Dynamics of vortices in a two-dimensional easy-plane antiferromagnet

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Dynamical behavior of vortices is investigated for a two-dimensional classical Heisenberg antiferromagnet with easy-plane anisotropy. Equations of motion for the vortices are derived with the use of the Lagrangian formulation. The effect of a magnetic field applied in the XY plane is considered. The implications of the theory developed here to the calculations of dynamical correlation functions are presented. However, since pair interactions are neglected, these calculations are relevant only for temperatures below the Kosterlitz-Thouless transition temperature.

I. INTRODUCTION

In the last years there has been continual interest in phase transitions of topological origin in various two-dimensional systems. By now it is well known that two-dimensional (2D) systems with SO(2) symmetry such as XY ferromagnets or superfluid ^{4}He films exhibit a phase transition at a finite temperature without conventional long-range order, the essential ingredients of this transition being a vortex, a topological stable structure of the system.[1] Kosterlitz and Thouless[2,3] have shown that vortices can be generated freely at temperatures higher than some critical temperature, while they exist only as bounded vortex-antivortex pairs at lower temperatures, thus giving rise to a well-defined vortex binding-unbinding transition at a temperature $T=T_{\rm KT}$. The Kosterlitz-Thouless transition may be characterized in the following way with the aid of two infinitesimal test charges inserted into the two-dimensional Coulomb gas, equivalent to the vortex system: the separation energy between the test charges falls off exponentially with separation for $T>T_{\rm KT}$, whereas it grows logarithmically with separation for $T<T_{\rm KT}$.

Although the thermodynamic properties of these models have been successfully and extensively studied there are still various aspects of the dynamics of vortices needing to be understood. Various authors have studied the dynamical problem for the two-dimensional anisotropic ferromagnetic Heisenberg model.[4–6] The starting point is the Landau-Lifshitz equation of motion. Then the static solution describing a topological excitation is introduced and an equation of motion for the center of the excitation is extracted by methods that differ in detail from one author to another. Côté and Griffin[7] have studied a dynamical version of a 2D planar model. They derive the coupled equation of motion for the spin-wave and vortex fields using a Lagrangian analogous to that used in classical electrodynamics of a continuous medium. Beck and Ariosa,[8] starting from a microscopic time evolution of the phase angles of the classical two-dimensional XY model have derived an equation for the center of a vortex configuration, but they did not consider explicitly the coupling of the vortex motion to the spin-wave excitations.

To calculate the dynamical correlation functions from mobile vortices in two-dimensional easy-plane ferromagnets, two approaches have been used: approximate analytic methods, based on a continuum description, where it is assumed an ideal gas of unbound vortices above $T_{\rm KT}$ and direct numerical simulations on a discrete lattice.[9–12]

For the ferromagnet, the equation of motion can be reduced to canonical equations of motion for vortices that are identical in form to those for incompressible fluids in hydrodynamics.[13] Analogy between easy-plane ferromagnets and the superfluid system have also been discussed from different points of view.[14] In contrast to the static properties, the dynamics of vortices in a fluid is distinctly different from those of Newtonian particles.[15] In a ferromagnet a single vortex remains fixed in position (although in reality it will undergo Brownian motion due to its interaction with spin waves). A uniform rectilinear motion can only be acquired by association with a second vortex of equal strength or under the action of a wall at rest.[5] On the other hand, in a planar antiferromagnet in zero external field, vortex motion with arbitrary velocity is possible. Also the equations of motion do not have an analogy with fluid mechanics, and particular traveling vortex solutions are obtained by the Lorentz transformation to the static vortex solutions, the equation of motion having a behavior similar to topological vortices in the two-dimensional Higgs equation of particle physics studied by Ishimori.[16]

In this paper we investigate the dynamics of vortices in a two-dimensional classical anisotropic antiferromagnetic model. In Sec. II, we review some aspects for the equations of motion for the vortices in an antiferromagnet, and obtain an effective mass for the planar vortex-antivortex pair. We also discuss the contribution of the vortex to the dynamical correlation functions. In Sec. III, we study the vortex spin-wave interaction, and use the Lagrangian approach to obtain equations of motion for the vortex centers. In Sec. IV, we study the effect of an applied field in the XY plane, and finally in Sec. V we present our conclusions.

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II. EQUATIONS OF MOTIONS

We will start from the Hamiltonian of the system given by

$$H=-J\sum_n \mathbf{S}_n\cdot\mathbf{S}_{n+1}+D\sum_n (S_n^z)^2-\gamma B\sum_n S_n^x\ , \quad (2.1)$$

where $\gamma=g\mu_B$ and B is a magnetic field applied in the easy plane. This Hamiltonian with $J=-11$ K and $D=7.3$ K describes the 2D compound $BaNi_2(PO_4)_2$ a good candidate for a 2D easy-plane antiferromagnet.

After obtaining the equations of motion by using

$$i\dot{\mathbf{S}}=[\mathbf{S},H]\ , \quad (2.2)$$

we treat the spin components as classical vectors with spherical components[17]

$$\mathbf{S}_n=(-1)^n S\{\sin[\theta_n+(-1)^n\nu_n]\cos[\phi_n+(-1)^n\alpha_n]\ ,\ \sin[\theta_n+(-1)^n\nu_n]\sin[\phi_n+(-1)^n\alpha_n],\ \cos[\theta_n+(-1)^n\nu_n]\}\ , \quad (2.3)$$

where the even n describes one sublattice, the odd n the other one. The angles θ and ϕ describe the perfect antiferromagnetic structure, while the small angles ν and α describe the deviations from this state. After taking the continuum limit we assume that the spatial variation of the angles is small, such that differences can be replaced by gradients. The equations of motion are then given by[18]

$$\nabla^2\theta-\frac{1}{c^2}\frac{\partial^2\theta}{\partial t^2}=\sin\theta\cos\theta\left[(\nabla\phi)^2-\frac{1}{c^2}\left[\frac{\partial\phi}{\partial t}\right]^2\right]+h^2\sin\theta\cos\theta\cos^2\phi+2\delta\sin\theta\cos\theta-\frac{h}{JS}\left[\frac{\partial\phi}{\partial t}\right]\cos\phi\sin^2\theta\ , \quad (2.4)$$

$$\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=-2\cot\theta\left[(\nabla\theta)\cdot(\nabla\phi)-\frac{1}{c^2}\left[\frac{\partial\theta}{\partial t}\right]\left[\frac{\partial\phi}{\partial t}\right]\right]-h^2\sin\phi\cos\phi+\frac{h}{JS}\left[\frac{\partial\theta}{\partial t}\right]\cos\phi\ , \quad (2.5)$$

$$2\alpha\sin\theta=\frac{1}{4JS}\left[\frac{\partial\theta}{\partial t}\right]-h\sin\phi\ ,$$
$$2\nu=-\frac{1}{4JS}\sin\theta\left[\frac{\partial\phi}{\partial t}\right]+h\cos\phi\cos\theta\ , \quad (2.6)$$

where $c=2JS$, $\delta=D/J$, and $h=\gamma B/(4JS)$. As we can see, for $B=0$ the equations of motion are Lorentz invariant.

On the other hand, using the classical and continuum approximations directly on Eq. (2.1) we obtain

$$H=\frac{JS^2}{2}\int d\mathbf{r}\left\{(\nabla\theta)^2+\sin^2\theta(\nabla\phi)^2+\frac{1}{c^2}\left[\left[\frac{\partial\theta}{\partial t}\right]^2+\sin^2\theta\left[\frac{\partial\phi}{\partial t}\right]^2\right]+2\delta\cos^2\theta+h^2\sin^2\theta\cos^2\phi\right\}\ . \quad (2.7)$$

The continuum equations (2.4) and (2.5) yield two types of static vortices,[10] viz. "planar," in which spin components are confined to the XY plane, and "out of plane," in which there is a pulse-shaped S_z distribution accompanying the vortex shape in S_x and S_y. There is a critical D denoted by D_c ($D_c=0.5$ J for the square lattice) such that for $D<D_c$, the planar vortex is the stable configuration; the out-of-plane vortex being stable only for $D>D_c$.

Our system, besides the energy E, has an integral of motion, a momentum, that generates the translation of the spin subsystem, given by

$$\mathbf{p}=-\frac{JS^2}{c^2}\int d\mathbf{r}\left[\nabla\theta\left[\frac{\partial\theta}{\partial t}\right]+\sin^2\theta\nabla\phi\left[\frac{\partial\phi}{\partial t}\right]\right]-\frac{Sh}{4}\int d\mathbf{r}\{\sin\phi\nabla\theta-\sin\theta\cos\theta\cos\phi\nabla\phi\}\ . \quad (2.8)$$

For solutions of the form $\phi(\mathbf{r}-\mathbf{v}t,t)$, $\theta(\mathbf{r}-\mathbf{v}t,t)$, Eq. (2.8) leads to a magnetic force acting on the vortex given by

$$\mathbf{F}_m=-G(\mathbf{v}\times\hat{\mathbf{z}})\ , \quad (2.9)$$

where

$$G=\left|Sh\int d\mathbf{r}\cos\phi\cos^2\theta(\nabla\phi\times\nabla\theta)\right| \quad (2.10)$$

and $\hat{\mathbf{z}}$ is a unit vector in the z direction.

In this section let us take $B=0$. From the momentum conservation law we find that the force exerted on a single vortex (or a single bound pair) by the spin subsystem is equal to zero. This means that the vortex (or pair) can move at any constant velocity and it is not frozen into the medium, as is a vortex (or pair) in the ferromagnet. In the next section we will consider the interaction force between members of a pair.

For simplicity let us consider the planar vortex with $\theta=\pi/2$. Equation (2.5) then becomes

$$\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=0 \quad (2.11)$$

with the energy given by

$$E=\frac{JS^2}{2}\int d\mathbf{r}\left[(\nabla\phi)^2+\frac{1}{c^2}\left[\frac{\partial\phi}{\partial t}\right]^2\right]\ . \quad (2.12)$$

The one-vortex solution centered at the origin is characterized by choosing the azimuthal angle as $\phi=\arctan(y/x)$. Since the Laplace equation for $\phi(r)$ is linear, stationary N-vortex configurations are obtained simply by linear combination:

$$\phi(\mathbf{r})=\sum_{j=1}^{N} e_j \arctan\left[\frac{y-y_j}{x-x_j}\right], \tag{2.13}$$

where $\mathbf{r}_j=(x_j,y_j)$ is the center of the jth vortex with vorticity e_j. (We will consider only the case $e_j=\pm1$.) Finiteness of the energy demands $\sum e_j=0$, i.e., N has to be even. The vortices are always created, as thermal excitations, in pairs of opposite vorticities. A vortex moving with constant velocity $\mathbf{v}_j$ can be obtained from the static solution by the Lorentz transformation. We will then have the condition[4,15] $\sum e_j \mathbf{v}_j=0$.

For a bound vortex-antivortex pair Eq. (2.13) becomes

$$\phi(\mathbf{r})=\arctan\left[\frac{y-y_1}{x-x_1}\right]-\arctan\left[\frac{y-y_2}{x-x_2}\right] \tag{2.14}$$

with the following boundary condition at infinity: $|\mathbf{r}|\to\infty$, $\mathbf{S}(\mathbf{r})\to(S,0,0)$. For a pair moving with velocity v, Eq. (2.12) gives

$$E=E_0(1-v^2/c^2)^{-1/2}, \tag{2.15}$$

where E_0, the energy of the pair at rest, can be calculated taking $\nabla\phi$ in Eq. (2.14) for a bound pair with the vortex and antivortex centers separated by a distance d, integrating $(\nabla\phi)^2$ from a distance beginning with a region of order of the radius of the vortex core[2] r_0 ($r_0=ae^{-\gamma}/(2\sqrt{2})$ where a is the lattice spacing), up to a large value R, a macroscopic distance on the order of the dimension of the system. After performing the integration we can take $R=\infty$. We find

$$E_0=2JS^2\pi\ln(d/r_0). \tag{2.16}$$

This result leads us to define an effective mass for the pair, independent of the system size, but dependent on the separation between the members of the pair, by

$$M=(2\pi JS^2/c^2)\ln(d/r_0). \tag{2.17}$$

The momentum of the pair is given by $\mathbf{p}=E\mathbf{v}/c^2$.

The Kosterlitz-Thouless theory predicts that, well below T_{KT}, all vortices will be tightly bound in pairs, with the mean separation between members of a pair, d, being around one lattice spacing. As T_{KT} is approached d increases. At T_{KT} the first pair unbinds. However, for the discrete lattice d does not vary continuously but in steps of the lattice parameter a. Monte Carlo simulation[19] shows that a pair with d greater than the lattice spacing appears only at $T=0.95\ T_{KT}$. Then at low temperatures we can take $d=a$ in Eq. (2.17) and the effective mass will become

$$M\approx0.46\pi JS^2/c^2. \tag{2.18}$$

The contribution of bound pairs to the in-plane dynamical correlation function is given by[20]

$$S^{\alpha\alpha}(\mathbf{q},\omega)=n|f^{\alpha\alpha}(\mathbf{q})|^2F(\mathbf{q},\omega)\quad(\alpha=x,y), \tag{2.19}$$

where $f^{\alpha\alpha}(\mathbf{q})$ is the form factor, n the pair density, and $F(\mathbf{q},\omega)$, under the phenomenological assumption of a gas of noninteracting bound pairs, which are thermally activated, is given by

$$F(\mathbf{q},\omega)=\rho(v)|_{v=\omega/q}, \tag{2.20}$$

where $\rho(v)$ is a velocity distribution function. Within the Boltzmann statistics we have, in the limit $v\ll c$,

$$\rho(v)=Ce^{-\beta Mv^2/2}, \tag{2.21}$$

where C is a normalization constant and M is given by (2.18), thus leading to

$$F(q,\omega)=(2\pi M\beta/q^2)^{1/2}\exp\left[-\frac{\beta M\omega^2}{2q^2}\right], \tag{2.22}$$

a Gaussian central peak centered about $\omega=0$.

It is important to note that isolated vortices and antivortices slowly drift towards one another and annihilate. Occasionally thermal fluctuations produce vortex-antivortex pairs; at equilibrium, of course the annihilation rate equals the production rate. This process will probably modify Eq. (2.22).

For higher temperatures we would have to take into account the interaction between bound pairs and the process would become very complicated. However, for temperatures above the Kosterlitz-Thouless temperature we have "free" vortices besides the bound pairs and their contribution to the dynamical spin-correlation functions would become dominant.[9] Note that by "free" vortices we mean vortices interacting through a potential that falls off exponentially with separation.[1] Of course for a free vortex with its center located at $\mathbf{r}_1$ the field configuration is not given by (2.13) (although this equation should give a good approximation for $|\mathbf{r}-\mathbf{r}_1|$ small) since for a static "free" vortex the energy is given by[1]

$$\frac{1}{2}\int_a^R(\nabla\phi)^2d\mathbf{r}=\pi[K_0(a/\lambda)-K_0(R/\lambda)], \tag{2.23}$$

where K_0, the modified Bessel function of order 0, has the following asymptotic behavior:

$$K_0(x)\approx e^{-\pi/\lambda}/\sqrt{\lambda},\quad x\to\infty \tag{2.24}$$

and λ is the screening length. The effective mass of a "free" vortex, in the limit $R\to\infty$ is then given by

$$M=(\pi/4J)K_0(a/\lambda). \tag{2.25}$$

Although being finite, M depends on the mean separation between "free" vortices.

At sufficiently high temperatures the mean spacing between unbound vortices approaches the vortex core size and diffusion spin-dynamics results. However, close to T_{KT}, the unbound vortex density is small enough that a phenomenology built on weakly interacting vortices moving ballistically between interactions is possible.[9] Let us consider the effect of using a "free" vortex instead of a "single" vortex [where by a "single" vortex we mean a vortex described by $\phi=\arctan(y/x)$] in the Mertens *et al.*[9] theory. The theory for the in-plane correlation is not affected at all because they have only used the fact that a moving vortex passing a lattice site changes the sign of the spin at this place (this correlation function is only globally sensitive to the presence of vortices). However the out-of-plane correlation is directly influenced by the vortex size and shape. To obtain the out-of-plane vortex structure we use Eq. (2.4) (with $\dot{\theta}=0$, $\dot{\phi}=0$) and take $(\nabla\phi)^2=2\pi K_1^2(r/\lambda)$ [instead of $(\nabla\phi)^2=1/r^2$, as was done for the single vortex]. For $|\mathbf{r}|\to 0$ the shape of a "free" vortex is the same as the single vortex. The asymptotic behavior for $|\mathbf{r}|\to\infty$ will also be the same for the two cases (considering that the term $1/r^2$ was neglected before for large values of r). However the vortex core (size of the out-of-plane structure) r_v will be different in the two circumstances. However since in the Mertens *et al.* theory they have not used the explicit form of $\theta(r)$, but only asymptotic behaviors and the parameter r_v (which could be obtained by fitting numerical data from computer simulations) we see that qualitatively the theory should hold the same for a "free" vortex.

III. LAGRANGIAN FORMULATION AND VORTEX-SPIN-WAVE INTERACTION

In the last section we have considered the case where a vortex, or a bound pair, moved with constant velocity. In this section we shall consider more general solutions to the equation of motion (2.11). We begin with the ansatz

$$\phi_v(\mathbf{r},t)=\sum_j e_j \arctan\left[\frac{y-y_j(t)}{x-x_j(t)}\right], \tag{3.1}$$

where $\mathbf{r}_j(t)=[x_j(t),y_j(t)]$ is the time-dependent center of the jth vortex. This ansatz has already been used by Ishimori,[16] in his study of dynamics of vortices of the two-dimensional Gross-Pitaevskii equation of superfluidity, and the Higgs equation of particle physics. Of course (3.1) implies that $\nabla^2\phi_v=0$ for all t, so that we have $\phi_v(\mathbf{r},t)=\Omega(\mathbf{r})t$ with $\nabla^2\Omega=0$. We shall consider $v\ll c$, otherwise we would have to take into account the Lorentz contraction of vortices.

Now we use the Lagrangian formulation to derive the equations of motion for the vortex positions for the in-plane vortices. From Eq. (2.12) we have the Lagrangian

$$L\{\phi\}=T\{\phi\}+U\{\phi\}, \tag{3.2}$$

where

$$T=\frac{J}{2c^2}\int d\mathbf{r}\left[\frac{\partial\phi}{\partial t}\right]^2, \quad U=\frac{J}{2}\int d\mathbf{r}(\nabla\phi)^2, \tag{3.3}$$

T and U are the kinetic and potential pairs of the Lagrangian, respectively. The potential-energy term U, assuming that the radius of the system R is much larger than the separation r_{ij} between vortices, is the well-known result

$$U=-\pi J\sum_{i\neq j} e_i e_j \ln(r_{ij}/r_0). \tag{3.4}$$

For the kinetic-energy part we find

$$T=\frac{\pi J}{2c^2}\left[\ln(R/r_0)\sum_j e_j\dot{\mathbf{r}}_j^2+\sum_{i\neq j}e_ie_j\left[\dot{\mathbf{r}}_i\cdot\dot{\mathbf{r}}_j\ln(R/r_0)-\dot{\mathbf{r}}_i\cdot\dot{\mathbf{r}}_j\ln(r_{ij}/r_0)+\frac{1}{\pi}(\dot{\mathbf{r}}_i\cdot\mathbf{r}_{ij})(\dot{\mathbf{r}}_j\cdot\mathbf{r}_{ij})\right]\right]. \tag{3.5}$$

Since the Lagrangian formulation is invariant with respect to the choice of variables describing the system, the equation of motion for the vortex position $\mathbf{r}_j$ can be derived then using the Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\mathbf{r}}_j}-\frac{\partial L}{\partial\mathbf{r}_j}=0. \tag{3.6}$$

We obtain, for the vortex i,

$$\ln(R/r_0)\sum_j e_j\ddot{\mathbf{r}}_j-\sum_{j(\neq i)}e_j[\ddot{\mathbf{r}}_j\ln(r_{ij}/r_0)-\mathbf{r}_{ij}(\ddot{\mathbf{r}}_j\cdot\mathbf{r}_{ij})/r_{ij}^2]$$
$$-\sum_{j(\neq i)}e_j[2\dot{\mathbf{r}}_j(\dot{\mathbf{r}}_i\cdot\mathbf{r}_{ij})-2r_{ij}(\dot{\mathbf{r}}_i\cdot\dot{\mathbf{r}}_j)+\mathbf{r}_{ij}(\dot{\mathbf{r}}_j)^2]/r_{ij}^2+\sum_{j(\neq i)}e_j\mathbf{r}_{ij}(\dot{\mathbf{r}}_j\cdot\dot{\mathbf{r}}_{ij})/r_{ij}^4=c^2\sum_{j(\neq i)}e_j\mathbf{r}_{ij}/r_{ij}^2. \tag{3.7}$$

The presence of quadratic terms in the velocities suggests that the vortex cannot be considered a Newtonian particle.

For a finite result in the limit $R\Longrightarrow\infty$, Eqs. (3.5) and (3.7) require that

$$\sum e_j\dot{\mathbf{r}}_j=0 \quad\text{and}\quad \sum e_j\ddot{\mathbf{r}}_j=0,$$

respectively. Thus for a bound pair we should have $\dot{\mathbf{r}}_1=\dot{\mathbf{r}}_2$, $\ddot{\mathbf{r}}_1=\ddot{\mathbf{r}}_2$, leading the pair to behave like a rigid structure. The equation of motion for a "free" vortex, above the Kosterlitz-Thouless temperature, certainly would have a form different from Eq. (3.7).

Our next step is to take into account the coupling of the vortex motion to the spin-wave oscillations. We write

$$\phi(\mathbf{r},t)=\phi_v(\mathbf{r},t)+\xi(\mathbf{r},t), \tag{3.8}$$

where ξ is the spin-wave contribution and ϕ_v is given by Eq. (3.1). Inserting (3.8) into (3.11) yields the following linear inhomogeneous equation for ξ:

$$\nabla^2\xi-\frac{1}{c^2}\frac{\partial^2\xi}{\partial t^2}=\frac{1}{c^2}\frac{\partial^2\phi_v}{\partial t^2} . \quad (3.9)$$

Without vortices ξ is just the spin-wave solution with the dispersion relation given by $\omega=ck$. The inhomogeneous solution of Eq. (3.9) can be written as

$$\xi(\mathbf{r},t)=\frac{1}{4\pi}\int\int g(\mathbf{r},t;\mathbf{r}',t')\left[\frac{\partial^2\phi_v}{\partial t^2}\right]_{r't'}d\mathbf{r}'\cdot dt' , \quad (3.10)$$

where g, the Green function for the two-dimensional wave equation, is given by

$$g(\mathbf{r},t;\mathbf{r}',t')=\begin{cases}2c\,[c^2(t-t')^2-|\mathbf{r}-\mathbf{r}'|^2]^{-1/2}, & |\mathbf{r}-\mathbf{r}'|<c(t-t') ,\\ 0, & |\mathbf{r}-\mathbf{r}'|>c(t-t') .\end{cases}$$

From (3.1) and (3.10) we obtain

$$\xi(\mathbf{r},t)=\frac{1}{4\pi}\sum_j e_j\int\int d\mathbf{r}'dt'g(\mathbf{r},t;\mathbf{r}',t')\left[\frac{\mathbf{r}'-\mathbf{r}_j}{(\mathbf{r}'-\mathbf{r}_j)^2}\times\hat{\mathbf{z}}\cdot\left[\ddot{\mathbf{r}}_j+2\dot{\mathbf{r}}_j\frac{(\mathbf{r}'-\mathbf{r}_j)\cdot\dot{\mathbf{r}}_j}{(\mathbf{r}'-\mathbf{r}_j)^2}\right]\right] . \quad (3.12)$$

Equation (3.12) implies that energy can be transferred from the vortex motion to the spin-wave modes. Let us discuss briefly the implications of this fact to the calculation of the dynamic correlation function. The vortex part is not affected in form because, as we have mentioned before, the phenomenological theory of Mertens *et al.*[9] uses only the fact that a moving vortex passing a lattice site changes the sign of the spin at this place. The effect of the coupling would appear only in the calculation of the mean vortex velocity, and this calculation has not been performed up to now. The spin-wave component evidently exhibits peaks at the frequencies $\omega=\pm cq$. These peaks are very sharp and exhibit a broadening dependent on the vortex motion.

Of course the vortex-spin-wave coupling affects the vortex velocity. We would have to rederive Eq. (3.7) using the ϕ function given by expression (3.8), a very difficult task considering that the whole problem would have to be solved self-consistently. However we can understand what this energy transfer means using a simple argument. As was pointed out by Eckern and Schmid[21] when we put a vortex into motion we create a disturbance that will propagate with velocity c (the spin-wave velocity) so that at a time t it has propagated to a region of size ct. So, the size of the structure that we have to move increases with time, as if the vortex had a time-dependent mass or, physically what is the same, as if we had a viscous drag.

IV. MAGNETIC FIELD APPLIED IN THE XY PLANE

One expects the effect of a field B to be small if it is applied perpendicular to the magnetic plane. In that case, at least for moderate field values ($h\ll 1$) the effect of a field is essentially to reinforce the planar character of the spin system.[22] However if the magnetic field is applied in the XY plane it acts as an effective anisotropy breaking the XY symmetry. Equation (2.5), in the static limit, for the in-plane vortices becomes

$$\nabla^2\phi=-h^2\sin\phi\cos\phi , \quad (4.1)$$

which is the classical two-dimensional sine-Gordon equation. The symmetry of the model is now of the Ising type. Then there is a transition towards a 2D ordered phase[23] (for $h\to\infty$ the value of T_c would correspond to the value for the 2D Ising lattice).

José *et al.*[24] have argued that there would be no topologically ordered state between the paramagnetic state and the state of 2D long-range order, in the anisotropic XY system. The absence of the topologically ordered state, even for very weak Ising anisotropy, has also been confirmed by Monte Carlo calculations.[25] However for extremely small anisotropy the behavior remains unclear.[23]

For the system described by Eq. (4.1) the topological excitations may form complicated line patterns, which can be divided between two types:[23] (i) small topological excitations, and (ii) large domain walls that meander on a large scale and may eventually cross the system. However, in the limit $h\to 0$ it may be expected that the vortex configuration of system (4.1) should become identical to the vortex configuration of the XY model. The vortex-type solution of Eq. (4.1) has the form[26]

$$\phi(\mathbf{r})=\pm 2\tan^{-1}\left\{\sinh\left[\frac{x-x_0}{h}\right]\left[\sinh\left[\frac{y-y_0}{h}\right]\right]^{-1}\right\} \quad (4.2)$$

with the vorticity of the above solution being $e=\pm 2$. The energy of configuration (4.2), in the limit $R\to\infty$, is linearly proportional to R, instead of logarithmically as for the isotropic XY model. An energy-entropy argument shows that the energy contribution to the free energy always dominates the entropy contribution,[26] showing that the vortices (4.2) are always bounded with their antivortices for all T. Hudak,[26] however, suggests that the high-temperature phase of the anisotropic XY model, for small values of the anisotropy parameter, may be expected to contain vortices with vorticities $e=\pm 1$, similar to the vortices in the isotropic XY model. As for the experiments, nuclear spin-lattice relaxation time measurements[27] performed on the compound $BaNi_2(PO_4)_2$ supports the validity of the vortex model, at least for small magnetic fields ($H \approx 1$ T).

Let us do a general, but simplified, analysis of the problem. As we saw in Sec. III a topological excitation (and here we do not need to restrict to in-plane vortices) moving in the system under the influence of an external magnetic field B will feel a magnetic force given by Eq. (2.9). If we take into account the effect of dissipation, and consider the limit $v \ll c$, the most simple effective equation, in the steady-state case, that we can write should have the form

$$-G(\mathbf{v}_j \times \hat{\mathbf{z}}) + \eta \mathbf{v}_j = \mathbf{F}_j \ , \qquad (4.3)$$

where $\mathbf{v}_j$ is the jth vortex center velocity, η is a viscous coefficient,[28] and $\mathbf{F}_j$ describes the interaction between the topological excitations. The root-mean-square (rms) velocity is then given by

$$\mathbf{v} = \frac{\langle F^2 \rangle^{1/2}}{(\alpha B^2 + \eta^2)^{1/2}} \ , \qquad (4.4)$$

where

$$\alpha = (\gamma/4J) \left| \int d\mathbf{r} \cos\phi \cos^2\theta (\nabla\phi \times \nabla\theta) \right|^2 . \qquad (4.5)$$

The calculation of $\langle F^2 \rangle$ and α depends on the explicit shape of the topological excitations. For the "free" vortex the calculation of $\langle F^2 \rangle$ has been performed by Huber,[4] and Ivanov and Sheka[22] by the introduction of a self-consistent effective "electric field" describing the interaction between vortices.

Equation (4.4) shows that a field in the XY plane would reduce appreciably the vortex velocity, leading to a narrowing of the central peak. This conclusion, which differs from that of the ferromagnetic case, agrees with experimental data of Gaveau *et al.*[27] For zero magnetic field the rms velocity of a vortex in an antiferromagnet is much greater than in a ferromagnet because it is determined by viscosity, and not by a gyroforce (as in a ferromagnet). This result agrees with numerical simulation of magnetic models.[20]

V. CONCLUSIONS

We have studied the dynamics of vortices in a two-dimensional XY antiferromagnet. We have shown that the equation of motion for the vortex core is very complex showing a non-Newtonian behavior and a substantial difference from the corresponding equations for a ferromagnet. To arrive at definite conclusions about the time evolution of these equations, extensive numerical analysis is required. With some simplifying assumptions we have obtained an expression for the root-mean-square velocity of the topological excitations when a magnetic field is applied in the XY plane, our result being in agreement with experimental data.[27] However these experiments have probed mainly the in-plane correlation function that is only globally sensitive to the presence of vortices—the main effect of a vortex passing a lattice site is to rotate the spin at this place about 180°. (The phenomenological theory[9] developed to calculate the in-plane correlation function was derived treating the vortices as pointlike excitations, neglecting any internal structure.) It is the out-of-plane correlation function that is sensitive to the explicit shape of the vortices.

In the Mertens *et al.*[9] theory the "free" vortices have been considered to follow a simple ballistic behavior. However from the complexity of our equations we conclude that it is possible that we have a more complicated type of motion. More developments by taking into account that a vortex is not a Newtonian particle are needed to get a complete description of the dynamics.

The fluctuations in the particle number, associated with the creation-annihilation phenomenon and discreteness effect (lattice pinning) of the lattice may also be important to a better understanding of the problem. We hope more experiments and simulations will clarify these points in the near future.

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