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# Effect of strong electromagnetic radiation on the screening of the Coulomb potential in a quantized magnetic field

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We study the influence of strong electromagnetic radiation and a quantized magnetic field on the potential of a point charge screened by a nondegenerate electron gas in the ultraquantum limit. We find that the presence of radiation in the frequency region far from the cyclotron resonance leads to the appearance of the screening breakdown terms (in addition to the screening potential), which for quantized magnetic fields retains the same structure as for the classical case. The screening potential for such a system has an oscillatory structure in the direction transverse to the magnetic field and an exponential decay in the longitudinal direction. The frequency regime close to the cyclotron resonance is also considered.

## I. INTRODUCTION

Screening of a Coulomb field by an electron gas in the presence of intense electromagnetic radiation (EMR) and/or a constant magnetic field has been studied by a number of authors.[1–7] The interest in this topic is particularly due to the fact that the potential of the static charge, screened by free carriers and subjected to external fields, manifests an essential modification. For example, the presence of a high-frequency electric field can cause the screening breakdown (SB), whereby the potential of a point charge does not decay exponentially with the distance ($\mathbf{r}$), but, in accordance with a power law, in the same way that the potential of a quadrupole behaves.[1,2] The SB is a universal phenomenon that can be observed (for example, by NMR methods) both in infrared and ultrahigh regimes as long as the frequency of the EMR ($\Omega$) exceeds the characteristic frequency of an electron gas.[2] The magnitude of the SB is determined by the amplitude of the electron oscillation in the field of the EMR ($a$) and in its lowest order this effect is proportional to $a^2$. The above-mentioned modification of the Coulomb field has many different consequences, e.g., it affects Mott transitions[8] and results in the reversal of the anisotropic photoconductivity and the sign change of the odd magnetoresistance in semiconductors, when electrons are scattered by the charged impurities.[9] This effect may also be pertinent to the diffusion of the impurities, with the Coulomb interaction.[4]

On the other hand, it is known that in the absence of EMR a strong quantized magnetic field also modifies the screening of the Coulomb potential[5–7] (note that this does not happen in the classical case). In particular, it was shown that the presence of the quantized magnetic field results in the renormalization and spatial anisotropy of the static shielding law,[5] which explains the anomalous dependence of magnetoresistance in the strong magnetic fields.[6] Let us note that the knowledge of how the Coulomb field is spatially distributed may be also relevant to the problem of the binding of two heavy fermions with a third different particle.[10]

In view of these effects we proceed with the further study of the screening of the Coulomb field by the free carriers in the presence of an intense EMR as well as a constant quantized magnetic field. The statement of the problem adopted here combines the two cases mentioned above, and considers the simultaneous influence of both external factors. However, some results, particularly the spatial distribution of the Coulomb potential in the quantized magnetic field, have not been discussed in the literature and therefore are considered in this paper. The following analysis, including the numerical data and estimates, is based on the solid-state plasma model, but these results may also be applicable for the gas plasma. It should be mentioned that the classical version of the similar effect was studied in Refs. 3 and 4, where dependence of the SB on the direction and the absolute value of the magnetic field was demonstrated.

## II. SCREENED COULOMB POTENTIAL IN THE PRESENCE OF EXTERNAL FIELDS

We shall assume that the energy of the incident photons exceeds the average kinetic energy of the electrons ($\Omega > T; \hbar = 1; k_B = 1$) and is such that only intraband transitions are induced. Then utilizing the standard dipole approximation for the field of the EMR [$\mathbf{F}(t) = \mathbf{F}_0 \sin(\Omega t)$],[2] and considering the unknown Coulomb potential $\varphi(\mathbf{r}, t)$ as a perturbation in the first order, the wave function takes the form

$$\psi^{(1)}_{l,p_x,p_z} = \psi^{(0)}_{l,p_x,p_z} - e \sum_{l',p'_x,p'_z} \int \frac{d\mathbf{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \tilde{\varphi}(\mathbf{k},\omega) \times \langle \psi_{l',p'_x,p'_z} \mid e^{i\mathbf{k}\cdot\mathbf{r}} \mid \psi_{l,p_x,p_z} \rangle_{\mathrm{st}} \times \frac{\exp\left[i\left(\frac{p'^2_z}{2m} - \frac{p^2_z}{2m} + \omega_c(l'-l) + \omega\right)t\right]}{\frac{p'^2_z}{2m} - \frac{p^2_z}{2m} + \omega_c(l'-l) + \omega} \times \psi^{(0)}_{l',p'_x,p'_z}, \tag{1}$$

where

 

$$\tilde{\varphi}(\mathbf{k},\omega) = \int_{-\infty}^{\infty} dt\tilde{\varphi}(\mathbf{k},t)e^{-i\omega t}, \tag{2}$$

$$\tilde{\varphi}(\mathbf{k},t) = \varphi(\mathbf{k},t)\exp\left[ia\left(\frac{\omega_c}{\Omega}k_y\cos\Omega t + k_x\sin\Omega t\right)\right], \tag{3}$$

$a = |\frac{eF}{m(\omega_c^2-\Omega^2)}|$, $\omega_c = eH/(mc)$ is the electron cyclotron frequency, $\varphi(\mathbf{k},t)$ is the Fourier transform of $\varphi(\mathbf{r},t)$, $\langle\psi_{l',p_x',p_z'}|e^{i\mathbf{k}\cdot\mathbf{r}}|\psi_{l,p_x,p_z}\rangle_{st}$ is a matrix element with respect to the stationary Landau states, and $e$ and $m$ are the charge and the effective mass of the carriers. $\psi^{(0)}_{l,p_x,p_z}$ is the exact unperturbed solution of the Schrödinger equation for an electron in the presence of a high-frequency electric field ($\mathbf{F}\|\hat{x}$) and a constant magnetic field ($\mathbf{H}\|\hat{z}$),[11] applicable only for the collisionless electron plasma ($|\omega_c-\Omega|\tau$; $\Omega\tau$; $\omega_c\tau \gg 1$, $\tau$ is the mean free time of the electron between collisions).

Using (1) we find the linear response of the electron density $[\delta\rho(\mathbf{q},t)]$,[12] which, in turn, determines the screened Coulomb potential, given by the Poisson equation:

$$\varphi(\mathbf{k},t) = \frac{4\pi\,\rho(\mathbf{k})}{\varepsilon_0 k^2} + \frac{4\pi\,\delta\rho(\mathbf{k},t)}{\varepsilon_0 k^2}. \tag{4}$$

Here $\rho(\mathbf{k})$ is a Fourier component of the density of the static charge and $\varepsilon_0$ is the dielectric constant. Let us note that this approach is self-consistent. On the one hand the distribution of the electron density is caused by the external potential and on the other hand the screening of the Coulomb field is defined by the electron density (4). The solution of (4) is a set of harmonics with the frequency $\Omega$. However, in this paper only a stationary component of the Coulomb potential will be considered:

$$\varphi(\mathbf{k}) = \frac{4\pi\rho(\mathbf{k})}{\varepsilon_0 k^2}\sum_{s=-\infty}^{\infty}\frac{J_s^2\left(a\sqrt{k_x^2+\frac{\omega_c^2}{\Omega^2}k_y^2}\right)}{1-\frac{4\pi e^2}{\varepsilon_0 k^2}\prod(\mathbf{k},s\Omega)}, \tag{5}$$

where

$$\prod(\mathbf{k},s\Omega) = \frac{m\omega_c}{2\pi^2}\sum_{l,l'=0}^{\infty}Q_{l,l'}^2(\tfrac{1}{2}l_H^2k_\perp^2)\int_{-\infty}^{\infty}dp_z\frac{f(l',p_z+k_z)-f(l,p_z)}{\frac{(p_z+k_z)^2}{2m}-\frac{p_z^2}{2m}+\omega_c(l'-l)+s\Omega} \tag{6}$$

is the polarization function,

$$Q_{l,l'}^2(x) = \frac{l'!}{l!}\exp(-x)\,x^{l-l'}[L_{l'}^{l-l'}(x)]^2 \quad (l>l'),$$
$$Q_{l,l'}(x) = (-1)^{l-l'}Q_{l',l}(x) \quad (l<l'),$$

$J_n(x)$ is the Bessel function of the real argument, $L_n^m(\xi)$ is the Laguerre polynomial, $l_H = (m\omega_c)^{-\frac{1}{2}}$ is the magnetic length, $k_\perp = \sqrt{k_x^2+k_y^2}$, and $f(l,p_z)$ is the distribution function of the electrons on the $n$th Landau level. Let us note that in the absence of the EMR ($a=0$) expression, (5) takes the familiar form given in Refs. 6 and 7. Result (5) is obtained in a quite general form and further calculations of the spatial distribution of the Coulomb potential require some specifications. We shall study the field of the point charge [$\rho(\mathbf{k}) = q$] screened by the nondegenerate electron gas in the ultraquantum limit ($\omega_c/T = \xi \gg 1$). The frequency of the radiation is supposed to be sufficiently high to cause the SB, i.e., $\Omega \gg \omega_{\rm pl}$ ($\omega_{\rm pl}$ is the plasma frequency, which is the characteristic frequency of the electron gas for collisionless electrons). We also confine ourselves to the consideration of the two frequency regimes: far from the cyclotron resonance ($|n_1\Omega+n_2\omega_c| \gg \omega_{\rm pl}, n_1 \neq n_2 \neq 0, n_1$ and $n_2 \in Z$) and close to the cyclotron resonance ($|\omega_c-\Omega| \ll \omega_{\rm pl}$).

### A. Nonresonance frequency regime

With these assumptions the polarization function (6) may be simplified:

$$\prod(\mathbf{k},s\Omega) = -\frac{n}{T}\,\exp(-\tfrac{1}{2}l_H^2k_\perp^2)\,\phi(k_z\lambda_T)\,\delta_{s,0}\,, \tag{7}$$

where

$$\phi(y) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}dx\frac{\exp[-(x+y)^2]-\exp(-x^2)}{x^2-(x+y)^2}, \tag{8}$$

$n$ is the carrier density, and $\lambda_T = 1/\sqrt{2mT}$ is the thermal length. Considering the nonresonance frequency region, we also suppose that the amplitude of the electron oscillation is much less compared to the Debye screening length ($æa \ll 1, æ = \sqrt{\frac{4\pi ne^2}{\varepsilon_0 T}}$ is the reciprocal of the Debye screening length). This condition allows us to neglect the effect of the EMR on the screening of the Coulomb field and to take into account only the influence of radiation (up to $a^2$) on the nonscreened part of the potential, which is independent of æ. Then, substituting (7) in (5) and Fourier transforming the result, we get

$$\varphi(\mathbf{r}) = \varphi_S(\mathbf{r}) + \varphi_{\rm SB}(\mathbf{r}), \tag{9}$$

$$\varphi_S(\mathbf{r}) = \frac{2q}{\pi\varepsilon_0}\int_0^{\infty}dk_\perp k_\perp J_0(k_\perp r_\perp) \times\int_0^{\infty}dk_z\frac{\cos(k_zz)}{k^2+æ^2\exp(-\frac{1}{2}l_H^2k_\perp^2)\phi(k_z\lambda_T)}\,, \tag{10}$$

$$\varphi_{\rm SB}(\mathbf{r}) = \frac{q}{2\varepsilon_0}\frac{a^2}{r^3}\left[1-\frac{3r_\perp^2}{r^2}\cos^2\alpha +\frac{\omega_c^2}{\Omega^2}\left(1-\frac{3r_\perp^2}{r^2}\sin^2\alpha\right)\right]. \tag{11}$$

Here $\alpha$ is the angle between $\mathbf{r}_\perp$ and the $x$ axis. The resulting potential (9) is obtained as a sum of two terms: the screened part of the Coulomb potential $\varphi_S(\mathbf{r})$, which under condition $æa \ll 1$ does not depend on the EMR, and a term characterizing the SB (11). Let us note that the SB $[\varphi_{\rm SB}(\mathbf{r})]$ in the quantized magnetic field preserves the same structure as in the classical one,[3,4] which is one more illustration of the universality of this phenomenon. Expression (10) is different from its classical counterpart in that the magnetic and the thermal lengths occur along with æ and in the limit $æl_H; æ\lambda_T \to 0$, $\varphi_S(\mathbf{r})$ assumes the standard form of the Debye screened potential. The low-wave-number description of (10) (which corresponds to $æl_H; æ\lambda_T < 1$) results in the appearance of quantum corrections, which, in turn, lead to the modification and spatial anisotropy of the static shielding law.[5]

We analyzed (10) numerically for arbitrary values of $æl_H$, including $æl_H \geq 1$ (this also implies $æ\lambda_T \geq 1$, since $\lambda_T/l_H = \sqrt{\xi/2}$, $\xi \gg 1$). In the direction along the magnetic field, the potential retains an exponential decay with the effective screening length $æ_{\rm eff}^{-1}$ (see Fig. 1), which at $æ\lambda_T \geq 1$ turns out to be a linear function of the magnetic length. This circumstance justifies the results of Ref. 13, where the the conductivity parallel to the magnetic field was calculated by employing a cutoff of the Coulomb potential at $r = l_H$. In the direction transverse to the manetic field the potential (10) acquires an oscillatory structure at $æl_H \geq 1$. This kind of oscillation of the Coulomb potential and their impact on the distribution of the second moments were noticed in Ref. 6; however, in that paper the author did not consider the nondegenerate electron gas. In order to illustrate this oscillating behavior of the screened potential we present in Fig. 2 the positions of the first minimum ($r_\perp^{\rm min}$, lower curves) and the maximum ($r_\perp^{\rm max}$, upper curves) of $\varphi_S(r_\perp, z=0)$ as a function of the magnetic length. Figure 3 shows the minimum value of the Coulomb potential $[\tilde{\varphi}_S(r_\perp^{\rm min}) = \frac{\varepsilon_0}{qæ}\varphi_S(r_\perp^{\rm min}, z=0)]$ as a function of $l_H$ and the inset exhibits the exponent for the decaying amplitude:

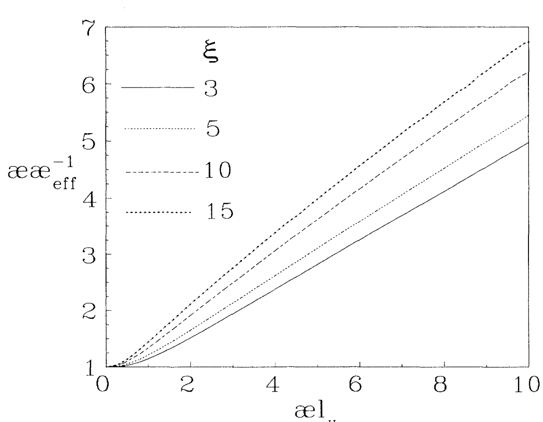


FIG. 1. The effective screening length $æ_{\rm eff}^{-1}$ in the direction along the magnetic field is illustrated as a function of $l_H$ for different $\xi = \omega_c/T$.

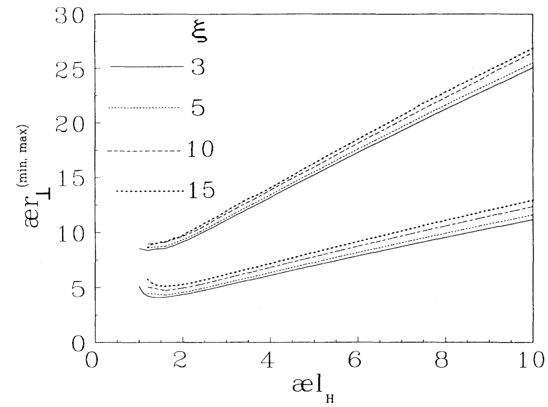


FIG. 2. The location of the potential's extremes is shown as a function of $l_H$ for different $\xi = \omega_c/T$; lower curves correspond to the minimum of the potential, upper curves to its maximum.

$$\beta = \ln\left(-\frac{\varphi_S(r_\perp^{\rm max}, z=0)}{\varphi_S(r_\perp^{\rm min}, z=0)}\right) \Big/ \ln\left(\frac{r_\perp^{\rm max}}{r_\perp^{\rm min}}\right). \qquad (12)$$

The oscillating Coulomb potential is also shown in Fig. 4 (see solid curve, corresponding to the absence of the EMR). These numerical results demonstrate the linear dependence of the location of the extremes on $l_H$, whereas $\tilde{\varphi}_S^{\rm min}$ does not change in an essential way for a wide range of $æl_H \geq 4$. The inset of Fig. 3 shows that for $æl_H \geq 4$ the exponent of the decaying amplitude of the Coulomb field is close to 3, which on the one hand makes the screening potential comparable on the large distances with the SB, and on the other hand points out an analogy with the Friedel oscillations.[12] Numerical estimates indicate that the condition $æl_H \geq 1$ is quite realistic, i.e., in $p$-GaAs, $T = 2$ K, $n = 5.6 \times 10^{15}$ cm$^{-3}$, $\xi = 5$ ($H \simeq 4$ T), the electron gas is nondegenerate ($4\pi^{3/2}n\lambda_T^3/\xi \ll 1$), and $æl_H \sim 3$.

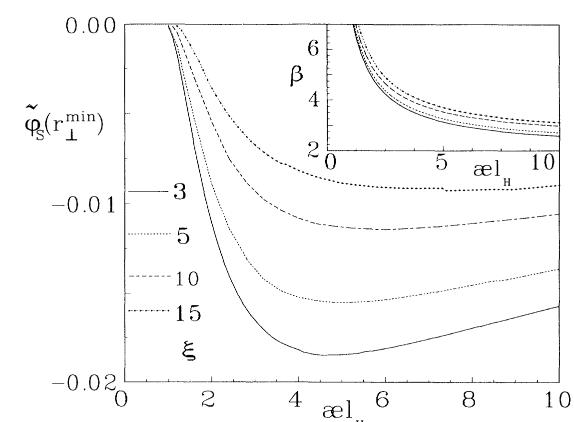


FIG. 3. The minimum value of the Coulomb potential $[\tilde{\varphi}_S(r_\perp^{\rm min})]$ is illustrated as a function of $l_H$ for different $\xi = \omega_c/T$. The inset shows the exponent of the decaying amplitude decaying: $\beta = \ln\left(-\frac{\varphi_S(r_\perp^{\rm max}, z=0)}{\varphi_S(r_\perp^{\rm min}, z=0)}\right) \Big/ \ln\left(\frac{r_\perp^{\rm max}}{r_\perp^{\rm min}}\right)$.

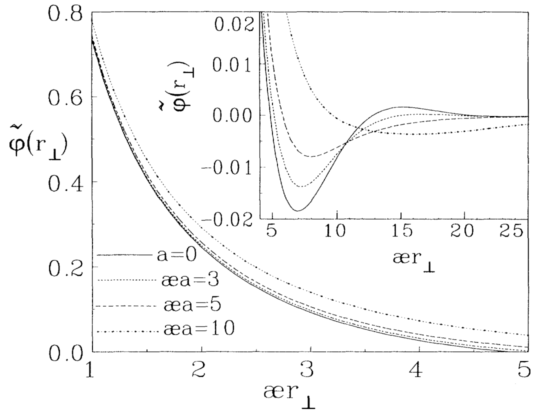


FIG. 4. The spatial distribution of the Coulomb potential in the direction transverse to the magnetic field is shown for different values of $a$. The inset is to demonstrate the oscillating structure of the potential $[\tilde{\varphi}(r_\perp)]$; $\xi = 3$.

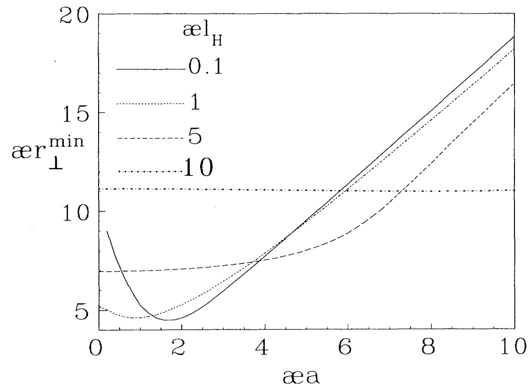


FIG. 5. The location of the potential's minimum ($r_\perp^{\min}$) is shown as a function of $a$ for different values of $l_H$; $\xi = 3$.

### B. Frequencies close to the cyclotron resonance

Close to the cyclotron resonance ($|\omega_c - \Omega| \ll \omega_{\rm pl}$) we still assume the collisionless conditions ($|\omega_c - \Omega|\tau \gg 1$), which allows us to employ the same approach as in the nonresonance case. Then, taking into account only the the resonance terms in the polarization function (6), the Coulomb potential for the arbitrary values of $a$ has the form

$$\varphi(\mathbf{r}) = \frac{2q}{\pi\varepsilon_0}\int_0^\infty dk_\perp k_\perp J_0(k_\perp r_\perp)\int_0^\infty dk_z \cos(k_z z) \times \sum_{n=0}^{\infty}\frac{1}{\gamma_n}\frac{J_n^2(ak_\perp)}{k^2 + \gamma_n æ^2 Q_{n,0}^2(\frac{1}{2}l_H^2 k_\perp^2)\phi(k_z\lambda_T)}. \quad (13)$$

Here $\gamma_n = (1 + \delta_{n,0})/2$. In the limit $æl_H; æ\lambda_T \to 0$, expression (13) can be integrated analytically up to the second order in the amplitude of the electron oscillation ($a^2$), and the result in the quantized magnetic field again coincides with the classical case.[3,4] Consideration of the low-wave-number description in (13) leads to the quantum corrections that renormalize the length scale, but in the final analysis results only in small corrections.

Qualitative changes in the structure of the Coulomb potential appear at arbitrary values of $æl_H$ and $æ\lambda_T$, which requires a numerical analysis of (13). In Fig. 4 we show the modification of the oscillating potential $[\tilde{\varphi}(r_\perp) = \frac{\varepsilon_0}{qæ}\varphi(r_\perp, z = 0)]$ for $æl_H = 5$ and $\xi = 3$ under the influence of the EMR. It illustrates that the presence of the EMR tends to destroy the screening of the Coulomb field at the distance $r < a$, i.e., for $æa = 10, \tilde{\varphi}(5æ^{-1})$ is just a factor of 5 smaller than the nonscreened potential, whereas the Debye screening potential is 30 times less. This result reflects the destruction of the halos of charge on a distance of order $a$ in the vicinity of the static charge. Figure 4 also demonstrates that the oscillating structure of the Coulomb field is suppressed in the presence of the strong EMR, i.e., at $æa \simeq 5$ there is only the minimum of the potential, while the maximum has disappeared (see curves corresponding to $æa = 5; 10$). In Fig. 5 we present the dependence of the location of the potential's minimum as a function of the amplitude of the electron oscillation. It is interesting to note that the position of the minimum is defined by the larger of the two length scales: $a; l_H$, when $a > l_H, æa > 1$ depends linearly on $a$. In the case of $a < l_H$ the EMR does not affect the position of the minimum, but does change the value of the potential $[\tilde{\varphi}(r_\perp^{\min})]$, which is illustrated in Fig. 6. At $æa; æl_H \le 1$ Fig. 6 does not show the substantial value of the potential in its minimum, which on the one hand is due to the absence of the oscillatory behavior (see Fig. 3), and on the other hand reflects the fact that the SB is not essential at $æa < 1$. It is worth mentioning that at $æl_H = 5$ the value of the potential does not depend on $a$ for $æa > 7$ (see Fig. 6), whereas the position of the minimum is a linear function of $a$ (Fig. 5). In the case of $æl_H = 10$ the situation is different: $\tilde{\varphi}(r_\perp^{\min})$ depends on $a$, but the location of the minimum does not.

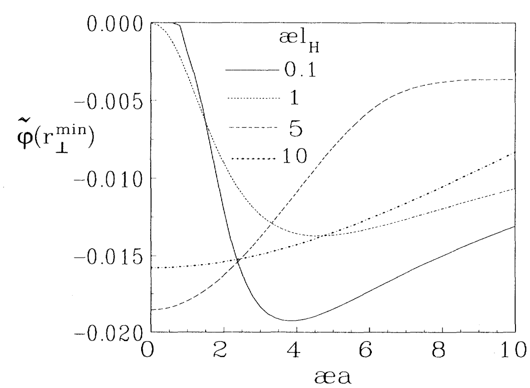


FIG. 6. The dependence of the minimum value of the Coulomb potential $[\tilde{\varphi}(r_\perp^{\min}, z = 0)]$ on the amplitude of the electron oscillation ($a$) is shown for different $l_H$; $\xi = 3$.

## III. CONCLUSION

In this paper we have demonstrated that the structure of the Coulomb potential screened by the nondegenerate electron gas in the ultraquantum limit is defined by the magnetic field orientation, i.e., along the magnetic field the potential decays exponentially and in the direction transverse to the magnetic field the potential oscillates when $\ae l_H \geq 1$. The effect of the strong EMR in the region far off the cyclotron resonance frequency is the appearance of the SB, whereas in the regime close to the cyclotron resonance the oscillating structure of the potential is suppressed at $\ae a \gg 1$. Generally we may conclude that in such a system the screening is determined by the largest of the lengths $(\ae^{-1}, l_H, a)$.

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