74$. From Fig. 10, we conclude that $\varepsilon_{int}/\varepsilon_{ex}$ tends to saturate for large r_s and we anticipate that $\varepsilon_{cor}(r_s \to \infty) \propto -\ln(r_s)/r_s$ and $\varepsilon_{int}(r_s \to \infty)/\varepsilon_{ex}(r_s \to \infty)$ becomes constant in the low-density region as found for d=2 and 3.^{8,9}

In order to show that the Coulomb interaction is very important for quasi-one-dimensional systems, we show in Fig. 11 the ratio $\varepsilon_{int}(r_s)/\varepsilon_{kin}(r_s)$ versus r_s for $R_0 = a^*/5$, $a^*/2$, a^* , $2a^*$, and $4a^*$. Note, that already for $r_s > 0.5$, the interaction energy is larger than the kinetic energy. A similar figure has been published recently for two-dimensional systems and three-dimensional systems in order to show that with increasing valley degeneracy Coulomb effects increase.²³

TABLE VII. Inverse compressibility $1/\kappa(R_0)$ (in units of the inverse compressibility of the free-electron gas $1/\kappa_0$) for $R_0 = a^*/5$, $a^*/2$, $a^*, 2a^*$, and $4a^*$ and for various values of r_s .

				and and an	
r _s	$\kappa_0/\kappa(a^*/5)$	$\kappa_0/\kappa(a^*/2)$	$\kappa_0/\kappa(a^*)$	$\kappa_0/\kappa(2a^*)$	$\kappa_0/\kappa(4a^*)$
0.5	0.5474	0.7804	0.9015	0.9661	0.9904
1	-0.2666	0.2931	0.5968	0.8120	0.9325
1.5	-1.381	-0.3247	0.2079	0.5751	0.8161
2	-2.830	-1.110	-0.2455	0.2983	0.6555
3	-6.612	-3.321	-1.476	-0.3503	0.2745
4	-11.34	-6.385	-3.271	-1.214	-0.1586
5	-16.69	-10.14	-5.689	-2.402	-0.6677
6	-22.61	-14.44	-8.671	-3.966	-1.296
7	-28.87	-19.16	-12.11	- 5.990	-2.098
8	-35.42	-24.24	-15.94	-8.391	-3.116
9	-42.38	-29.61	-20.13	-11.19	-4.409
10	-49.55	-35.22	-24.58	-14.31	- 5.965



FIG. 9. Correlation energy ε_{cor}/r_s^2 versus RPA parameter r_s for $R_0 = a^*$ in the two-sum-rule approach as the solid line. The dashed line represents the RPA-like results calculated using $S_{MSA}(q)$. The dotted line corresponds to Eq. (26).

V. APPLICATION

In this section, we apply our results to calculate the plasmon dispersion, the compressibility, and the chemical potential in quasi-one-dimensional systems.

A. Plasmon dispersion

The collective modes $\omega_p(q)$ are determined by $1+V(q)[1-G(q)]\mathbf{X}_0(\omega_p(q),q)=0$ with $\mathbf{X}_0(\omega,q)$ as the Lindhard function of the free-electron gas in one dimension. The plasmon dispersion $\omega_p(q)$ in one-dimensional systems is given by an analytical formula, as described already before, ²⁴ and given by

$$\frac{\omega_p(q)}{\mathrm{Ry}^*} = qa^* \left[\frac{B_+(q)^2 A(q) - B_-(q)^2}{A(q) - 1} \right]^{1/2}, \quad (27a)$$



FIG. 10. Interaction energy ε_{int} (in units of the exchange energy ε_{ex}) versus RPA parameter r_s for $R_0 = 0.2a^*$, $0.5a^*$, a^* , $2a^*$, and $4a^*$ calculated within the two-sum-rule approach.

with

$$A(q) = \exp\{2\pi q a^* / [\mathbf{g}_v f(q R_0) [1 - G(q)]]\}$$
(27b)

and $B_{\pm}(q) = \pi/2g_v r_s \pm qa^*$. (27) was originally derived for a short-range potential with $f(q) = \text{const.}^{25} B_{\pm}(q)$ describes the electron-hole (eh) spectrum via

$$\omega_{\pm eh} / \mathbf{Ry}^* = qa^* |B_{\pm}(q)| \ge 0$$
 (28)

The electron-hole spectrum $\omega_{\rm eh}$ is characterized by $\omega_{-\rm eh} \leq \omega_{\rm eh} \leq \omega_{+\rm eh}$. For small wave numbers one finds $\omega_{+\rm eh}/{\rm Ry}^* = \pi q a^*/2g_v r_s \propto q$. For large wave numbers, the asymptotic law is given by $\omega_{\pm \rm eh}/{\rm Ry}^* = (qa^*)^2$.

For small wave numbers, a more transparent result for the plasmon dispersion is given by

$$\frac{\omega_p(q)}{\mathrm{Ry}^*} = \frac{qa^*}{r_s^{1/2}} \left\{ \frac{f_G(q)}{2} \left[1 + \left[1 + \frac{\pi^2/g_v^2 r_s + 4r_s(qa^*)^2}{f_G(q)} \right]^{1/2} \right] \right\}^{1/2},$$
(29a)

with

$$f_{G}(q) = f(qR_{0})[1 - G(q)]$$
. (29b)

For small wave numbers, we can rewrite (29) as

$$\frac{\omega_p(q)}{\mathbf{Ry}^*} = \frac{1}{r_s^{1/2}} q a^* \mathbf{C}_F(q) , \qquad (30a)$$

with

$$\mathbf{C}_{F}(q) = \left[f(qR_{0})[1 - G(q)] + \frac{\pi^{2}}{4g_{v}^{2}r_{s}} \right]^{1/2}.$$
 (30b)

Note, that in Eq. (30b) the dominant term is $f(qR_0)$ and a finite LFC reduces the plasmon energy. The recent measurements of the plasmon dispersion were done for doped semiconductor quantum wires with $r_s = 0.58$ and $R_0 = 2.6a^*$ and $0.05 < qa^* < 0.15$.^{15,16} Many-body effects are not yet important in such structures, as discussed in Ref. 24. However, the importance of the LFC for the plasmon dispersion increases for decreasing wire radius and decreasing density.

Numerical results for $\omega_p(q)$ versus wave number q in different approximations are shown in Fig. 12 for (i) $r_s = 4$ and $R_0 = a^*$ and (ii) $r_s = 10$ and $R_0 = a^*$. Our results confirm quantitatively that many-body effects reduce the



FIG. 11. Interaction energy ε_{int} (in units of the kinetic energy ε_{kin}) versus RPA parameter r_s for $R_0=0.2a^*$, $0.5a^*$, a^* , $2a^*$, and $4a^*$ calculated within the two-sum-rule approach.

plasmon energy. We note that for $r_s = 4$, many-body effects reduce the plasmon energy by about 30% for $0.05 < qa^* < 0.15$. We claim that the many-body effects can be measured by available techniques for quantum wires with $R_0 < a^*$ and $r_s > 1$. For $r_s = 10$, we find that many-body effects strongly reduce the plasmon energy (100%) compared with the RPA calculation. The analytical result according to Eq. (30) is a very good approximation to the numerical result, see the dotted lines in Fig. 12.

In Fig. 13, we present numerical results for $\omega_p(q)$ versus wave number q for $R_0 = a^*$ and different values of r_s : $r_s = 1$, 4, 10, and 20. Note that the wave-number



FIG. 12. Plasmon energy ω_p (in units of the effective Rydberg) versus wave number q for (i) $r_s = 4$, $R_0 = a^*$ and (ii) $r_s = 10$, $R_0 = a^*$ as solid lines according to Eq. (27). The dashed and dashed-dotted lines represent G(q)=0 and $G(q)=G_{\rm HFA}(q)$, respectively. The dotted line represents the analytical result according to Eq. (30). The shaded area represents the particle-hole spectrum.



FIG. 13. Plasmon energy ω_p (in units of the Fermi energy ε_F of the free electron gas) versus wave number q for $r_s = 1, 4, 10$, and 20 for $R_0 = a^*$. The shaded area represents the particle-hole spectrum.

range in Fig. 13 is much larger than in Fig. 12. The plasmon energy is given in terms of the Fermi energy $\varepsilon_F \propto 1/r_s^2$ of the free-electron gas. The regime of electron-hole excitations is also shown as the shaded area. With increasing r_s the energy difference between the collective modes and the one-particle excitations increases. This effect might help to discriminate in experiments the collective modes from the electron-hole excitations.

B. Compressibility

The compressibility κ is given as²

$$\frac{\kappa_0}{\kappa} = 1 + \frac{8g_v^2 r_s^4}{\pi^2 \text{Ry}^*} \frac{d^2(\varepsilon_c + \varepsilon_{\text{int}})}{dr_s^2}$$
$$= 1 + \frac{\kappa_0}{\kappa_c} + \frac{\kappa_0}{\kappa_{\text{ex}}} + \frac{\kappa_0}{\kappa_{\text{cor}}} , \qquad (31)$$

with $\kappa_0 = 16g_v^2 r_s^3 a^* / \pi^2 Ry^*$ as the compressibility of the free-electron gas. In the following, we use $\varepsilon_C = 0$ and $\kappa_0 / \kappa_C = 0$: some comments concerning κ_C are given in the Appendix A.

With our analytical results for $\varepsilon_{ex}(r_s)$, we can calculate κ_0/κ_{ex} . For $R_s \ll 1$, we find

$$\frac{\kappa_0}{\kappa_{\rm HFA}} = 1 - \frac{9R_0}{5\pi a^*} R_s^3 , \qquad (32)$$

and for $R_s \gg 1$, we get

$$\frac{\kappa_0}{\kappa_{\rm HFA}} = 1 - \frac{2R_0}{\pi a^*} R_s [\ln(R_s)[1 + 1/2R_s^2] + C - 73/120 + [1949/1680 - C]/2R_s^2]. (33)$$

In the same way, Eq. (26) leads for $R_s \ll 1$ to

$$\frac{\kappa_0}{\kappa_{\rm cor}} = -B_{\rm cor} \frac{16g_v^5}{\pi^6} \frac{a^{*2}}{R_0^2} r_s^4 .$$
(34)

 κ_0/κ versus r_s is shown in Fig. 14 for $R_0 = a^*$, together with results within the HFA. The fact that



FIG. 14. Inverse compressibility $1/\kappa$ (in units of the inverse compressibility of the free-electron gas $1/\kappa_0$) versus RPA parameter r_s for $R_0 = a^*$. The solid line represents the two-sum-rule approach. The dashed line represents the HFA. The dotted lines represent the analytical results for $r_s \ll 1$ and $r_s \gg 1$, according to Eq. (32) and Eq. (33), respectively. In the inset, we show the correlation contribution to the inverse compressibility $\kappa_0/\kappa_{\rm cor}$ at large density and the dashed-dotted line represents Eq. (34). The direct Coulomb term is neglected.

 $\kappa_0/\kappa_{\rm HFA} > \kappa_0/\kappa$ for $r_s < 0.5$ (see the inset) and for $r_s > 7$ is due to the nonmonotonous behavior of the correlation energy versus r_s , see Fig. 8. For small density, we find that $\kappa_0/\kappa < 0$. In the literature, a negative κ_0/κ for large r_s has been interpreted as an instability.^{4,5} However, experimentally there are no indications of such an instability: the positive background charge for global neutrality stabilizes the system.²⁶ For $R_0 = a^*$, we find numerically that $\kappa_0/\kappa = 0$ for $r_s = 1.75$, while $\kappa_0/\kappa_{\rm HFA} = 0$ for $r_s = 1.45$. A negative compressibility in a two-dimensional electron system has been recently found in experiments²⁷ in the low-density regime. It has been shown that the Hartree term²⁷ and the finite extension effects²⁰ strongly modify the compressibility. We note that our results for the compressibility in two dimensions¹² were in very good agreement with Monte Carlo calculations.9 Numerical results for the compressibility are given in Table VII for $R_0 = 0.2a^*, 0.5a^*, a^*, 2a^*, and 4a^*.$

C. Chemical potential

The chemical potential μ at zero temperature is given in terms of the ground-state energy (theorem of Seitz) as $\mu = 3\varepsilon_{kin} + \partial(N\varepsilon_{int})/\partial N$. The factor with the kinetic energy corresponds to $\varepsilon_F = k_F^2/2m^* = 3\varepsilon_{kin}$ with the Fermi energy ε_F of the noninteracting electron gas. Seitz's theorem implies that²

$$\mu = \varepsilon_F + \Sigma(k_F) , \qquad (35)$$

with

$$\Sigma(k_F) = \varepsilon_C + \varepsilon_{\rm int} - r_s \partial(\varepsilon_C + \varepsilon_{\rm int}) / \partial r_s . \qquad (36)$$

We conclude that $\Sigma(k_F)$ describes interaction effects for the chemical potential.

We can write $\Sigma(k_F) = \Sigma_C(k_F) + \Sigma_{ex}(k_F) + \Sigma_{cor}(k_F)$. For the direct Coulomb term $\Sigma_C(k_F)$, see the Appendix A. Numerical results for $\Sigma(k_F)$ and $\Sigma_{ex}(k_F)$ versus r_s are shown in Fig. 15 for $R_0 = a^*$: the nonmonotonous behavior of the correlation energy versus r_s gives rise to $\Sigma_{ex}(k_F) < \Sigma(k_F)$ for large density and $\Sigma_{ex}(k_F) > \Sigma(k_F)$ for small density. Analytical results for the exchange contribution are given as

$$\Sigma_{ex}(R_s \to 0) / Ry^* = -\frac{a^*}{R_0} \left[2.4082 - \frac{18}{5\pi} R_s \right]$$
 (37)

and

$$\Sigma_{ex}(R_s \to \infty) / Ry^* = -\frac{4a^*}{\pi R_0 R_s} [\ln(R_s)[1+1/6R_s^2] + C + 1 - 73/120 + [1949/1680 - C + 1/3]/6R_s^2].$$
(38)

We note that $\sum_{ex}(R_s)R_0/a^*$ is only a function of R_s . For the contribution of the correlation, we obtain

$$\Sigma_{\rm cor}(r_s <<1)/{\rm Ry}^* = B_{\rm cor} \frac{g_v^3 a^{*2}}{\pi^4 R_0^2} r_s^2 .$$
(39)

Numerical results for the chemical potential μ versus r_s are shown in Fig. 16 for $R_0 = a^*$. The chemical potential is strongly renormalized by many-body effects when the carrier density becomes small. Within the HFA, we get the analytical results

$$\mu_{\rm HFA}(R_s \to 0)/\varepsilon_F = 1 - 2.4082 \frac{R_0}{a^*} R_s^2 + \frac{18}{5\pi} \frac{R_0^2}{a^{*2}} R_s^3$$
(40)

and



FIG. 15. $\Sigma(k_F)$ versus RPA parameter r_s for $R_0 = a^*$ according to Eq. (36). The arrow indicates $\Sigma(k_F) = \Sigma_{ex}(k_F)$ For $r_s = 0$, see Eq. (37). The solid line represents the two-sum-rule approach. The dashed line represents the HFA. The dotted lines represent the analytical results for $r_s \ll 1$ and $r_s \gg 1$ according to Eq. (37) and Eq. (38), respectively.



FIG. 16. Chemical potential μ (in units of the Fermi energy of the free electron gas ε_F) versus r_s for $R_0 = a^*$. The solid line represents the two-sum-rule approach. The dashed line represents the HFA. The dotted lines represent the analytical results for $r_s \ll 1$ and $r_s \gg 1$ according to Eq. (40) and Eq. (41), respectively.

$$\mu_{\rm HFA}(R_s \to \infty)/\varepsilon_F$$

$$= 1 - \frac{4R_0}{\pi a^*} R_s [\ln(R_s)[1 + 1/6R_s^2] + C + 1 - 73/120 + [1949/1680 - C + 1/3]/6R_s^2].$$
(41)

These analytical results for the exchange contribution to the chemical potential can be used by experimenters to estimate the importance of many-body effects in a given structure with known r_s and R_0 . Note that for $R_0 = a^*$, we obtain $\mu/\varepsilon_F = 0$ for $r_s = 0.6$.

VI. DISCUSSION

A. Comparison with other work

In this paper, we used a sum-rule version of the STLS approach to calculate the LFC of the quasi-onedimensional electron gas. A cylindrical symmetry has been used for the confinement. It is well known that the STLS approach is not exact, because the pair-correlation function g(z=0) is negative for large values of r_s . We found in the two-sum-rule version of the STLS approach a slightly negative value of g(z=0), even for very large r_s . In Ref. 28, we studied this sum-rule approximation of the STLS approach for the quasi-one-dimensional electron gas, where the Coulomb interaction potential is given by $V(q)=2e^2K_0(qb)/\varepsilon_L$ and with b as the extension parameter. Some analytical results for this model are given in the Appendix B.

Earlier numerical results for the STLS approach concern one-dimensional systems with an oscillator confinement.⁵ In Ref. 5, the LFC and the plasmon dispersion have been calculated and given in numerical form: the general trend for the LFC in this model is in agreement with the results presented in this paper and we have checked that the LFC given in Ref. 5 is in agreement with our generalized approximating expression for large and small wave numbers. Our detailed results for the oscillator confinement will be presented elsewhere.²⁹ A square-well confinement was studied in Refs. 6 and 30 within the STLS approach for $r_s = 1.5$: the results obtained for the plasmon dispersion and the paircorrelation functions are in qualitative agreement with our results. Our intention was to supply an analytical expression for the LFC and we suggest checking some of our results by more accurate calculations as, for instance, Monte Carlo calculations.

The calculation of the correlation energy in interacting systems is a topic of considerable interest (for a recent review, see Ref. 31). In this paper, we calculated the correlation energy in quasi-one-dimensional systems. The calculations of the exchange energy, using the selfenergy,^{17,32} are in agreement with our results. In two and three dimensions, we found¹² that the sum-rule approach is in good agreement with quasiexact results from Monte Carlo calculations. However, Monte Carlo calculations have not yet been published for quasi-onedimensional systems. A Monte Carlo calculation would be extremely useful in order to test the validity range of our approach. Due to the small negative pair-correlation function at large r_s , we expect that our approach is quantitatively correct for $r_s < 10$.¹² For $r_s > 10$, we expect at least a qualitative correct result.

Many-body effects using the self-energy have been calculated in Ref. 33. However, within this approach, no information about the dielectric function is available and the ground-state energy was not discussed. At this point it should be noted that even Monte Carlo calculations for the ground-state energy do not immediately supply information on the dielectric function. Only recently some progress in this direction was reported for the threedimensional electron gas.³⁴

It is known^{8,9} that the STLS approach gives a correct description of the ground-state energy of the threedimensional³ and two-dimensional electron gas⁴ for $r_s < 15$. We expect for quasi-one-dimensional systems reliable numerical results concerning the correlation energy, the chemical potential and the compressibility. We note, however, that the local-field correction, when calculated using the STLS approximation, does not fulfill the compressibility sum rule:² G(q) is overestimated for $q \rightarrow 0$ and for long wavelengths and we expect that our results for the dielectric function and the plasmon dispersion are only qualitatively correct. The Vashishta-Singwi approach³⁵ offers the possibility of fulfilling the compressibility sum rule using a modified STLS approach.

The LFC determines the susceptibility $\mathbf{X}(q)$, given in terms of the Lindhard function $\mathbf{X}_0(q)$ of the free-electron gas, by

$$\mathbf{X}(q) = \frac{\mathbf{X}_{0}(q)}{1 + V(q)[1 - G(q)]\mathbf{X}_{0}(q)}$$
 (42)

The dielectric function $\varepsilon(q)$ is expressed as

$$1/\varepsilon(q) = 1 - V(q)\mathbf{X}(q) . \tag{43}$$



FIG. 17. Inverse dielectric function $1/\epsilon(q)$ versus wave number q for $r_s = 0.5$, 1, 4, and 10 and for $R_0 = a^*$ according to Eq. (43).

Equation (43) represents the fundamental relation between $\varepsilon(q)$ and $\mathbf{X}(q)$.¹ The inverse dielectric function $1/\varepsilon(q)$ is represented in Fig. 17 for $r_s = 0.5$, 1, 4, 10, and for $R_0 = a^*$. Note that $1/\varepsilon(q)$ has a singular behavior for $q = 2k_F$, due to the logarithmic divergence of $\mathbf{X}_0(q)$: $1/\varepsilon(2k_F) = -G(2k_F)/[1-G(2k_F)]$. We see that the inverse dielectric function goes to zero for a small wave number and that $1/\varepsilon(q)$ is negative for small q and low density, due to the finite LFC. Such a behavior has been found in three-dimensional systems within a Monte Carlo calculation.³⁵ Within the RPA with G(q)=0, one finds $0 \le 1/\varepsilon(q) < 1$.

It was shown before that the mobility at zero temperature is determined by the susceptibility X(q) of the interacting electron gas.³⁶ Our analytical results for the LFC have been used for the calculation of the mobility³⁷ and of the screened potential³⁸ in the real space, which is determined by the dielectric function. It was shown that Friedel oscillations are strongly enhanced by the LFC in the low-density range.³⁸ We hope that our analytical form for the LFC can be used in more complex calculations as an input function.

B. Model, experiments, and real systems

We used in our calculation a one-subband model. In this model, the Fermi energy must be smaller than the intersubband energy distance between the first and the second subband. One gets as condition $R_s > 0.3$ $(r_s > 0.25R_0/g_v a^*)$.¹⁴ Clearly, in the low-density regime, where many-body effects are most important, this condition can always be fulfilled.

It was predicted by theory³⁹ that the interacting onedimensional electron gas is not a normal Fermi liquid with a well-defined Fermi surface. However, experiments¹⁵ clearly point to a Fermi liquid description and it was recently^{16,40} shown that electron-hole excitations are seen in inelastic light-scattering experiments made with artificial quantum wires based on doped GaAs. The existence of electron-hole excitations shows that a Fermiliquid description is applicable, at least at the finite temperatures used in experiments.

We believe that the results presented in this paper are not only important for *artificial* quasi-one-dimensional systems (as realized with semiconductor heterostructures^{15,16,40,41}), but also for *natural* quasi-onedimensional systems (as realized with organic materials or polymers⁴²). In the presently studied systems of semiconductor quantum wires, ^{15,16} the many-body effects are still small because of $r_s < 1$ (and $R_0 > 2a^*$).²⁴ However, we expect that the predictions made in this paper can be checked in the near future when existing technology is used to produce semiconductor wire structures with a gate, where the electron density can be varied.⁴¹

In Eq. (23), we have introduced the parameter $R_s = 4g_v a^* r_s / \pi R_0$ as the relevant parameter for the exchange energy. The correlation energy in Eq. (26) is also given by $\varepsilon_{cor}(R_s \ll 1) \propto -R_s^2$. This indicates that the parameter for weak coupling in quasi-one-dimensional systems is $R_s < 1$ (and not $r_s < 1$). R_s depends on the confinement. Strongly correlated systems in quasi-one-dimensional systems are characterized by $R_s > 1$. We believe that the importance of this parameter was not yet identified in previous work. In this paper, we presented numerically results for $g_v = 1$. Our analytical results include the valley degeneracy. In fact, it was shown recently that this parameter is quite important. With increasing valley degeneracy, the many-body effects increase in the two-dimensional and three-dimensional electron gas.²³ Note that $r_s \propto g_v$.

VII. CONCLUSION

In this paper, we calculated the local-field correction of quasi-one-dimensional systems confined in cylindrical wires. The local-field correction is given in an analytical form and characterized by two coefficients, which are calculated numerically and which depend on the carrier density and the wire radius. We found that for decreasing density and decreasing wire radius, many-body effects increase. The ground-state energy has been calculated as function of the wire radius and the electron density. Predictions have been made for experimentally measurable quantities as to the dispersion of the collective modes and the compressibility. A parameter R_s characterizing interaction effects in one-dimensional systems has been identified.

The results for many-body effects in the quasi-onedimensional electron gas presented in this paper complete text-book discussions for the three-dimensional^{1,2} and the two-dimensional electron gas.⁷ Our numerical and analytical results are important from a theoretical and practical point of view and should be of help to experimenters for the design and the analysis of experimental results. Our theory has predictive power, which was demonstrated by analytical results in the relevant parameter space (r_s , R_0) of quasi-one-dimensional systems. In recent work, the results of the present paper have been used in order to study transport properties³⁶ and screening effects³⁷ modified by many-body effects.

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APPENDIX

1. Direct Coulomb energy

In this section, we give some arguments concerning the effect of the Hartree contribution. Let us assume that the donors are distributed uniformly in the volume (and do not contribute to the charge density in Poisson's equation). The electrons confined in the wire give rise via Poisson's equation to a band-bending potential $V_{\rm BB}(r)$ of the form¹⁴

$$V_{\rm BB}(r)/{\rm Ry}^* = 6Na^* \left[\frac{11}{18} - \frac{r^2}{R_0^2} + \frac{r^4}{2R_0^4} - \frac{r^6}{9R_0^6} \right].$$
 (A1)

This band-bending potential increases with increasing density and shifts the subband energy E_{10} for N=0 to a larger energy E_1 . In the lowest order perturbation theory $(N \rightarrow 0)$, one finds¹⁴

$$E_1 = E_{10} + \delta E_1 \tag{A2a}$$

with

$$\delta E_1 = 2.4 \text{Ry}^* N a^* = 1.2 \text{Ry}^* / r_s$$
 (A2b)

It was recently shown for quantum wells that in the lowdensity limit (with only one subband occupied), the lowest order approximation is already a very good approximation of the full Hartree solution.⁴³

The shift of the subband energy induces a shift of the chemical potential $\mu = \varepsilon_F + \delta E_1$ even if many-body effects (described by the LFC) are neglected. E_1 can be written as $E_1 = E_{10}(1+\eta)$ and η is the small parameter, which is expected to be proportional to $\eta \propto Ne^2 \propto N/a^*$. In order to get a parameter without dimensions, we conclude that $\eta \propto NR_0^2/a^*$. With $E_{10} \propto 1/R_0^2$, we derive $\delta E_1 = \eta E_{10} \propto N$ and δE_1 is independent of R_0 , the length scale of the confinement, in agreement with Eq. (A2b). In quantum wells of width L, one gets $\delta E_1 \propto NL$ with $\eta = NL^3/a^*$ and δE_1 depends on the length scale of the confinement.⁴³ We note that our calculation of δE_1 does not take into account the exchange contribution, which also modifies the subband energy.⁴⁴

With Eq. (A2b), the Coulomb energy $\varepsilon_C = \delta E_1/2$ in the lowest order perturbation theory is given by

$$\epsilon_C / Ry^* = 1.2Na^* = 0.6/r_s$$
, (A3)

and the contribution to the compressibility is expressed as

$$\frac{\kappa_0}{\kappa_C} = 1.2 \frac{8g_v^2}{\pi^2} r_s . \tag{A4}$$

We conclude that the direct Coulomb energy increases the inverse compressibility, while exchange and correla-



FIG. 18. Inverse compressibility $1/\kappa$ (in units of the inverse compressibility $1/\kappa_0$ of the free-electron gas) versus electron density N for $R_0 = a^*$ when the Hartree term [Eq. (A4)] is included as the solid line. For the dashed line, the Hartree term was neglected. The dotted line $\kappa_0/\kappa=1$ represents the inverse compressibility of the free-electron gas.

tion decrease the inverse compressibility. In Fig. 18, we show the inverse compressibility versus the electron density when the Hartree term according to Eq. (A4) is taken into account. For large densities, the Hartree term dominates and $\kappa_0/\kappa > 1$. At small densities, the interaction effects described by the LFC are larger than the Hartree contribution and the inverse compressibility is negative: $\kappa_0/\kappa < 0$. For the contribution to $\Sigma(k_F)$, we find with Eq. (A3) in the lowest order perturbation theory,

$$\Sigma_C(k_F)/\mathrm{Ry}^* = \frac{1.2}{r_s} . \tag{A5}$$

In Ref. 27, the compressibility of quantum wells has been measured and it was argued that the direct Coulomb energy gives rise to a negative contribution to the inverse compressibility. This negative sign is the consequence of the experimental setup, where an electric field is applied and which *reduces*, apparently, the subband energy $\delta E_1 < 0$.

2. Simplified models

Within a simplified model, the Coulomb interaction potential $V(q)=2e^2K_0(qb)/\varepsilon_L$ is described by the form factor $f(x)=4K_0(x)$ with x=qb and this is the Fourier transform of $V(z)=e^2/\varepsilon_L(z^2+b^2)$. We note that this interaction potential has the wrong limit for large wave numbers, $K_0(x \to \infty) = (\pi/2x)^{1/2} \exp(-x)$, however, the small wave number behavior is correct: $K_0(x\to 0)$ $= \ln(2/x)$. Some numerical results of the sum-rule approach for the simplified model can be found in Ref. 28.

In the HFA, we find that $C_{11,\text{HFA}}$ is given by the following transcendental equation:

$$K_{0}(q_{0}b/C_{11,\text{HFA}}) = 2K_{0}(2k_{F}b) + K_{1}(2k_{F}b)/k_{F}b$$
$$-2K_{2}(2k_{F}b) + 2/(2k_{F}b)^{2}.$$
(A6)

The coefficients $C_{i1,\text{HFA}}$ are given by

$$\exp[-2b/r_s^{1/2}a^*C_{11,\text{HFA}}(r_s \to 0)]$$

= $\pi (2/\pi)^{7/2} (g_v r_s a^*/b)^2$
 $\times [2b/r_s^{1/2}a^*C_{11,\text{HFA}}(r_s \to 0)]^{1/2}$, (A7a)

 $C_{11,\text{HFA}}(r_s \to \infty) = 4g_v \exp[-3/2] r_s^{1/2} / \pi,$ (A7b)

and

$$C_{21,\rm HFA} = 2g_v r_s a^* / b \pi$$
 (A8)

Equation (21) determines the exchange energy and we get

$$\varepsilon_{ex}(r_s)/\mathbf{Ry}^* = -[\pi K_0(2k_F b)\mathbf{L}_1(2k_F b) + \pi K_1(2k_F b)\mathbf{L}_0(2k_F b) + 2K_0(2k_F b) + K_1(2k_F b)/k_F b - 2/(2k_F b)^2]/2g_v r_s .$$
(A9)

 $L_n(x)$ is the modified Struve function.⁴⁵ With Eq. (A9), we conclude that in the simplified model, the exchange energy can be calculated *analytically*; however, only in terms of Bessel and modified Struve functions. The limiting behavior of the exchange energy is given by

$$\epsilon_{\rm ex}(R_s \to 0)/{\rm Ry}^* = -\frac{a^*}{b}$$
 (A10a)

for small $R_s = 4g_v a^* r_s / \pi b$ and by

$$\varepsilon_{\rm ex}(R_s \to \infty)/{\rm Ry}^* = -\frac{2a^*}{\pi b R_s} \ln(R_s)$$
 (A10b)

for large R_s . We note that $\varepsilon_{ex}(R_s \to 0)$ for the simplified model is smaller than for a cylindrical wire, where $\varepsilon_{ex}(R_s \to 0)/Ry^* = -2.4a^*/R_0$, if b is identified with R_0 , which is suggested by the comparison of the behavior of the interaction potentials for small wave numbers. This difference (the factor 2.4) is due to the fact that for large wave numbers, the real potential in a cylindrical wire is larger than in the simplified model. We mention that an universal scaling exists for the exchange energy $\varepsilon_{ex}(R_s)b/a^* = H(R_s)$ and $H(R_s)$ is a function of R_s only. This is easily seen from Eq. (A9), where $2k_Fb \propto b/g_p r_s \propto 1/R_s$.

For the correlation energy in the high-density range, we find

$$\varepsilon_{\rm cor}(R_s \ll 1)/{\rm Ry}^* = -B_{\rm cor} \frac{g_v^3}{\pi^4} \frac{a^{*2}}{b^2} r_s^2$$
, (A11)

with $B_{cor} = 8$ and we conclude that $\varepsilon_{cor}(R_s \ll 1) \propto g_v R_s^2$. We want to mention that the simplified model and the cylindrical wire model give similar results for the groundstate energy at large density $[\varepsilon_{cor}(r_s \ll 1) \propto r_s^2]$ and $\varepsilon_{ex}(r_s \ll 1) \propto r_s^0]$, due to the fact that $f(x \to \infty)$ in both models decreases more rapidly than 1/x.

A different behavior is found in the model used in Ref. 46, where $f(x)=2\exp(x^2/4)K_0(x^2/4)$, with x=qc and c is the confinement parameter: $f(x \to \infty) \propto 1/x$. For this model, we derive for the exchange energy

$$\varepsilon_{\rm ex}(r_s \ll 1) / \mathbf{Ry}^* = a^* [(2/\pi)^{1/2} \ln(2g_v a^* r_s / \pi c) - 0.262] / c$$
(A12)

and

$$\varepsilon_{\text{ex}}(r_s \gg 1) / \mathbf{Ry}^* = - [\ln(2^{1/2} 4g_v a^* r_s / \pi c) + (3 - \mathbf{C})/2] / 2g_v r_s , \qquad (A13)$$

and for the correlation energy,

$$\varepsilon_{\rm cor}(r_s \ll 1)/{\rm Ry}^* = 8g_v^3 a^{*2} r_s^2 \ln(2g_v a^* r_s / \pi c) / \pi^3 c^2$$
 (A14)

Note that this model is less realistic:²⁴ it corresponds to a wire which has a zero confinement width in one direction and the confinement area vanishes.

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