Phase diagrams of the diluted and random-bond Ising model

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The quenched-bond disordered Ising model is studied by means of the mean-field renormalization-group approach. Besides dilution and ferromagnetic random bond, the probability distribution function also includes two extra competing random bonds which can be ferromagnetic or antiferromagnetic, depending on the sign of some competing parameters. The critical temperature against ferromagnetic bond concentration phase diagrams are analyzed for several values of the theoretical parameters.

The mean-field renormalized-group method (MFRG) was proposed by Indekeu, Maritan, and Stella[1] in order to calculate critical properties of lattice spin systems. This method was applied by Droz, Maritan, and Stella[2] to the study of dilute, random fields and the symmetric random-bond Ising model. The asymmetric random-bond Ising model has been treated by Lyra and Coutinho[3] using a phenomenological competition parameter α. For $\alpha=1$ they obtained the symmetric phase diagram and asymmetric ones with reentrants in the ferromagnetic or antiferromagnetic boundaries depending on the α value. More recently, Rosales Rivera, Pérez Alcázar, and Plascak[4] have studied a diluted and random-bond Ising model with a probability distribution for the coupling constant between the pair of nearest-neighbor spins which includes the concentration p, x, and q for the ferromagnetic, antiferromagnetic, and diluted bonds, respectively. For $q=0$, the results are identical to those obtained by Lyra and Coutinho[3] in the Ising limit and for $x=0$ the equation for the diluted model studied by Droz, Maritan, and Stella[2] are recovered. These studies were applied to the magnetic properties of the Fe-Mn-Al disordered alloys considering only one type of ferromagnetic or antiferromagnetic bond.

In this paper we study the phase diagrams of a diluted and random-bond Ising model by extending that proposed by Rosales Rivera, Pérez Alcázar, and Plascak[4] to that case in which all the possible bonds in a ternary magnetic system with two magnetic atoms and one diluted atom are considered.

The present model is defined by the Hamiltonian

$$H=-\sum_{\langle i,j\rangle} K_{ij}\sigma_i\sigma_j \ , \qquad (1)$$

where K_{ij} is the reduced random bond between nearest-neighbor spins $\langle i,j\rangle$ and $\sigma_i=\pm 1$. The probability distribution for each bond K_{ij} is given by

$$P(K_{ij})=p\delta(K_{ij}-K)+x\delta(K_{ij}+\alpha K)+y\delta(K_{ij}+\gamma K)+q\delta(K_{ij}) \ , \qquad (2)$$

where p is the concentration of ferromagnetic bonds ($K>0$), x and y are, respectively, the concentration of bonds αK and γK (α and γ being the competition parameters), q is the concentration of diluted bonds, and $p+x+y+q=1$.

For α and/or γ positive the expected phase diagram of the above model presents paramagnetic (P), ferromagnetic (F), antiferromagnetic (AF), and spin-glass (SG) phases. The mean magnetization $[\langle\sigma\rangle]$ and the staggered magnetization $[\langle\sigma\rangle]_s$ are the ordered parameters for the ferromagnetic and antiferromagnetic phases, respectively, and associated with the spin-glass quenched disorder phase we have the Edwards-Anderson order parameter[5] defined as $[\langle\sigma\rangle^2]$, where the angular brackets stand for the thermal average and the square brackets for the configurational average. In order to get the phase diagram we employ here the MFRG procedure[1] by using the simplest choice for the clusters namely, the one- and two-spin clusters. The critical lines are then obtained by evaluating, for each cluster, these order parameters (assuming appropriated boundary conditions for the corresponding phase) and renormalizing them according to the MFRG method. This model, in some limiting cases (which will be shortly discussed below), has been previously treated by means of the MFRG approach[2,4] and it has been shown that, with one- and two-spin clusters, the results are equivalent to the Bethe-Peierls approximation.

By following the same procedure outlined in Refs. 3 and 4 we obtain for the present model

$$z=(z-1)[p(1\pm\tanh K_c)+x(1\mp\tanh\alpha K_c)+y(1\mp\tanh\gamma K_c)+q] \ , \qquad (3)$$

which gives the ferromagnetic (upper sign) and the antiferromagnetic (lower sign) phase boundaries, and

$$z=(z-1)[p(1+\tanh^2 K_c)+x(1+\tanh^2\alpha K_c)+y(1+\tanh^2\gamma K_c)+q] \ , \qquad (4)$$

for the spin-glass boundary, with z being the coordination number of the lattice and K_c the reduced critical coupling. The above equations are a simple generalization of Eqs. (11) and (12) of Ref. 4 for the probability distribution given by (2).

It can be easily shown that Eqs. (3) and (4) recover the results already obtained in previous MFRG studies (with the same cluster size) in the following special cases: (i) Bond diluted Ising model[2] ($x=y=0$), where the reduced critical temperature $T_c(p)=K_c(p)^{-1}$ goes to zero at the critical ferromagnetic bond concentration $p_c=1/(z-1)$; (ii) symmetric random-bond Ising model[3] (for example, $q=0$, and $\alpha=\gamma=1$), where the phase diagram in the $T_c(p)\times p$ plane is symmetric and the spin-glass boundary is independent of p; (iii) asymmetric random-bond Ising model[3] (for example, $q=y=0$ and $\alpha\neq 1$). Here, depending on the values of α, reentrants appear in the ferromagnetic or antiferromagnetic phase boundaries; and (iv) diluted and random-bond Ising model[4] ($y=0$), where the main effect of dilution is to reduce the antiferromagnetic and spin-glass critical temperatures. In this particular case the model has also been proposed to simulate some magnetic properties of the Fe-Mn-Al alloys in the disordered phase.[4] The more general definition of the probability distribution given by Eq. (2) could then be seen as an attempt to include (besides the dilution) three different exchange interactions that appear, for instance, in ternary alloys composed of ions A, B, and C, where C is nonmagnetic and the couplings come from the interactions of A-A, A-B, and B-B ions.

Before treating the more general model let us first analyze some limiting cases of the probability distribution (2) (in particular, for $y=0$). Figure 1 shows the phase diagram in the $[T_c(p)/T_c(1)]\times p$ plane with $T_c(p)=K_c(p)^{-1}$ and $T_c(1)=2/\ln[z/(z-2)]$, for $z=8$, $\alpha=\gamma=1$, $y=0.0$, and different values of q (this is the symmetric diluted and random-bond Ising model). In these diagrams the maximum concentration of ferromagnetic bonds, p_m, is given by $p_m=1-q(x=0)$ and they are symmetric with respect to $p_m/2$. The spin-glass transition temperature is independent of the ferromagnetic bond concentrations for a fixed q and decreases as q increases. For $q\geq q_c=(z-2)/(z-1)$ all ordered phases disappear. It should be mentioned that wrong results regarding the phase diagram and this concentration has been reported in Ref. 4. The correct q_c is the one listed above which is, moreover, independent of α and γ. The width of the SG phase at $T=0$ is independent of the value of q, as well as of α and γ. When $\alpha\neq 1$ (keeping $y=0$) the corresponding phase diagrams are asymmetric with reentrants appearing in some phase boundaries (for further details, see Refs. 3 and 4).

Let us now discuss the global phase diagrams for $y\neq 0$. In what follows we will consider $y=0.1$ and $z=8$. Figure 2 shows the phase diagrams for some selected values of q in the case $\alpha=\gamma=1.5$. In this case reentrants appear in the AF boundary. Due to the extra concentration of antiferromagnetic bonds (given by $y=0.1$) the phase diagram is now nonsymmetric with the area of the AF region being greater than the corresponding area of the F one. Apart from the asymmetry, for $0<q<q^*$ (not shown in Fig. 2) the phase diagram is similar to those depicted in Fig. 1 where a transition line from F to P phases is present. As q is increased from 0 this transition line shrinks and at $q=q^*=0.44$ it disappears once the ferromagnetic and spin-glass critical temperatures are equal at $p=p_m^*=1-q^*-y=0.46(x=0)$. For $q^*<q<q'$, such as $q=0.600$ for which $p_m=0.3$, a small region of F phase still persists and at $q=q'=0.657$ for which $p_m'=0.243$, the system just can no longer be ferromagnetically ordered. For $q'<q<q_c$ only the AF and SG or-

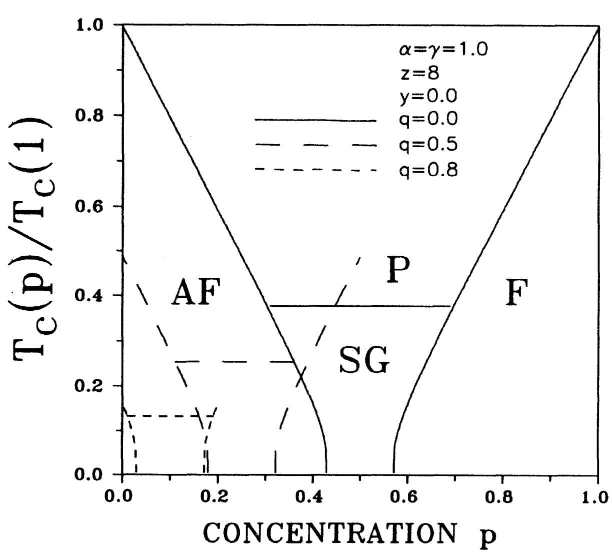

FIG. 1. Phase diagram in the $T_C(p)/T_C(1)\times p$ plane for different values of q. The phases are denoted by (F) ferromagnetic; (AF) antiferromagnetic; (SG) spin-glass, and (P) paramagnetic, and they refer to the $q=0$ phase diagram. The same pattern, however, applies to the other values of q.

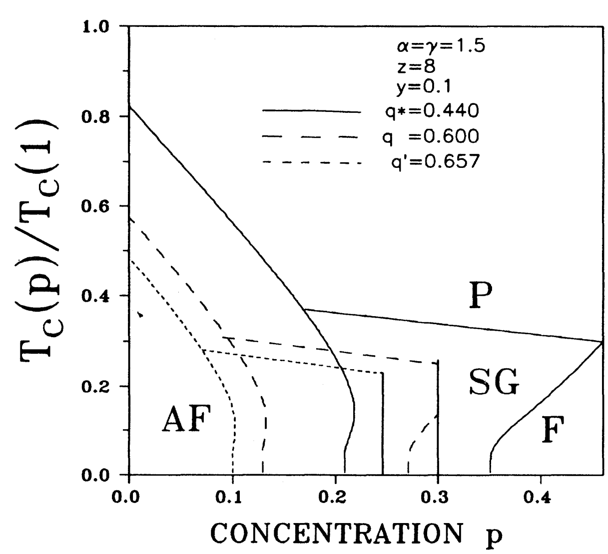

FIG. 2. The same as Fig. 1. In this case, for $q=0.6$ a small region of F phase at low temperatures is present and for $q=q'$ just the F ordered phase disappears and only AF and SG ordered phases are present. For q^*, q, and q' the phase diagrams go up to $p_m^*=0.46$, $p_m=0.30$, and $p_m'=0.243$, respectively.

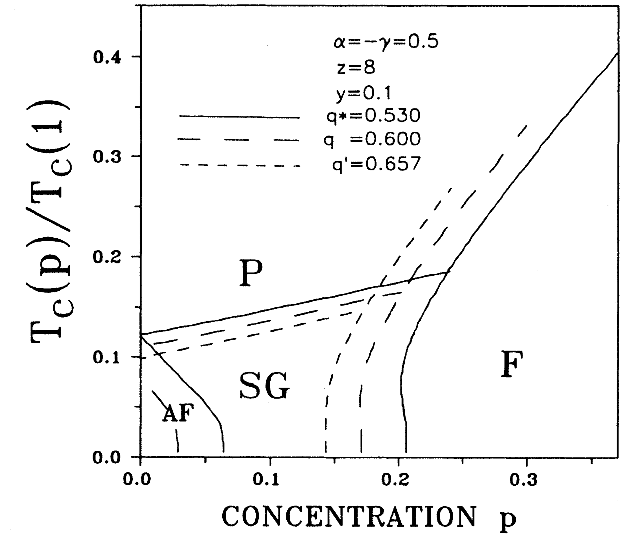

FIG. 3. The same as Fig. 1. In contrast to Fig. 2, in this case, for $q=0.6$ a small region of AF phase at low temperatures is present and for $q=q'$ just F and SG ordered phases are present and the AF ordered phase disappears.

dered phase are present and finally, for $q>q_c$ the system is completely disordered. An identical behavior is achieved concerning the AF phase by taking negative values of γ, as depicted in Fig. 3 for $\alpha=-\gamma=0.5$ once, in this case, we have an extra concentration of ferromagnetic bonds. Similar results are obtained for $\alpha\neq|\gamma|$.

We can see from Figs. 2 and 3 that different slopes for the SG transition line are obtained by changing the value of α. Indeed, it can be shown from Eq. (4) that this slope is negative for $\alpha>1$, null for $\alpha=1$ and positive for $\alpha<1$, irrespective to the value of γ. This behavior is better seen in Figs. 4 and 5 for q and γ fixed and varying α. For those values of the parameters the antiferromagnetic crit-

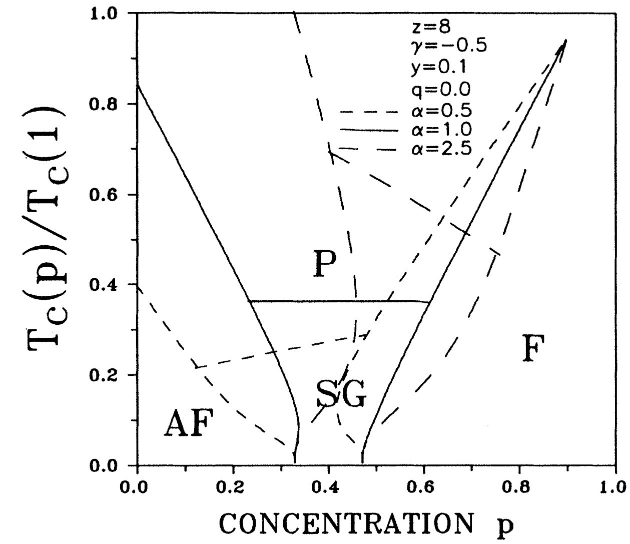

FIG. 5. The same as Fig. 1 for $\gamma=-0.5$, $q=0$, $y=0.1$ and different values of α.

ical temperature at $p=0$ is an increasing function of α (as a consequence of the enhancing of antiferromagnetic bonds) while the ferromagnetic critical temperature at $p=p_m$, as well as the SG phase at $T=0$, is independent of α. We also note that, in both figures, depending on α, a reentrant can appear in the ferromagnetic or in the antiferromagnetic boundary. The presence of such reentrants can be determined from Eq. (3) by locating the temperature T_R at which $(dp/dT)_{T=T_R}=0$. The solid line in Fig. 6 shows $T_R(p)/T_C(1)$ as a function of α for the parameters listed in Fig. 4. In this case, for $\alpha<1$ the ferromagnetic line is reentrant, for $\alpha=1$ it is not reentrant, and for $\alpha>1$ the antiferromagnetic line is reentrant with T_R being a continuous increasing function of α. On

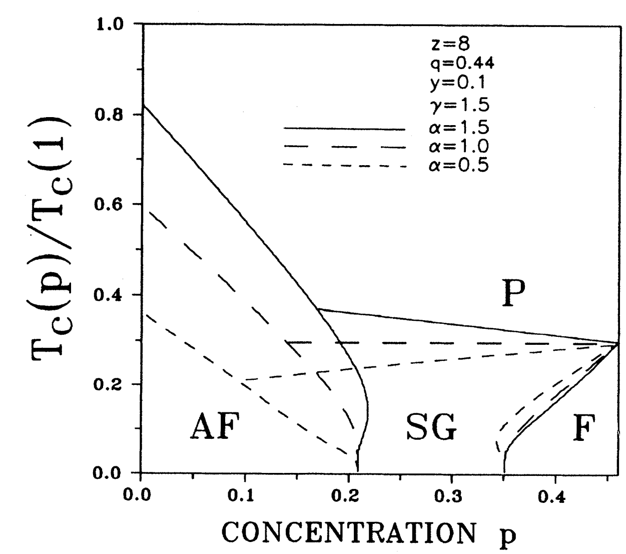

FIG. 4. The same as Fig. 1 for q and γ fixed and different values of α.

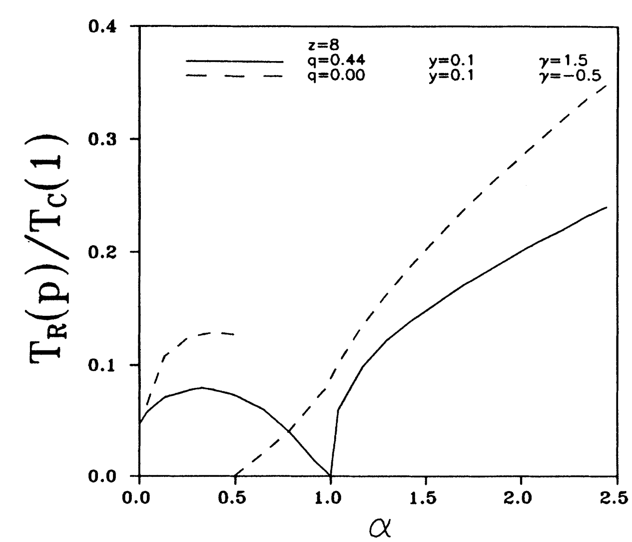

FIG. 6. $T_R(p)/T_C(1)$ as a function of α for the parameters of Fig. 4 (solid line) and Fig. 5 (dashed line).

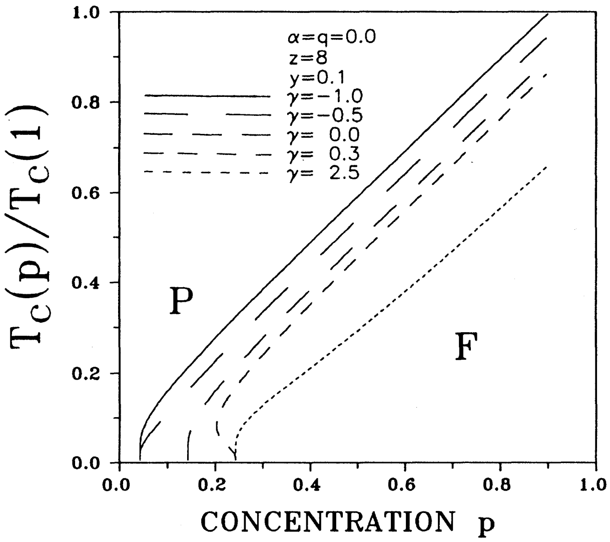

FIG. 7. The same as Fig. 1 for $\alpha=q=0$, $y=0.1$ and different values of γ.

the other hand, for the parameters of Fig. 5, the ferromagnetic line is reentrant for $\alpha<0.5$ with T_R increasing up to a finite value for $\alpha=0.5$; then for $\alpha>0.5$ the antiferromagnetic line is reentrant beginning at $T_R=0$ and then increasing continuously with α. In this way, T_R as a function of α shows a sudden discontinuity at $\alpha=0.5$ for which the reentrant passes from the F to AF boundary (see the dashed line in Fig. 6).

Finally, in Fig. 7 we have phase diagrams for $\alpha=q=0$, $y=0.1$ and different values of γ, where now x plays the rule of dilution. In this case, irrespective to the value of γ, we only obtain ferromagnetic and paramagnetic phases once the concentration of antiferromagnetic bonds (for $\gamma>0$) is not sufficient to stabilize the antiferromagnetic phase nor to produce the necessary competitive bonds for stabilizing the spin-glass phase. For $\gamma=0$ we recover the simple bond-diluted Ising model with the known Bethe-Peierls critical concentration $p_c=1/(z-1)$ ($p_c=0.143$). For $\gamma<0$ we also have a bond-diluted model but with two different ferromagnetic couplings. For this reason, the critical concentration is given by p_c-y ($p_c=0.043$) which is independent of γ. For $\gamma>0$ we have the diluted and random-bond model where the critical concentration is now given by p_c+y ($p_c=0.243$). So, a kind of triple crossover is observed in the ground state when γ changes from negative, to zero, and to positive values. Besides, reentrants are observed in the ferromagnetic boundary just for values of γ in the range $0<\gamma<1$. A typical one is shown in Fig. 7 for $\gamma=0.3$.

In summary, the diluted and random-bond Ising model given by Eqs. (1) and (2) presents a variety of phase diagrams depending on the values of the theoretical parameters defined in Eq. (2) for the probability distribution. Although in the present work we have restricted the analysis to the special $y=0.1$ case, similar phase diagrams can also be drawn by considering other values of y. Applying this model to the magnetic properties of Fe-Mn-Al disordered alloys gives similar results to that reported by Rosales, Pérez Alcázar, and Plascak.[4] Recently this model was applied by Bohórquez, Zamora, and Pérez Alcázar[6] in the calculation of the magnetic phase diagram for the $(Fe_{0.65}Ni_{0.35})_{1-x}Mn_x$ system and their reported phase diagram is in good agreement with previous experimental one reported by Hesse.[7] For this case the distribution has six types of bonds and these bonds are related with the atom concentrations.

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[1] J. O. Indekeu, A. Maritan, and A. L. Stella, J. Phys. A **15**, L291 (1982).

[2] M. Droz, A. Maritan, and A. L. Stella, Phys. Lett. **92A**, 287 (1982).

[3] M. L. Lyra and S. Coutinho, Physica A **155**, 232 (1989).

[4] A. Rosales Rivera, G. A. Pérez Alcázar, and J. A. Plascak, Phys. Rev. B **41**, 4774 (1991).

[5] S. F. Edwards and P. W. Anderson, J. Phys. A **20**, 5667 (1975).

[6] A. Bohórquez, Ligia E. Zamora, and G. A. Pérez Alcázar, Phys. Rev. B **49**, 16035 (1994).

[7] J. Hesse, Hyperfine Interact. **47**, 357 (1989).