Observation of the integer quantum Hall effect by magnetic coupling to a Corbino ring

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Observation of the integer quantum Hall effect by magnetic coupling to a Corbino ring is reported. A small solenoid is used to induce an ac azimuthal electric field in a high-mobility GaAs-$Al_xGa_{1-x}As$ Corbino ring. The Hall voltage is measured between the inner and outer edges of the disk. The presence of plateaus observed in the response is a clear indication that quantization in this geometry is the consequence of bulk transport. The ac response and its frequency dependence are described by a model taking into account the effect of the self-inductance and the capacitance of the Corbino disk.

I. INTRODUCTION

It is generally accepted that most four-terminal resistance measurements on Hall bar devices operating in the quantum Hall regime (e.g., Refs. 1 and 2) can be described using the edge-channel model of the quantum Hall effect (QHE) proposed by Büttiker.[3] In this model, the transport properties of the sample are described in terms of one-dimensional channels running parallel to the sample edges. These channels are a consequence of the upward bending of the Landau levels at the sample edges, due to the confinement potential there.[4] However, the Büttiker model does not actually exclude the existence of extended states away from the boundaries of a two-dimensional electron gas (2DEG), rather, it asserts that only edge states affect the accuracy of four-terminal resistance measurements of the QHE in Hall bar topologies. Therefore, experiments in which the edge channel model is applicable will be unable to provide evidence for or against the existence of states away from the boundaries of a 2DEG.

Since the discovery of the QHE, there has been considerable interest in the role of current carrying states in the interior of a 2DEG. Early studies attempted to measure the potential within a Hall bar by placing contacts across the channel.[5] However, such measurements can only be performed at relatively high temperatures (above 1 K), because the interior contact effectively decouples from the 2DEG due to the low value of σ_{xx} in the quantum Hall regime.

If the topology of the 2DEG is changed from a Hall bar to a Corbino ring, experimental situations can be contrived in which charge transport takes place in the absence of edge states and can be measured. Applying a time-varying normal magnetic flux to a Corbino ring will give rise, by Faraday's law, to an azimuthal electric field around the ring's channel. Because of the tensorial relationship of current to electric field in the quantum Hall regime, the induced field causes a radial current to flow across the channel. This current, because of the ring topology, is clearly edgeless.

The Corbino geometry was first used in gedanken experiments proposed by Laughlin[6] and Halperin,[4] in which a gauge invariance argument was put forward to explain the QHE. It has been used in early experimental studies by Syphers, Martin, and Higgins[7] and Fontein *et al.*[8] Recently, renewed interest in the existence of bulk states in the QHE, both experimentally[9] and theoretically,[10] has highlighted the importance of the Corbino geometry. It has been successfully used to measure the Hall conductivity, σ_{xy}, under QHE conditions by Dolgopolov *et al.*[9] However, when compared with the wealth of experimental work that has been performed on Hall bar devices, it is surprising how little effort has been invested in the Corbino ring where there is the possibility of observing edge-free charge transport.

In this paper, we apply an ac technique to a Corbino sample and present data, which clearly shows quantization of the 2DEG under QHE conditions. Our experiment creates a time-varying azimuthal electric field by using a small solenoid to modulate the flux linking the ring. A potential difference, arising from the displacement of charge across the ring, is then measurable between the inside and outside edges. We observe well-defined voltage plateaus at static magnetic-field values coinciding with the Hall resistance plateaus in dc measurements. The measurements are interpreted with the aid of a phenomenological model, that quantitatively describes the response of the solenoid-Corbino system to an ac signal by taking into account the self-inductance and capacitance of the ring. It shows that the voltage plateaus observed are a manifestation of the dissipation-free state of the quantum Hall regime and that their amplitude is evidence for the quantization of the conductivity of the 2DEG. Furthermore, it is shown that the ac technique is highly sensitive to small values of the longitudinal resistivity, ρ_{xx}, and can be used to probe the onset of breakdown in the QHE.

II. SAMPLE AND EXPERIMENTAL SETUP

The experimental details are as follows: The Corbino ring is etched into a GaAs/$Al_xGa_{1-x}As$ heterostructure

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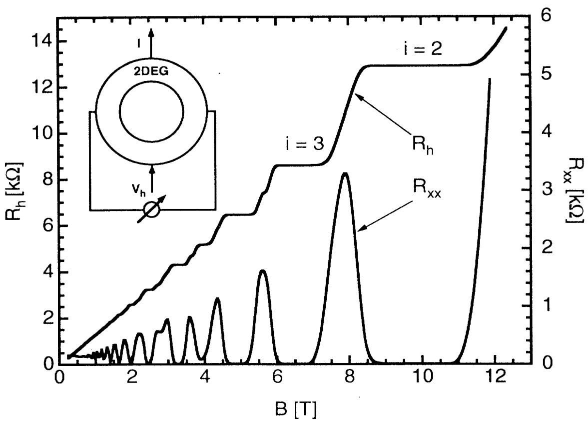

FIG. 1. dc measurements of Hall R_h and longitudinal R_{xx} resistances as a function of the magnetic induction B at $T=0.3$ K. The measurement configuration is shown in the inset.

with internal and external radii of, respectively, $r_1=4$ mm and $r_2=5$ mm. Four pairs of AuGeNi contacts[11] are arranged at 90° intervals, around the ring's periphery. At a temperature of 4 K, the 2DEG mobility was $\mu=42$ T^{-1} and the charge density $n=4.8\times10^{15}$ m^{-2}. The Corbino sample is mounted coaxially in the middle of a 1000-turn superconducting solenoid, which produces a field of $B_m=53$ μT per mA. The whole setup is located in the center of a 14-T superconducting magnet (static field B), in a bath of ^{3}He at a temperature of 0.3 K.

As a test of the sample quality, dc measurements of the longitudinal resistance, R_{xx}, and the Hall resistance, R_h, were made as a function of B and are shown in Fig. 1. These measurements used current contacts and voltage probes located on the outer edge of the Corbino disk, as shown in the inset of Fig. 1. This configuration is similar to the usual four-terminal configuration for measurements on a Hall-bar sample. The presence of the inner edge is of no importance: it is decoupled from the outer edge and the resistance is quantized at the usual values.[12] These measurements attest to the high degree of homogeneity in the sample in spite of its large size.

III. RESULTS AND INTERPRETATION

Figures 2, 3, 4 display ac measurements made on the sample. In these figures, the voltage V_r is measured between the edges of the Corbino ring and is represented in terms of two phase components: Re(V_r), the real component, which is in phase with the solenoid current I_m; and Im(V_r), the imaginary component of V_r, in quadrature with I_m. In all measurements, the amplitude of I_m was kept at 4.0 mA, producing a field of $B_m=212$ μT. The resulting induced azimuthal electric field was on the order of $E_\varphi=5$ nV/cm at a frequency of $\nu=1$ Hz.

It is apparent in Fig. 2 that for certain values of B, V_r becomes entirely real (i.e., in-phase with I_m) and exhibits plateaus, which coincide with those of R_h in the dc measurements of Fig. 1. It will be shown below that these

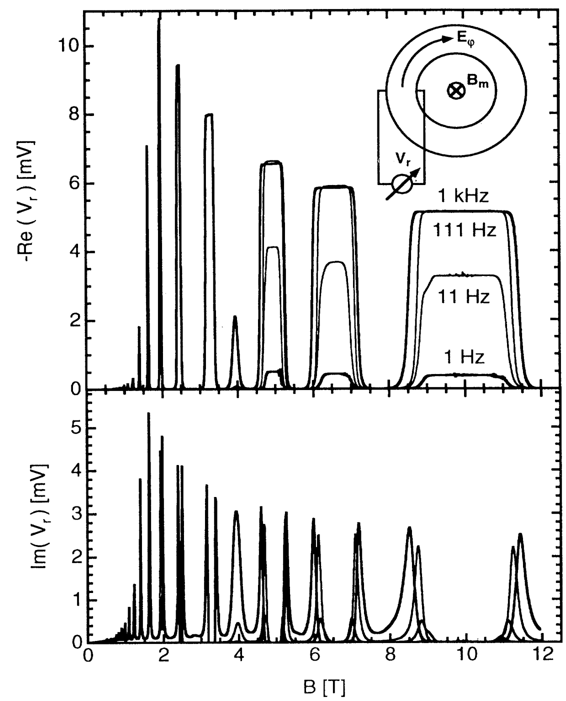

FIG. 2. Real and imaginary components of the voltage V_r as a function of the magnetic induction B for a range of frequencies from 1 Hz to 1 kHz. The measurements configuration is shown in the inset. The modulating field is $B_m=212$ μT. For clarity, only the 1-kHz data have been displayed below $B=4$ T.

plateaus are manifestations of the dissipation-free state of the quantum Hall regime ($R_{xx}=0$). The Corbino ring is, in this case, behaving as a pure inductance and the voltage V_r is proportional to an azimuthal current via the Hall resistance R_h. The amplitude of a plateau may also be thought of as a measure of the quantity of charge crossing the ring due to the modulation in the magnetic

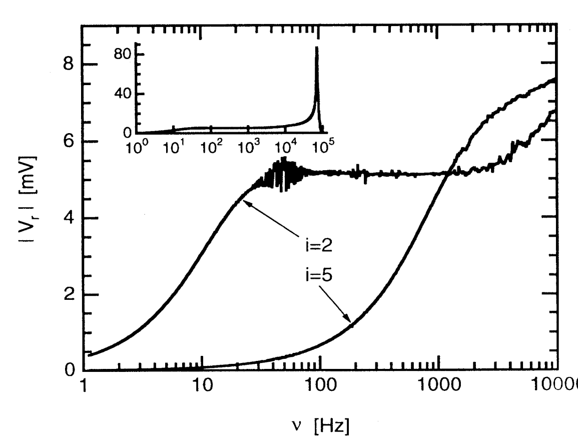

FIG. 3. Frequency dependence of the $i=2$ and $i=5$ plateaus at $T=0.3$ K. The inset shows an expanded scan of the $i=2$ plateau up to 100 kHz.

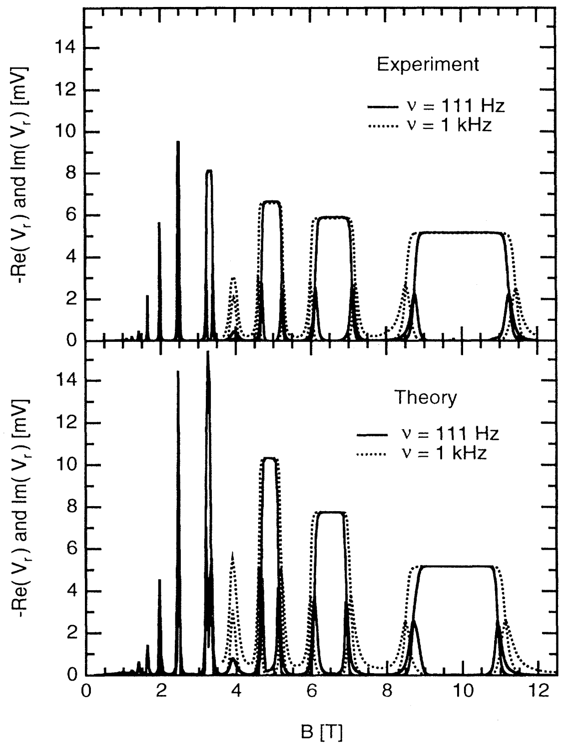

FIG. 4. Comparison between the experimental data and model predictions. The upper part shows the real and imaginary components of the experimentally measured V_r. The lower part shows calculated values of the same quantity. The comparison has been made at two different frequencies: 111 Hz and 1 kHz. For clarity, only the 111-Hz data have been displayed below $B=4$ T.

induction. This charge is measured as a potential difference, which will depend on the capacitance at the sample edges.[4]

According to Laughlin, an integer number of charges cross the channel for every quantum of flux change, $\phi=h/e$.[6] This explains the increasing height of the low index plateaus in Fig. 2. However, as B decreases plateaus become narrower, as do the dc Shubnikov–de Haas oscillations in Fig. 1, and the higher index peaks show sharply decreasing amplitude with decreasing B. We attribute this to breakdown of the QHE at lower critical current values for these plateaus, which leads to significant values of R_{xx} and to a reduction in the amplitude of V_r.

In Fig. 2, Im(V_r) is seen to have peaks that flank the plateaus in Re(V_r). The position of these peaks is frequency dependent, tending to increase separation as the frequency is increased. Also, both Re(V_r) and Im(V_r) clearly show greater detail of the high-order plateaus when the frequency is increased. For example, at $\nu=1$ Hz no signature of plateaus with index larger than $i=20$ has been seen, whereas at $\nu=10$ kHz (not shown in Fig. 2) we have observed peaks up to $i=60$.

A better understanding of these measurements can be gained by considering a model of the Corbino ring-solenoid system. For the Corbino, voltage to current relationships can be written as

$$V_\varphi=R_{xx}I_\varphi+R_hI_r\ , \tag{1}$$

$$V_r=-R_hI_\varphi+R_{xx}I_r\ , \tag{2}$$

where I_r and I_φ are the radial and azimuthal current, V_r and V_φ are the radial and azimuthal voltages. V_φ is related to the total flux threading the annulus Φ by $V_\varphi=-d\Phi/dt$ and the total flux can be written as $\Phi=\Phi_m+LI_\varphi$, where Φ_m is the external modulating flux, LI_φ is the flux due to the diamagnetic currents in the 2DEG, and L is the self-inductance of the Corbino ring. The external flux Φ_m linking the Corbino is related to I_m by the mutual inductance M: $\Phi_m=MI_m$. The capacitance, C, of the ring edges and the attached voltage cables plays a crucial role. It is incorporated in the model by writing

$$V_r=ZI_r \tag{3}$$

with $Z=1/j\omega C$, which represents a capacitive element connected between the edges of the ring. To obtain an expression describing the voltage measured across C, we solve expressions (1), (2), (3), and, assuming an $e^{j\omega t}$ time dependence for all the fields, we obtain

$$V_r=\frac{j\omega MR_h}{(R_{xx}+j\omega L)}\frac{I_m}{(1-j\omega CZ_\alpha)}\ , \tag{4}$$

where

$$Z_\alpha=\frac{R_{xx}^2+R_h^2+j\omega LR_{xx}}{R_{xx}+j\omega L}\ . \tag{5}$$

In the limit $R_{xx}\ll\omega L$, which occurs in the region of the QHE resistance plateaus, expression (4) can be simplified to

$$V_r=-\frac{M}{C}\frac{1}{R_h}I_m\ . \tag{6}$$

This result shows that V_r will be in antiphase and directly proprotional to I_m. Given that $1/R_h=ie^2/h$, where i is an integer, the incremental change in plateau height, as i increases, should be constant. Also, the independence of V_r on frequency is worth noting. This perfectly agrees with Laughlin's description in which the quantity of charge transferred across the channel depends on the change in flux $\Delta\Phi$, not on the rate of change of flux, $d\Phi/dt$.[6]

As R_{xx} increases, the magnitude of V_r decreases and at the same time the dissipation introduces a component in quadrature with I_m. It is evident from Eqs. (4) and (5) that a change over from purely real to mixed real-imaginary behavior in V_r will occur when $R_{xx}\approx\omega L$. This explains the increasing separation of the peaks in Im(V_r) that flank the plateau in Re(V_r). It also explains the frequency dependence of the $i=5$ peak in Fig. 2.

This peak is barely visible at 1 Hz, but clearly emerges at 1 kHz, the effect being more pronounced in Im(V_r). In this case, R_{xx} is comparable to ωL at these frequencies. Similar changes in plateau width have also been observed when V_r was measured as a function of temperature at fixed frequency. The plateaus became narrower as the temperature increased, although their amplitude remained constant. In this case, the change in plateau width can be ascribed to increasing R_{xx} with temperature.

The frequency dependence of the $i=2$ and $i=5$ plateaus is presented in Fig. 3. For $i=2$, the most striking feature is the frequency independence of V_r. In perfect agreement with the prediction of Eq. (6), this behavior is observed over almost two decades of frequency, namely, between 50 Hz and 2 kHz. The frequency dependence occurring above 2 kHz is due to the first resonance of the solenoid coil at 70 kHz, as shown in the inset of Fig. 3. The attenuation of the signal below 50 Hz is due to the finite input impedance R_L of the lock-in amplifier used. This can be seen by considering that the voltage measured at the Corbino edges is shunted by the capacitance C, by R_L and by the Corbino source-drain impedance $Z_c=(2\pi\sigma_{xx})^{-1}\ln(r_2/r_1)$.[8] The last term may be neglected on plateau $i=2$, where we have measured $Z_c > 10^{14}$ Ω at $T=0.3$ K. At low enough frequencies, when $R_L < 1/\omega C$, the input impedance of the lock-in modifies the response of the circuit and the measured voltage decreases. With $R_L = 100$ MΩ and $C=0.25$ nF, deviations are expected at around 40 Hz, in very good agreement with the behavior depicted in Fig. 3. For the $i=5$ plateau the situation is rather different. The condition $R_{xx} \ll \omega L$ is not satisfied over the whole range of measuring frequencies and as a result no saturation of the measured voltage is observed.

The change over from a frequency-independent regime to one in which the amplitude of the signal becomes proportional to the frequency can be readily understood by considering the effect that a nonzero value of R_{xx} will have on the azimuthal current flowing around the ring. In this case, there will be a component of the azimuthal current that depends on the induced azimuthal electric field, and hence on frequency through $d\Phi/dt$. The change over will occur when $\omega \approx R_{xx}/L$, which, at 100 Hz and with $L=15$ nH, probes resistance values on the order of $\mu\Omega$.

The high sensitivity of this magnetic coupling method, to changes in R_{xx}, makes it an ideal probe into the early stages of breakdown of the 2DEG. The method allows I_φ to be varied by changing the amplitude of I_m, and to investigate small resistance changes by frequency-dependent measurements. Furthermore, the ring geometry requires no contacts for the injection of current in the 2DEG, which eliminates the strong dissipative regions that always occur at the diagonal corners of a Hall bar sample, due to squeezing of equipotentials near the metal-2DEG interface.

Figure 4 shows a comparison, at two different frequencies, between the theoretical expressions (4) and (5) and the ac measurements of Fig. 2. The values for R_{xx} and R_h used to calculate V_r were taken from the dc measurements of Fig. 1 and the ratio M/C has been adjusted to reproduce the value measured for V_r on the $i=2$ plateau. With only one adjustable parameter, the agreement between the model predictions and the experimental data is quite remarkable. The plateaus in Re(V_r) and the associated peaks in Im(V_r) are very well described: The increasing plateau height with plateau index is seen at high field, as is the change over to decreasing peak height below $B=2$ T. The frequency dependence of Re(V_r) and Im(V_r) is also well reproduced, including the striking behavior of the $i=5$ peak. It is apparent in Fig. 4 that broadening of the plateau occurs with increasing frequency and that the peak height, of those regions that are not fully developed as plateaus, increases.

There is, however, a clear discrepancy between the experimental data and the model predictions for the magnitude of Re(V_r) on the plateaus in the high-field region. In agreement with our analysis, the experimental data do display a constant incremental change of amplitude as the plateau index increases. However, this increment should, if our model is complete, be equal to one-half of the amplitude of the $i=2$ plateau. Clearly, the experimental data show a value that is significantly less than this. The origin of this constant offset is not yet understood and deserves further study.

IV. CONCLUSIONS

In conclusion, the observation of the QHE by magnetic coupling to a Corbino ring has been reported. A phenomenological model has been developed to describe the ac response of the Corbino ring and measurement system, and a clear signature of the dissipation-free regime of the QHE has been observed in the form of voltage plateaus. These plateaus result from charge movement across the ring giving evidence that the QHE can exist as a consequence of bulk charge transport. Moreover, it has been shown that the self-inductance of the Corbino disk plays a fundamental role in interpreting the frequency response of the system. The magnetic coupling method, because it is highly sensitive to small changes in the longitudinal resistance of the 2DEG, is potentially a powerful tool for the study of the onset of breakdown phenomena in the quantum Hall regime.

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[1] B. W. Alphenaar, P. L. McEuen, R. G. Wheeler, and R. N. Sacks, Phys. Rev. Lett. **64**, 677 (1990).
[2] G. Müller, D. Weiss, S. Koch, K. von Klitzing, H. Nickel, W. Schlapp, and R. Lösch, Phys. Rev. B **42**, 7633 (1990).
[3] M. Büttiker, Phys. Rev. B **38**, 9375 (1988).
[4] B. I. Halperin, Phys. Rev. B **25**, 2185 (1982).
[5] H. Z. Zheng, D. C. Tsui, and A. M. Chang, Phys. Rev. B **32**, 5506 (1985).
[6] R. B. Laughlin, Phys. Rev. B **23**, 5632 (1981).
[7] D. A. Syphers, K. P. Martin, and R. J. Higgins, Appl. Phys. Lett. **48**, 293 (1986).
[8] P. F. Fontein, J. M. Lagemaat, J. Wolter, and J. P. André, Semicond. Sci. Technol. **3**, 915 (1988).
[9] V. T. Dolgopolov, A. A. Shashkin, N. B. Zhitenev, S. I. Dorozhkin, and K. von Klizing, Phys. Rev. B **46**, 12 560 (1992).
[10] D. J. Thouless, Phys. Rev. Lett. **71**, 1879 (1993).
[11] D. Jucknischke, H.-J. Bühlmann, R. Houdré, M. Ilegems, M. A. Py, B. Jeckelmann, and W. Schwitz, IEEE Trans. Instrum. Meas. **IM-40**, 228 (1991).
[12] H. Wolf, G. Hein, L. Bliek, G. Weimann, and W. Schlapp, Semicond. Sci. Technol. **5**, 1046 (1990).