Manifestation of spin degrees of freedom in the double fractional quantum Hall system

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The double fractional quantum Hall system of spin-$\frac{1}{2}$ electrons is studied numerically. We predict that the inclusion of spin degrees of freedom gives rise to a spin-unpolarized quantum liquid with the simplest example being at $\nu=\frac{4}{7}$. The state, which emerges from an intimate link between the spin state and the interlayer electron correlation, and also survives the Zeeman effect up to a critical g factor, illustrates that the fractional quantum Hall liquid is versatile enough to accommodate both the doubled fermion species and spin. Even when the ground state is spin polarized as in $\nu=1$, the lowest charge-excitation mode can involve the spin when the interlayer tunneling is considered.

Recently, much attention is focused on the double fractional quantum Hall (FQH) system, in which two layers interact with each other as realized in double quantum wells[1] or in wide single quantum wells.[2] Specifically, Einstein *et al.* have observed a FQH state at a total Landau-level filling of $\nu=\frac{1}{2}$ in a structure in which the interlayer tunneling is prohibited due to a barrier separating the two layers and yet the two layers are coupled via Coulomb interactions.[3] A usual practice in considering a double FQH system is to introduce a pseudospin describing the layer degrees of freedom, while the real spins are neglected under the assumption that they are fully polarized. Then the spin-polarized double FQH system mimics the single FQH system of spin-$\frac{1}{2}$ electrons, apart from the differences in the symmetry [U(1) vs SU(2)] and in the controllability of the "Zeeman energy."

However, already in single-layer FQH systems,[4] the real spin degrees of freedom fundamentally affect the electron correlation via Pauli's principle. Namely, the ground state is spin fully polarized for some odd-denominator filling fractions, while the ground state is spin unpolarized for other fractions as detected in tilted-field experiments. We must then question if the spin should be included in the double-layer problem, and we can indeed envisage that the total spin should crucially affect the intimate link between the intralayer and interlayer electron correlations. Now the real question is as follows: will the inclusion of the spin degrees of freedom give rise to a new quantum liquid state?

Motivated by this, we shall show from a numerical study for finite double-layer FQH systems of $\frac{1}{2}$ electrons that the Laughlin's quantum liquid is, rather surprisingly, versatile enough to accommodate both the double-layer degrees of freedom and spin degrees of freedom. The resultant spin-unpolarized FQH state specific to the system occurs at $\nu=\frac{4}{7}$, the simplest fraction predicted from the double Greek-Roman wave function, for a certain range of d/l (the layer separation normalized by the magnetic length, $l=\sqrt{c\hbar/eB}$), which survives the Zeeman energy up to a critical g factor. Another motivation of the present paper is to look into the low-lying excitations in the coexistence of real and pseudospins in view of the recently emerged measurements of the excitations in the FQH system from the inelastic light scattering.[5,6] We shall show that, even when the ground state is spin polarized as in $\nu=1$, the lowest charge-excitation mode can involve the spin.

We consider the Hamiltonian, first in the absence of interlayer tunneling, given by

$$H=\frac{1}{2}\sum_{m_1\sim m_4}\sum_{\lambda_1\sim\lambda_4}\sum_{\sigma,\sigma'}\left\langle m_1\lambda_1,m_2\lambda_2\left|\frac{e^2}{\epsilon\sqrt{|\mathbf{r}_1-\mathbf{r}_2|^2+(z_1-z_2)^2}}\right|m_4\lambda_4,m_3\lambda_3\right\rangle c^{\sigma\dagger}_{m_1\lambda_1}c^{\sigma'\dagger}_{m_2\lambda_2}c^{\sigma'}_{m_3\lambda_3}c^{\sigma}_{m_4\lambda_4}\ , \tag{1}$$

where $c^{\sigma\dagger}_{m\lambda}$ is the creation operator for the mth orbit of real spin σ in layer (pseudospin) λ ($=1,2$), $\mathbf{r}$ the in-plane position, and ϵ the dielectric constant.

We have obtained the ground-state wave functions from the exact diagonalization of finite systems in both torus and spherical geometries. Since the total spin, S_{tot}, of the system is conserved, we concentrate on the subspace of $S_z^{\text{tot}}=(N_\uparrow-N_\downarrow)/2=0$. Still, the inclusion of the spin in the double-layer system enormously increases the dimension of the Hamiltonian matrix (to typically 8×10^5 for $\nu=\frac{4}{7}$ with four electrons per layer), which has been diagonalized by the Lanczos method. We have determined S_{tot} and the intralayer and interlayer electron correlations for various values of total ν and d.

To characterize the numerically obtained wave functions we have looked into, in addition to the radial distribution function, the overlap between the exact and trial wave functions: we can extend the Greek-Roman wave

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function[7] proposed to accommodate the spin degrees of freedom to the double-layer system of spin-$\frac{1}{2}$ electrons, which is feasible in the absence of interlayer tunneling with the fixed number of electrons in each layer.[8] The "double Greek-Roman" wave function is given (in the symmetric gauge) for N electrons as

$$\Psi_{lmn} = \hat{A}[\Phi_{lmn}(z)(u\alpha)_1 \cdots (u\alpha)_{N/4}(u\beta)_1 \cdots (u\beta)_{N/4} \times (d\alpha)_1 \cdots (d\alpha)_{N/4}(d\beta)_1 \cdots (d\beta)_{N/4}] , \quad (2)$$

$$\Phi_{lmn}(z) = \Phi^{\rm intra}_{\uparrow\uparrow}\Phi^{\rm intra}_{\uparrow\downarrow}\Phi^{\rm inter}\exp\left[-\sum_{i=1}^{N}|z_i|^2/4l^2\right] , \quad (3)$$

$$\Phi^{\rm intra}_{\uparrow\uparrow} = \prod_{1\le i<j\le(N/4)} (z_i - z_j)^l (Z_i - Z_j)^l (\xi_i - \xi_j)^l (\Xi_i - \Xi_j)^l , \quad (4)$$

$$\Phi^{\rm intra}_{\uparrow\downarrow} = \prod_{1\le i,j\le(N/4)} (z_i - Z_j)^m (\xi_i - \Xi_j)^m , \quad (5)$$

$$\Phi^{\rm inter} = \prod_{1\le i,j\le(N/4)} (z_i - \xi_j)^n (z_i - \Xi_j)^n (Z_i - \xi_j)^n (Z_i - \Xi_j)^n . \quad (6)$$

Here, $z_i = x_i - iy_i$ is the position of an ↑-spin electron in layer 1, Z_i for a ↓ spin in layer 1, ξ_i for an ↑ spin in layer 2 and Ξ_i for a ↓ spin in layer 2. $\hat{A}$ is the antisymmetrization operator, u/d are the spinors for layer 1/2, α/β are the spinor for real spin up/down. The exponents (l,m,n) in the Jastrow factors specify the orbital correlation (minimum relative angular momentum) for intralayer like spins (l), intralayer unlike spins (m), and interlayer electrons (n).

Fermi statistics requires l to be odd. In addition, a wave function must be an eigenstate of $S_{\rm tot}$. Since the total spin of each layer is conserved in the absence of interlayer tunneling, we should impose the usual Fock condition on each layer, which is satisfied only when $m = l$ (spin polarized) or $m = l - 1$ (spin unpolarized) for $S_z^{\rm tot} = 0$. The filling factor is given by $\nu = 4/(l + m + 2n)$, since we have $N_\phi = (l + m + 2n)N/4 - l$ in a spherical system having N_ϕ flux quanta going out of the sphere, which is in turn related to the Landau-level filling via $N_\phi = \nu^{-1}N -$ (integer).

We have previously obtained the result for total $\nu = 1$.[8] The result shows an existence of a spin-polarized/spin-unpolarized transition at a critical distance, $(d/l)_c = 1.43$. A change in the interlayer radial distribution function, which is quantitatively slight but discontinuous, signals the transition. The spin-polarized ground state for $d < d_c$ has a large overlap with Ψ_{111}. This is in fact expected, since we have an obvious limit of $d = 0$ at which both the pseudospin and real spin should be polarized and all the correlations between like/unlike layers and like/unlike spins become equivalent as realized in Ψ_{mmm}.

Now we turn to the case where the role of spin is truly dramatic. The double Greek-Roman trial function predicts the simplest spin-unpolarized state to be Ψ_{321}, which has $\nu = \frac{4}{7}$. This state is thus in sharp contrast with the extensively studied $\nu = \frac{1}{2}$ state, in which case the only eligible function among Ψ_{lmn}'s with a finite interlayer correlation ($n \neq 0$) is spin-polarized Ψ_{331}. For $\nu = \frac{1}{2}$, Yoshioka *et al.* have pointed out, for spinless electrons, a large overlap between the ground state and (the spinless counterpart of) Ψ_{331} around $d/l \simeq 1.5$ from the numerical calculation,[9] followed by an experimental identification of the state by Einstein *et al.*[3] Here, the $\nu = \frac{4}{7}$ state exemplifies a spin unpolarized class in the coexistence of real and pseudo spins.

The numerical result for the overlap between the exact ground state at $\nu = \frac{4}{7}$ and Ψ_{321} calculated in the spherical geometry in Fig. 1(a), preliminarily reported in Ref. 8, has indeed a maximum value, 0.968, around $d/l \simeq 1.0$. Ψ_{321} has the intralayer correlation similar to that of the $\nu = \frac{2}{5}$ single-layer state with $S_{\rm tot} = 0$ proposed by Halperin,[7] but incorporates a significant interlayer correlation as well. Namely, given the fact that three of the intralayer correlation of parallel spins, intralayer correlation of antiparallel spins and interlayer correlation can-

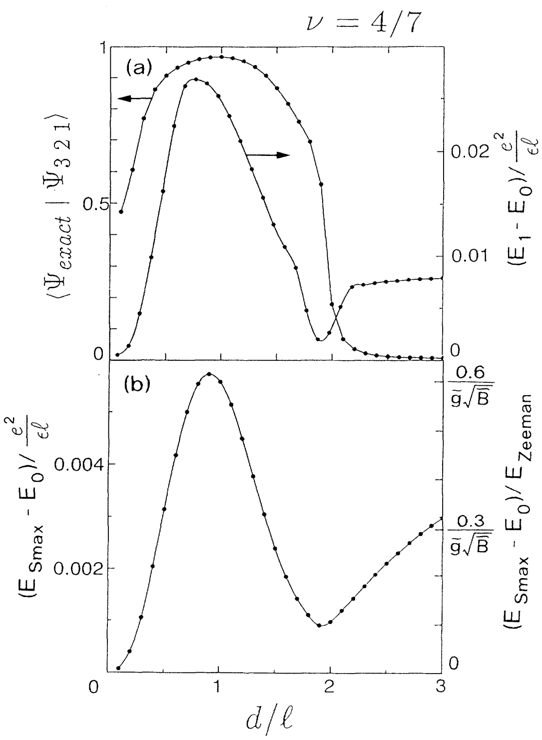

FIG. 1. The overlap between the exact ground state at $\nu = \frac{4}{7}$ and Ψ_{321} against the layer separation, d, for eight (four per layer) electrons in the spherical geometry [(a) left scale]. The energy difference, ΔE, per particle between the ground state (which is spin unpolarized) and the lowest spin-polarized state (b) and the gap between the ground state and the first excited state [(a) right scale] are also plotted. The lines are guides to the eye. The right scale in (b) represents the ratio, $\Delta E/E_{\rm Zeeman}$, in units of $1/\tilde{g}\sqrt{\tilde{B}}$, where $\tilde{g} \equiv g/g_{\rm GaAs}$ and $\tilde{B} \equiv B/10$ T: in the region where the line exceeds unity, the spin-unpolarized ground state survives the Zeeman effect.

not vary independently in a quantum liquid, Ψ_{321} provides a simplest example in which all the three correlations are distinct. The state is realized for a finite range of d, because, as d is increased, the difference between the intralayer and interlayer Coulomb interactions (or the Haldane pseudopotentials in the spherical geometry) increases, thereby giving a chance for the interlayer correlation to deviate from the intralayer correlation, while the system will eventually reduce to independent layers when d becomes too large.

Experimentally, the quantization at $\nu=\frac{4}{7}$ in a double FQH system has not been observed so far. This may be because the layer separation has not been made small enough. Another factor is that a spin-unpolarized state will be unfavored when the Zeeman energy, E_{Zeeman}, is taken into account. Hence it is imperative to check whether the $\nu=\frac{4}{7}$ state can survive the Zeeman effect. We have calculated the energy difference per particle, ΔE, between the lowest of the spin-polarized states and the ground state as a function of d/l. The result in Fig. 1(b) shows that it has a peak of $0.0058e^2/\epsilon l$, which is $\simeq 1.0$ K for GaAs (with $\epsilon=12.6$) in a magnetic field of $B=10$ T and is quite comparable with the Zeeman energy $g\mu_B Bs \simeq 1.5$ K, where g ($=0.44$ for GaAs) is Lande's g factor.[10] The figure may also serve as a phase diagram, if we normalize the vertical axis to regard it as the ratio, $\Delta E/E_{\text{Zeeman}}$ (the right scale in the figure). Since $\Delta E/E_{\text{Zeeman}} \propto (g\sqrt{B})^{-1}$, the peak value of $\Delta E/E_{\text{Zeeman}}$ exceeds unity for smaller B (with smaller density of electrons to retain $\nu=\frac{4}{7}$), or for a smaller g factor possibly realized in high-pressure experiments.[11] Then, in the region where the curve exceeds unity, the spin-unpolarized ground state supersedes the Zeeman energy to realize the $\nu=\frac{4}{7}$ FQH state.

We have also calculated the energy gap between the ground state and the first excited state as a function of d/l. The result in Fig. 1(a) has a peak of $0.027e^2/\epsilon l$, which has a magnitude similar to the gap for the single-layer $\nu=\frac{1}{5}$ state. The overlap, ΔE, and the gap in Fig. 1 are peaked in the same region of d, which confirms the existence of an intrinsic state in this region.

We can further show that, even when the ground state is real-spin polarized, the discussion of charge excitations has to include the spin degrees of freedom when we take the interlayer tunneling into account. The tunneling adds a term, $H_t=-(\Delta_{\text{SAS}}/2)\sum_{m,\sigma}(c_{m1}^{\sigma+}c_{m2}^{\sigma}+\text{H.c.})$, to the Hamiltonian. The single-particle wave functions split into symmetric and antisymmetric (SAS) ones about the center of the system, and the gap, Δ_{SAS}, enters as another energy scale. For spin-$\frac{1}{2}$ electrons in a double-layer system, we have then to consider the excitation mode in which both pseudospin flip and real-spin flip take place simultaneously (which we call SPS mode) in addition to the spin-wave (S) and pseudospin-wave (PS) excitations.

The necessity of considering SPS modes is in fact expected from the effective spin/pseudospin Hamiltonian for the system, which comprises the pair creation/annihilation of the PS mode, the $S\leftrightarrow$PS+SPS and SPS$\leftrightarrow S$+PS processes on top of three free-boson pieces. Because of these processes, the effective Hamiltonian cannot be diagonalized by a Bogoliubov transformation, unlike the spinless case where the PS mode for $\nu=1$ may be well described with the single-mode approximation (SMA).[12,13] The SPS excitation has been discussed by Brey for $\nu=1$ in the Hartree-Fock approximation.[14] However, this problem has to be investigated

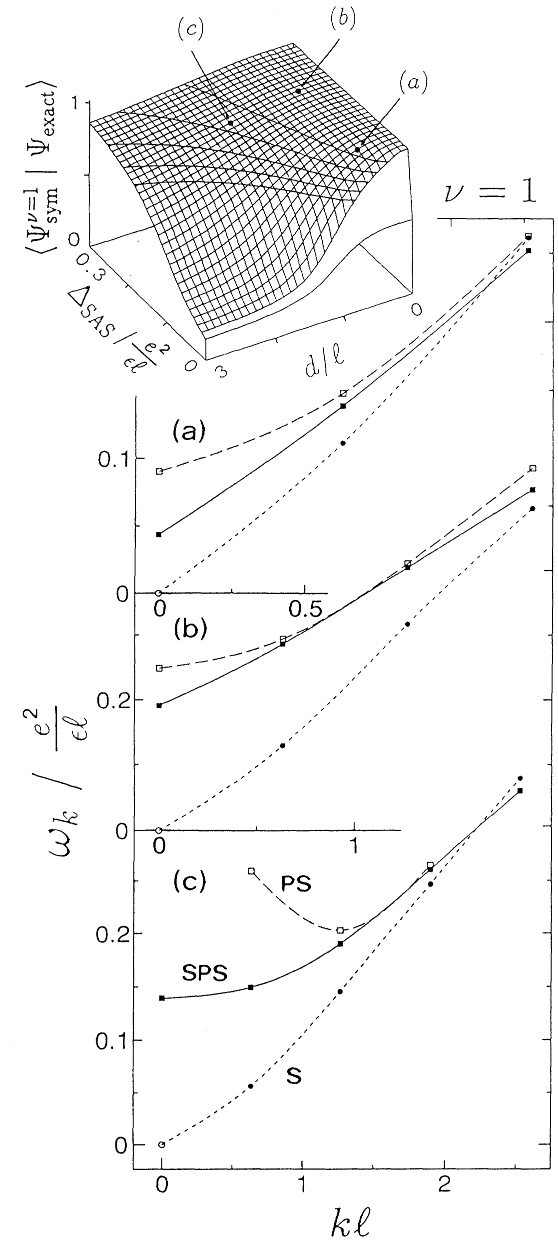

FIG. 2. The excitation modes for the double-layer system with six electrons at $\nu=1$, in the absence of the Zeeman energy, for $[d/l, \Delta_{\text{SAS}}/(e^2/\epsilon l)]=(0.5,0.05)$ (a), (0.5,0.20) (b), and (1.5, 0.20) (c). The modes comprise the spin-wave excitation (S, dotted line), pseudospin-wave excitation (PS, broken line), and pseudospin-wave excitation accompanied by one-spin-flip (SPS, solid line). The lines are guides to the eye. The inset indicates the positions of the three sets of parameters on a plot of the numerically obtained overlap between the exact wave function and the fully-occupied symmetric state against d/l and Δ_{SAS}.

rigorously, since we are dealing with a strongly correlated system, in which the SMA for the PS mode indeed tends to underestimate the energy for larger d and for finite wave numbers according to a numerical finite-size study.[13]

Here, we look into the case of $\nu=1$, because of the recent interest for this situation in the presence of interlayer tunneling. Murphy *et al.* have experimentally investigated the double-layer system to probe the phase diagram,[15] in which the $\nu=1$ "QHE region" exists with the ground state being both real-spin polarized and pseudospin polarized throughout. There the nature of the ground state evolves continuously,[13] as Δ_{SAS} is increased, from the Ψ_{111} state dominated by the interlayer Coulomb interaction [(a) in the inset of Fig. 2] down to the fully-occupied symmetric state, $\Psi_{sym}^{\nu=1}$, dominated by single-particle tunneling [(b) in Fig. 2].

We present in Fig. 2 the numerical result, in the absence of the Zeeman energy, for the low-lying excitations at three typical points in the QHE region, which include the case (c) with a large d/l for which a dip evolves in the pseudospin-wave dispersion, a precursor of an instability of $\Psi_{sym}^{\nu=1}$. When $\Delta_{SAS}\neq 0$, the spin-wave excitation amounts to a gapless Goldstone mode restoring the SU(2) symmetry of the spin if the Zeeman shift is neglected, while both the PS mode and the SPS mode have gaps.

For long wavelengths ($k\sim 0$), the energies of the three modes satisfy an inequality $E_S < E_{SPS} < E_{PS}$ for all the cases (a)–(c), which persists when the Zeeman energy is considered, in agreement with Brey.[14] Namely, the pseudospin excitation expends less energy ($<\Delta_{SAS}$ for $k\sim 0$) when the spin flip is exploited simultaneously. One finding here is that, for small Zeeman energies, the inequality $E_{SPS} < E_{PS}$ persists for *finite* wavelengths with $k\sim l^{-1}$, which applies even when there is a dip in the dispersion associated with the collapse of the QHE gap (i.e., softening of the "roton").[12] The SPS gap for finite wave vectors can thus be a candidate for the thermal gap, although a study of the sample-size effect will be required for its quantitative estimate.

As for the multispin flips, the pseudospin-wave excitations from the spin-polarized ground state have been shown to have multiplet structures of weakly-interacting bosons (pseudomagnons) for small d.[13] Extension of this picture to include the real-spin degrees of freedom serves as another future problem.

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