Quantum oscillations in the cyclotron phonon emission from a heated two-dimensional electron gas

J. Cooper,* F.F. Ouali, L.J. Challis, and C.J. Mellor
Physics Department, University of Nottingham, Nottingham NG7 2RD, United Kingdom
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Oscillations have been observed in the phonon intensity I_N emitted normal to a heated two-dimensional electron gas bridge in a Si metal-oxide-semiconductor field-effect transistor in a quantizing magnetic field as the sheet density is varied at constant power input. These oscillations ($\sim$0.3% of the total signal at 6 T and filling factor i=12) are attributed to changes in the angular distribution of the emission arising from variations in the small fraction of the total energy emitted from higher Landau levels caused by changes in the screening of the electron-phonon interaction with Fermi-level position.

I. INTRODUCTION

In a quantizing magnetic field, a heated two-dimensional electron gas (2DEG) can lose energy by both intra- and inter-Landau-level transitions. Inter-Landau-level transitions give rise to acoustic-phonon emission at the cyclotron frequency and its harmonics $\omega = j\omega_c$ ($j = 1,2,3,\dots$ and $\omega_c = Be/m^*$) as well as to optic phonons at $\omega_{\rm opt}$ together with lower frequency acoustic phonons. In a well quantized system ($\mu B \gg 1$), intra-Landau-level transitions produce low-frequency phonons with $\omega \sim \Gamma/\hbar \ll \omega_c$, where Γ is the Landau level width. Three regimes may be distinguished depending on whether k_BT_e is much less than, approximately equal to, or much greater than $\hbar\omega_c$ (T_e is the electron temperature).

In the first case $k_BT_e \ll \hbar\omega_c$ (which is equivalent to the quantum Hall regime if the Fermi level E_F is midgap), there will be negligible cyclotron phonon emission ($\omega = \omega_c$) from the 2DEG apart from at the current entry and exit corners, and the effect of changing the filling factor on the energy relaxation has been studied previously.[1–3] When $k_BT_e \sim \hbar\omega_c$, the emission will be dominated by cyclotron phonons resulting from transitions between the n and $n-1$ Landau levels on either side of E_F. There will, however, also be several other processes contributing to some degree: cyclotron phonons from higher and lower Landau level transitions ($n+1$ to n, $n+2$ to $n+1$, $n-1$ to $n-2$, etc.), phonons at cyclotron harmonics ($2\omega_c$, $3\omega_c$, etc.), and optic phonons. We also note the possibility of two-phonon emission[4] at $\omega_c/2$ and emission from intra- and inter-edge-state transitions.[5] In the third case $k_BT_e \gg \hbar\omega_c$, energy relaxation should eventually be dominated by optic-phonon emission. To investigate these effects, measurements were made of the phonon intensity I_N emitted normal to an area of strongly heated 2DEG in a Si metal-oxide-semiconductor field-effect transistor (MOSFET). Section II describes the experimental arrangement and the samples used, Sec. III shows how I_N varies with power density and filling factor, and in Sec. IV mechanisms are suggested to account for the observed behavior, in particular the oscillations seen in I_N as E_F moves through the Landau-level spectrum.

II. EXPERIMENT

A schematic diagram of the experimental arrangement is shown in Fig. 1. The (001) Si MOSFET has a gate of width 1 mm and length 3 mm which was selectively etched in the center to leave 15×180 μm^2 of 2DEG between two larger areas each $\sim 1 \times 1.5$ mm^2. One end of the sample is mounted in a copper holder in good thermal contact with the 1 K heat sink. This geometry allows high power densities and so T_e values to be achieved for modest total power inputs. Before the bridge was etched the MOSFET had a peak mobility at 1 K of 1.2 m^2 V^{-1} s^{-1}. Since the aspect ratios of the whole device and of the bridge are 1:3 and 1:12, respectively, $\sim$80% of the power is dissipated in the bridge at low currents and zero magnetic field; this proportion rises with increasing current due to the increase in T_e in the bridge. The p-

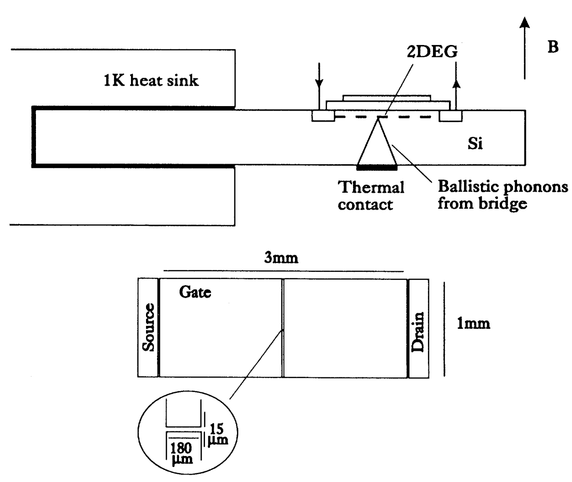

FIG. 1. Experimental arrangement.

type substrate is 0.4 mm thick with a room temperature resistivity of 1000 Ω cm and is transparent to phonons of frequency $\lesssim$1500 GHz, the limit being set by isotope scattering. When the 2DEG is heated, the phonon emission is mainly transverse acoustic[6] and, for the magnetic fields used ($\sim$6 T), the cyclotron phonon emission occurs at $\sim$900 GHz and so travels ballistically across the substrate. The emitted phonons travel mostly at angles close to the normal [001] direction due to the combined effects of in-plane momentum conservation[7] and phonon focusing.

The phonon intensity I_N along [001] is measured from the rise in temperature produced by the ballistic phonons of a contact placed opposite the bridge. The source-drain current is computer controlled to maintain constant power input to the device to $< 5 \times 10^{-4}$ as the sheet density n_s is varied by the gate voltage. Two thermometers T_M and T_R are used to avoid small changes in signal due to temperature drifts of the whole sample. They consist of a matched pair of carbon resistors which are held in cages $\sim$15 cm above the sample to minimize magnetoresistive effects and are thermally connected to the back surface of the substrate through strips of silver foil glued with Stycast or GE varnish. The contact for the reference thermometer T_R is a 3×0.5 mm^2 strip positioned to measure the temperature of the substrate $\sim$2 mm from the center of the device towards the 1 K heat sink. The contact for the main thermometer T_M is a square 0.25×0.25 mm^2 directly opposite the bridge and its temperature rise $\Delta T = T_M - T_R$ has two components: ΔT_{TR} due to the thermal resistance of the sample and a ballistic phonon signal $\Delta T_B \propto I_N$. For constant power input to the device, T_R and ΔT_{TR} are both fixed and any changes in T_M and so ΔT with gate voltage should be due entirely to ΔT_B and attributable to variations in I_N caused by changes in the location or angular distribution of the emission.

III. RESULTS

For a fixed sheet density n_s and a substrate temperature T_R of 1 K, ΔT is found to vary linearly with power as expected (in practice, T_R increases with power to $\sim$ 2.2 K for P=200 μW because of the thermal resistance between the substrate and the helium bath and, to correct for this, we assume ΔT_B and ΔT_{TR} to vary as T_R^{-3} due to the temperature dependence of the specific heat and thermal conductivity of the substrate). Measurements of ΔT_{TR} made using a heater on a similar sample[8] show that $\Delta T_{TR} \simeq 0.1\Delta T$. So the rise in temperature opposite the heated bridge is almost entirely due to ballistic phonons $\Delta T_B \simeq 0.9\Delta T$. This large ballistic signal is about 2.5 times larger than that produced by the same dissipation in a "hot spot" at the current entry or exit point in the quantum Hall regime,[1] indicating that the emission from the bridge is appreciably more tightly confined to directions approximately normal to the 2DEG.

Figure 2 shows the temperature rise ΔT as a function of n_s for powers of 1 μW and 100 μW, corresponding to power densities $\sim$ 0.3 to 30 mW mm^{-2} (data at other powers are given elsewhere[9]). Also shown is the source-drain resistance $R_{\rm SD}$ and at i = 12 (n_s = 1.7×10^{16} m^{-2}), $R_{\rm SD}$ is $\sim$ 4 times the Hall resistance (25 812/12 Ω) suggesting that most of the power is dissipated in the bridge. At 1 μW, ΔT and so I_N are in phase with $R_{\rm SD}$ and have minima when E_F is midgap. This behavior is similar to that seen in 2DEG devices without bridges[1] and, as there, is attributed to the fact that the dissipation in the center of the 2DEG is proportional to ρ_{xx} and so falls when E_F is midgap. Since the power is kept constant, this fall is balanced by an increase in dissipation at the current entry and exit points of the device. Thus ΔT_{TR} remains constant while ΔT_B falls leading to the drop in ΔT observed. However, as the power is increased, ΔT undergoes a phase change, as can be seen in Fig. 2, and now has maxima close to the points where E_F is midgap. At 6 T, the out-of-phase peak starts to appear at $\sim$ 5 μW and eventually becomes the dominant feature.

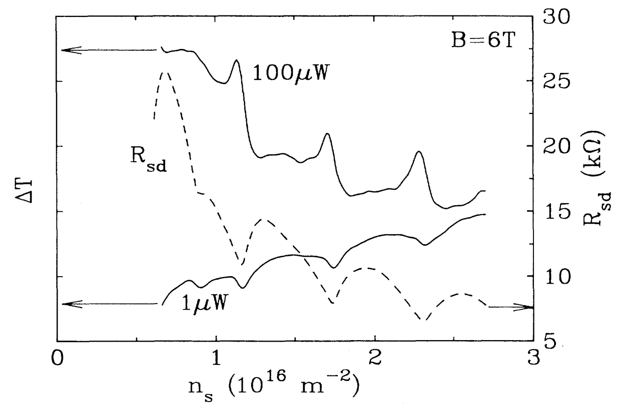

FIG. 2. Variation of ΔT with sheet density for powers of 1 μW and 100 μW. Also shown is the source-drain resistance $R_{\rm SD}$ measured at a power of 1 μW.

The magnitude of the oscillations $\delta(\Delta T)$ (peak to peak) increases approximately linearly with power over the range studied.[9] Since ΔT also increases linearly with power, the relative size of the oscillations $\delta(\Delta T)/\Delta T$ remains approximately constant with power for a particular field and filling factor. This value is $\sim$0.3% for B=6 T and i=12 and decreases with decreasing B.

IV. DISCUSSION

We first consider the possibility that the out-of-phase oscillations at higher powers are also due to the oscillations in ρ_{xx}. These latter oscillations will be appreciably larger in the bulk of the 2DEG than in the bridge, where the electron temperature is higher, so at constant power the dissipation in the bridge will oscillate out of phase with ρ_{xx}. In principle then, this could be the cause of the out-of-phase oscillations in I_N. It is clear, however, that it is not at the higher powers since the source-drain resistance only has points of inflection at the midgap positions and does not rise with n_s. So, while this effect may contribute to the out-of-phase oscillations at the lower powers used, it cannot be the principal cause and we attribute this to changes in the relaxation processes.

At low powers, the phase of the oscillations is consistent with dissipation proportional to ρ_{xx} and the phonon emission from the bridge should be largely by low-frequency intra-Landau-level transitions. However, at higher powers, cyclotron emission should be the dominant process and so the change in phase of the oscillations that occurs as the power is increased presumably signals the change to this process. For $B = 6$ T, the phase change occurs at ~ 5 μW and oscillations of this phase persist up to the highest power used of 200 μW. Since the total emission is constant, the fall in normal phonon intensity I_N when E_F moves from midgap must be due to transfer from cyclotron emission from low lying Landau levels (e.g., n to $n-1$) to a parallel relaxation process or processes with wider angular distribution. There are four likely parallel processes: low energy intra-Landau-level phonons, optic phonons, cyclotron phonons from higher Landau levels (e.g., $n+1$ to n, $n+2$ to $n+1$, etc.), and phonons at harmonics of the cyclotron frequency (e.g., $n+1$ to $n-1$, $n+2$ to $n-1$, etc.). Low energy intra-Landau-level phonons have frequencies $\omega \lesssim \Gamma/\hbar \ll \omega_c$ when $\mu B \gg 1$ and since emission via this process should saturate once $k_B T_e \gg \Gamma$, it seems unlikely to contribute significantly at high powers.[10] The relaxation rate by optic phonons should be very small for $T_e \ll 80$ K and its exponential temperature dependence seems inconsistent with the linear increase in oscillation amplitude with power input. We conclude therefore that the most likely parallel processes involve phonon emission from higher Landau levels. Cyclotron phonons from higher Landau levels have wider angular distributions[7] as should phonons emitted at harmonics of the cyclotron frequency because of the isotope scattering. Hence a relative increase in either process leads to a fall in I_N. The size of oscillations expected from transfer of emission to these two processes is examined in the following subsections.

A. Calculation of electron temperature

To analyze the processes leading to the oscillations in phonon intensity, we need to know how the mean electron temperature varies with power and sheet density in a quantizing magnetic field. No experimental information on this exists at present, although measurements[11] in zero magnetic field of the far infrared intensity emitted by 3×1 mm^2 samples processed from the same wafers and fitted with transparent gates indicate that T_e varies from 15 K to 30 K for power densities of 2–15 mW mm^{-2} at $n_s = 1.5\times10^{16}$ m^{-2} so that $P \mathrel{\tilde{\propto}} T_e^3$ (these values of T_e include an additional correction associated with the finite gate conductance[12]). Although these values may change significantly in magnetic fields, we take them as a guide for the present analysis. To examine the change in relative contribution to the emission of the various processes we use theoretical analysis of phonon emission in magnetic fields by Toombs *et al.*[7] In zero magnetic field it is known that similar calculations underestimate the phonon emission from 2DEG's in Si MOSFET's by nearly an order of magnitude (Cooper[9] and references therein). The present calculations could therefore provide a qualitative indication of the relative importance of the various emission processes, but for power densities ten times larger.

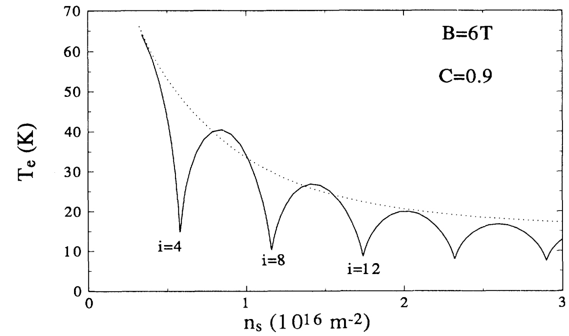

FIG. 3. Electron temperature T_e calculated as a function of n_s at a constant power. The solid and broken lines show the values with and without screening, respectively. (The powers in the two cases differ and were both chosen to give $T_e = 25$ K at n_s=1.5×10^{16} m^{-2}.) A value of C=0.9 was used to calculate the solid line.

Figure 3 shows calculated values of T_e for B=6 T as a function of n_s at two constant powers each chosen to give $T_e = 25$ K at $n_s = 1.5\times10^{16}$ m^{-2}. The broken curve shows the result when screening of the electron-phonon interaction is neglected: T_e decreases with n_s, but no oscillations are apparent. The solid curve shows the effect of including screening by dividing the integrand of the expression for the total phonon intensity $I(T_e)$ by $\epsilon^2/\epsilon_r{}^2$, where the dielectric function

$$\epsilon(q_{\|},\omega=0) = \epsilon_r\left(1+\frac{q_s}{q_{\|}}\right) .$$

q_s is the screening parameter and $q_{\|}$ the in-plane component of the phonon wave vector. Pronounced oscillations are now present which continue up to the highest powers examined. They arise because the efficiency of the screening increases with the density of states at the Fermi level $N(E_F)$ being greatest when E_F lies within a Landau level and falling to a minimum when E_F is midgap. In Fig. 3 this is illustrated using the Thomas-Fermi expression $q_s = \frac{e^2}{2\epsilon_0\epsilon_r}N(E_F)$,[13] with

$$N(E_F) = \frac{4\pi m^*}{\hbar^2}\left[1 - C\cos\left(\frac{2\pi E_F}{\hbar\omega_c}\right)\right] ,$$

where ϵ_r is the dielectric constant and C is a fitting parameter taken to be 0.9 in Fig. 3. The calculations are approximate since for simplicity the expression for cyclotron emission from unbroadened Landau levels is used and that for q_s neglects thermal smearing. It is, however, quite similar to that of Qin *et al.*,[14] although in the present work we neglect the effect of screening on the interaction with the normal component of the phonon wave vector due to the finite 2DEG thickness.

B. Calculation of phonon signal

An increase in T_e increases the population of the higher Landau levels and so the proportion of phonons arising from them. These phonons have different angular distributions than those from lower levels, leading to the changes with E_F seen in I_N and so ΔT_B. The dependence of ΔT_B on angular distribution, including the effect of phonon focusing, has been calculated theoretically,[15] where it was shown that for this geometry ΔT_B is mainly caused by phonons emitted with wave vectors making angles to the normal $\theta < 45°$. So, to obtain the changes in ΔT_B caused by any particular change in angular distribution, the power output $P(\theta)$ is multiplied by a weight function $W(\theta)$ that follows approximately the form obtained theoretically[15] and is shown as the inset in Fig. 4. This is the approach used for cyclotron phonons. For phonons of $2\omega_c$ and above, we make the very rough approximation that because of isotope scattering they make no contribution to ΔT_B. The sizes of the oscillations in the phonon signal and so $\delta(\Delta T)$ from these two effects are comparable; both have the phase seen in the experiments and they are shown in Fig. 4 together with their combined effect for C=0.9. A large value of C was used for illustrative convenience. $\delta(\Delta T)/\Delta T$ increases with C and for $C \ll 1$, the variation is approximately linear with C.

The increase in T_e with power has two effects on $\delta(\Delta T)/\Delta T$. It is increased by the increase in excited state population but decreased by the damping of the oscillations in q_s caused by thermal smearing. An approximate indication of the two effects can be obtained by considering them separately. If the thermal smearing is neglected by retaining the temperature-independent expression for q_s, $\delta(\Delta T)/\Delta T$ increases approximately linearly with power for $P \leq 1$ mW mm^{-2} (this depends on C), but the increase then becomes progressively weaker. The effect of thermal smearing can be allowed for by replacing $N(E_F)$ in the expression for q_s by[13] $\int N(E)(-dF/dE)dE$, where $F(E)$ is the Fermi function. For Gaussian linewidths with $\Gamma = 200$ GHz ($\sim 0.2\omega_c$ at 6 T), the relative amplitudes of the oscillations in q_s, C', decreases with T_e and so with P approximately as $\sim P^{-\frac{7}{4}}$ in the power range of interest. So since $\delta(\Delta T)/\delta T \propto C'$, the overall effect of increasing the power varies approximately as $\sim P^{-\frac{3}{4}}$. So the model predicts a stronger dependence of $\delta(\Delta T)/\Delta T$ on P than observed. The calculated values of $\delta(\Delta T)/\Delta T$ obtained including both effects are five times smaller than those observed experimentally for powers corresponding to $T_e = 15$ K and the difference increases as P and so T_e is increased. So while this simple model explains qualitatively several of the main experimental features, a more detailed analysis would clearly be needed to obtain quantitative agreement.

V. CONCLUSION

The variation with filling factor of the normal component of the ballistic phonon intensity I_N emitted from a magnetically quantized 2DEG has been observed over a wide range of powers. At low power inputs, the oscillations are in phase with ρ_{xx} as expected, but a phase change occurs as the power is raised and at higher powers the oscillations are approximately 180° out of phase. In this range, the emission should be dominated by cyclotron phonon emission. The amplitude of the oscillations varies linearly with power up to the highest power densities used, suggesting that they are not due solely to the effect of intra-Landau-level emission which would be expected to diminish relatively once $k_BT_e \gg \hbar\omega_c$. A more likely explanation is that oscillations in T_e arise from variations in the screening of the electron-phonon interaction leading to oscillations in the proportion of emission from higher Landau levels. Cyclotron phonon emission from these has a wider angular distribution than from lower levels and emission at harmonics of ω_c is scattered by isotopes so that an increase in either process leads to a fall in the normal intensity of the phonon emission. Several of the main features of the data can be explained qualitatively using a simple model, but quantitative agreement has not been achieved.

Finally, we note that the phonon emission from the narrow bridge appears to be appreciably more tightly confined to the normal than the emission from hot spots at the current entry and exit points of a Hall bar. This may suggest that, at the hot spots, a significant part of the potential drop and so dissipation is taking place just inside the three-dimensional contacts. The phonons arising from this part would have a wider angular distribution than from the 2DEG, in line with the observations.

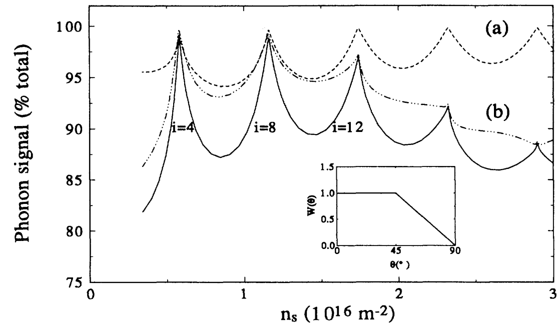

FIG. 4. Proportion of the total phonon intensity reaching the detector (ΔT_B) calculated as a function of n_s for the constant power used in Fig. 3. Screening is included with C=0.9. The two broken curves show the separate effects of (a) the omission of phonons of frequency $j\omega_c$ ($j = 2, 3, \ldots$) and (b) the reduction in intensity due to phonons emitted at $\theta > 45°$. The solid curve shows the combined effects of (a) and (b).

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[*] Present address: Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, 70569 Stuttgart, Germany.
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