Exact solution of the model of degenerate electrons interacting with an impurity

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A model of degenerate electrons interacting with an impurity via the spin-exchange and correlated-hybridized interactions is formulated. We present an exact solution of the SU(ν)-symmetric model Hamiltonian. Exact expressions for ground-state properties, e.g., the valence, the impurity magnetic susceptibility, and the linear temperature coefficient of the specific heat, are calculated on the basis of the Bethe ansatz solutions.

I. INTRODUCTION

An important step towards understanding an apparently simple physical situation, the behavior of localized spin in metals, was achieved by Wiegmann[1] and Andrei[2] who solved the Kondo problem and Anderson model.[3] Wiegmann and Andrei showed that the Hamiltonian of the s-d exchange and Anderson models can be diagonalized by the Bethe ansatz. The solutions were obtained in a continuous approach for a linearized spectrum of conduction electrons assuming the Fermi energy or the bandwidth to be largest of the energy scales. The Anderson model and Kondo problem picture electrons localized at an impurity coupled to the itinerant electrons of a metal. These models have great physical interest since the heavy fermion state of rare-earth and actinide systems has long been associated with the Kondo effect; the Anderson model describes the intermediate-valence state and the formation of the localized magnetic moment in a metal.

Solutions were formulated to the orbitally degenerate extensions of these models, namely, the Coqblin-Schrieffer[4,5] and infinite-U Anderson models.[6] The inclusion of realistic features such as the orbital degeneracy and crystal-field splitting in the models permits one to fit the experimental results to results of calculations. The results of calculations for the magnetization and specific heat were used to provide reasonable fits to compounds such as $Ce_xLa_{1-x}B_6$ and $CeAl_3$,[7] YbCuAl and to dilute systems such as ThCe.[8]

The modification of the Anderson model determined for two arbitrary configurations of an impurity shell, e.g., $4f^n$ and $4f^{n+1}$, was considered in Ref. 9. The model describing the exchange and the correlated-hybridized interactions between electron states localized at an impurity and conduction electrons of a metal was proposed in Ref. 10. In this paper we propose a modification of this model, namely, the model of conduction electrons of a metal interacting via the exchange and hybridized interactions with a orbital degenerate shell of an impurity atom. The inclusion of realistic features such as the orbital degeneracy of electrons in the Hamiltonian proposed by the author earlier[10] permits a more realistic single-impurity model to be built. Such a model has interest and direct relevance to rare-earth impurity systems.

II. BETHE ANSATZ EQUATIONS

Let us consider a model describing the behavior of a localized moment in a paramagnetic host metal in which highly correlated degenerated conduction states of a metal interact with a magnetic moment of an impurity. The model consists of a orbital degenerate level, which can be singly or doubly occupied, and is hybridized, and the exchange interacts with the conduction electron states. As shown in this paper for a linear dispersion of the conduction states the model is integrable by means of the Bethe ansatz. The Hamiltonian of the model considered is

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1, \tag{1}$$

$$\mathcal{H}_0 = \sum_{\sigma}\sum_{m}\int dx\left[-ic^{\dagger}_{\sigma m}(x)\frac{\partial}{\partial x}c_{\sigma m}(x) - \varepsilon_F c^{\dagger}_{\sigma m}(x)c_{\sigma m}(x)\right] + \varepsilon_d\sum_{\sigma}\sum_{m} n_{\sigma m} + U/2\sum_{\sigma,\sigma'}\sum_{m,m'} n_{\sigma m}n_{\sigma' m'} + \Delta\mathcal{H}, \tag{2}$$

$$\mathcal{H}_1 = J\sum_{\sigma,\sigma'}\sum_{m,m'}\int dx\delta(x)c^{\dagger}_{\sigma m}(x)c_{\sigma' m'}(x)d^{\dagger}_{\sigma' m'}d_{\sigma m} + \sum_{\sigma,\sigma'}\sum_{m,m'}\int dx\delta(x)[Wc^{\dagger}_{\sigma m}(x)d_{\sigma m}n_{\sigma' m'} + \text{H.c.}], \tag{3}$$

where $c^{\dagger}_{\sigma m}(x)$ and $c_{\sigma m}(x)$ are the creation and annihilation operators of the conduction electrons of spin σ ($\sigma = \uparrow, \downarrow$) at x in the band m ($m = 1, 2, ..., f$), $d^{\dagger}_{\sigma m}$ and $d_{\sigma m}$ are the operators of electrons localized at the impurity located at $x = 0$, $n_{\sigma m}$ is the number of spin-σ and orbital index-m electrons localized at an impurity, ε_d is the energy of the one-electron impurity level measured from the Fermi energy ε_F, U is the on-site Coulomb repulsion, and W is the constant of the correlated-hybridized interaction. The additional term $\Delta\mathcal{H}$ excludes all configurations of the impurity shell with more than two electrons. The addition of such a term in (2) is necessary for the factorization of a many-electron scattering process into

two-electron scattering matrices.

The kinetic term of the Hamiltonian (2) is written for the dispersion of band electrons linearized near the Fermi energy. In this approximation the conduction electrons interact only with the same sign of a momenta (right- or left-going wave numbers). By setting the Fermi velocity equal to unity, the coupling constants in (2) and (3) become dimensionless and the conduction electron density of states at the Fermi level $\rho_F = \frac{1}{2\pi}$ in our units.

Instead of directly considering the model Hamiltonian it turns out to be more fruitful to study a diagonalization of the T_j matrix related to (1)–(3). The construction of the T_j matrix is standard; it is built from two-particle scattering matrices: electrons on impurity and mutual scattering of electrons. After some manipulations we obtain the expression for the scattering matrix of electrons on an impurity:

$$R_{j0} = \frac{g(k_j) - P_{j0}}{g(k_j) - i} \exp(i\phi), \tag{4}$$

$$g(k) = \frac{k - \varepsilon_F - \varepsilon + |W|^2 c/2}{c(k - \varepsilon_F - \varepsilon - |W|^2/J)}, \tag{5}$$

where

$$c = \frac{J}{1 - J^2/4}, \quad \phi = -2\tan^{-1}(J/2),$$

and P_{j0} is the permutation operator, $P_{j0} = P^{\sigma}_{j0} P^{m}_{j0}$; here P^{σ}_{j0} and P^{m}_{j0} are the permutation operators for spin and orbital indices of the conduction and localized electrons, $\varepsilon = \varepsilon_d + U$.

Note that the phase shifts are additive at the scattering of electrons on impurities; thus the electrons are scattered independently and the result of scattering is independent of the relative positions of impurity atoms.

The form of the T_j matrix in our case is determined by the two-electron scattering matrix of the conduction electrons (the S matrix). Considering the scattering processes of two electrons on the impurity we get the factorization equations[1,2]

$$R_{i0} R_{j0} S_{ji} = S_{ji} R_{j0} R_{i0}, \quad S_{ij} S_{il} S_{jl} = S_{jl} S_{il} S_{ij}, \tag{6}$$

which are easily verified to hold for the S matrix. From Eqs. (6) we obtain the solution for the matrix of mutual scattering of band electrons,

$$S_{ij} = \frac{g(k_i) - g(k_j) - iP_{ij}}{g(k_i) - g(k_j) - i}, \tag{7}$$

where P_{ij} is the permutation operator for spin and orbital indices of the scattered electrons.

Taking into account the factorization equations (6) the Hamiltonian (1)–(3) is diagonalized following standard procedure.[11] On imposing the periodic boundary conditions the $(N_e + 1)$-particle problem is solved by means of the nested Bethe ansatz. The T_j matrix has the following form:

$$T_j = S_{jj+1} S_{jj+2} \cdots S_{jN_e} R_{j0} S_{j1} S_{j2} \cdots S_{jj-1}. \tag{8}$$

The band m and spin σ indices can be considered as one color index $r = 1, 2, ..., 2f$; in such a manner according to (4),(7),(8) the model Hamiltonian is SU(ν) invariant ($\nu = 2f$).

We may use the quantum method of the inverse problem to obtain the eigenvalues of the T_j matrix (8). All solutions are characterized by N_e momenta k_j ($j = 1, 2, ..., N_e$) of charge spinless excitations and sets of rapidities $\lambda^{(r)}_{\alpha}$ ($r = 1, 2, ..., \nu - 1$) which are coupled via the Bethe ansatz equations

$$\exp(ik_j L + i\phi) = \prod_{\alpha=1}^{M_1} \frac{g(k_j) - \lambda^{(1)}_{\alpha} - i/2}{g(k_j) - \lambda^{(1)}_{\alpha} + i/2}, \tag{9}$$

$$\prod_{j=1}^{N_e} \frac{\lambda^{(1)}_{\alpha} - g(k_j) + i/2}{\lambda^{(1)}_{\alpha} - g(k_j) - i/2} \prod_{\delta=1}^{M_2} \frac{\lambda^{(1)}_{\alpha} - \lambda^{(2)}_{\delta} + i/2}{\lambda^{(1)}_{\alpha} - \lambda^{(2)}_{\delta} - i/2} = -\prod_{\beta=1}^{M_1} \frac{\lambda^{(1)}_{\alpha} - \lambda^{(1)}_{\beta} + i}{\lambda^{(1)}_{\alpha} - \lambda^{(1)}_{\beta} - i}, \tag{10}$$

$$\prod_{\gamma=1}^{M_{r-1}} \frac{\lambda^{(r)}_{\alpha} - \lambda^{(r-1)}_{\gamma} + i/2}{\lambda^{(r)}_{\alpha} - \lambda^{(r-1)}_{\gamma} - i/2} \prod_{\delta=1}^{M_{r+1}} \frac{\lambda^{(r)}_{\alpha} - \lambda^{(r+1)}_{\delta} + i/2}{\lambda^{(r)}_{\alpha} - \lambda^{(r+1)}_{\delta} - i/2} = -\prod_{\beta=1}^{M_r} \frac{\lambda^{(r)}_{\alpha} - \lambda^{(r)}_{\beta} + i}{\lambda^{(r)}_{\alpha} - \lambda^{(r)}_{\beta} - i} \quad \text{for} \quad 1 < r < \nu, \tag{11}$$

where M_r is the number of rapidities in the set $\{\lambda^{(r)}_{\alpha}\}$, $M_{\nu} = 0$; the number of particles with color r is equal to $n_r = M_{r-1} - M_r$.

The energy of the $(N_e + 1)$-particle system is given by the sum of all momenta,

$$E = \sum_{j=1}^{N_e} k_j - \varepsilon_F N_e + \varepsilon_d. \tag{12}$$

The total magnetization can be found as

$$\mathcal{M} = \frac{\nu - 1}{2} N - \sum_{r=1}^{\nu-1} M_r; \tag{13}$$

here N is the total number of particles.

Equations (9)–(11) together with expressions (12),(13) for the energy and the magnetization describe explicitly the ground state of the model for an arbitrary density of electrons and coupling constants.

III. GROUND STATE

The Bethe ansatz equations have real for k_j and complex solutions for $\lambda^{(r)}_{\alpha}$. In the ground state, however, all $\lambda^{(r)}_{\alpha}$ take real values. In the thermodynamic limit the rapidities are closely spaced, so that we can introduce the distribution functions $\rho(k)$ and $\sigma^{(r)}(\lambda)$ for k and $\lambda^{(r)}$, respectively. The corresponding hole distribution functions are $\rho_h(k)$ and $\sigma^{(r)}_h(\lambda)$. Note that $\rho_h(k) \neq 0$ for $k_F < k < k_1$ (where k_F is the Fermi momentum and k_1 is the conduction bandwidth) and $\sigma^{(r)}_h(\lambda) \neq 0$ if $-\infty < \lambda < -B_r$. By taking the continuum limit of Eqs. (9)–(11) the Bethe equations corresponding to the ground state can be transform into linear integral equations

$$\rho(k)+\rho_h(k)=\frac{1}{2\pi}-g'(k)\int_{-B_r}^{\infty}d\lambda a_1[\lambda-g(k)]\sigma^{(1)}(\lambda), \tag{14}$$

$$\sigma^{(1)}(\lambda)+\sigma_h^{(1)}(\lambda)+a_2*\sigma^{(1)}(\lambda)=\int_0^{k_F}dk a_1[\lambda-g(k)]\rho(k)+\frac{1}{L}a_1(\lambda)+a_1*\sigma^{(2)}(\lambda), \tag{15}$$

$$\sigma^{(r)}(\lambda)+\sigma_h^{(r)}(\lambda)+a_2*\sigma^{(r)}(\lambda)=a_1*\sigma^{(r-1)}(\lambda)+a_1*\sigma^{(r+1)}(\lambda)\ \text{ for } 1<r<\nu, \tag{16}$$

where $\sigma^{(\nu)}(\lambda)=0$ and the function $a_n(\lambda)$ used in (14)–(16) is defined as

$$a_n(\lambda)=\frac{n}{2\pi}\frac{1}{\lambda^2+(n/2)^2}. \tag{17}$$

The symbol $a*f(\lambda)$ denotes the convolution of the functions $a(\lambda)$ and $f(\lambda)$,

$$a*f(\lambda)=\int_{-\infty}^{\infty}d\lambda' a(\lambda-\lambda')f(\lambda').$$

Equations (14)–(16) determine all the distribution functions as a function of the integration limits k_F and B_r $(r=1,2,...,\nu-1)$. The integration limits are determined from the numbers of conduction electrons with each spin and orbital component. Let n_r be the density of conduction electrons with color component r; then

$$n_1=\int_0^{k_F}dk\rho_e(k)-\int_{-B_1}^{\infty}d\lambda\sigma_e^{(1)}(\lambda),$$

$$n_r=\int_{-B_{r-1}}^{\infty}d\lambda\sigma_e^{(r-1)}(\lambda)-\int_{-B_r}^{\infty}d\lambda\sigma_e^{(r)}(\lambda)\quad\text{for } 1<r<\nu, \tag{18}$$

and

$$n_e=N_e/L=\int_0^{k_F}dk\rho_e(k), \tag{19}$$

where $\rho(k)=\rho_e(k)+\frac{1}{L}\rho_i(k)$ and $\sigma^{(r)}(\lambda)=\sigma_e^{(r)}(\lambda)+\frac{1}{L}\sigma_i^{(r)}(\lambda)$. The distribution functions can be divided into a host and impurity part; n_e is the density of the conduction electrons.

After some algebraic manipulations Bethe ansatz equations yield then the set of integral equations

$$\rho(k)+\rho_h(k)=\frac{1}{2\pi}-g'(k)\int_0^{k_F}dk'\mathcal{L}[g(k)-g(k')]\rho(k')-\frac{1}{L}g'(k)\mathcal{L}[g(k)]$$
$$+g'(k)\sum_{r=1}^{\nu-1}\int_{-\infty}^{-B_r}d\lambda\mathcal{F}_r[\lambda-g(k)]\sigma_h^{(r)}(\lambda), \tag{20}$$

$$\sigma^{(r)}(\lambda)+\sum_{m=1}^{\nu-1}\mathcal{G}_{rm}*\sigma_h^{(m)}(\lambda)=\int_0^{k_F}dk\mathcal{F}_r[\lambda-g(k)]\rho(k)+\frac{1}{L}\mathcal{F}_r(\lambda), \tag{21}$$

$$\mathcal{L}(\lambda)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\,\exp(-|\omega|/2)\frac{\sinh[\omega(\nu-1)/2]}{\sinh(\omega\nu/2)}\exp(i\omega\lambda), \tag{22}$$

$$\mathcal{F}_r(\lambda)=\frac{1}{\nu}\frac{\sin(\pi r/\nu)}{\cosh(2\pi\lambda/\nu)-\cos(\pi r/\nu)}, \tag{23}$$

where $\mathcal{G}_{rm}(\lambda)$ is an integral operator whose kernel, in momentum space, is

$$\mathcal{G}_{rm}(\omega)=\exp(|\omega|/2)\frac{\sinh[\min(r,m)\omega/2]\sinh\{[\nu-\max(r,m)]\omega/2\}}{\sinh(\omega/2)\sinh(\omega\nu/2)}. \tag{24}$$

Below we consider a valence of an impurity ion at $H=0$ (H is an external magnetic field). In the absence of a magnetic field it is easily seen that all $B_r=\infty$. Equation (20) is transformed into the following equation for the function $\rho(k)$ at $H=0$ denoted as $\rho_0(k)$:

$$\rho_0(k)+\rho_{h0}(k)+g'(k)\int_0^{k_F}dk'\mathcal{L}[g(k)-g(k')]\rho_0(k')$$
$$=\frac{1}{2\pi}-\frac{1}{L}g'(k)\mathcal{L}[g(k)]. \tag{25}$$

The value of $g(k)$ tends to zero for a momentum equal to $k_0=\varepsilon_F+\varepsilon-|W|^2c/2$ and tends to infinity at the point $k_\infty=\varepsilon_F+\varepsilon+|W|^2/J$. In contrast to the Anderson model a strong dispersion of $g(k)$ has taken place. Two branches of values of $g(k)$ are realized for $0<k<k_1$, namely, for $0<k<k_\infty$ and $k_\infty<k<k_1$. The valence of an impurity atom depends of the value of $g(k)$ at $k=k_F$; the state of an intermediate valence is determined by the region of values of k_F at close range of a strong dispersion of $g(k)$. Let us consider the solution of Eq. (25) for the state of an impurity with a localized moment

assuming that $\varepsilon = k_F - \varepsilon_F - |W|^2/J$ and $J < 0$. For this case we can solve the integral equation (25) using standard Wiener-Hopf technique. We find this solution in the weak interaction limit at $|W|^2/|(\varepsilon_F - \varepsilon)J| \ll 1$ and $|W|^2/|(k_1 - \varepsilon_F - \varepsilon)J| \ll 1$ approximation.[10] The key to the solution of this equation is to find a decomposition of the kernel of the integral equation (25) into factors $G_\pm(\omega)$ that are analytic in the upper and lower ω planes, respectively,

$$[1 - \mathcal{L}(\omega)]^{-1} = G_+(\omega)G_-(\omega), \quad \lim_{\omega\to\infty} G_\pm(\omega) = 1,$$

where

$$G_-(\omega) = G_+(-\omega) = \sqrt{\nu}\left(\frac{i\omega + 0}{2\pi e}\right)^{\frac{i\omega(\nu-1)}{2\pi}} \times \frac{\Gamma[1 + i\omega/(2\pi)]}{\Gamma[1 + i\omega\nu/(2\pi)]}\exp\left(\frac{i\omega}{2\pi}\nu\ln\nu\right). \quad (26)$$

$\Gamma(x)$ is the gamma function.

The density of the conduction electrons is related to the dimensionless parameter k_F via the following equation:

$$n_e = \frac{\nu k_F}{2\pi} - \frac{\sqrt{\nu}}{2\pi^2}\int_0^\infty \frac{d\omega}{\omega}\,\frac{\sin(\omega\nu/2)\sin[\omega(\nu-1)/2]}{\sin(\omega/2)G_+(i\omega)} \times \int_0^{k_F} dk\,\exp\{\omega[g(k) - 1/c]\}. \quad (27)$$

The valence of an impurity is obtained from the solution of Eq. (25):

$$n_i = 1 + \frac{\sqrt{\nu}}{\pi}\int_0^\infty \frac{\sin(\omega/2)\sin[\omega(\nu-1)/2]}{\sin(\omega\nu/2)} \times\exp(\omega/c)G_+(i\omega) - \frac{\sqrt{\nu}}{2\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2\pi n/\nu)\exp[2\pi n/(\nu c)]G_+(i2\pi n/\nu). \quad (28)$$

This solution is obtained for $\varepsilon > 0$ when a local magnetic moment is formed.

We calculate the behavior of a localized magnetic moment in a small magnetic field. According to Eq. (21) the driving terms for the host and impurity parts of the distribution functions have the same $\sin(\pi r/\nu)$ dependence on r and the exponential asymptotics for large λ; hence the impurity and host solutions for $\sigma_n^{(r)}(\lambda)$ are proportional for small magnetic fields. This leads to the following relation between the impurity and host magnetic susceptibilities:

$$\frac{\chi_{\rm imp}}{\chi_{\rm host}} = \frac{2}{LT_K}, \quad (29)$$

where T_K is the Kondo temperature,

$$T_K = 2\int_0^{k_F} dk\,\exp[-2\pi g(k)/\nu]\rho_0(k). \quad (30)$$

Since the magnetic field corrections to $\rho(k)$ are of order H^2, we use the zero-field solution for $\rho(k)$ in (30). The value of T_K is very small compared to ε_F, such that the function of $g(k)$ is positive for $0 < k < k_F$. This factor leads to the following inequalities, which were obtained in Ref. 10:

$$1 + \frac{|W|^2 c}{2(k_F - \varepsilon_F - \varepsilon)} > 0 \quad \text{and} \quad c - \frac{c|W|^2}{J(k_F - \varepsilon_F - \varepsilon)} > 0.$$

Note that the magnetic susceptibility is just the Coqblin-Schrieffer or degenerate exchange model[4,5] with the Kondo temperature defined above and the mixed-valence region corresponds to the strong dispersion of function of $g(k)$ near k_F.

IV. THERMODYNAMIC EQUATIONS

The asymptotic solutions of Eqs. (9)–(11), within $L \to \infty$, lie in the complex plane and form a string,

$$\lambda_{nk\alpha}^{(r)} = \lambda_{n\alpha}^{(r)} + i(n + 1 - 2k)/2 + 0[\exp(-\delta L)], \quad k = 1, 2, ..., n, \quad \delta > 0, \quad (31)$$

which is characterized by a common real abscissa $\lambda_{n\alpha}^{(r)}$ and the string length n.

The solutions for the excitations are the same as for the degenerate exchange model.

The equilibrium distribution functions at temperature T can be obtained by minimizing the thermodynamic potential $\Omega = E - H\mathcal{M} - \varepsilon_F N - T\mathcal{S}$ ($\mathcal{S}$ is the entropy) with respect to the distribution functions subject to the Bethe equations. The density of the thermodynamic potential in terms of the newly defined distribution functions is given by

$$\Omega/N = \mathcal{E}_0 + T\int_0^{k_1} dk\rho_0(k)\ln\{n[K(k)]\} + T\sum_{r=1}^{\nu-1}\int_{-\infty}^{\infty} d\lambda\sigma_0^{(r)}(\lambda)\ln\{n[\varepsilon_1^{(r)}(\lambda)]\}, \quad (32)$$

where $\mathcal{E}_0$ is the density of the ground-state energy at $H = 0$, $n(\varepsilon) = [1 + \exp(\varepsilon/T)]^{-1}$ is the Fermi distribution function, and $\sigma_0^{(r)}(\lambda)$ are the ground-state distribution functions independent of H and T.

The equations determining the thermodynamics of the model are conveniently written as

$$K(k) = k - \varepsilon_F - H(\nu - 1)(1 - \nu/2) + T\sum_{r=1}^{\nu-1}\int_{-\infty}^{\infty} d\lambda\mathcal{F}_r[g(k) - \lambda]\ln\{n[\varepsilon_1^{(r)}(\lambda)]\} - T\int_0^{k_1} dk'g'(k')\mathcal{L}[g(k) - g(k')]\ln\{n[-K(k')]\}, \quad (33)$$

$$\varepsilon_1^{(r)}(\lambda) = -Ts * \ln\{n[\varepsilon_2^{(r)}(\lambda)]\} + Ts * \ln\{n[-\varepsilon_1^{(r+1)}(\lambda)]n[-\varepsilon_1^{(r-1)}(\lambda)]\} - T\delta_{r1}\int_0^{k_1} dk g'(k) s[\lambda - g(k)]\ln\{n[-K(k)]\}, \tag{34}$$

$$\varepsilon_n^{(r)}(\lambda) = -Ts * \ln\{n[\varepsilon_{n+1}^{(r)}(\lambda)]n[(\varepsilon_{n-1}^{(r)}(\lambda)]\} + Ts * \ln\{n[-\varepsilon_n^{(r+1)}(\lambda)]n[-\varepsilon_n^{(r-1)}(\lambda)]\} \text{ for } n > 1, \tag{35}$$

where $s(\lambda) = \frac{1}{2\cosh(\pi\lambda)}$.

The quasienergies are required to satisfy the boundary conditions

$$\lim_{n\to\infty}[\varepsilon_{n+1}^{(r)}(\lambda) - \varepsilon_n^{(r)}(\lambda)] = H.$$

The ground-state configuration corresponds to the filling of all states with $K(k) < 0$ and $\varepsilon_1^{(r)}(\lambda) < 0$. Consequently, the conditions

$$K(k_F) = 0, \quad \varepsilon_1^{(r)}(B_r) = 0$$

provide another way to define the values of k_F and B_r for the ground state for the given magnetic field and the Fermi energy. The qualitative behavior of the quasienergies is most easily understood by writing (33)–(35) in the form

$$\ln\{n[-\varepsilon_1^{(r)}(\lambda)]\} = \sum_{m=1}^{\nu-1} \mathcal{G}_{rm} * (s^{-1} * \ln\{n[\varepsilon_1^{(r)}(\lambda)]\} - \ln\{n[\varepsilon_2^{(r)}(\lambda)]\}) - \int_0^{k_1} dk g'(k)\mathcal{F}_r[\lambda - g(k)]\ln\{n[-K(k)]\}, \tag{36}$$

$$\ln\{n[-\varepsilon_n^{(r)}(\lambda)]\} = \sum_{m=1}^{\nu-1} \mathcal{G}_{rm} * (s^{-1} * \ln\{n[\varepsilon_n^{(r)}(\lambda)]\} - \ln\{n[\varepsilon_{n+1}^{(r)}(\lambda)]n[\varepsilon_{n-1}^{(r)}(\lambda)]\}) \text{ for } n > 1. \tag{37}$$

We are interested in the thermodynamics in the scaling regime $H, T \ll \varepsilon_F, 1$. The Kondo limit is obtained by taking into account the spin excitations at low energies and suppressing the charge excitations. In this approximation Eq. (36) for the thermodynamic function $\varepsilon_1^{(r)}(\lambda)$ takes the form

$$\ln\{n[-\varepsilon_1^{(r)}(\xi)]\} = -\frac{2}{\nu}\sin(\pi r/\nu)\exp\left(\frac{2\pi}{\nu}\xi\right) + \sum_{m=1}^{\nu-1} \mathcal{G}_{rm} * (s^{-1} * \ln\{n[\varepsilon_1^{(r)}(\xi)]\} - \ln\{n[\varepsilon_2^{(r)}(\xi)]\}), \tag{38}$$

and Eqs. (37),(38) become universal, $\xi = \lambda - \frac{\nu}{2\pi}\ln(T/T_K)$. According to (32) the impurity part of the thermodynamic potential is then given by

$$\Omega_{\text{imp}}^{\text{sp}} = T\sum_{r=1}^{\nu-1}\int_{-\infty}^{\infty} d\xi \mathcal{F}_r\left[\xi + \frac{\nu}{2\pi}\ln(T/T_K)\right] \times \ln\{n[\varepsilon_1^{(r)}(\xi)]\}. \tag{39}$$

Equations (37)–(39) are just the thermodynamic equations for the Coqblin-Schrieffer model as derived by Tsvelick and Wiegmann.[4]

Let us consider the density of the thermodynamic potential in low-temperature limit in which $H \to 0, T \to 0$ and H/T is fixed. Write $K(k) = K_0(k) + T^2\delta K(k)$, where $K_0(k)$ is the ground-state solution and $\delta K(k)$ is the solution of the equation

$$\delta K(k) + \int_0^{k_F} dk' g'(k')\mathcal{L}[g(k) - g(k')]\delta K(k') = \frac{\pi^2}{6} g'(k_F)\left|\frac{dK_0(k)}{dk}\right|^{-1}\mathcal{L}[g(k) - g(k_F)] + \frac{1}{T}\sum_{r=1}^{\nu-1}\int_{-\infty}^{\infty} d\lambda \mathcal{F}_r[g(k) - \lambda]\ln\{n[\varepsilon_1^{(r)}(\lambda)]\}. \tag{40}$$

Substituting the low-temperature asymptotics of the quasienergies into (32) we obtain the impurity part of specific heat coefficient at $T \to 0$,

$$\gamma_{\text{imp}} = \gamma_{\text{imp}}^{\text{CS}} + \frac{2\pi^2}{3}\chi_{\text{imp}}^{\text{ch}}, \tag{41}$$

where $\gamma_{\text{imp}}^{\text{CS}}$ is the low-temperature special heat calculated in the Coqblin-Schrieffer model[4,11] and $\chi_{\text{imp}}^{\text{ch}}$ is the charge susceptibility.

The structure of this expression is quite natural from the point of view of Fermi liquid theory. In the case when the Dirac seas are filled by momenta and strings, there are two Fermi liquids with different sound velocities. The low-temperature specific heat is simply the sum of the specific heats of all Fermi liquids.

V. SUMMARY AND CONCLUSIONS

We obtained the ground-state properties of the SU(ν)-symmetric impurity model. The model Hamiltonian takes into account both the exchange interaction between degenerated conduction electrons of a metal and electrons localized at an impurity and the hybridization of

localized and band electronic states. The charge states in the system are determined by the real solutions of the momenta of electrons in contrast to the Anderson model. The solutions of the Bethe ansatz equations of the model and the ones in the Coqblin-Schrieffer model have a similar structure and the spin analogy between these models has frequently been invoked.

At low temperatures the infinite set of coupled nonlinear integral equations that arise from the Bethe ansatz solution have been solved using standard iteration procedure. The linear specific heat coefficient has been obtained analytically.

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