Critical behavior of vortices in layered superconductors

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The critical behavior of vortices in layered systems is studied. The vortex interactions that we use approximate those of vortices in a layered superconductor. We do a mathematically rigorous, real-space renormalization-group study on our model to derive the recursion relations. Terms not found by other studies on layered systems have been derived. It is found that one of the new terms contributes to layer decoupling just above the transition temperature, behavior which is consistent with Monte Carlo studies of this system. By analylzing the correlation length, we are able to study the dependence of the transition temperature and of the size of the three-dimensional (3D) critical region on the strength of the interlayer coupling. The size of the 3D critical region above the transition is found to depend on the interlayer coupling in a way which is different than that predicted by others. We study our results in the context of the high-temperature superconductors and find that they are in good accord with experiments on these materials.

I. INTRODUCTION

The effect of vortex fluctuations on the interlayer Josephson coupling and on the critical behavior of layered superconductors [such as the high-temperature superconductors (HTSC's)] has been under intense scrutiny. At issue is the effect of the coupling on the dimensionality of the critical behavior of vortices in these systems. The majority of theoretical studies predict a three-dimensional (3D) transition, whereas electronic transport experiments consistently observe two-dimensional (2D) signatures. In Monte Carlo studies on the other hand, it is found that the layers become decoupled just above the transition, thereby making the system 2D. These studies seem to help explain the discrepancy between theory and experiment and are part of a growing body of evidence that vortex fluctuations tend to decouple the layers near the transition. The case for this scenario was strengthened greatly by recent theoretical studies,[1,2] which confirmed that the interlayer coupling is renormalized to zero just above the transition due to the screening effect of vortices.

The new findings call for a modification of the conventional wisdom regarding the critical behavior of vortices in layered systems, an intuitive understanding of which can be achieved by considering[3] the correlation length. As the reader knows, the correlation length diverges at the critical temperature T_c. When the correlation length is smaller than the distance separating the layers, 2D behavior of the vortices is expected. When the correlation length becomes larger than the interlayer separation, the behavior should be 3D. Since the latter condition is met as the correlation length diverges near the transition, 3D behavior is expected[4] in a small temperature window around T_c. Outside of this window, the layers are expected to be uncoupled. Incorporating the effect of vortex fluctuations on this scenario, we will show that the interlayer coupling shrinks to zero more quickly, thereby making the size of the 3D temperature window above T_c smaller than in the conventional picture.

The study of layered systems and their critical behavior has received constant attention since the discovery of the HTSC's. Nevertheless, their study goes back much further. The earliest work[4,5] on layered systems was being done at the same time that Kosterlitz,[6] Thouless,[7] and Berezinskii[8] (KTB) were considering the critical behavior of purely 2D systems and the effect of vortices. While the early, layered system studies[4,5] did not consider the effects of vortices, they were instrumental in establishing the presence of long-range order, which Mermin[9] had shown to be absent in two dimensions.

The effect of vortices on the critical behavior of layered systems was considered by Hikami and Tsuneto.[10] In their work, which is based on a phenomenological argument and which is in agreement with the intuitive picture described above, they estimated the dependence of the size of the 3D temperature window and of the transition temperature on the interlayer coupling strength. More rigorous theoretical studies,[11–13] which are in agreement with this general picture, have been done since the discovery of the HTSC's. One such study[11] was a momentum-space renormalization-group (RG) study that was done on a sine-Gordon-like Hamiltonian.[15] In Ref. 12 an iterated mean-field approximation of the dielectric function is used to determine the recursion relations. In the RG study of Ref. 13, the integration of the recursion relations was extended into the 3D region by considering the scaling of vortex loops in the 3D *XY* model. While all of these studies were consistent in their findings of 3D behavior near the transition, none considered the effect of vortex fluctuations on the interlayer coupling.

The first indications of layer decoupling and the importance of the vortex fluctuations came from Monte Carlo studies.[16,17] In these studies, it was found[16] that the vortex interactions became essentially 2D just above the transition. It was suggested by those authors that the vortex fluctuations were responsible for this behavior. The effect of vortex fluctuations has also been considered in an analytical study.[18] It was found that vortex fluctuations can weaken the interlayer coupling for temperatures much lower than the critical temperature. As those authors point out, a consistent RG approach is needed to extend these results to the critical region.

The nature of the vortices and their interactions was one of the first major thrusts after the discovery of the HTSC's. The model upon which these studies were based is the Lawrence-Doniach system[19] in which the superconducting layers are coupled to one another via Josephson coupling. The effect of the Josephson coupling on the interlayer vortex interactions was studied by Cataudella and Minnhagen,[20] who found a minimal correction to the interactions for small separations and a linear term that dominates at large separations. The work has since been extended[21] to the interactions between vortices in neighboring layers. In the absence of the Josephson coupling the vortices interact logarithmically[22,23] due to the electromagnetic coupling as they do in a strictly 2D system.

Of course, vortex fluctuations are only part of the story in layered superconductors. Another type of topological[24] excitation, which is a close relative of the vortex fluctuation, is the fluxon. Friedel pointed out that a vortex loop whose core lies parallel to the layers (i.e., entirely between the layers) is also possible and is energetically favorable, since its normal core lies entirely in the insulating region separating the layers. He conjectured that such excitations, called fluxons, could drive a transition causing the layers to become decoupled. Further studies,[25] however, showed the transition to be 3D. In this paper we will not deal with the effect of fluxons. Another type of excitation receiving considerable study are vortex loops, which cut through the layers. These excitations are expected to be most dominant in the 3D temperature window, where the interlayer coupling is strong. As we will discuss below, this is expected to happen very close to the transition. The applicability of our model to the vortex loops will be discussed below.

Having reviewed the theoretical background to this work, we will now briefly describe the experimental work that has been done to study the effect of vortices. The behavior of vortices in 2D superconductors near the transition is well described by KTB theory and is characterized by an unbinding of pairs of vortices of opposite vorticity. Signatures of the KTB transition have been observed in electrical transport measurements on several types of HTSC's.[26,27] Yet, there has been no evidence of 3D behavior in these measurements.[28] There is a more recent study[29] on $Bi_2Sr_2CaCu_2O_{8-\delta}$ in the presence of a finite current, which suggests that the layers decouple at a temperature slightly higher than a temperature at which there is evidence of unbinding. So while evidence of the presence of KTB behavior in the HTSC's has long been established, only recently has there been experimental evidence of layer decoupling, and there remain many questions about why the 2D signatures are predominant.

In this work, we study a model[30] for vortex fluctuations in a layered system based on two-body interactions, which approximate those of vortices in a layered superconductor. A mathematically rigorous real-space RG study is performed on this model to find the recursion relations. The recursion relations are analyzed to find the size of the 3D critical region and the critical temperature as functions of the interlayer coupling. It is found that the vortex fluctuations do renormalize the interlayer coupling to zero just above the transition and that the 3D critical region is substantially smaller above T_c than below. Finally, we address how our results may explain the prevalence of 2D signatures.

This paper is organized as follows. In Sec. II, we describe our model for vortex fluctuations in a layered system. In Sec. III, we derive and explain the recursion relations for this model. In Sec. IV, we analyze the recursion relations and discuss the results. Our conclusions are presented in Sec. V.

II. MODEL FOR A LAYERED VORTEX GAS

In this section we will explain our model for vortex fluctuations in a layered system. As we mentioned in Sec. I, the bare interactions between vortices in a layered superconductor are well studied. The intralayer[20] and interlayer[21] vortex interactions [$V(R,0)$ and $V(R,1)$, respectively] can be summarized as follows:

$$V(R,0)=\begin{cases}-\ln(R/\tau)+(\lambda R^2/4\tau^2)\ln(\lambda R^2/\tau^2), & \tau \ll R \ll R_\lambda \\ -(\pi\sqrt{\lambda}R/\tau\sqrt{2}), & R \gg R_\lambda ,\end{cases} \tag{2.1}$$

$$V(R,1)\propto\begin{cases}-\lambda R^2/\tau^2\ln(\lambda R^2/\tau^2), & R \ll R_\lambda \\ \sqrt{\lambda}R/\tau, & R \gg R_\lambda ,\end{cases} \tag{2.2}$$

where λ is the ratio of interlayer coupling to intralayer coupling, $R_\lambda=\tau/\sqrt{\lambda}$ is an interlayer length scale, $V(R,l)$ is written in units of the intralayer interaction strength, R is the in-plane separation, l is the number of layers separating the vortices, and $V(R,0)$ smoothly approaches zero as $R\rightarrow 0$. τ is an in-plane ultraviolet cutoff, which we will take to be the core size of the vortex. The origin of the linear dependence in the interactions is the Josephson coupling between the layers and can be thought of as the energy of a flux line connecting two vortices, i.e., Josephson vortex loops.[31] For more details about the interactions of vortices in layered superconductors, we refer the reader to the literature.[20,21]

Given that the vortices interactions are known, it is natural to treat the system of vortices as a gas of charged particles. The scheme of our model is depicted in Fig. 1(a), where we show vortices located throughout a layered system. A vortex with positive (negative) vorticity will be treated as a positive (negative) charge. We will consider only thermally induced vortices, implying that the total vorticity of the system must be zero. The grand partition function[14] for our layered, neutral gas of charges is

$$Z=\sum_N y^{2N}\frac{1}{(N!)^2}\sum_{l_1}\int_{D_1}d^2r_1\sum_{l_2}\int_{D_2}d^2r_2\cdots\sum_{l_{2N}}\int_{D_{2N}}d^2r_{2N}\exp\left[-\frac{\beta}{2}\sum_{i\neq j}p_ip_jV(|\mathbf{r}_i-\mathbf{r}_j|,l_i-l_j)\right], \tag{2.3}$$

where $2N$ is the total number of particles, N of which have a positive (negative) charge $p_i=+p\,(p_i=-p)$, and $(\mathbf{r}_i,l_i)$ are the coordinates of the ith charge corresponding to the in-plane coordinates $\mathbf{r}$ and the lth layer. $\beta^{-1}=k_BT$, where T is the temperature and k_B is the Boltzmann constant. $y=\exp(\beta\mu)/\tau^2$ is the fugacity, where $\mu=-E_c$ and E_c is the "core energy." $V(R,l)$ is the interaction between two vortices[33] expressed in the units p^2. The integrals are over an area D_i which is all of the layer l_i except for disks $d_i(j)$ of radius τ around the charges $j<i$, which lie in the same layer. The area D_i is illustrated in Fig. 1(b) and can be written

$$D_i=A-\sum_{j<i}d_i(j)\delta_{l_i,l_j}, \tag{2.4}$$

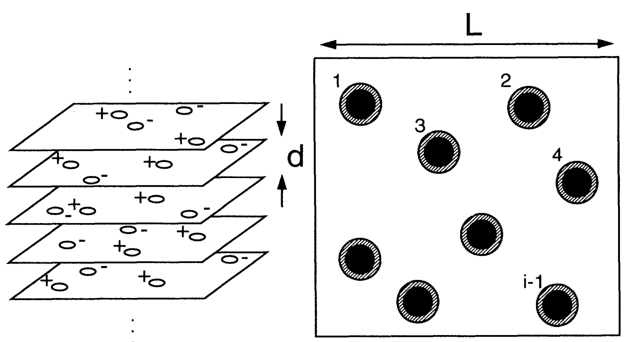
where A is the area of the layer.

For the vortex interactions, we approximate Eqs. (2.1) and (2.2) by

$$V(R,0)=-\ln(R/\tau)-\sqrt{\lambda}(R-\tau)/\tau, \tag{2.5}$$

$$V(R,1)=b\sqrt{\lambda}R/\tau, \tag{2.6}$$

where λ is the ratio of the interlayer coupling to the intralayer coupling p^2 and is assumed to be small. The intralayer interaction term Eq. (2.5), which neglects the small-R Josephson correction, has been shown[16(a)] to be a good approximation for the intralayer interactions of vortices in layered superconductors. The effect of the small-R interaction on the renormalization of the interaction strengths, λ and p^2, has been found to be insignificant and so will be neglected to keep the calculation tractable. The effect of the small-R intralayer interaction does, however, affect the renormalization of the fugacity in an important way and will be included there. The constant $\sqrt{\lambda}$ has been added to Eq. (2.5) to be consistent with the definition of E_c (which must be one half the energy of an intralayer vortex pair at smallest separation). The interlayer interaction, Eq. (2.6), approximates the short-range interaction by the long-range interaction of vortices in a layered superconductor. Interactions between vortices separated by more than one layer ($l\geq 2$) are very small relative to the intralayer and interlayer interactions and will be neglected.



(a) (b)

FIG. 1. (a) An illustration of our model, where vortices are located throughout a stack of layers and interact with each other if they are in the same layer or in neighboring layers. (b) The region over which the integrals in our model and our RG study are done. All but the solid disks represent the area D_i, Eq. (2.4). The shaded region represents the annuli in Eq. (A1). L is the width of the layer, which we take to be infinite.

This model emphasizes the 2D aspect of the problem. This approach is justified for two reasons, the first of which can be seen in terms of crossover and RG behavior. In our problem, the critical behavior crosses over from 2D (KTB) behavior away from the critical temperature to 3D (anisotropic 3D X-Y model) behavior near the critical temperature. This phenomenon is termed crossover behavior. In terms of RG terminology, the RG flows (which are determined by integrating the recursion relations) are first controlled by one fixed point, which characterizes the critical behavior and then, under the influence of a "relevant" parameter, flow to a second fixed point, which determines the ultimate critical behavior. In our case, the first fixed point is that of the KTB model and the second is the 3D system, and the new parameter is the interlayer coupling. In RG theory it has been shown[34] that one can determine the dependence of the size of the critical region controlled by the second fixed point and of the critical temperature on the new parameter by the recursion relations in the vicinity of the first fixed point. Therefore, finding the first-order corrections of the KTB relations due to the interlayer coupling will enable us to study these two quantities, the size of the 3D critical region, and the critical temperature.

The second reason for which the 2D approach is justified involves how one addresses the 3D aspect of the problem. In Ref. 13, the behavior of the system was examined by considering the scaling of vortex loops. There, the RG flows are determined first by the 2D recursion relations and then are "handed over" to the 3D recursion relations at a certain value of the interlayer coupling. Plainly, a complete understanding of the 2D aspect of the problem is very important because it is the behavior of the two-dimensional recursion relations, which determines when to switch[13] to the 3D recursion relations. Therefore, not only is it fruitful to understand the 2D recursion relations; it is also essential. Furthermore, it is clear that our model is valid even into the 3D region, where 3D vortex loops are the dominant excitation. This is because the energy of a vortex loop of diameter a in our model (which is found by summing the two-body interactions) is a good approximation for the correct energy[35] of the vortex loop, $E_{vl}=a\ \ln a$. Equation (2.3) is expected to break down deep into the 3D region.

III. RENORMALIZATION-GROUP STUDY

In this section, we will do a renormalization-group[36] analysis on Eq. (2.3) to derive the recursion relations for this model. There are three principal steps in a RG study. The first step is to integrate out the small scale structure incorporating it into the parameters of the system. This step makes the system appear larger. The second step is to arrange the resulting terms into the form of the original partition function so that one will be able to identify the renormalized parameters of the system. The final step is to rescale the lengths so that the system is "shrunk" back to its original size. The iteration of these steps allows one to study the fixed point and the behavior of the system near it. This is because the RG steps of integrating out small scale structure and rescaling the lengths does not change the system parameters at the fixed point where the correlation length has diverged and where the system is scale invariant.

We have done a RG analysis on the partition function Eq. (2.3) in Appendix A and have derived equations for the renormalized parameters, Eqs. (A20)–(A22). Below we write the equations in differential form,[37]

$$dx/d\epsilon=2y^2(1-A\lambda)+O(y^2\lambda^{3/2})\ , \tag{3.1}$$

$$dy/d\epsilon=2y(x+\tfrac{1}{2}\lambda\ln\lambda)/(1+x)+O(xy^3\lambda)\ , \tag{3.2}$$

$$d\lambda/d\epsilon=2\lambda[1-4y^2/(1+x)]+O(y^2\lambda^{3/2})\ , \tag{3.3}$$

where $\epsilon=\ln(\tau/\xi_0)$, $x=4/(\beta p^2)-1$, $A=(1+b^2)/32$, and where ξ_0 is the zero-temperature correlation length. Note that a factor of $2\pi\tau^2$ has been absorbed into y (see Appendix A), and that, in the $\lambda=0$ limit, our equations reduce to the recursion relations of Kosterlitz.[6] Because λ is in both the intralayer and interlayer interaction, its renormalized value appears in two different places [see Eqs. (A17)–(A19)]. That both corrections lead to identical recursion relations for λ demonstrates the robustness of our results.

Our recursion relations, Eqs. (3.1)–(3.2), contain two terms that recursion relations derived by others[12,13] for the layered system do not have. In Eq. (3.1), we include the lowest-order λ term. This term takes into account the Josephson coupling on the renormalization of the intralayer interaction strength. An even more important term is $-8\lambda y^2$ in the recursion relation for λ. This term represents the effect of vortex fluctuations on the interlayer coupling. As we shall show in the next section, this term plays an important role in the decoupling of the layers above T_c. Our λ correction to the recursion relation for y differs from those references because we have incorporated the small R Josephson correction into the fugacity [see discussion after Eqs. (A20)–(A22)]. Equation (3.3) is qualitatively similar to that derived for the single-layer analog of our system.[2] In that work, where vortices interact with potential $V(R,0)$, the recursion relation for λ is derived by a phenomenological RG study of a current conservation equation.

IV. RESULTS

In this section, we analyze the recursion relations Eqs. (3.1)–(3.3). We will begin with a qualitative discussion of the recursion relations and of the results. These results will be made more quantitative by examining the behavior of the correlation length. We will study the dependence of the size of the 3D critical region and of the critical temperature on the strength of the interlayer coupling. Finally, we will examine the critical behavior of two related layered systems.

A. Analysis of the recursion relations

Better insight into Eqs. (3.1)–(3.3) is attained by knowing the origin of each of the terms. Any term that is of $O(y^2)$ originates from the first step of the RG process, the averaging out of small-scale structure. This can be seen by inspecting Eq. (A18), the last equation in the coarse graining step of the RG analysis and the last equation before rescaling the lengths, where all the corrections are of $O(y^2)$. A more intuitive understanding of this is reached by recalling that the density of vortex pairs is proportional to the square of the fugacity. Therefore terms that depend upon y^2 must be due to small vortex pairs. The rest of the terms in the recursion relations are due to the rescaling step.

In Eq. (3.1), the λ correction to the recursion relation is due to the first step of the RG process and causes x to grow more slowly. In other words it causes the intralayer coupling to be weakened less rapidly. This is because in the presence of the interlayer coupling the vortex pairs are more tightly bound and therefore more resilient to fluctuations. With x growing more slowly, y will not grow as quickly and therefore fewer flows will go to the high-temperature limit. (Recall that the flows for which $y\rightarrow\infty$ correspond to $T>T_c$, since the density of vortex pairs is proportional to y^2.) As we shall see, this leads to a larger transition temperature.

The λ correction to the recursion relation for y, Eq. (3.1), is due to the rescaling step and, more specifically, to the Josephson correction to the small R interaction. It can be significant, since $\lambda\ln\lambda$ can be of the same order as x. Its effect is to make y grow more slowly, which means that more flows will go towards $y=0$, the same effect that the λ correction has in the recursion relation for x. This will be discussed further in the context of T_c.

In the recursion relation for λ, the first term is due to the rescaling step and makes λ grow. Counteracting this is the second term, which is the effect of small pairs and which weakens the interlayer coupling, as one would expect. For small y, the first effect wins out, and λ grows. For larger y, the latter term dominates, and λ gets smaller. In RG parlance, λ is a relevant parameter in one regime and irrelevant in another. This is reflected in Fig. 1 of Ref. 1, where we have plotted the RG flows for certain initial values (denoted by subscript i) of x and λ and various initial values of y. For small enough values of y_i, the flows move towards $y=0$ and take off to a large value of λ. For larger values of y_i the flows follow approximately the associated 2D flows, never attaining a large value of λ and ultimately moving toward the $\lambda=0$ plane. The implications are clear: because the flows for which $y\to 0$ correspond to $T<T_c$ and because $\lambda\to 1$ for these flows, 3D behavior is expected for a small temperature window below the transition. The flows for which $y\to\infty$ correspond to $T>T_c$, and, because $\lambda\to 0$ for these flows, one expects that the layers become decoupled above T_c. This is in contrast to prior RG studies,[11–13] where $\lambda\to 1$ as $y\to\infty$. Our results are in accord with the Monte Carlo studies and discussions[38] of Minnhagen and Olsson. How far above T_c the layers become uncoupled will be discussed in more depth below.

The behavior of T_c can be examined in light of the relations (3.1) and (3.2). As we mentioned earlier, the effect of the λ term is to decrease the tendency of the flows to go to the high-temperature limit. This increases the low-temperature "parameter space," which corresponds to a higher transition temperature. A better understanding of the behavior of T_c is attained by taking the definition[39] of T_c as the maximum T such that

$$\lim_{\epsilon\to\infty} y(\epsilon)=0 \ . \tag{4.1}$$

Given an x_i, the values of y_i which satisfy the above equations for our recursion relations, grow with increasing values of λ_i. Larger values of y_i correspond to an increased temperature and therefore a higher T_c.

There are two reasons for which the transition temperature increases, each being associated with either the λ correction to the recursion relation for x or to the λ correction the recursion relation for y. The first reason involves Eq. (3.1). There are actually two λ corrections to this recursion relation, one that does not depend upon b and one that does. Because the b enters through the interlayer interaction, the first correction refers to the intralayer effect and the second to an interlayer effect. The intralayer effect is due to the fact that the vortex pairs are more strongly attracted with the Josephson coupling, and therefore the binding of the vortex loop in the in-plane direction is much stronger. The b^2 term is present because the interlayer interaction serves to bind the vortex pairs and loops even more strongly in the in-plane and out-of-plane directions. For these reasons, the vortex excitations will not unbind until temperatures greater than the 2D transition temperature are attained. This explains the origin of the two terms in Eq. (3.1) and why they contribute to a higher T_c. The second reason for which T_c increases is associated with the recursion relation for the fugacity, Eq. (3.2). In the presence of the Josephson coupling, more energy is needed to create a vortex pair. Therefore, higher temperatures are needed to generate enough vortex pairs to cause an unbinding. This is the reason for which the λ correction to the recursion relation for the fugacity contributes to a higher T_c.

In two dimensions, the renormalized interaction strength, p^2, jumps discontinuously to zero at the transition.[39] In the layered system, the size of this jump is greatly reduced, although we cannot determine if it is destroyed because its ultimate behavior is determined by the 3D recursion relations. The origin of the reduction in the size of the jump can be traced to the λ correction to the recursion relation for y, which causes flow to be stopped at finite positive values of x. The behavior of this quantity has been investigated more thoroughly in Refs. 2 and 13, although the latter reference does not take into account the effect of the vortex fluctuations on the interlayer coupling.

B. Correlation length

We can make our observations more quantitative by examining the correlation length. We will first review how one derives the temperature dependence of the correlation length from the recursion relations in the 2D case. Using the first integral of the 2D recursion relations, $y^2+2\ln(1+x)-2x=c=y_i^2+2\ln(1+x_i)-2x_i$, for small x and y, one can derive the following equation by integrating the recursion relation for x[40]

$$\frac{1}{\sqrt{c}}\tan^{-1}\left[\frac{x}{\sqrt{c}}\right]\Bigg|_{x_i}^{x}=2\epsilon \ , \tag{4.2}$$

where $c\propto t\equiv(T-T_{KT})/T_{KT}$ and T_{KT} is the 2D transition temperature. This equation defines x as a function of ϵ. One integrates the recursion relations until $y=0$ or until the variables become so large that the recursion relations are no longer valid. The value of ϵ when the recursion relations are stopped defines $\epsilon_{\max}$ and in turn $x_{\max}$. Since $\epsilon_{\max}$ is related to a length scale, and since the correlation length is the only length scale in the problem, one makes the following association

$$\epsilon_{\max}=\ln[\xi(T)/\xi_0] \ . \tag{4.3}$$

It is this equation that we will use to study the behavior of the correlation length for finite $\lambda, \xi_\lambda(T)$.

Using Eq. (4.3) with Eq. (4.2), one obtains the temperature dependence of the 2D correlation length

$$\xi_{\lambda=0}(T)\equiv\xi_{2D}(T)\propto\exp(\alpha_\pm t^{-1/2}) \ ,$$

where $\alpha_\pm$ is a nonuniversal constant whose value for $T>T_{KT}$ α_+, differs from that for $T<T_{KT}$, α_-. In Fig. 2, we have plotted $\ln[\xi_{2D}(T)/\xi_0]$, which we have determined by numerically integrating Eqs. (3.1)–(3.3) for $\lambda_i=0$. For small x_i, small y_i, and $\lambda_i=0$, we have verified the temperature dependence of $\xi_{2D}(T)$. Note that $\xi_{2D}(T)$ is defined below T_{KT} and that it has the same temperature dependence in that regime as it does above the critical temperature. This length scale is related to the average size of a vortex pair, in contrast to the $\xi_{2D}(T<T_{KT})$ defined in Ref. 6, where the quantity is associated with the susceptibility and is infinite. Note also that $\xi_{2D}(T)$ tends to be much larger below the critical temperature than above.

We have also calculated the correlation lengths for

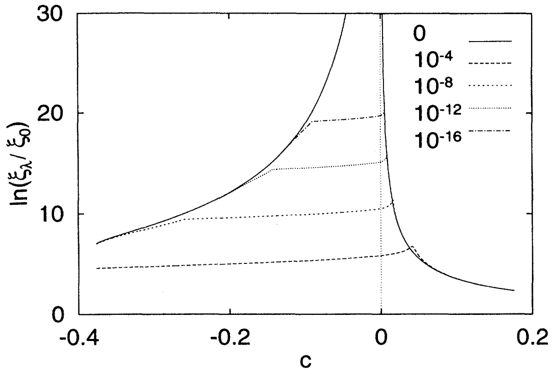


FIG. 2. The logarithm of the correlation length, $\ln[\xi_\lambda(T)/\xi_0]$, vs $c \propto T/T_{KT}-1$, for several values of λ, which are listed in the corner beside the corresponding line type. For each value of λ, the temperatures where $\xi_\lambda(T)$ is different than $\xi_{2D}(T)$ is interpreted as a region where the critical behavior is 3D. The temperature where $\xi_\lambda(T)$ peaks is associated with the critical temperature. See discussion in text and Fig. 3.

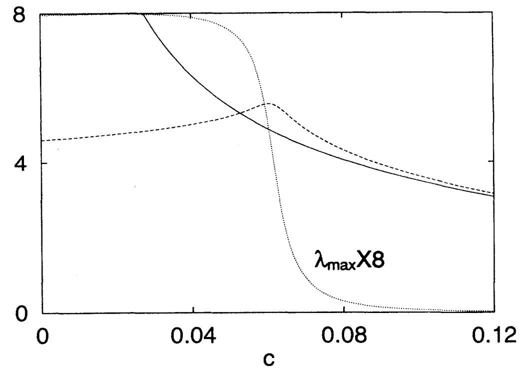


FIG. 3. The logarithm of the correlation length for $\lambda=0.0$ (solid line) and $\lambda=1.0e-3$ (dashed line), and the renormalized ratio of the interlayer coupling to the intralayer coupling λ_{max} (dotted line) vs c, where $c \propto t \equiv T/T_{KT}-1$. Notice that the latter quantity is magnified by a factor of 8 for purposes of comparison. One can see that the quantity t_c divides the regime where λ_{max} is large and where it is small.

finite λ and have plotted them in Fig. 2. The method we use for determining this quantity is to fix x_i and λ_i at certain starting points and then to numerically integrate the recursion for various values of y_i, which correspond to temperatures above and below T_c. We stop the integration of the recursion relations when $y(\epsilon) \leq 10^{-7}$ or when $x^2(\epsilon)+y^2(\epsilon)+\lambda^2(\epsilon) \geq 1$ and record the value of ϵ, ϵ_{max}. This quantity is then plotted versus $c \propto t \equiv T/T_{KT}-1$. This process is repeated for several values of λ_i.

Several quantities can be determined by studying the behavior of the correlation length for various values of λ_i and comparing it with the 2D correlation length. This is apparent in Fig. 2. There is a temperature, larger than the 2D transition temperature T_{KT} $(c=0)$, where the correlation length peaks and which seems to separate two temperature regimes. We have studied the behavior of the renormalized interlayer coupling $\lambda_{max}=\lambda(\epsilon_{max})$ around this temperature and have verified that it separates the regime where λ_{max} is large and where it is small (see Fig. 3). It is also the temperature that separates small values of $y_{max}=y(\epsilon_{max})$ from large values of y_{max}. Because one can derive the T_c from the recursion relations describing the behavior around the 2D fixed point (see the discussion at the end of Sec. II), we believe that T_c should also be reflected in the correlation length, and we will therefore relate the temperature of the peak with the transition temperature $t_c \equiv T_c/T_{KT}-1$ of the system. One can see that t_c does indeed increase with larger interlayer coupling, and we find the following dependence on λ:

$$t_c(\lambda) \propto 1/(\ln\lambda)^2 \ . \tag{4.4}$$

This is the same dependence found in Refs. 10 and 11. In this equation (and the following two equations), λ corresponds to the initial value of this quantity used in the integration of the recursion relations.

We have also studied the λ dependence of the size of the 3D critical region, which we identify as the temperatures where the correlation length for finite λ is different than the 2D correlation length. Because the vortex fluctuations are expected to affect the size of this region more above the transition temperature than below it, we differentiate the 3D temperature window above T_c, τ_{3D}^{+}, from that below, τ_{3D}^{-}. From inspection of Fig. 2, it is apparent that τ_{3D}^{-} tends to be larger than τ_{3D}^{+} and that both quantities get larger with increasing λ. The primary λ dependence of τ_{3D}^{-} is found to be

$$\tau_{3D}^{-} \propto 1/(\ln\lambda)^2 \ . \tag{4.5}$$

This is the same λ dependence as $t_c(\lambda)$ and is in agreement with Refs. 10 and 11. The result for τ_{3D}^{+}, the size of the 3D critical region above T_c, is more interesting. The dominant λ dependence is found to be

$$\tau_{3D}^{+} \propto \lambda^{1/4} \ . \tag{4.6}$$

This is a result contrary to what has been derived by previous studies. We attribute this unique behavior to the effect of vortex fluctuations on the interlayer coupling. Equation (4.6) was found to hold over eight orders of magnitude, while Eqs. (4.4) and (4.5) held over at least twelve orders of magnitude. In the small window above T_c, where the correlation length for finite λ is different than the 2D correlation length, λ_{max} is actually quite small. We believe then that in this region the system is only weakly three dimensional, i.e., that the vortices are weakly correlated in the direction perpendicular to the layers and that vortex lines are not well defined. This is in contrast to the 3D window below T_c, where λ_{max} is large and correlations between vortices in neighboring layers are much stronger.

The anisotropy of the behavior above and below T_c is quite evident in Eqs. (4.5) and (4.6). Not only is the λ dependencies of the critical regions different but also the size of the critical regions varies dramatically. There are two reasons for this. One reason appears to be that the correlation length below T_c is much larger than that above. This is the case in Fig. 2, and it is also the scenario depicted in Fig. 7(a) of Ref. 38. The second reason deals with the effect of vortex fluctuations. Not only do these fluctuations cause the layers to become decoupled at a temperature closer to T_c than would be the case if the fluctuations were not taken into account; they cause the T_c to be slightly lower than in that case.

The constants of proportionality in Eqs. (4.4)–(4.6) cannot be determined with certainty in this analysis. This is because they depend upon the cutoffs that one uses in the integration of the recursion relations. Another point is whether the anisotropy of the 2D correlation length is due to our choice of cutoffs. When we adjust the cutoffs so that the temperature dependence of the 2D correlation length is isotropic around T_{KT}, we find that the relation $\xi_{2D}(t) \propto \exp(1/t^{1/2})$ no longer holds. We therefore believe that the anisotropy of the 2D correlation length is intrinsic. Furthermore, the relative sizes of the 3D critical regions and the λ dependencies of those quantities and T_c are not affected when we vary the cutoffs.

To summarize, we have verified that the critical behavior of this system does have a 3D character for a temperature window around T_c, crossing over from 2D behavior away from T_c. We have found that the 3D region above T_c is much smaller than below and that this is at least partially due to the effect of vortex pair fluctuations.

C. Related layered systems

Further insights into the behavior of vortices in layered systems are gained by changing the interactions Eqs. (2.5) and (2.6) of the vortices slightly. For example, if one uses a purely logarithmic intralayer interaction, $V(R,0) = -\ln R/\tau$, and the same interlayer interaction, one would find the following recursion relations $dx/d\epsilon = 2y^2(1-b^2\lambda/32)$, $dy/d\epsilon = 2xy$, and $d\lambda/d\epsilon = 2\lambda[1-4y^2/(1+x)]$. Two points are clear: (1) Vortex pair fluctuations weaken the interlayer coupling. (2) T_c increases with larger values of λ in this system. This is because the vortex loops that form are more stable to vortex fluctuations than their 2D counterparts, which can be seen the first recursion relation. One could infer that these properties tend to be general properties of layered systems.

One can also derive the recursion relations for a purely 2D system of vortices whose interaction is $V(R,0)$ [Eq. (2.5)]. This is the model studied in Ref. 2, where the recursion relations are derived by a phenomenological RG study. The recursion relations derived there for y and λ include the first-order effect due to coarse graining and rescaling, and are qualitatively the same as Eqs. (3.2) and (3.3). The recursion relations we derive for that system are virtually the same as those for Eq. (2.3) and are given by Eqs. (3.1)–(3.3) with $A = 1/32$. This implies that the critical behavior is the same as for the layered system (except possibly for the small temperature regime, where λ is large and our recursion relations are no longer valid). The main difference is that the b^2 term is absent for the 2D analog system, which means that T_c increases less in this system for a given λ. It appears then that the large-R interactions are more important in determining the critical behavior than the interlayer vortex interactions.

V. CONCLUSIONS

We have studied the critical behavior of vortices interacting in a layered system in zero-field and more specifically, the dependence of the transition temperature and the size of the 3D critical region on the interlayer coupling strength. In our study, we have included the effect of vortex pair fluctuations on the interlayer coupling, an effect neglected by earlier studies on layered systems. We find that the 3D region is much smaller above T_c than below. This is not only because of the effect of vortex pairs but also because of the anisotropy of the 2D correlation length. The λ dependence of the size of the 3D region and of T_c have also been found. The effect of the vortex fluctuations on the interlayer coupling is found to significantly influence the size and λ dependence of the size of the 3D critical region above T_c.

Since Eq. (2.3) can be taken as a simple model for the behavior of vortices in the HTSC's, let us now address why KTB signatures are prevalent in electrical transport measurements on these materials. (Recall that electrical transport properties are heavily influenced by vortex behavior, since free vortices cause resistance in the presence of a current. For specific-heat measurements on the other hand, vortices are not believed to have a significant effect.) There are several possible reasons that can explain this, all or some of which may play a role. The most prominent reason for the lack of 3D signatures in these experiments is a very small (and experimentally unobservable) 3D window above T_c. In this paper we have rigorous evidence that vortex fluctuations and an anisotropic correlation length can make the 3D window small, making this reason very viable. It has been suggested[18] that the current used to probe these materials could make τ_{3D}^{+} small, although recent[41] self-consistent calculations of the effect of the current cast doubt on this scenario. Another possibility for why 2D signatures are predominant is that the transport measurements do not reveal the 3D aspects of this region. In other words, in the window above T_c where the behavior is 3D, the *I-V* characteristics and resistivity could behave in the same way as in a 2D system. This seems probable if the current is applied evenly through each layer, or if the vortices in neighboring layers are only weakly correlated in this narrow window (which, based on our results, we believe to be the case), or both of the above. While preliminary analysis[41] of the *I-V* measurements suggest that the *I-V* curves in the window τ_{3D}^{+} are the same as in the 2D case for temperatures above the transition temperature, a more rigorous experimental check can be made using the techniques of Wan *et al.*,[29] where in-plane transport measurements can be taken while monitoring correlations between layers.

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APPENDIX A: RENORMALIZATION OF THE PARTITION FUNCTION

In this appendix, we detail the steps of the renormalization-group method of the partition function Eq. (2.3) culminating in the recursion relations Eqs. (3.1)–(3.3). This calculation is a generalization of that done on the 2D Coulomb gas by Kosterlitz[6] to the layered system.

The first step in the RG calculation is to integrate out the small scale structure. In our case, this amounts to increasing the minimum separation between the vortices τ to $\tau+d\tau$ and determining the effect of the vortex pairs with a separation in that range on the interactions of other vortices. This is realized by first increasing slightly the allowed size of the disks around each vortex D_i in Eq. (2.4) from τ to $\tau+d\tau$:

$$D_i'=A-\sum_{n<i} d_n(i)\delta_{l_i,l_n}-\sum_{n<i}\delta_n(i)\delta_{l_i,l_n} , \tag{A1}$$

where $d_n(i)$ is a disk of radius τ centered around charge n, and $\delta_n(i)$ is an annulus of radius τ and width $d\tau$ centered around charge n. This is illustrated in Fig. 1(b), where the shaded regions represent the annuli, $\delta_n(i)$. One then substitutes this into the integrals of the partition function. Expanding this resulting expression to $O(d\tau)$, we obtain

$$\begin{aligned}\sum_{l_i}\int_{D_1}d^2r_1\cdots\sum_{l_{2N}}\int_{D_{2N}}d^2r_{2N}=&\sum_{l_1}\int_{D_1'}d^2r_1\cdots\sum_{l_{2N}}\int_{D_{2N}'}d^2r_{2N}\\&+\tfrac{1}{2}\sum_{i\neq j}\sum_{l_1}\int_{D_1'}d^2r_1\cdots\sum_{l_{i-1}}\int_{D_{i-1}'}d^2r_{i-1}\sum_{l_{i+1}}\int_{D_{i+1}'}d^2r_{i+1}\cdots\sum_{l_{j-1}}\int_{D_{j-1}'}d^2r_{j-1}\\&\times\sum_{l_{j+1}}\int_{D_{j+1}'}d^2r_{j+1}\cdots\sum_{l_{2N}}\int_{D_{2N}'}d^2r_{2N}\sum_{l_j}\int_{D_j''}d^2r_j\sum_{l_i}\int_{\delta_i(j)}d^2r_i+O(d\tau^2) .\end{aligned} \tag{A2}$$

Here D_j'' is all of the layer l_j except for the disks around the other vortices.

It is the last two sums and integrals of the second term of the right-hand side of Eq. (A2) which concern us. Physically, the integral over $\delta_i(j)$ puts the charge i in an annulus around charge j to form a pair of smallest separation. The integral over D_j'' and sum over l_j then move this pair through all possible positions in the system. By doing these integrals, the effect of the small pairs is integrated out.

To do the stated integrals, we separate out all terms in the partition function that include i and j:

$$\sum_{l_j}\int_{D_j''}d^2r_j\sum_{l_i}\int_{\delta_i(j)}d^2r_i\exp\left[-\beta\sum_k p_ip_kV(r_{ik},l_i-l_k)-\beta\sum_k p_jp_kV(r_{jk},l_j-l_k)\right] . \tag{A3}$$

As in the 2D case, we assume that only vortices of opposite "charge" can form pairs and therefore $p_i=-p_j$. We also assume that only intralayer vortices can form pairs, which is the reason for the delta function in the last term of Eq. (A1). As a consequence of the integral over r_i, $\mathbf{r}_i=\mathbf{r}_j+\tau$. The sum over l_i results in $l_i=l_j$ because the ith and jth charges must lie in the same layer [see Eq. (A1)]. Equation (A3) can then be written

$$\tau d\tau\sum_{l_j}\int_{D_j''}d^2r_j\int_0^{2\pi}d\theta\exp\left[-\beta\sum_k p_jp_k(M_k+N_k-O_k)\right] , \tag{A4}$$

where

$$M_k=[\mathbf{r}\cdot\mathbf{r}_{jk}/r_{jk}^2+\tau^2/(2r_{jk}^2)-(\tau\cdot\mathbf{r}_{jk})^2/r_{jk}^4]\delta_{l_j,l_k} , \tag{A5}$$

$$N_k=\sqrt{\lambda}\left[\frac{\tau\cdot\mathbf{r}_{jk}}{r_{jk}\tau}+\frac{\tau}{2r_{jk}}-\frac{(\tau\cdot\mathbf{r}_{jk})^2}{2r_{jk}^3\tau}-\frac{\tau\tau\cdot\mathbf{r}_{jk}}{2r_{jk}^3}+\frac{(\tau\cdot\mathbf{r}_{jk})^3}{2\tau r_{jk}^5}\right]\delta_{l_j,l_k} , \tag{A6}$$

and

$$O_k=b\sqrt{\lambda}\left[\frac{\tau\cdot\mathbf{r}_{jk}}{r_{jk}\tau}+\frac{\tau}{2r_{jk}}-\frac{(\tau\cdot\mathbf{r}_{jk})^2}{2r_{jk}^3\tau}-\frac{\tau\tau\cdot\mathbf{r}_{jk}}{2r_{jk}^3}+\frac{(\tau\cdot\mathbf{r}_{jk})^3}{2\tau r_{jk}^5}\right]\delta_{l_j+1,l_k} . \tag{A7}$$

Note that M_k contains the two-dimensional terms resulting from the logarithmic interaction. Also N_k and O_k are virtually identical except that the latter pertains to the interlayer interaction.

The exponential in Eq. (A4) can now be expanded, since the leading terms in M_k and N_k [$O(\tau/r_{jk})$ and $O(\sqrt{\lambda})$, respectively] are both small:

$$\tau\,d\tau \sum_{l_j} \int_{D_j''} d^2r_j \int_0^{2\pi} d\theta \left[1-\beta p_j \sum_k p_k(M_k+N_k)+(\beta p)^2/2 \sum_{k\neq l} p_k p_l[(M_k+N_k)(M_l+N_l)-(M_k+N_k)^2]\right]$$
$$\times\left[1+\beta p_j \sum_k p_k O_k+(\beta p)^2/2 \sum_{k\neq l} p_k p_l[O_k O_l - O_k^2]\right] . \qquad \text{(A8)}$$

Not all the terms in Eq. (A8) contribute to the renormalization of the interactions. For example, the terms that are linear in p are found to be zero either because the integral over θ is zero or because the integral over M_k, N_k, or O_k is independent of k leaving the sum over p_k, which, in turn, vanishes.

Performing the integral over θ, we find, including only the relevant terms,

$$2\pi\tau\,d\tau \sum_{l_j} \int_{D_j''} d^2r_j \left[1+\frac{\beta^2p^2\tau^2}{4} \sum_{k\neq l} p_k p_l \left\{\left[\left[\frac{\mathbf{r}_{jk}\cdot\mathbf{r}_{jl}}{r_{jk}^2 r_{jl}^2}-\frac{1}{r_{jk}^2}\right]\right.\right.\right.$$
$$+\lambda(1+b^2)\left[\frac{1}{16}\frac{(\mathbf{r}_{jk}\cdot\mathbf{r}_{jl})^2}{r_{jk}^3 r_{jl}^3}+\frac{1}{32}\frac{1}{r_{jk}r_{jl}}-\frac{1}{8}\frac{\mathbf{r}_{jk}\cdot\mathbf{r}_{jl}}{r_{jk}r_{jl}^3}+\frac{1}{32}\frac{1}{r_{jk}^2}\right]\Bigg]\delta_{l_j,l_k}\delta_{l_j,l_l}$$
$$\left.\left.\left.+2\sqrt{\lambda}\left[\frac{\mathbf{r}_{jk}\cdot\mathbf{r}_{jl}}{r_{jk}^2 r_{jl}\tau}-\frac{1}{\tau r_{jk}}\right](\delta_{l_j,l_k}\delta_{l_j,l_l}-b\,\delta_{l_j,l_k}\delta_{l_j,l_l+1})\right\}\right] . \qquad \text{(A9)}$$

We have defined the relevant terms as those that will contribute to the renormalization of the logarithmic or linear interaction. It can be shown in our case (but not in general) that the terms that contribute to the renormalization of the logarithmic terms must be $O(1/r_j^2)$ (in the limit $r_j \gg r_k, r_l$). Thus, all the terms on the first and second lines of Eq. (A9) will renormalize the logarithmic interaction. The first two such terms can be seen to be the strictly 2D pieces and match those of Eq. (A7) in Ref. 1. The next such terms of $O(\lambda)$ result from products of $N_k N_l - N_k^2$ that match this criteria. The factor $(1+b^2)$ in front of the λ takes into account the terms $O_k O_l - O_k^2$, which are of the same form. It should be noted that the terms that are $O(b^2\lambda)$ arise from an approximation, which breaks down in certain cases. In the expansion of Eq. (A4) we assume that all of the terms in O_k were small. However, since the vortices in neighboring layers can sit on top of each other, r_{jk} can, in fact, be very small, causing O_k to be very large. This breakdown does not have great consequences, since the term in the recursion relations [see Eq. (3.1)] that it affects is the least significant λ correction and, since the breakdown only applies when $r_{jk} < \sqrt{\lambda}\tau$, a very small region.

Similarly it can be shown that the terms that contribute to the renormalization of the linear interaction must be $O(1/r_j)$. These are the terms on the third line of Eq. (A9). The first set of δ functions correspond to the product $M_k N_k$ and renormalize the intralayer linear interaction, and the second set to $M_k O_k$, which renormalizes the interlayer linear interaction. We have included only terms that contribute to the renormalization of the linear and logarithmic interactions to lowest order in λ.

To proceed with this step, we do the integral over D_j'', which is all of the area in layer l_j except the disks $d_j(n)$ centered at each of the other vortices. This integral is done by separately doing the integral over all of the plane and the integrals over each of the disks. It runs out that only two disks are important, $d_j(k)$ and $d_j(l)$. Because we are working in the low density limit, three vortices cannot be in the same vicinity of each other and therefore, given three vortices in the same layer, say n, k, and l, $r_{nk} \gg r_{nl}, \tau$, $r_{nl} \gg r_{nk}, \tau$, or $r_{nk}, r_{nl} \gg \tau$. Because one of these criteria is satisfied when considering the integral over $d_j(n\neq k,l)$, it can be shown that the integrals are of $O(\tau/\max[r_{nk}, r_{nj}])$ and that therefore they do not contribute to the renormalization of the interactions.

Another note should be made for the integrals to be done. Any term that includes only r_{jk} (and not r_{jl}) will diverge as a function of system size L. There are also divergencies that arise in the integrals that depend on both r_{jk} and r_{jl}. It can be shown that these divergent pieces will cancel each other.

To give the reader a feel for the integrals, we will perform two of them. The first one that we will do is the first term of the second line of Eq. (A9). We will combine the integral over all of the area and the integral over $d_j(k)$:

$$\int_{A-d_j(k)} d^2r_j \frac{(\mathbf{r}_{jk}\cdot\mathbf{r}_{jl})^2}{(r_{jk}r_{jl})^3} . \qquad \text{(A10)}$$

Changing variables to $\mathbf{r}=\mathbf{r}_{jk}$, we find

$$\int_0^{2\pi} d\theta \int_\tau^\infty r\,dr \frac{r^2(r+r_{kl}\cos\theta)^2}{r^3(r^2+2rr_{kl}\cos\theta+r_{kl}^2)^{3/2}} , \qquad \text{(A11)}$$

where θ is the angle between $\mathbf{r}$ and $\mathbf{r}_{kl}$. This integral can be evaluated using integration by parts:

$$\int_0^{2\pi} d\theta \left[1 - \frac{\tau + r_{kl}\cos\theta}{\sqrt{\tau^2 + 2\tau r_{kl}\cos\theta + r_{kl}^2}} + \ln\left[\frac{2L}{\sqrt{\tau^2 + 2\tau r_{kl}\cos\theta + r_{kl}^2} + \tau + r_{kl}\cos\theta} \right] \right], \tag{A12}$$

where L is the size of the system. We can assume[42] that $r_{kl} \gg \tau$, since we are most interested in that regime. In that case, we find

$$= -2\pi \ln(r_{kl}/\tau) . \tag{A13}$$

We have neglected the term dependent on L because, as we discussed above, it will be canceled by the term that depends only on r_{kl}. We have also neglected the constant terms, since they do not contribute to the renormalization of the interactions. Using similar methods, we can evaluate the integral over $d_j(l)$. One finds that it does not depend upon r_{kl}, and therefore Eq. (A13) is the main product of Eq. (A10).

The next integral that we will do is from the third line of Eq. (A9):

$$\int_{A-d_j(k)} d^2 r_j \frac{\mathbf{r}_{jk} \cdot \mathbf{r}_{jl}}{r_{jk}^2 r_{jl}} . \tag{A14}$$

Using the change of variables $\mathbf{r} = \mathbf{r}_{jk}$, we find

$$= \int_0^{2\pi} d\theta \int_\tau^\infty \frac{dr}{\tau} \frac{r + r_{kl}\cos\theta}{\sqrt{r^2 + 2 r r_{kl}\cos\theta + r_{kl}^2}} .$$

Making the same assumptions as we did above, we find that the dominant contribution of Eq. (A14) is

$$-2\pi\sqrt{\lambda}\, r_{kl}/\tau . \tag{A15}$$

Performing the integrals over the rest of the terms in Eq. (A9), we find

$$2\pi\tau\, d\tau \left\{ A - \frac{2\pi\tau^2\beta^2 p^2}{4} \sum_{k\neq l} p_k p_l \left[\left[[1-\lambda(1+b^2)/32] \ln\frac{r_{kl}}{\tau} + 2\sqrt{\lambda}\frac{r_{kl}}{\tau} \right] \delta_{l_k, l_l} - 2b\sqrt{\lambda}\frac{r_{kl}}{\tau}\delta_{l_k, l_l+1} \right] \right\} . \tag{A16}$$

Notice that, in the 2D limit, this equation is the same as Eq. (A8) of Ref. 6.

We have now removed the i and j dependence from Eq. (A2) and can now proceed to regroup the right-hand side of that equation (and the accompanying integrands) into the form of the left-hand side. Rearranging the sum over n so that we again have $2N$ integrals in the second term on the right-hand side and performing the sum over i and j, we arrive at

$$\begin{aligned} Z = &\sum_N y^{2N} \frac{1}{(N!)^2} \sum_{l_1} \int_{D_1'} d^2 r_1 \sum_{l_2} \int_{D_2'} d^2 r_2 \cdots \sum_{l_{2N}} \int_{D_{2N}'} d^2 r_{2N} \\ &\times \left[1 + 2\pi y^2 \tau\, d\tau \left\{ A - \frac{2\pi\tau^2\beta^2 p^2}{4} \sum_{k\neq l} p_k p_l \left[\left[\ln\frac{r_{kl}}{\tau}[1-\lambda(1+b^2)/32] + 2\sqrt{\lambda}\frac{r_{kl}}{\tau} \right] \delta_{l_k, l_l} - 2b\sqrt{\lambda}\frac{r_{kl}}{\tau}\delta_{l_k, l_l+1} \right] \right\} \right] \\ &\times \exp\left[-\frac{\beta}{2} \sum_{i\neq j} p_i p_j V(|\mathbf{r}_i - \mathbf{r}_j|, l_i - l_j) \right] . \end{aligned} \tag{A17}$$

The next step is to put the renormalization terms into the exponential. Doing so, we find

$$\begin{aligned} Z = Z' &\sum_N y^{2N} \frac{1}{(N!)^2} \sum_{l_1} \int_{D_1'} d^2 r_1 \sum_{l_2} \int_{D_2'} d^2 r_2 \cdots \sum_{l_{2N}} \int_{D_{2N}'} d^2 r_{2N} \\ &\times \exp\left[-\frac{\beta}{2} \sum_{i\neq j} p_i p_j \left\{ -\left[1 - (2\pi y \tau^2)^2 \frac{\beta p^2}{2} \frac{d\tau}{\tau} \left[1 - \frac{\lambda}{32}(1+b^2) \right] \right] \ln\frac{r_{ij}}{\tau}\delta_{l_i, l_j} \right.\right. \\ &\left.\left. - \left[1 - 2(2\pi y \tau^2)^2 \frac{\beta p^2}{2} \frac{d\tau}{\tau} \right] \sqrt{\lambda} \left[\frac{r_{ij} - \tau}{\tau} \delta_{l_i, l_j} - b\frac{r_{ij}}{\tau} \delta l_i, l_{j+1} \right] \right\} \right] , \end{aligned} \tag{A18}$$

where Z' includes the terms that renormalize the overall free energy. For example, the constant terms, which we neglected in the integrations of Eq. (A9), would contribute to Z'.

The final step in our renormalization is to rescale the lengths so that the limits of the integration in Eq. (A18) match those of Eq. (2.3). Making the substitution $r' = r/(1+d\tau/\tau)$ in Eq. (A18), we obtain

$$Z=Z'\sum_N \{y[1+d\tau/\tau(2-\beta p^2/2-\beta p^2\sqrt{\lambda}/2)]\}^{2N}\frac{1}{(N!)^2}\sum_{l_1}\int_{D_1}d^2r_1\sum_{l_2}\int_{D_2}d^2r_2\cdots\sum_{l_{2N}}\int_{D_{2N}}d^2r_{2N}$$
$$\times\exp\left[-\frac{\beta}{2}\sum_{i\neq j}p_ip_j\left\{-\left[1-(2\pi y\tau^2)^2\frac{\beta p^2}{2}\frac{d\tau}{\tau}\left[1-\frac{\lambda}{32}(1+b^2)\right]\right]\ln\frac{r_{ij}}{\tau}\delta_{l_i,l_j}\right.\right.$$
$$\left.\left.-\left[1-2(2\pi y\tau^2)^2\frac{\beta p^2}{2}\frac{d\tau}{\tau}+\frac{d\tau}{\tau}\right]\sqrt{\lambda}\left[\frac{r_{ij}-\tau}{\tau}\delta_{l_i,l_j}-b\frac{r_{ij}}{\tau}\delta_{l_i,l_j+1}\right]\right\}\right], \quad (A19)$$

where we have dropped the primes. The partition function now has the same form as Eq. (2.3), $Z(y,x,\lambda)=Z_0Z(y',(\beta p^2)',\lambda')$, but the parameters have been renormalized:

$$(\beta p^2)'=\beta p^2[1-(2\pi y\tau^2)^2\beta p^2(1-A\lambda)d\tau/2\tau], \quad (A20)$$

$$y'=y\{1+[2-\beta p^2(1-\tfrac{1}{2}\lambda\ln\lambda)/2]d\tau/\tau\}, \quad (A21)$$

$$\sqrt{\lambda'}=\sqrt{\lambda}\{1+[1-\beta p^2(2\pi y\tau^2)^2]d\tau/\tau\}, \quad (A22)$$

where the primed variables are the renormalized parameters and $A=(1+b^2)/32$. In Eq. (A21), $1/2\lambda\ln\lambda$ was used in place of $-\sqrt{\lambda}$ to account for the correct small-R Josephson term in Eq. (2.1). As we have discussed in Secs. II and III, this terms does not have a significant effect on the other recursion relations. This completes our RG study of Eq. (2.3). The final steps to arrive at Eqs. (3.1)–(3.3) are to convert Eqs. (A20)–(A22) to differential form, to make the substitution $x=4/(\beta p^2)-1$, and to absorb the factor $2\pi\tau^2$ into y.

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