Structure of fractional edge states: A composite-fermion approach

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I study the structure of the two-dimensional electron gas edge in the quantum Hall regime using the composite-fermion approach. The electron density distribution and the composite-fermion energy spectrum are obtained numerically in the Hartree approximation for bulk filling factors $\nu = 1, 1/3, 2/3, 1/5$. For a very sharp edge of the $\nu = 1$ state the one-electron picture is valid. As the edge width a is increased the density distribution shows features related to the fractional states and new fractional channels appear in pairs. For a very smooth edge I find, within a quasiclassical approximation, that the number of channels $p \sim \sqrt{a/l_H}$, where l_H is the magnetic length.

I. INTRODUCTION

The concept of edge states was introduced originally in the framework of the integer quantum Hall effect (IQHE).[1] It is based on the fact that the two-dimensional electron gas (2DEG) at an integer Landau level filling factor ν can support gapless excitations only at the edge. Hence the transport properties can be understood without explicitly considering the bulk of the sample. The edge of the 2DEG is created by the confinement potential which bends the Landau levels up in energy. The intersection of each Landau level with the Fermi energy gives origin to a chiral one-dimensional edge channel. Naturally, the total number of channels within the one-electron theory is given by the bulk filling factor. Experiments with nonideal contacts[2–4] have confirmed the existence of separate edge channels corresponding to different Landau levels. The quantized multiprobe resistance was found to be in agreement with the predictions of the Landauer-Büttiker formalism[5] based on the one-electron picture. However, a number of more sophisticated experiments were impossible to explain on the one-electron footing.[6,7]

Chklovskii *et al.*,[8] developing earlier works,[9–14] have shown that in a realistic confinement potential edge channels (compressible strips) have finite width as a result of electron-electron interactions. The edge channels being not strictly one dimensional brings up the problem of their internal structure. In the two limits of very sharp and very smooth confinement this problem can be tackled. When the confinement potential is very sharp it dominates over the electron-electron interactions and the single-electron theory must be valid. However, it is unclear how to implement this situation experimentally. In a very smooth confinement potential a compressible strip breaks up into a number of fractional states with fractional edge channels between them.[12,13] This explains the observation of the fractionally quantized Hall conductance for integer bulk filling factor in devices with nonideal contacts.[7]

The purpose of this paper is to study the evolution of the edge channel structure as the smoothness of the confinement is changed continuously. The complexity of the problem arises from the strongly correlated nature of the electron state. Any theory that resolves this problem should be able to describe the one-electron limit of very sharp confinement as well as the formation of the incompressible fractional states in a very smooth confinement. In this paper I present such a theory, which also predicts the number of fractional edge channels as a function of the confinement sharpness. The theory is based on the composite-fermion approach.[15–18] The advantage of this approach is that the important electron-electron correlations are automatically built into the single-particle (Hartree) approximation for composite fermions.

The main idea can be demonstrated by considering the evolution of the $\nu = 1$ edge as the confinement potential varies from the very sharp to the very smooth limit (see Fig. 1). The confinement potential can be characterized

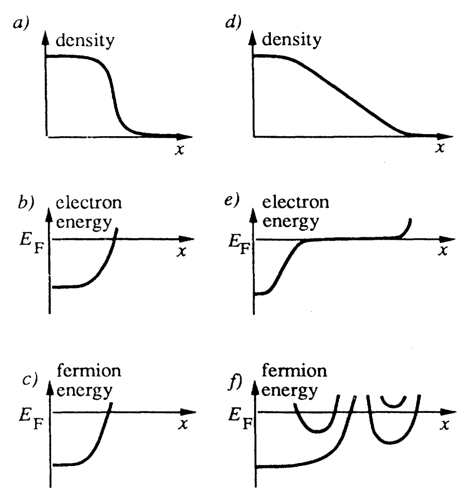

FIG. 1. Schematic density distribution and energy spectra for the sharp and smooth edge of the $\nu = 1$ state. Energy is plotted as a function of the guiding center position of momentum eigenstates. (a) Density distribution for the sharp edge. (b) Electron energy spectrum for the sharp edge. (c) Composite-fermion energy spectrum for the sharp edge. (d) Density distribution for the smooth edge. (e) Electron energy spectrum for the smooth edge. (f) Composite-fermion energy spectrum for the smooth edge.

by a single parameter a which represents the width of the compressible liquid strip where the filling factor drops from 1 to zero.

When the confinement potential is very sharp ($a \sim l_H$) the one-electron theory must be valid, implying that there is just a single one-dimensional channel, Fig. 1(b). This result can also be understood in terms of composite fermions. Attachment of two flux quanta maps the $\nu = 1$ electron state on a single filled Landau level for composite fermions with the magnetic field direction reversed. At the edge this level bends up and intersects the Fermi energy forming a single channel, Fig. 1(c). Higher Landau levels for composite fermions remain empty.

As the confinement gets smoother (larger a) a finite width region with partially filled Landau level emerges, Fig. 1(d). One-electron theory is useless in this case since it predicts a huge degeneracy, Fig. 1(e). The composite-fermion approach, on the other hand, yields an interesting picture, Fig. 1(f).[19] The effective magnetic field experienced by composite fermions is proportional to the deviation of the filling factor from 1/2. Hence the compressible strip with the filling factor dropping from 1 to zero results in the effective magnetic field varying between minus and plus the external magnetic field value. The problem is reduced then to a particle in a varying effective magnetic field.[20,19,21] Composite-fermion energy bands descend in the regions of weak effective magnetic field because of the reduction in the effective cyclotron energy. As parameter a is increased the gradient of the effective magnetic field gets smaller and higher energy bands intersect the Fermi energy, Fig. 1(f). When each consecutive band touches the Fermi energy a pair of fractional channels is formed. They are composed of states moving in the opposite directions. When the channels are just created it is impossible to identify them with any particular fraction, but for sufficiently large a, the channels become separated by the fractional filling factor regions.[12,13]

In this paper I show that the above picture is self-consistent within the Hartree approximation for composite fermions. The electron density distribution and the composite-fermion energy spectrum are found for several bulk filling factors. The model used in the numerical solution and the formalism of the Hartree approximation are explained in Sec. II. In Sec. III results of the Hartree approximation for simple fractions are compared against the existing Laughlin wave function calculations to check the reliability of the method. Encouraged by the ability of the Hartree calculation to reproduce the essential features of the density distribution I apply the method to study the evolution of the $\nu = 1$ edge in Sec. IV. In Sec. V the composite-fermion approach is used to address the long-standing problem of the $\nu = 2/3$ edge. In short, I find that the MacDonald[22] and Chang-Beenakker[13,12] models describe sharp and smooth confinement, respectively, in agreement with the conclusions of Meir[23] and Brey.[24]

In the limit of very smooth confinement the number of occupied composite-fermion bands is large, calling for the quasiclassical approximation. This approach is used in Sec. VI to find the number of fractional channels in the large-a limit. It also gives the highest fractional Hall state in the principal sequence $p/(2p+1)$ (Ref. 15) that survives a density gradient determined by the length scale a. The dependence of the highest p on the Landau level number is found.

II. MODEL AND METHOD

In GaAs heterostructures the 2DEG edge is most commonly defined either by a negatively biased gate on top of the device or by the chemical etching process.[25] In both confinement schemes the electron density distribution at the edge has to be determined self-consistently.[8,26] For the purpose of studying the structure of the edge channels I consider a simplified model instead of the actual confinement scheme. The 2DEG is placed on a positive background, the density of which mimics the electron density in the absence of a magnetic field. This model is supported by the smallness of the magnetic-field-caused electron density redistribution.[8] For computational reasons I consider a quantum wire with two edges. Yet another simplification has to be made in order to keep the bulk filling factor constant while the confinement potential is varied. Instead of a realistic density profile.[27] I consider the positive background of trapezoidal form in the x-z plane and translationally invariant in the y direction (Fig. 2). The density in the center corresponds to various bulk filling factors. The separation between the two edges in the x direction is chosen large enough to keep the interference between them negligible. In order to apply results of the calculation to a particular experimental situation one has to estimate the typical distance a at which the electron density drops from its bulk value to zero. The trapezoidal background model with the width a of the gradient regions is a good approximation for most purposes.

The above model has been used to study integer edge channels in the Hartree approximation,[28] and the edge reconstruction of the $\nu = 1$ state.[29] Neither work treated electron-electron correlations which are explicitly present in this calculation. The model used by Brey[24] is essentially the same except that he places an infinite potential wall at the edge of the trapezoid. I believe that such a boundary condition imposes an unrealistically strong confinement. In my model the infinite potential wall is

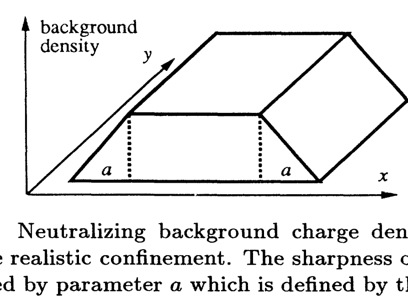

FIG. 2. Neutralizing background charge density used to model the realistic confinement. The sharpness of the edge is determined by parameter a which is defined by the zero magnetic field electron density distribution. The distance between the two edges is chosen large enough to eliminate interference between them.

positioned far enough from the edge of the trapezoid to make the solution insensitive to its exact location.

The advantage of using the composite-fermion approach is in transforming a system of strongly correlated electrons to a system of weakly interacting quasiparticles. Formally this can be realized in the Hartree approximation for *composite fermions.* The starting point is the electron Hamiltonian,

$$H = \sum_{i=1}^{N} \frac{(\mathbf{p}_i + e/c\mathbf{A}_i)^2}{2m_B} + \sum_{i\neq j} V(\mathbf{r}_i - \mathbf{r}_j) + \sum_{i=1}^{N} U(\mathbf{r}_i), \tag{1}$$

where $\mathbf{A}$ is the external vector potential, m_B is the electron band mass, V is the electron-electron interaction, and U is the external potential. After a singular gauge transformation the Hamiltonian has the form

$$H = \sum_{i=1}^{N} \frac{(\mathbf{p}_i + e/c\mathbf{A}_i - \mathbf{a}_i)^2}{2m_B} + \sum_{i\neq j} V(\mathbf{r}_i - \mathbf{r}_j) + \sum_{i=1}^{N} U(\mathbf{r}_i), \tag{2}$$

where

$$\mathbf{a}_i = 2\hbar \sum_{j\neq i} \frac{\hat{\mathbf{z}} \times (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_i - \mathbf{r}_j|^2} \tag{3}$$

is the Chern-Simons vector potential. This Hamiltonian acts on the composite-fermion wave function Ψ which is related to the electron wave function Ψ_e through the phase factor:

$$\Psi_e = \prod_{i<j} \frac{(z_i - z_j)^2}{|z_i - z_j|^2} \Psi, \tag{4}$$

where $z = x + iy$ is a complex coordinate. The self-consistent Hamiltonian for the gauge field problem has been derived in the context of anyon superconductivity.[30,31] Following the derivation of Halperin[31] the Hartree Hamiltonian can be written as

$$H = \frac{[\mathbf{p} + e/c\mathbf{A}(\mathbf{r}) - \langle \mathbf{a}(\mathbf{r})\rangle]^2}{2m^*} + U_j(\mathbf{r}) + U_c(\mathbf{r}), \tag{5}$$

where m^* is the effective mass. Although in the Hartree approximation $m^* = m_B$,[18] I use a different value of m^* that yields correct energy gaps for the fractional quantized Hall states (see below). This simple substitution should capture correctly short-range features in the electron density related to the fractional channels. A deficiency at large length scales can be fixed by an insignificant monotonic redistribution of the background charge.

The expectation value of the Chern-Simons vector potential is given by

$$\langle \mathbf{a}(\mathbf{r})\rangle = 2\hbar \int d^2\mathbf{r}' \frac{\hat{\mathbf{z}} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^2} \langle \rho(\mathbf{r}')\rangle, \tag{6}$$

with the particle density operator

$$\rho(\mathbf{r}) = \sum_i \delta(\mathbf{r} - \mathbf{r}_i). \tag{7}$$

The Chern-Simons contribution to the scalar potential is

$$U_j(\mathbf{r}) = 2\frac{\hbar}{e} \int d^2\mathbf{r}' \frac{\hat{\mathbf{z}} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^2} \mathbf{j}_H(\mathbf{r}'), \tag{8}$$

where the Hartree approximation to the expectation value of the current density is

$$\mathbf{j}_H(\mathbf{r}) = \langle \mathbf{j}_p(\mathbf{r})\rangle - \frac{e}{m^*} \langle \rho(\mathbf{r})\rangle [\langle \mathbf{a}(\mathbf{r})\rangle - \mathbf{A}(\mathbf{r})], \tag{9}$$

while

$$\mathbf{j}_p(\mathbf{r}) = \frac{e}{2m^*} \{\mathbf{p}_i, \delta(\mathbf{r} - \mathbf{r}_i)\}. \tag{10}$$

Finally the Coulomb interaction is included in

$$U_c(\mathbf{r}) = \frac{e^2}{\epsilon} \int d^2\mathbf{r}' \frac{\langle \rho(\mathbf{r}')\rangle - \rho_+(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}, \tag{11}$$

where $\rho_+(\mathbf{r})$ is the positive background density.

In the chosen geometry the Hamiltonian can be simplified further by taking advantage of the translational invariance in the y direction and using dimensionless units it can be written as

$$H = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}\left(k_m + x - \int dx' \mathrm{sgn}(x - x')\nu(x')\right)^2 + \int dx' \mathrm{sgn}(x - x') \sum_{m,l} \nu_{m,l}(x') \times \left(k_m + x' - \int dx'' \mathrm{sgn}(x' - x'')\nu(x'')\right) + \alpha \int dx' \ln|x - x'| [\nu(x') - \nu_+(x')], \tag{12}$$

where k_m is the momentum of the mth state, l is the Landau level index, and ν is the electron filling factor. The distances are in units of the magnetic length l_H. The energy is measured in units of the cyclotron frequency for composite fermions at $\nu = 1$. The strength of the Coulomb interaction compared to the kinetic energy is determined by

$$\alpha = \frac{1}{\pi} \frac{m^*}{m_B} \frac{l_H}{a_B}. \tag{13}$$

Large α makes deviations of the electron density from the positive background very costly. This means that despite the high kinetic energy the electron density mimics the positive background. For small values of α the kinetic energy of composite fermions becomes important, leading to the formation of the fractional plateaus. For the experimentally relevant parameters $m^* = 4m_B$, $l_H = a_B = 100$ Å , the value of $\alpha \approx 1$ is found. I use $\alpha = 1$ throughout this paper except for Sec. III. Introduction of a short-distance cutoff in the logarithm in Eq. (12) reflecting a finite z extent of the wave functions was found to have a similar effect as the reduction of α.

This self-consistent Hamiltonian (12) was solved numerically in this work and independently by Brey.[24] Alternatively, one may view the solution as a variational electron wave function of the form (4) where Ψ is a product of one-particle wave functions and all the correlations are taken care of by the phase factor.

III. RESULTS FOR SIMPLE FRACTIONS $\nu = 1/(2K+1)$

The special role of the filling fractions $\nu = 1/(2k+1)$ in the fractional quantum Hall effect (FQHE) has been recognized ever since Laughlin[32] proposed his wave functions. The knowledge of the explicit wave functions made possible the calculation of the density profile for a $\nu = 1/(2k+1)$ state.[33,34] In this section I compare results of the Hartree calculation with the calculations for the Laughlin wave function.[34]

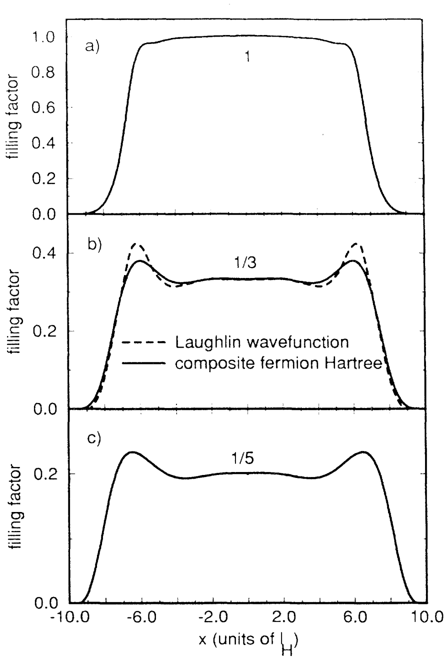

FIG. 3. Electron density distribution obtained from the composite-fermion Hartree calculation for simple filling fractions in very sharp confinement. (a) Filling factor $\nu = 1$ state. (b) Filling factor $\nu = 1/3$ state. Dashed line: Rezayi-Haldane calculation for Laughlin wave function; full line: composite-fermion Hartree calculation. (c) Filling factor $\nu = 1/5$ state.

As shown by Wen,[35] a $\nu = 1/(2k+1)$ state in very sharp confinement supports a single branch of edge excitations. In terms of composite fermions this implies that there is a single energy band, giving rise to a single edge channel. Since the composite-fermion energy spectrum does not have an intuitive meaning in the electron representation I can only compare the density distribution.

In Fig. 3 I present the electron density distribution for three simple fractions in the case of the sharp edge. In the case of $\nu = 1$ the density is almost featureless and is very close to the profile expected for the lowest Landau level filled up to the Fermi momentum. The density distribution at fractional filling factors shows damped oscillations near the edge. These oscillations have been first found numerically.[36,37] Their period corresponds to the interparticle spacing. The oscillations were argued to be the precursor of Wigner solid formation and are reproduced in the single-mode approximation to the density-density response function.[38]

It is clear that the composite-fermion Hartree calculation is able to reproduce the essential features in the density distribution such as the period and the damping of the oscillations. In the following sections the method is used to study the edge structure when the exact wave function is not available.

The composite-fermion approach allows oscillations in density to be related to a pole in the Fourier transform of the density-density response function (static susceptibility) $K_{00}(q)$. To demonstrate this I consider linear response to a potential of the form

$$V(x,y) = V_0\delta(x). \tag{14}$$

The induced change in the electron density is then independent of y and expressed as

$$\delta\rho(x) = V_0 \int_{-\infty}^{+\infty} dq K_{00}(q) e^{iqx}. \tag{15}$$

The large-x behavior of $\delta\rho(x)$ is dominated by a pole in the complex q plane closest to the real axis. For a pole at $q = q' + iq''$ the asymptotic form of the induced change in the electron density is

$$\delta\rho(x) \sim e^{-q''x}\cos(q'x). \tag{16}$$

The electromagnetic response function for the fractional states has been calculated by Simon and Halperin[39] by using the random-phase approximation (RPA) for composite fermions. I use their results, neglecting the effect of Coulomb interaction and the renormalization of the composite-fermion mass. When there is only one filled band for composite fermions the static density-density response function is

$$K_{00}(q) = \frac{-q^2\Sigma_0}{2\pi\hbar\Delta\omega_c[(\tilde{\phi}\Sigma_1 + 1)^2 - \tilde{\phi}^2\Sigma_0(\Sigma_2 + 1)]}, \tag{17}$$

where the number of attached flux quanta $\tilde{\phi} = -2k$

for the fractions $1/(2k+1)$, $\Delta\omega_c = \omega_c/(2k+1)$ is the cyclotron frequency for composite fermions, and Σ_j are functions of q^2 as defined in Ref. 39. By performing an analytic continuation of Eq. (17) to the complex plane one can see that this function has a pole at $q \approx (1.3 + 0.6i)l_H^{-1}$ for $\tilde{\phi} = -2$ corresponding to $\nu = 1/3$ and $q \approx (1.1 + 0.5i)l_H^{-1}$ for $\tilde{\phi} = -4$ corresponding to $\nu = 1/5$. This pole leads to damped oscillations in the induced change in the electron density of the form (16). The inclusion of Coulomb interaction and the renormalization of the composite-fermion mass shift the position of the pole in the complex plane but the qualitative picture remains the same.

IV. RESULTS FOR THE $\nu = 1$ EDGE

The properties of the electron state at the Landau level filling factor $\nu = 1$ in the bulk can be understood mostly from the standpoint of noninteracting electrons. This may not be the case at the edge of the system where the filling factor falls slowly to zero as demonstrated in experiments with nonideal contacts.[7] The detailed structure of the edge is determined by the strength of the confinement potential, since it dictates the filling factor gradient at the edge. The approach outlined in Sec. II enables me to study the structure of the $\nu = 1$ edge in terms of noninteracting composite fermions. Here I give a physical interpretation to the results of the Hartree calculation shown in Fig. 4. (Plots are restricted to the right edge only because of the symmetry of the problem.)

In the case of a very sharp edge ($a \approx l_H$) the confinement energy dominates over the electron-electron interactions. Consequently, the one-electron picture gives an adequate description of the edge: a single one-dimensional edge channel is formed at the intersection of the Landau level and the Fermi level. In the composite-fermion description the $\nu = 1$ state corresponds to a single filled Landau level in the reversed effective magnetic field. At the edge this level intersects the Fermi level, forming a single channel as illustrated in Fig. 4(a). The higher composite-fermion Landau levels remain unfilled. The self-consistent energy spectrum obtained in the Hartree calculation can be understood by solving a one-particle problem.[19,21] Since the effective magnetic field follows the electron density profile, composite fermions find themselves in a steplike magnetic field. In the Landau gauge the vector potential is $A = B_0|x|$, where x is the distance from the edge. The problem is reduced to a one-dimensional Schrödinger equation similar to the uniform magnetic field case. However, the effective potential in this equation has two potential wells at large momenta. This leads to the existence of a degenerate pair of states centered at positive and negative x. The degeneracy is lifted by the confinement potential. But it is important to remember that the momentum of a given state only determines its distance from the zero magnetic field line. States with the same momentum may be on different sides of this line. To get an idea of the energy spectrum in real space it is useful to reflect every other energy band around the intersection of the lowest one with the Fermi level.

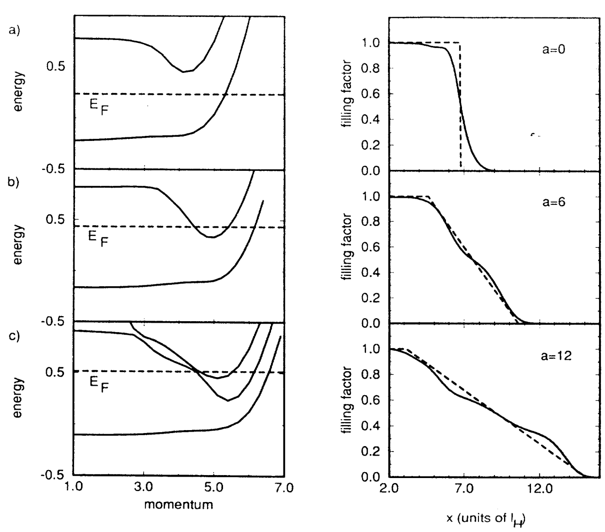

FIG. 4. Results of the Hartree calculation for the $\nu = 1$ edge for different sharpness of the confinement potential parametrized by a. Composite-fermion energy spectrum is in the left column (dashed line denotes the Fermi level), and the electron density distribution is in the right column (dashed line shows the neutralizing background). Energy is given in the units of effective cyclotron frequency. (a) $a = 0$. (b) $a = 6l_H$. (c) $a = 12l_H$.

For larger a there must be a finite width region with the filling factor between 1 and zero. In the one-electron approximation this implies a huge degeneracy of the states at the edge. The composite-fermion approach allows me to resolve this problem. The self-consistent Hartree energy spectrum for $a = 6l_H$ is presented in Fig. 4(b). The main difference from the sharp confinement case is the descent of the second Landau level below the Fermi energy. Again, this result follows from the solution of the one-particle problem for composite fermions. The effective magnetic field variation may be approximated by a constant gradient. The spectrum obtained by solving this problem[20,19,21] with the confinement potential is very close to the one in Fig. 4(b). The descent of the Landau level is because of the reduction of the cyclotron energy in the vicinity of $\nu = 1/2$. As the composite-fermion band touches the Fermi level a pair of fractional channels is formed. It is not clear how to identify these channels with any particular filling fraction since there is no incompressible state in between.

For larger a higher composite fermion bands become occupied, giving rise to additional pairs of channels. Incompressible states are formed between the channels as shown in Fig. 4(c). When the incompressible regions are wide the energy spectrum in momentum space can be mapped to real space. Since the effective magnetic field for composite fermions changes sign at $\nu = 1/2$ the dependence of coordinate on momentum is double valued. The position of the states belonging to the odd-numbered bands is given by their momentum up to a factor of the magnetic length. The position of the states in the even-numbered bands can be obtained roughly by reflecting these bands around the $\nu = 1/2$ line (see Fig. 1). This allows me to identify the channels by the filling fractions between them. The second band gives rise to the 1/3 channel (between $\nu = 0$ and $\nu = 1/3$) while the third band produces the 2/3 channel (between $\nu = 2/3$ and $\nu = 1$). According to the composite-fermion picture there are also three channels between $\nu = 2/3$ and $\nu = 1/3$.

As shown in Fig. 4 the composite-fermion approach fractional channels are formed in pairs as the confinement is relaxed, in agreement with the prediction by Wen.[40] The formation of the first pair of fractional edge channels is reminiscent of the edge reconstruction proposed by Chamon and Wen.[29] However, the physics is very different here. Electron-electron *correlations* are responsible for the formation of the *fractional* channels as opposed to the *exchange* interaction producing a pair of *integer* channels in the edge reconstruction model.[29] In order to determine the relationship between the two effects a solution that takes into account both exchange and correlations is needed.

As the edge becomes very smooth the one-particle composite-fermion approach must break down because the filling factor is expected to vary smoothly between any two principal sequence filling factors. This calls for the transformation to the next generation of composite fermions. A simple calculation shows that a range of confinement strength exists where the one-electron description fails while the one-particle approach for composite fermions of the principal sequence is still valid. The main reason is that composite fermions of the higher generations experience a stronger gradient of the effective magnetic field at the same density gradient. The calculation is similar to the one given for higher Landau levels in Sec. VI.

At this point I would like to address the effect of disorder on the edge structure. In the case of short-range disorder fermions may scatter between different channels, making it a complicated problem. However, when disorder is long ranged a simple model emerges. A superposition of the slowly varying disorder potential and the confinement potential can be modeled by a background charge with width a varying slowly along the edge. By treating the problem adiabatically one gets the number of channels varying along the edge. It is clear that the channels corresponding to the higher effective Landau levels become localized in the regions of smooth edge (large a). The number of delocalized channels is determined by the steepest confinement region (smallest a).

V. RESULTS FOR THE $\nu = 2/3$ EDGE

Wen has shown[35] that the $\nu = 2/3$ quantum Hall state must support more than one branch of edge excitations. Two, seemingly incompatible, theoretical models have been proposed to explain the detailed structure of the composite edge. The first model, due to MacDonald,[22] claims the existence of two excitation branches and the absence of the $\nu = 1/3$ state at the edge. The second model was proposed by Beenakker[12] and Chang.[13] In their picture there is a transition from the 2/3 state to the 1/3 state and from the 1/3 state to 0 with compressible regions in between.

I use the composite-fermion approach to address this controversy. Results of the calculation are shown in Fig. 5 for different sharpness of the confinement potential. In the case of the sharp edge, $a = 0$, there are clearly two edge channels, in agreement with MacDonald's model. However, I do not find any region with the filling factor close to 1. Therefore it is not clear how to identify these channels.

As the confinement is relaxed, Fig. 5(b), the third composite-fermion energy band descends and crosses the Fermi level. This signals the formation of a pair of edge channels. Further smoothing of the confinement leads to the formation of the incompressible $\nu = 1/3$ state, Fig. 5(c). The electron density distribution is in agreement with the Chang-Beenakker picture. One can see incompressible regions corresponding to $\nu = 1/3$ and $\nu = 2/3$ and compressible regions between them. However, according to the composite-fermion approach there are four channels in this case: one of them is located between $\nu = 1/3$ and $\nu = 0$, and the other three are between $\nu = 1/3$ and $\nu = 2/3$. As the edge becomes smoother higher bands must become occupied, reflecting the formation of new fractions at the edge in analogy with the $\nu = 1$ edge discussed in Secs. IV and VI.

VI. QUASICLASSICAL APPROXIMATION

For a very smooth edge, $a \gg l_H$, the number of filled composite-fermion bands is large, allowing me to use the

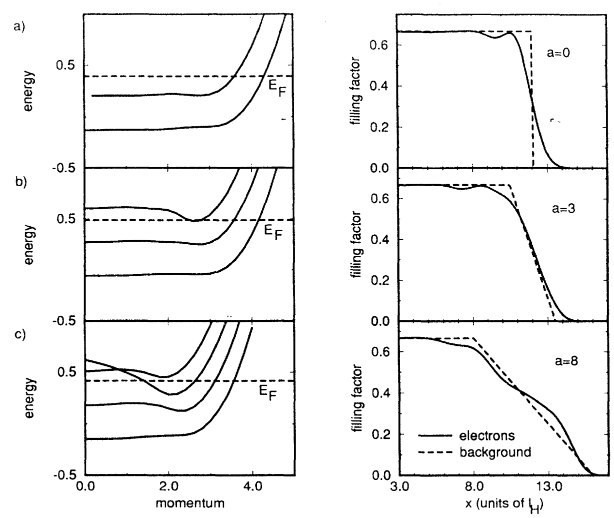

FIG. 5. Results of the Hartree calculations for the $\nu = 2/3$ edge for different sharpness of the confinement potential parametrized by a. Note the transition from the MacDonald to the Chang-Beenakker picture as the confinement is relaxed. Composite-fermion single-particle spectrum is in the left column (dashed line denotes the Fermi level), and the electron density distribution is in the right column (dashed line shows the neutralizing background). Energy is given in the units of effective cyclotron frequency. (a) $a = 0$. (b) $a = 3l_H$. (c) $a = 8l_H$.

quasiclassical approximation to find the number of fractional channels. To be specific I consider the edge of the $\nu = 1$ state and restrict myself to fractions of the principal sequence $p/(2p+1)$.[15] The question may be restated as: what is the value of p for the highest fractional Hall state that survives the density gradient determined by a? The value of p is given by the highest filled energy band of composite fermions. To answer this question I look for the classical orbit of composite fermions moving with the Fermi velocity and enclosing the largest number of effective magnetic field flux quanta. The number of enclosed flux quanta gives the value of p.

This argument is similar to the estimate of the last integer Hall plateau in a quantum wire by Roukes *et al.*[41] They have argued that the maximum number of edge channels is given by the number of flux quanta encircled by the largest cyclotron orbit that fits in the wire.

Assuming that the electron density roughly follows the positive background [Fig. 6(a)], composite fermions experience a roughly linearly varying effective magnetic field, Fig. 6(b):

$$B(x) \approx B_0 x/a, \tag{18}$$

where x is the distance from the zero effective field ($\nu = 1/2$) line. There are three types of classical orbits for composite fermions in this field:[20,19,21] drifting orbits that move along the edge in the direction of electron drift, snake orbits that move in the opposite direction, and closed orbits that do not drift along the edge, Fig. 6(c). I focus on the closed orbits which are the classical analogue of the states at the minima of the energy bands and must be filled first.

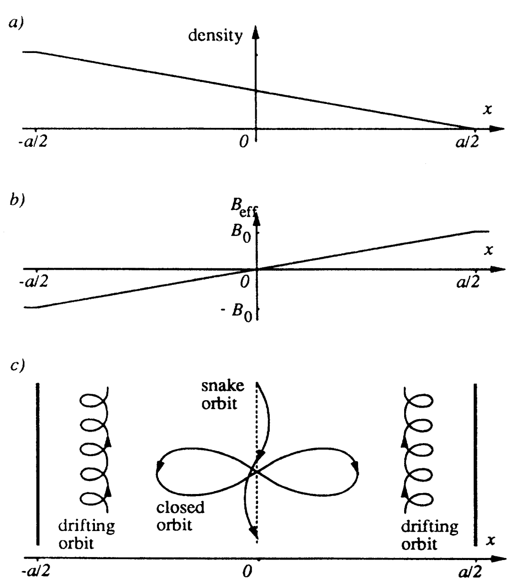

FIG. 6. Quasiclassical approximation (dependence of the Fermi velocity on the density is neglected). (a) Electron density profile in the approximation of the constant gradient. (b) The effective magnetic field experienced by composite fermions. (c) Three kinds of classical orbits possible in the varying effective magnetic field.

Neglecting the dependence of the Fermi velocity on the deviation of the filling factor from $\nu = 1/2$ the composite-fermion cyclotron radius as a function of x,

$$R_c(x) \sim l_H a/|x|, \tag{19}$$

where I used the fact that the cyclotron radius at the boundary of the compressible strip ($x = -a$) is l_H. The closed orbit should have the extent in the x direction such that R_c at x is equal to x. By using Eq. (19) I find from this condition that

$$R_c \sim \sqrt{l_H a}. \tag{20}$$

The number of channels is equal to the number of effective flux quanta enclosed by the orbit. The area of the orbit is proportional to R_c^2. The typical area corresponding to one flux quantum at $|x| = R_c$ is $l_H^2 a/R_c$. Then the number of flux quanta enclosed by the orbit is

$$p \sim \frac{R_c^2}{l_H^2 a/R_c}. \tag{21}$$

By using the value of R_c from Eq. (20) I find the number of channels for the smooth edge of width a:

$$p \sim \sqrt{\frac{a}{l_H}}. \tag{22}$$

This result agrees with the work of Lee, Chalker, and Ko[21] who used the WKB approximation to study the motion of a particle in a slowly varying magnetic field.

Equation (22) counts only the channels for composite fermions of the principal sequence. In the case of a very smooth edge, composite-fermion channels must become wide. Then one should make a transformation to the composite fermions of the next generation and follow the same line of argument.

The above argument can also be generalized to higher Landau levels. Then a denotes the distance where the filling factor changes by 1. Therefore, if the magnetic field is varied while density is fixed, the number of channels in each Landau level is

$$p_N = p_1/N^{3/4}, \tag{23}$$

where p_1 is given by Eq. (22) and N is the bulk filling factor. This result may account for difficulties in observing fractional Hall states in higher Landau levels.

VII. SUMMARY

In this paper I have presented a theory of the compressible liquid strip at the edge of an incompressible quantum

Hall state. The theory is based on the composite-fermion approach and describes the evolution of the edge as the steepness of the confinement potential is varied. When the width of the compressible strip is of the order of several magnetic lengths, the composite-fermion Hartree approximation is used to find numerically the density distribution and the energy spectrum for composite fermions. This allows me to predict the number of fractional channels for a given confinement potential. The direction of propagation and the detailed structure of the excitations in these channels require further study.

In the asymptotic limit of a wide compressible strip the number of channels can be found quasiclassically. It is given by Eq. (22), assuming that the density gradient is roughly constant. The result gives an upper bound on the highest fraction that can be observed in the FQHE for a given strength of long-range disorder. Damped density oscillations at the edge are found for filling factors $\nu = 1/(2k+1)$ in the case of sharp confinement. They are related to the pole in the linear response static susceptibility derived in the RPA for composite fermions. The $\nu = 2/3$ edge structure is found to depend strongly on the steepness of the confinement potential. The transition from the MacDonald to the Chang-Beenakker picture is observed when the width of the edge is $\sim 3l_H$.

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