PHYSICAL REVIEW B VOLUME 52, NUMBER 14 1 OCTOBER 1995-II

Interlayer charge dynamics of the uniform resonating-valence-bond state

Naoto Nagaosa
Department of Applied Physics, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan
(Received 4 January 1995; revised manuscript received 7 June 1995)

The physical properties along the c axis of the uniform resonating-valence-bond state are studied. The physical electron, i.e., the composite particle of a spinon and a holon, hops between the layers incoherently. Conductivity, thermopower, thermal conductivity, and Hall constant are calculated and compared with the experiments in high-T_c cuprates. The renormalization group equation is obtained for the interlayer hopping t_c which is found to be marginally relevant. The effects of the spin gap are also discussed.

I. INTRODUCTION

Recently there appeared several experiments on the physical properties of the high-T_c cuprates along the c axis, i.e., perpendicular to CuO_2 planes, revealing anomalous features.[1–8] The dc resistivity ρ_c behaves as insulating for small x (x: hole concentration) while metallic in overdoped region .[1] The infrared optical conductivity $\sigma_c(\omega)$ along c axis has been measured by several groups .[2–6] In the underdoped region $\sigma_c(\omega)$ is broad and featureless in the normal state showing the incoherent nature of the c axis transport. Below the superconducting transition temperature T_c, on the other hand, the plasma edge appears suddenly in the infrared response implying the onset of the coherent motion along the c axis.[2] The thermopower S_c is positive and increases with T and tends to saturate around $S_c \sim 80$ μV/K for $T > 250$ K at $x = 0.15$ ($La_{2-x}Sr_xCuO_4$).[7] The Hall constant R_H^c with the magnetic field H parallel to the plane has been measured on $YBa_2Cu_3O_y$ (Ref. 8) and on $La_{2-x}Sr_xCuO_4$.[5] They found the negative and small values which are almost temperature independent in a sharp contrast with the in-plane R_H^{ab} which is positive and depends strongly on the temperature.

At an early stage, Anderson[9] proposed that the insulating ρ_c with the metallic ρ_{ab} invalidates the Fermi-liquid picture in high-T_c cuprates. The localization occurs simultaneously along all the directions because the scaling equation can be mapped to the isotropic one by the appropriate change of the scales along a, b, and c axis. These considerations are for the cases of static disorder, i.e., the elastic scattering of the electrons. In the case of inelastic disorder, on the other hand, one should distinguish between the two cases, i.e., the scatterings within the layers and between the layers. In the former case, one should compare the interlayer hopping t_c with the inverse of the inelastic lifetime $\hbar/\tau$. When $t_c \gg \hbar/\tau$ the coherent motion along all the directions occurs with the anisotropic band. When $t_c \ll \hbar/\tau$, on the other hand, the thermalization occurs before the interplane hopping, and the motion along c axis is incoherent.[10] However the dc conductivities have the similar expressions in both cases when t_c is much less than the intralayer transfer t_a. Both the inplane σ_a and c axis σ_c are proportional to τ and their ratio is of the order of $(t_a/t_c)^2$.[10] When the scatterings occur between the layers and give rise to σ_c, the inplane σ_a is proportional to τ while σ_c to $1/\tau$. Hence their product $\sigma_a\sigma_c \cong [e^2N(\mu_F)v_Fa]^2$ [$N(\mu_F)$: density of states at Fermi level μ_F; v_F: Fermi velocity in the plane; a: lattice constant] is temperature independent.[11] Recently, Rojo and Levin[12] studied models including elastic and inelastic scatterings between the layers and claimed that there appears some temperature range where the c-axis resistivity is insulating while the inplane one is metallic.

According to resonating-valence-bond (RVB) scenarios with spin-charge separation,[9,13–19] a spinon and a holon have to recombine to form a physical electron in order to hop between adjacent layers, and dissociate again into a spinon and a holon.[14] Hence the conduction mechanism is expected to be quite different between inter- and intralayer. Recently Anderson *et al.* proposed "charge confinement" within each layer with the vanishing oscillator strength along the c axis at zero temperature.[18] This is coming from the infrared catastrophe due to the rearrangements of the whole electronic system due to the charge transfer between the layers. They also demonstrated "confinement" for the two coupled one-dimensional Tomonaga-Luttinger liquids.[19]

Considering the situation that many scenarios have been proposed to explain the c-axis conductivity, it is important to study various transport properties besides the conductivity. In this paper we calculate several physical properties, i.e., conductivity $\sigma_c = \rho_c^{-1}$, thermopower S_c, thermal conductivity κ_c, and Hall constant R_H^c, based on the uniform RVB state.[14,15] This means that the calculations in this paper are restricted to the region where the temperature is higher than the Bose condensation temperature of the holon T_{BE} and also the spinon pairing temperature T_D. The rough estimate of the condensation temperature T_{BE} is given by $T_{BE} \sim xJ$ with J being the spin exchange interaction. T_D is estimated to be around one-fifth of J at half-filling ($x = 0$) and decreases as a function of x. Therefore our considerations can be

0163-1829/95/52(14)/10561(8)/$06.00

applied to the normal state near the optimal doping.[13,15] We will treat the hopping matrix element t_c between adjacent layers as a small perturbation. When the temperature T is not very low, the lowest order terms with respect to t_c give the dominant contributions. As T is lowered, the higher order terms grow and effectively renormalize t_c. We will also discuss this renormalization later, but first restrict ourselves to the lowest order contributions.

The plan of this paper is the following. In Sec. II the model is introduced and the conductivity sum rule is derived. In Sec. III the conductivity, thermopower, and thermal conductivity along the c axis are studied, and the Hall constant is calculated in Sec. IV. The renormalization of the interlayer hopping t_c is studied in terms of the effective action for the boson and the gauge field in Sec. V. Section VI is devoted to discussion and conclusions. We take the units where $\hbar = c = k_B = 1$, and the lattice constant of the system is taken to be unity.

II. MODEL AND SUM RULE

The essential difficulty in the theories of strongly correlated systems is the treatment of the strong repulsive interaction between the electrons. A useful trick to treat the strong on-site repulsion is the slave boson method where the double occupation of the electrons at one site is excluded.[13] Hence the three states for each site x is expressed as (i) up-spin electron, (ii) down-spin electron, and (iii) hole. Then we introduce the operators for each state $f^\dagger_\uparrow(R), f_\uparrow(R)$, $f^\dagger_\downarrow(R), f_\downarrow(R)$, and $b^\dagger(R), b(R)$, respectively. f operators satisfy the fermion anticommutation relations and b operators boson commutation relations. The constraint that all the states for each site R is exhausted by the three states above is expressed as

$$f^\dagger_\uparrow(R)f_\uparrow(R) + f^\dagger_\downarrow(R)f_\downarrow(R) + b^\dagger(R)b(R) = 1. \qquad (1)$$

The creation operator of the physical electron $C^\dagger_\sigma(R)$ with spin σ at site R is expressed in terms of the boson annihilation operator $b(R)$ and fermion creation operator $f^\dagger_\sigma(R)$ as

$$C^\dagger_\sigma(R) = f^\dagger_\sigma(R)b(R). \qquad (2)$$

Effective Hamiltonian for the strongly correlated system is written in terms of the constrained operators $C^\dagger_\sigma$ and C_σ and hence in terms of b (holon) and f (spinon) operators by setting Eq. (2). Introducing the mean-field decoupling for the fermions and bosons and replacing the local constraint Eq. (1) by the global and averaged one, we obtain the mean-field theory of RVB state.[13] The low lying fluctuation around this mean-field state is the gauge field which recovers the local constraint Eq. (1). It has been shown that this gauge field plays important roles when studying the physical properties of RVB states [15]. For the details of this derivation the readers are referred to Ref. 15 and we start with the following Hamiltonian for the uniform RVB state:

$$\begin{aligned} H &= H_0 + V \\ &= -\sum_{\ell,r,\sigma}\sum_{\alpha=a,b}\left\{t_F\left[f^\dagger_{\ell\sigma}(r)f_{\ell\sigma}(r+e_\alpha)e^{i[a_\alpha(r,\ell)+(1+\zeta)eA_\alpha(r,\ell)]} + \text{H.c.}\right] + \mu_F f^\dagger_{\ell\sigma}(r)f_{\ell\sigma}(r)\right\} \\ &\quad -\sum_{\ell,r}\sum_{\alpha=a,b}\left\{t_B\left[b^\dagger_\ell(r)b_\ell(r+e_\alpha)e^{i[a_\alpha(r,\ell)+\zeta eA_\alpha(r,\ell)]} + \text{H.c.}\right] + \mu_B b^\dagger_\ell(r)b_\ell(r)\right\} \\ &\quad -\sum_{\ell,r,\sigma}\left[t_c C^\dagger_{\ell\sigma}(r)C_{\ell+1\sigma}(r)e^{ieA_c(r,\ell)} + \text{H.c.}\right], \end{aligned} \qquad (3)$$

where H_0 is the effective Hamiltonian describing the independent layers while V is the interlayer hopping Hamiltonian. In deriving Eq. (3) we have assumed that the interlayer hopping t_c is small and does not modify the two-dimensional RVB state in each layer. The fermion transfer energy t_F is of the order of $xt_a + J$, while that of boson is estimated as $t_B \sim J$. $t_a = t_b, t_c$ are the transfer energy of the electron along the three directions and J is the spin exchange energy appearing in the original t-J model. In Eq. (3) the site index R is explicitly specified by the layer index ℓ and the site index $r = (r_x, r_y)$ within the plane. ζ is an arbitrary constant which does not appear in the physical quantities as discussed in Ref. 15. Each layer is the square lattice and $e_a = (1,0,0)$, $e_b = (0,1,0)$, and $e_c = (0,0,1)$ are the three unit vectors. We have employed the Peierls approximation for the external electromagnetic field, i.e.,

$$A_\mu(r,\ell) = \int_{R=(r,\ell)}^{R'=R+e_\mu} d\vec{r}\cdot\tilde{\mathbf{A}}(\vec{r})$$

where $\mu = a,b,c$, $\vec{r}$ is the three-dimensional space vector, $\tilde{\mathbf{A}}(\vec{r})$ is the vector potential for the external electromagnetic field, and the integral is along the straight line connecting the two end points R and R'. As has been discussed in Ref. 15 the phase of the order parameters, i.e., $\langle f^\dagger_{\ell\sigma}(r)f_{\ell\sigma}(r+e_\alpha)\rangle$, which is assumed to be independent spin σ, and $\langle b^\dagger_\ell(r)b_\ell(r+e_\alpha)\rangle$, constitutes the spatial component of the gauge field $a_\alpha(r,\ell)$. It has been known that only the transverse part of the gauge field gives the singular contribution in the small frequency and wave number.[15] Then we consider only the transverse part for $a_\alpha(r,\ell)$. Then our Hamiltonian is gauge invariant for the following two kinds of gauge transformation. One is the internal one which leaves the physical electron invariant, i.e.,

$$f_{\ell\sigma}(r) \to f_{\ell\sigma}(r)e^{i\theta(r,\ell)},$$

$$b_\ell(r) \to b_\ell(r)e^{i\theta(r,\ell)},$$

$$a_\alpha(r,\ell) \to a_\alpha(r,\ell) + \theta(r,\ell) - \theta(r+e_\alpha,\ell).$$

The other is the usual one related to the electromagnetic field,

$$f_{\ell\sigma}(r) \to f_{\ell\sigma}(r)e^{i(1+\zeta)\phi(r,\ell)},$$

$$b_\ell(r) \to b_\ell(r)e^{i\zeta\phi(r,\ell)},$$

$$C_{\ell\sigma}(r) \to C_{\ell\sigma}(r)e^{i\phi(r,\ell)},$$

$$A_\alpha(r,\ell) \to A_\alpha(r,\ell) + [\phi(r,\ell) - \phi(r+e_\alpha,\ell)]/e,$$

$$A_c(r,\ell) \to A_c(r,\ell) + [\phi(r,\ell) - \phi(r,\ell+1)]/e.$$

For a moment we will neglect this gauge field, which corresponds to the mean-field treatment. In this case the Hamiltonian for each plane ℓ becomes

$$H_{0\ell} = \sum_{k,\sigma} \xi_k f^\dagger_{\ell,k,\sigma} f_{\ell,k,\sigma} + \sum_k \varepsilon_k b^\dagger_{\ell,k} b_{\ell,k} \ , \qquad (4)$$

where ξ_k (ε_k) is the energy dispersion of the fermion (boson) measured from the chemical potential μ_F (μ_B). In the continuum approximation these dispersions are given by $\xi_k = k^2/(2m_F) - \mu_F$ and $\varepsilon_k = k^2/(2m_B) - \mu_B$. Although the physical electron operators $C^\dagger$ and C with the constraint of no double occupancy do not obey the usual fermion anticommutation relations, the definition of the currents which satisfy the conservation law remains the same and J_c along the c axis is given by

$$J_c(r,\ell) = -it_c \sum_\sigma [C^\dagger_{\ell\sigma}(r)C_{\ell+1\sigma}(r) - C^\dagger_{\ell+1\sigma}(r)C_{\ell\sigma}(r)]. \qquad (5)$$

On the other hand the energy current $J_{Ec}(\ell,r)$ cannot be written in terms of the electron operators, and its integral over the plane $J_{Ec}(\ell)$ is given by

$$\begin{aligned} J_{Ec}(\ell) &= \sum_r J_{Ec}(r,\ell) \\ &= -i\frac{t_c}{N_{2D}} \sum_{k,q,q',\sigma} (\xi_{k+q} - \varepsilon_k)[f^\dagger_{\ell,k+q,\sigma} b_{\ell,q} f_{\ell+1,k+q',\sigma} b^\dagger_{\ell+1,q'} - f^\dagger_{\ell+1,k+q',\sigma} b_{\ell+1,q'} f_{\ell,k+q,\sigma} b^\dagger_{\ell,q}] \ , \end{aligned} \qquad (6)$$

where N_{2D} is the number of lattice site per plane.

Now we discuss the sum rule for the conductivity $\sigma_c(\omega)$. It is well known that the conductivity sum rule can be derived for quite a general Hamiltonian just using the commutation relation between the coordinate $x_{i\mu}$ and the momentum $p_{i\mu}$ of the ith particle in the first quantization scheme. (Here we treat the three directions in a unified way by using the notation $\mu = a,b,c$.) The result is

$$I_{\mu\nu} \equiv \frac{2}{\pi}\int_0^\infty \mathrm{Re}\sigma_{\mu\nu}(\omega)d\omega = \frac{e^2N_e}{m}\delta_{\mu\nu} \ , \qquad (7)$$

where N_e is the electron number and m is the bare mass of the electron. This result is too general to be useful, because we are usually interested in the energy scales less than some characteristic cutoff. The t-J model offers exactly this type of description, i.e., the effective model for energy cutoff E_c less than the on-site repulsion U and the band gaps between conduction band and any other bands. This leads to the tight-binding model with the constraint of no double occupancy. Then the sum rule for the energy up to E_c is derived by the following general formula:

$$I_{\mu\nu} = i\langle[J_\mu, \bar{R}_\nu]\rangle, \qquad (8)$$

where $\bar{R}_\nu = -e\sum_{R,\sigma} R_\nu C^\dagger_\sigma(R)C_\sigma(R)$ is the center of mass of the charge. Using the (anti)commutation relation of the holons and spinons, the right-hand side of Eq. (8) can be easily calculated and the result remains the same form as in the case of noninteracting fermions on the lattice,

$$\begin{aligned} I_{\mu\nu} &= \frac{2}{\pi}\int_0^{E_c} \mathrm{Re}\sigma_{\mu\nu}(\omega)d\omega \\ &= \delta_{\mu\nu}t_\mu \sum_{R,\sigma}\langle C^\dagger_\sigma(R)C_\sigma(R+e_\mu) + \mathrm{H.c.}\rangle, \end{aligned} \qquad (9)$$

where $\mu = a, b, c$ and $R = (r, \ell)$ specifies the lattice point in 3D. Applying the mean-field theory for the spinons and holons, Eq. (9) gives

$$I_{\mu\nu} \cong \delta_{\mu\nu} t_\mu N_{3D} \sum_\sigma \langle f_\sigma^\dagger(R) f_\sigma(R+e_\mu)\rangle \times \langle b(R) b^\dagger(R+e_\mu)\rangle + \text{c.c.}, \quad (10)$$

where N_{3D} is the total number of the lattice sites. Here the RVB order parameters $\chi_{R+e_\mu,R} \equiv \langle f_\sigma^\dagger(R) f_\sigma(R+e_\mu)\rangle$ and $b_{R,R+e_\mu} \equiv \langle b(R) b^\dagger(R+e_\mu)\rangle$ appear. We have assumed that $\chi_{R+e_\mu,R}$ is independent of spin σ. It should be noted that the arguments of the operators are different, i.e., R and $R+e_\mu$. In the continuum model leading to Eq. (7), the products of the operators have the same argument and give the electron number. In the present case, the holon part vanishes like $\sim x$ as $x \to 0$. Actually its order of magnitude is roughly estimated as

$$I_{aa} = I_{bb} \sim N_{3D} t_a x \ . \quad (11)$$

Another comment on Eq. (10) is that the existence of the RVB order parameters between the layers is not a trivial issue. We assume that the interlayer coupling t_c is weak enough and $\chi_{R\pm e_c,R} = b_{R,R\pm e_c} = 0$ and the hopping between the layers is incoherent in the normal phase, while the coherency sets in and $\chi_{R\pm e_c,R}$ becomes nonzero in the superconducting phase. However this does not lead to the conclusion that $I_{cc} = 0$ in the normal phase. In this case the approximation Eq. (10) is not accurate enough because it describes only the coherent motion, and we estimate I_{cc} in terms of the incoherent hopping via the perturbation theory with respect to t_c (V). The result is

$$I_{cc} = 2N_{3D} t_c^2 \sum_k \sum_{q_1,q_2} f_{k+q_1}(1 - f_{k+q_2})(1 + n_{q_1}) n_{q_2} \times \frac{e^{\beta(\xi_{k+q_1} - \epsilon_{q_1} - \xi_{k+q_2} + \epsilon_{q_2})} - 1}{\xi_{k+q_1} - \epsilon_{q_1} - \xi_{k+q_2} + \epsilon_{q_2}}, \quad (12)$$

which is roughly estimated as

$$I_{cc} \sim N_{3D} t_c^2 N(\mu_F) x \sim N_{3D} \frac{t_c^2}{t_a} x. \quad (13)$$

Therefore the ratio of the integrated oscillator strengths is estimated as

$$\frac{I_{cc}}{I_{aa}} \sim \left(\frac{t_c}{t_a}\right)^2. \quad (14)$$

This result is not characteristic of the incoherent hopping between the layers, but is the generic feature for the open orbit along the c axis. Actually assuming the three-dimensional band structure with the open Fermi surface along the c axis, the ratio of the integrated oscillator strengths is the same order as Eq. (14) for small $t_c/t_a \ll 1$. Therefore we conclude that the integrated oscillator strength does not distinguish between the coherent and incoherent motion between the layers.

III. TRANSPORT PROPERTIES ALONG c AXIS

We now calculate various transport properties along the c axis in terms of the perturbation theory in second order with respect to t_c. Because the physical electron hops between layers, the c axis conductivity σ_c, thermopower S_c, and thermal conductivity κ_c are expressed in terms of the physical electron Green's function $G(k, \varepsilon)$. Using the tunneling Hamiltonian formalism we calculate the following three correlation functions.

$$\sigma_c = \langle J_c; J_c\rangle \sum_{\vec{k}} \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi} 2t_c^2(-e)^2 \left[-\frac{\partial f(\varepsilon)}{\partial \varepsilon}\right] [A_{\vec{k}}(\varepsilon)]^2, \quad (15)$$

$$\varphi_c = \langle J_c; J_{Ec}\rangle = \sum_{\vec{k}} \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi} 2t_c^2(-e)\varepsilon \left[-\frac{\partial f(\varepsilon)}{\partial \varepsilon}\right] [A_{\vec{k}}(\varepsilon)]^2, \quad (16)$$

$$\sigma_{Ec} = \langle J_{Ec}; J_{Ec}\rangle = \sum_{\vec{k}} \int_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi} 2t_c^2 \varepsilon^2 \left[-\frac{\partial f(\varepsilon)}{\partial \varepsilon}\right] [A_{\vec{k}}(\varepsilon)]^2, \quad (17)$$

where $A_{\vec{k}}(\varepsilon) = -\frac{1}{\pi}\text{Im}G^R(\vec{k}, \varepsilon)$ is the spectral function of the physical electron and $f(\varepsilon) = [e^{\beta\varepsilon} + 1]^{-1}$ is the Fermi distribution function. These expressions show that these three correlation functions reveal the temperature and energy dependence as well as the asymmetry of the spectral function $A_{\vec{k}}(\varepsilon)$. In the uniform RVB state, the spectral function $A_{\vec{k}}(\varepsilon)$ consists of the quasi-particle-like peak [quasiparticle part $A_{\vec{k}}^{(\text{q.p.})}(\varepsilon)$] and the incoherent background [incoherent part $A_{\vec{k}}^{(\text{incoh})}(\varepsilon)$].[15] The quasiparticle-like peak is centered at $\varepsilon = \xi_{\vec{k}} - |\mu_B|$ ($\xi_{\vec{k}} = \frac{\vec{k}^2}{2m_F} - \mu_F$ with μ_F, μ_B being the chemical potentials for spinons and holons, respectively). Its width $\Gamma_{\text{q.p.}} \sim \sqrt{JT}$ comes from the momentum distribution of the holons, its peak height P_c is $\sim x/\Gamma_{\text{q.p.}}$, and the integrated weight is x. The incoherent part, on the other hand, extends below $\varepsilon = \varepsilon_c(\vec{k}) = -|\mu_B| - \frac{\xi_k^2}{2m_B v_F^2}$, with $A_{\vec{k}}^{(\text{incoh})}(\varepsilon) \sim 2m_B\sqrt{\varepsilon_c(\vec{k}) - \varepsilon}$ near the edge. The integrated weight of the incoherent part is $(1-x)/2$. Putting this spectral function $A_{\vec{k}}(\varepsilon) = A_{\vec{k}}^{(\text{q.p.})}(\varepsilon) + A_{\vec{k}}^{(\text{incoh})}(\varepsilon)$ into Eq. (15), σ_c is decomposed into three parts as

$$\sigma_c = \sigma_c^{(\text{q.p.})} + \sigma_c^{(\text{incoh})} + \sigma_c^{(\text{cross})}, \quad (18)$$

where $\sigma_c^{(\text{q.p.})}$, $\sigma_c^{(\text{incoh})}$, and $\sigma_c^{(\text{cross})}$ correspond to $[A_{\vec{k}}^{(\text{q.p.})}]^2$, $[A_{\vec{k}}^{(\text{incoh})}]^2$, and the cross term $2A_{\vec{k}}^{(\text{q.p.})} A_{\vec{k}}^{(\text{incoh})}$, respectively. For the $\vec{k}$ integration in Eq. (15), $[A_{\vec{k}}^{(\text{q.p.})}(\varepsilon)]^2$ can be replaced by $\frac{x}{\Gamma_{\text{q.p.}}}\delta(x - \xi_k + |\mu_B|)$ and $\sigma_c^{(\text{q.p.})}$ is obtained as

$$\sigma_c^{(\mathrm{q.p.})} = \frac{e}{\pi} t_c^2 N(\mu_F) \frac{x^2}{\Gamma_{\mathrm{q.p.}}} , \tag{19}$$

where $N(\mu_F)$ is the density of states at the Fermi level of the spinon. Considering $N(\mu_F) \sim \frac{1}{J}$ and $\Gamma_{\mathrm{q.p.}} \sim \sqrt{JT}$, $\sigma_c^{(\mathrm{q.p.})}$ is estimated as $\sigma_c^{(\mathrm{q.p.})} \sim e^2 \frac{t_c^2}{J^{3/2}} x^2 T^{-1/2}$. Although we have assumed that the motion between the layers is of hopping type which is justified when $t_c \ll \Gamma_{\mathrm{q.p.}}$, its temperature dependence is metallic. The incoherent part $\sigma_c^{(\mathrm{incoh})}$ comes from the overlap between $A_k^{(\mathrm{incoh})}(\varepsilon)^2$ and the tail of $-\partial f(\varepsilon)/\partial\varepsilon$. Approximating $-\partial f(\varepsilon)/\partial\varepsilon$ by $\beta e^{\beta\varepsilon}$ for $-\varepsilon \gg T$ and using $\mu_B = T \ln \frac{2\pi x}{m_B T}$ assuming the Boltzmann distribution, we obtain $\sigma_c^{(\mathrm{incoh})}$ as

$$\sigma_c^{(\mathrm{incoh})} \cong 8e^2 t_c^2 m_B N(\mu_F) x v_F \sqrt{2\pi m_B T} \sim e^2 \frac{t_c^2}{J^{5/2}} x T^{1/2}. \tag{20}$$

The cross part $\sigma_c^{(\mathrm{cross})}$ is similarly estimated as

$$\begin{aligned} \sigma_c^{(\mathrm{cross})} &\sim 4\sqrt{\pi} e^2 t_c^2 N(\mu_F) x^2 \frac{v_F \sqrt{2\pi m_B T}}{\Gamma_{\mathrm{q.p.}}} \\ &\sim e^2 \frac{t_c^2}{J} x^2 T^{-1/2}, \end{aligned} \tag{21}$$

giving the same order contribution as $\sigma_c^{(\mathrm{q.p.})}$. Summarizing the above

$$\sigma_c \sim e^2 \left(\frac{t_c}{J}\right)^2 x \left[a x \left(\frac{J}{T}\right)^{1/2} + b \left(\frac{T}{J}\right)^{1/2} \right], \tag{22}$$

where a and b are dimensionless constants of order unity. The first term in Eq. (22) might give the metallic temperature dependence but reduced by the factor x compared with the second. Because our calculation is valid only for $T > T_{\mathrm{BE}} \sim xJ$, the metallic dependence of the $\sigma_c^{(\mathrm{q.p.})}$ does not mean that $\sigma_c \to \infty$ as $T \to 0$. Actually the ratio of the first to the second term is of the order of $T_{\mathrm{BE}}/T \sim xJ/T$; σ_c shows insulating behavior $\sigma_c \sim T^{1/2}$ for $T \gg T_{\mathrm{BE}}$. Whether the metallic temperature dependence is observed or not at lower temperature depends on the ratio a/b.

The other two correlation functions φ_c and σ_{Ec} are estimated similarly as

$$\begin{aligned} \varphi_c &= \varphi_c^{(\mathrm{q.p.})} + \varphi_c^{(\mathrm{incoh})} + \varphi_c^{(\mathrm{cross})} \\ &\simeq 0 + \frac{|\mu_B|}{e} \sigma_c^{(\mathrm{incoh})} + \frac{|\mu_B|}{e} \sigma_c^{(\mathrm{cross})}, \end{aligned} \tag{23}$$

$$\begin{aligned} \sigma_c^E &= \sigma_c^{E(\mathrm{q.p.})} + \sigma_c^{E(\mathrm{incoh})} + \sigma_c^{E(\mathrm{cross})} \\ &\simeq \frac{T^2}{e^2} \sigma_c^{(\mathrm{q.p.})} + \frac{\mu_B}{e} \sigma_c^{(\mathrm{incoh})} + \frac{\mu_B}{e} \sigma_c^{(\mathrm{cross})}. \end{aligned} \tag{24}$$

Then the thermopower S_c and the thermal conductivity κ_c are obtained as

$$S_c = \frac{1}{T} \frac{\varphi_c}{\sigma_c} \simeq \frac{1}{T} \frac{|\mu_B|}{e} = \frac{1}{e} \ln \frac{m_B T}{2\pi x}, \tag{25}$$

$$\kappa_c = \frac{1}{T} \left(\sigma_c^E - \frac{\varphi_c^2}{\sigma_c} \right) \sim \frac{T^{3/2}}{e^2} \left[\ln \frac{m_B T}{2\pi x} \right]^2, \tag{26}$$

where we have assumed $T \gg T_{\mathrm{BE}}$ and hence $|\mu_B| \gg T$, i.e., the incoherent and the cross parts dominate. The thermopower is positive and its magnitude is of the order of $k_B/e \sim 90\ \mu\mathrm{V/K}$ when we make the Boltzmann constant k_B explicit in Eq. (25), which is in qualitative agreement with the experiment.[7] This positive sign comes from the incoherent background which extends only for $\varepsilon < 0$, which means that the "hole" carries the energy between the layers.

IV. HALL EFFECT

We now turn to the Hall constant R_H^c with the electric field $E(\| a)$ and the magnetic field $H(\| b)$ parallel to the layer. R_H^c cannot be expressed in terms of only the physical electron Green's function because it comes from the loops enclosing the magnetic flux and the motion within the layer is in the form of spin-charge separation. Then we must first construct the composition rule for R_H^c. This can be easily achieved by noticing that the effective electric fields E_a^B, E_a^F which the bosons and fermions are experiencing are the screened ones by the gauge field as given by $E_a^B = -\frac{\sigma_F}{\sigma_F+\sigma_B} E_a$, $E_a^F = \frac{\sigma_B}{\sigma_F+\sigma_B} E_a$. Hence the off-diagonal conductivity σ_{ca} is composed as

$$\begin{aligned} \sigma_{ca} &= \langle J_c; J_a \rangle \\ &= \langle J_c; J_a^B \rangle \left(-\frac{\sigma_F}{\sigma_F + \sigma_B} \right) + \langle J_c; J_a^F \rangle \left(\frac{\sigma_B}{\sigma_F + \sigma_B} \right) \\ &= \frac{\sigma_{ca}^B \sigma_F + \sigma_{ca}^F \sigma_B}{\sigma_F + \sigma_B}, \end{aligned} \tag{27}$$

where $\sigma_{ca}^B (\sigma_{ca}^F)$ is the off-diagonal conductivity with the current in the layer being carried by the positively charged bosons (negatively charged fermions). Then it is straightforward to derive

$$R_H^c = \frac{\sigma_{ca}}{\sigma_{aa}\sigma_{cc} H} = \frac{\sigma_{ca}^B}{\sigma_B \sigma_{cc} H} + \frac{\sigma_{ca}^F}{\sigma_F \sigma_{cc} H} \equiv R_H^{cB} + R_H^{cF} , \tag{28}$$

where the composition rule $\sigma_{aa} = \sigma_F \sigma_B / (\sigma_F + \sigma_B)$ [$\sigma_{F(B)}$ is the conductivity of the spinon (holon)] has been used. R_H^{cB} (R_H^{cF}) is the Hall constant assuming that the current in the plane is carried by the boson (fermion). Here some comments are in order. First, both σ_{ca} and σ_{cc} are of the order of t_c^2 and $R_H^c \sim O(t_c^0)$. Therefore the small value of R_H^c does not come out from the weak interlayer coupling. Secondly the cancellation of the relaxation rate $1/\tau$ occurs between the denominator and the numerator to result in the finite R_H^c, and σ_{ca} diverges when one uses the bare Green's functions of fermions and bosons. Instead of going to the detailed analysis of $\sigma_{ca}^{B,F}$ taking into account the relaxation, let us be content with the high frequency limit where R_H^c is determined solely by the kinematics and the interaction of the system. Following the argument by Shastry *et al.*,[20] R_H^{cB} and R_H^{cF}

are expressed as

$$R_H^{cB(F)}(\omega) = \frac{R_H^{cB(F)^*}}{1-\sum_H^{B(F)}(\omega)}, \tag{29}$$

with

$$R_H^{cB(F)^*} = \lim_{H\to 0} -\frac{i\langle[J_c, J_a^{B(F)}]\rangle}{HI_{cc}I_{aa}^{B(F)}} \quad , \tag{30}$$

where $I_{aa}^{B(F)}$ is the integrated oscillator strength of the boson (fermion) within the layer.

The self-energy part $\sum_H^{B(F)}(\omega)$ contains the information about the inelastic scattering while $R_H^{cB(F)^*}$ about the kinematics and interaction of the system. Because $\sum_H^{B(F)}(\omega) \to \frac{1}{\omega^2}$ as $\omega \to \infty$, $R_H^{cB(F)^*}$ is regarded as the high frequency limit of $R_H^{cB(F)}$. The commutator in the numerator of Eq. (30) is evaluated up to the second order in t_c and after some calculations we arrive at

$$R_H^{cB^*} = \frac{2\eta_B}{ex}, \tag{31}$$

$$R_H^{cF^*} = -\frac{2\eta_F}{e(1-x)}, \tag{32}$$

where the dimensionless factors η_B, η_F are defined as

$$\eta_B = \frac{\int_0^\beta d\tau \sum_r r_x \frac{\partial}{\partial r_x}[G_{B1}(r,\tau)G_{B2}(r,\tau)]G_{F1}(r,\tau)G_{F2}(r,\tau)}{\int_0^\beta d\tau \sum_r G_{B1}(r,\tau)G_{B2}(r,\tau)G_{F1}(r,\tau)G_{F2}(r,\tau)}, \tag{33}$$

$$\eta_F = \frac{\int_0^\beta d\tau \sum_r \frac{1}{2}[G_{F1}(r,\tau)G_{F2}(r+e_x,\tau) + G_{F1}(r+e_x,\tau)G_{F2}(r,\tau)]}{\int_0^\beta d\tau \sum_r G_{F1}(r,\tau)G_{F2}(r,\tau)}. \tag{34}$$

The boson (fermion) Green's functions $G_{B1}(r,\tau)$, $G_{B2}(r,\tau)$ $[G_{F1}(r,\tau), G_{F2}(r,\tau)]$ are defined as $G_{B1}(r,\tau) = \langle b(r,\tau)b^\dagger(0,0)\rangle$, $G_{B2}(r,\tau) = \langle b^\dagger(r,\tau)b(0,0)\rangle$ $[G_{F1}(r,\tau) = \langle f(r,\tau)f^\dagger(0,0)\rangle$, $G_{F2}(r,\tau) = \langle f^\dagger(r,\tau)f(0,0)\rangle]$. In deriving Eqs. (31)–(34) we have assumed that the characteristic momentum and energy scales of $G_{B1,2}$ are much smaller than those of $G_{F1,2}$ because the fermions are Fermi degenerate with a large Fermi surface and large Fermi energy μ_F while bosons are distributed thermally near the bottom of the dispersion with small momentum ($\sim \sqrt{T}$) and energy ($\sim T$). Therefore η_B is estimated to be $\eta_B \sim \frac{\langle k^2\rangle_{\rm boson}}{k_F^2} \sim \frac{T}{J} \ll 1$ assuming the Boltzmann distribution. η_F, on the other hand, is determined solely by the fermion Green's function, and is estimated as

$$\eta_F = \frac{\sum_k\left[-\frac{\partial f(\xi_k)}{\partial \xi_k}\right]e^{ik_x}}{\sum_k\left[-\frac{\partial f(\xi_k)}{\partial \xi_k}\right]} \simeq \frac{\sum_k \delta(\xi_k)e^{ik_x}}{\sum_k \delta(\xi_k)} = \langle e^{ik_x}\rangle_{\rm FS}, \tag{35}$$

where $\langle\ \rangle_{\rm FS}$ is the average over the Fermi surface. η_F is almost temperature independent but depends on the hole concentration x. For example $\eta_F \to 0$ as $x \to 0$ assuming the tight-binding band with only nearest-neighbor hopping. Although one cannot discuss the dc Hall constant R_H^c, it is noted that the positive contribution R_H^{cB} is suppressed by the small factor $\eta_B \sim T/J \ll 1$ in contrast with the in-plane R_H which is almost determined by the boson Hall constant $R_H^B \cong \frac{1}{ex}$. Therefore $R_H^{c^*} = R_H^{cB^*} + R_H^{cF^*}$ is determined by the subtle balance between the reduced positive contribution $R_H^{cB^*}$ and the negative one $R_H^{cF^*}$, and its magnitude is expected to be small and less than $\frac{1}{e}$ which is consistent with experiments.[5,8]

V. RENORMALIZATION-GROUP EQUATION FOR INTERLAYER HOPPING

In the previous sections we have studied the effects of interlayer hopping up to the lowest order in t_c. We now investigate the renormalization of t_c due to the higher order contributions. For this purpose, we first integrate over the fermions from the Hamiltonian Eq. (3) and obtain the effective action S_B for the bosons:

$$\begin{aligned} S_B = &\sum_\ell \int_0^\beta d\tau \int d^2\vec{r}\bar{b}_\ell(r,\tau)\left(\frac{\partial}{\partial\tau} - \mu_B + \frac{1}{2m_B}(i\nabla - \vec{a})^2\right)b_\ell(r,\tau) \\ &+ \int_0^\beta d\tau \int d^2\vec{r}\left\{\sum_\ell \frac{1}{2}u[\bar{b}_\ell(r,\tau)b_\ell(r,\tau)]^2\right\} \\ &-2{\rm Tr}\left[\ln\left(\left(\partial_\tau - \frac{(\nabla + i\vec{a})^2}{2m_F} - \mu_F\right)\delta_{\ell,\ell'} + t_c b_\ell(r)b_{\ell'}^\dagger(r)\delta_{\ell+1,\ell'} + t_c b_\ell^\dagger(r)b_{\ell'}(r)\delta_{\ell,\ell'+1}\right)\right], \end{aligned} \tag{36}$$

where $\vec{a}=(a_a,a_b)$ is the gauge field and $\bar{b}$ and b are the functional integral variables (c-number fields) corresponding to the operators $b^\dagger$ and b, respectively. In Eq. (36) we have introduced the repulsive interaction u within the layer. This repulsion originally comes from the constraint that only one boson or one fermion can occupy the site, and hence the bosons obey the hard core condition. In terms of the T-matrix approximation which is valid in the dilute limit, the renormalized interaction u in the effective theory describing the low energy states is estimated to be of the order of the kinetic energy of the boson on the lattice, i.e., $u \sim t_B \sim 1/m_B$. Expanding Tr ln in Eq. (36) with respect to t_c we obtain the following effective action for the boson and the gauge field as

$$\begin{aligned} S_B = {} & \sum_\ell \int_0^\beta d\tau \int d^2r \bar{b}_\ell(r,\tau)\left(\frac{\partial}{\partial\tau}-\mu_B+\frac{1}{2m_B}(i\nabla-\vec{a})^2\right)b_\ell(r,\tau) \\ & + \int_0^\beta d\tau \int d^2\vec{r}\left\{\sum_\ell \frac{1}{2}u[\bar{b}_\ell(r,\tau)b_\ell(r,\tau)]^2+\sum_\ell \frac{1}{2}v[\bar{b}_\ell(r,\tau)b_\ell(r,\tau)][\bar{b}_{\ell+1}(r,\tau)b_{\ell+1}(r,\tau)]\right\} \\ & + \sum_{q,\omega}\left(\chi_F q^2+\gamma_F\frac{|\omega|}{q}\right)a_\alpha(q,\omega)a_\alpha(-q,-\omega). \end{aligned} \qquad (37)$$

We have chosen the Coulomb gauge, i.e., $\partial_x a_x + \partial_y a_y = 0$ and a_α represents the transverse part. Its dynamics is given by the current-current correlation function of the fermions. χ_F is the Landau's diamagnetic susceptibility of the fermions and γ_F is the dissipation constant of the order of the Fermi wave number k_F. The interlayer interaction v is generated from the interlayer hopping and is given by $v=-2t_c^2N(\mu_F)<0$ (attractive interaction). It is noted that there is no direct hopping term between the layers in S_B because we are interested in the normal state above T_c and assume $\chi_{R\pm e_c,R}=\langle f_\sigma^\dagger(R)f_\sigma(R+e_c)\rangle=0$ as has been discussed below Eq. (11). Higher order terms with respect to t_c are also higher order in b and $b^\dagger$, and hence are irrelevant in the following renormalization-group (RG) argument. Here we redefine a_α as $a_\alpha/\sqrt{\chi_F}$. Then the coefficient of $a_\alpha(q,\omega)a_\alpha(-q,-\omega)$ in the last term of Eq. (37) becomes $q^2+\gamma|\omega|/q$ with $\gamma=\gamma_F/\chi_F$, and $i\nabla-\vec{a}$ should be replaced by $i\nabla-e\vec{a}$ with the gauge charge e being $1/\sqrt{\chi_F}$.

Now we treat u, v, and e in terms of RG.[21–23] We successively integrate over the rapidly varying part of the boson field down to the energy scale of the order of T, which corresponds to the "quantum regime" in the terminology of Fisher and Hohenberg.[22] In this quantum regime the recursion formula up to the one-loop order is obtained following the similar procedures in Refs. 22,23 as

$$\frac{d\tilde{u}(\eta)}{d\eta}=-\frac{1}{4\pi}\tilde{u}^2(\eta)-\frac{e^4(\eta)}{\pi^2\gamma}, \qquad (38)$$

$$\frac{d\tilde{v}(\eta)}{d\eta}=-\frac{1}{8\pi}\tilde{v}^2(\eta), \qquad (39)$$

$$\frac{de^2(\eta)}{d\eta}=0, \qquad (40)$$

where $\tilde{u}(\eta)=2m_Bu(\eta), \tilde{v}(\eta)=2m_Bv(\eta)$, and $\eta=\ln\frac{1}{\Lambda}$ with Λ being the cutoff energy. As is described in Ref. 23, there is no scaling of $e^2(\eta)$ in the quantum regime within this approximation by choosing the appropriate dynamical exponent $z(\eta)$. It is noted that Eq. (39) does not include the coupling constant $e^2(\eta)$ with the gauge field and either the intralayer interaction $\tilde{u}$ up to this order, and which can be easily solved. The attractive interaction $v(\eta)=-2t_c(\eta)^2N(\mu_F)$ scales to larger value, i.e., marginally relevant, as

$$t_c(\Lambda)=\frac{t_c}{\left[1-\frac{t_c^2N(\mu_F)m_B}{4\pi}\ln\frac{\Lambda_0}{\Lambda}\right]^{1/2}}, \qquad (41)$$

which blows up at $\Lambda_c=\Lambda_0 e^{-\frac{4\pi}{t_c^2N(\mu_F)m_B}}$. We stop the renormalization at $\Lambda=T$, and the effective interlayer coupling at T is $t_c(\Lambda=T)$. As long as the temperature T is higher than Λ_c, the renormalization effect can be neglected and $t_c(T)$ remains nearly t_c. This slow increase of t_c should be compared with that of the direct hopping $\tilde{t}_c b_\ell^\dagger b_{\ell+1}$ + H.c. which is relevant and behaves like $\tilde{t}_c(\Lambda)=\tilde{t}_c(\Lambda_0)(\Lambda_0/\Lambda)^2$. Therefore we expect the interlayer charge dynamics remains incoherent as long as $T>\Lambda_c$ and $T>T_{\rm BE}$ in the underdoped region. For $t_c/J\sim 10^{-1}$, $\Lambda_c\sim\Lambda_0e^{-10^2}$ which is extremely small energy. It should be noted that Clarke, Strong, and Anderson[19] claimed that the incoherent nature of the interlayer (interchain) motion cannot be captured by the RG equation and is a more subtle issue.

VI. DISCUSSION AND CONCLUSION

We have studied the interlayer charge dynamics of the uniform RVB state in terms of the perturbation theory in t_c. In the underdoped region we should take into account the spin gap formation.[15–17] The effects of this spin gap are essentially different between the intralayer and interlayer charge dynamics.[16] Within each layer the charge dynamics is largely determined by holons, and the formation of the spin gap in the spinon system affects the holon only indirectly through the gauge field, i.e., the fluctuation of the gauge field is suppressed and also the relaxation rate of the holon which leads to the suppression of the resistivity ρ_a from the T-linear behavior.[24] This naturally explains why ρ_a is affected so little in spite of the pseudogap formation observed in the neutron scatterings[25] and NMR.[26] The interlayer hopping, on the other hand, occurs through the physical electron whose spectrum has a (pseudo) gap and the conductivity $\sigma_c = \rho_c^{-1}$ is reduced and shows insulating behavior as a function of temperature. Recently Homes *et al.*[4] and Tajima *et al.*[6] measured the optical conductivity $\sigma_c(\omega)$ of $YBa_2Cu_3O_{6.70}$, and found the clear evidence of the gap from $T \sim 150$ K in contrast with $\sigma_{ab}(\omega)$ which shows no symptom of the gap. The Josephson process does not contribute to σ_c because the holons are not condensed in the spin gap phase, i.e., $\langle b_\ell b_{\ell+1} \rangle = 0$. Then the integrated oscillator strength I_{cc} is suppressed by the pseudospin gap through the reduction of $N(\mu_F)$ in Eq. (13). Very recently Ong *et al.*[27] observed the negative magnetoresistance for semiconducting ρ_c in $Bi_2Sr_2CaCu_2O_{8+\delta}$ and oxygen-reduced $YBa_2Cu_3O_{6+x}$. The magnetoresistance is weakly anisotropic, and activated in temperature. This result is consistent with the idea that the magnetic field reduces the spin gap and increases the conductivity along the c axis. However, the existence of the spin gap in $La_{2-x}Sr_xCuO_4$, especially at room temperature, is not well established and further studies are needed for this material.

ACKNOWLEDGMENTS

The author acknowledges S. Uchida, H. Takagi, N. P. Ong, P.A. Lee, and H. Fukuyama for fruitful discussions. This work is supported by Grant-in-Aid for Scientific Research No. 05044037, No. 04240103, and No. 04231105 from the Ministry of Education, Science, and Culture of Japan.

1 T. Ito *et al.*, Nature (London) **350**, 596 (1991).

2 K. Tamasaku, Y. Nakamura, and S. Uchida, Phys. Rev. Lett. **69**, 1455 (1992).

3 S.L. Cooper *et al.*, Phys. Rev. Lett. **70**, 1533 (1993).

4 C.C. Homes *et al.*, Phys. Rev. Lett. **71**, 1645 (1993).

5 K. Tamasaku, Y. Nakamura, and S. Uchida, Phys. Rev. Lett. **70**, 1533 (1993).

6 S. Tajima, J. Scutzmann, and S. Miyamoto, Solid State Commun. **95**, 759 (1995).

7 Y. Nakamura and S. Uchida, Phys. Rev. B **47**, 8369 (1993).

8 J.M. Harris, Y.F. Yan, and N.P. Ong, Phys. Rev. B **46**, 14 239 (1992).

9 P.W. Anderson, Science **235**, 1196 (1987); in *Strongly Correlation and Superconductivity*, edited by H. Fukuyama, S. Maekawa, and A.P. Malozemoff (Springer-Verlag, Berlin, 1989), p. 2.

10 N. Kumar and A.M. Jayannavar, Phys. Rev. B **45**, 5001 (1992).

11 N. Kumar, P.A. Lee, and B. Shapiro, Physica A **168**, 447 (1990).

12 A.G. Rojo and K. Levin, Phys. Rev. B **48**, 16 861 (1993).

13 G. Baskaran, Z. Zou, and P.W. Anderson, Solid State Commun. **63**, 973 (1987); Y. Isawa, S. Maekawa, and H. Ebisawa, Physica B **148**, 391 (1987); P.W. Anderson, Z. Zou, and T. Hsu, Phys. Rev. Lett. **58**, 2790 (1987); Y. Suzumura, Y. Hasegawa, and H. Fukuyama, J. Phys. Soc. Jpn. **57**, 2768 (1988).

14 P.W. Anderson and Z. Zou, Phys. Rev. Lett. **60**, 132 (1988).

15 N. Nagaosa and P.A. Lee, Phys. Rev. Lett. **64**, 2450 (1992); Phys. Rev. B **45**, 966 (1992); P.A. Lee and N. Nagaosa, *ibid.* **46**, 5621 (1992).

16 N. Nagaosa, J. Chem. Phys. Solids **53**, 1493 (1992).

17 B.L. Altshuler and L.B. Ioffe, Solid State Commun. **82**, 253 (1992).

18 P.W. Anderson, Science **256**, 1526 (1992).

19 D.G. Clarke, S.P. Strong, and P.W. Anderson, Phys. Rev. Lett. **72**, 3218 (1994).

20 B.S. Shastry, B.I. Shraiman, and R.R.P. Singh, Phys. Rev. Lett. **70**, 2004 (1993).

21 B.I. Halperin, T.C. Lubensky, and S.-K. Ma, Phys. Rev. Lett. **32**, 292 (1974).

22 D.S. Fisher and P.C. Hohenberg, Phys. Rev. B **37**, 4936 (1988).

23 M.U. Ubbens, P.A. Lee, and N. Nagaosa, Phys. Rev. B **48**, 13 762 (1993).

24 T. Ito, K. Takenaka, and S. Uchida, Phys. Rev. Lett. **70**, 3995 (1993).

25 J. Rossat-Mignod *et al.*, Physica C **185-189**, 86 (1991).

26 M. Takigawa *et al.*, Phys. Rev. B **43**, 247 (1991).

27 Y.F. Yan, P. Matl, J.M. Harris, and N.P. Ong, Phys. Rev. B **52**, R751 (1995).