# Normal-state transport properties in the flux-binding phase of the $t$-$J$ model

Z.Y. Weng, D.N. Sheng, and C.S. Ting
*Texas Center for Superconductivity and Department of Physics, University of Houston, Houston, Texas 77204-5506*
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The normal-state transport properties are studied in the flux-binding phase of the $t$-$J$ model within the framework of gauge theory. We find that the resistivity is linearly temperature dependent with a relaxation rate $\hbar/\tau \simeq 2k_BT$. We also show that the Hall coefficient involves a second relaxation rate and the cotangent Hall angle follows a $T^2$ law. Furthermore, the thermopower exhibits a strong-doping dependence. All these features are in good agreement with the transport measurements of the high-$T_c$ copper-oxide compounds.

After several years efforts since the discovery of the high-$T_c$ copper-oxide superconductors, the experiments now have achieved a great deal of consensus about the anomalous normal-state transport properties in these materials. For example, it is well known that the resistivity in the $CuO_2$ layers increases linearly in temperature for all the hole-doped compounds in the optimal $T_c$ regime. Combined with the optical measurements,[1] such a temperature dependence of resistivity has been related to a linear-$T$ dependence of the scattering rate $\hbar/\tau \simeq 2k_BT$. A linear-frequency dependence of $\hbar/\tau(\omega)$ at $\omega > T$ has also been implied in the infrared spectroscopy[1] up to 0.15 eV. The Hall measurements show the hole characteristic with a strong temperature anomaly.[2] The recent Hall angle concept proposed by the Princeton group,[3,4] with the involvement of a second scattering rate, has given an excellent account for the temperature dependence as well as the impurity effects.[4] The thermopower in these compounds exhibits[5] a monotonic decrease with increasing doping and its sign could even change in the overdoped regime.

Several strong-correlation-based normal-state theories have been developed under the inspiration of the experiments. Among them the gauge field theory[6,7] for the uniform resonant-valence-bond (RVB) state and Anderson's two-dimensional (2D) Tomonaga-Luttinger liquid theory[8,3] have attracted much attention. Nevertheless, a full and systematic understanding of the aforementioned transport properties has not yet been attained within the framework of these approaches.

Recently a so-called flux-binding phase in the $t$-$J$ model has been investigated.[9] The physical origin of this phase shares some similarities with the gauge theory of the commensurate-flux phase.[10] Such a phase exhibits the superconducting condensate in the ground state just like in a semion system. But the time-reversal (T) and parity (P) symmetries, which are usually violated in a fractional statistics system, are found to be restored due to the cancellation between the charge and spin degrees of freedom.[9]

In this paper we study the transport properties of the normal state in the flux-binding phase. We shall find a simple and consistent explanation of all the universal anomalous phenomena mentioned above by using the gauge-theory method.

The flux-binding phase[9] is composed of three subsystems: The spin degree of freedom is described by a semion gas with spin (spinon), while the charge is associated with a boson gas (holon); a third degree of freedom also involves a semion gas (eon), which may be interpreted as the backflow of holons and takes care of the frustration imposed on holons after optimizing the spin correlations by flux binding. These subsystems are connected by gauge fields which are well known[6,7] for enforcing the density and current constraints in a decomposition formulation. The real excitations can be dramatically different from these constituents due to the constraints. It turns out that charge and spin excitations are separated in the flux-binding phase. Their dynamics become rather simple in a normal state where the Bose condensation is gone for holons while the spinons and eons remain condensed. As only the normal-state transport properties are concerned in this paper, we will start by giving a brief description of such charge excitations and leave the details of demonstration to a separate publication.[11]

A spinless charge excitation in the flux-binding phase is a composite particle made up of holelike excitations $p$ and $q$ in the spinon and eon subsystems, which are bound together with an excited holon due to the density constraint. The $p$ and $q$ species are similar to the charge-fluxoid excitations[12] in a semion gas, except that the quantized flux tubes bound to the charges are produced by the internal gauge fields instead of the external magnetic field. The usual charge-vortex excitations[13] in a semion gas are forbidden in the present phase due to the violations of the density and current constraints.[11] The normal state is obtained when enough holons get excited as bound to these composite excitations and no residual holons of a macroscopic number stay in the Bose condensation. Above the transition temperature, no more charge excitations are possible and the semionic condensations in the spinon and eon subsystems can last a wide range of temperature which sustain the present exotic normal state. The effective Lagrangian for such a composite charge fluid in the normal state is described by $\mathcal{L}_n = \mathcal{L}_n^0 + \mathcal{L}_n^1$ with

$$\mathcal{L}_n^0 = \int d^2\mathbf{r}\{h^\dagger[\partial_\tau + a_0^{\rm ext} - (\lambda+\beta)]h + p^\dagger[\partial_\tau + \lambda]p + q^\dagger[\partial_\tau + \beta]q\}, \quad (1a)$$

 

$$\mathcal{L}_n^1 = \int d^2\mathbf{r} \left\{ h^\dagger \frac{(-i\nabla - \boldsymbol{a}^{\rm ext})^2}{2m_h} h + p^\dagger \frac{(-i\nabla - \mathbf{A})^2}{2m_p} p + q^\dagger \frac{(-i\nabla + \mathbf{A})^2}{2m_q} q \right\}. \tag{1b}$$

The $p$ and $q$ fields describing the charge-fluxoid excitations[11,12] will always "see" the fictitious flux tubes carried by the background spinons and eons as well as the fluxoids bound to other $p$'s and $q$'s, respectively. The total fluxes are described by the vector potential $\pm\mathbf{A}$ in Eq. (1b). Since $p$ and $q$ species have finite sizes which in general are larger than the lattice spacing, a continuum description of $p$ and $q$ should be appropriate as given in the Lagrangian (1b), where $m_h \simeq (2t_h a^2)^{-1}$, $m_p \simeq (2Ja^2)^{-1}$, and $m_q \simeq (2\delta t_h a^2)^{-1}$ with $\delta$ as the doping concentration, $a$ as the lattice constant, and $t_h \sim t$. In the normal state with the numbers of $p$ and $q$ equal to the total hole number, a direct counting, with incorporating the density constraint,[14] shows that the fictitious magnetic fields corresponding to $\pm\mathbf{A}$ become uniform with a strength $H = \pi/a^2$. The degeneracy of the Landau level is then equal to the half of the total electron number and is large enough to accommodate $p$ or $q$ species at small doping, which are confined in the lowest Landau level (LLL) as the hole-type excitations.

In Eq. (1a) three species $h$, $p$, and $q$ are bound together as a composite particle by the density constraint

$$h^\dagger(r)h(r) = p^\dagger(r)p(r) = q^\dagger(r)q(r), \tag{2}$$

through the Lagrangian multipliers $\lambda$ and $\beta$. [A minimum scale for the constraint (2) to hold is the size of the $p$ and $q$ species which, in terms of (1b), is approximately in the order of the magnetic length $a_0 = H^{-1/2}$, and thus an ultraviolet cutoff $q_c \sim a_0^{-1}$ in the momentum space is needed in (1a)]. Equation (2) is a natural consequence of the original density constraint between the holon and the other two subsystems and leads to the longitudinal current constraint[11]

$$\mathbf{J}_h^l = \mathbf{J}_p^l = \mathbf{J}_q^l \ , \tag{3}$$

where the superscript $l$ denotes the longitudinal components of the currents. An important feature in Eq. (1b) is that the transverse external electromagnetic field $\boldsymbol{a}^{\rm ext}$ is solely applied to the holon part, as a consequence of the fact that the background spinons and eons are still condensed as "superfluids" which show the Meissner effect and expel out the magnetic field. Because of the same reason, the transverse gauge fields remain suppressed[9] whose strengths on the normal components $h$, $p$, and $q$ will be negligible. This leads to the decoupling of the charge composite particles in (1) from the spinon and eon backgrounds, which also implies the charge-spin separation. With the absence of the transverse spatial gauge fields in (1b), there is no constraint similar to Eq. (3) holding for the transverse currents of $h$, $p$, and $q$. This is certainly not contradictory to the original current constraint because a total current in the spinon or eon subsystem is composed of both a superfluid component and a normal one (of $p$ or $q$), and in order to balance a finite transverse current of $\mathbf{J}_h$, a fictitious transverse electric field with an infinitesimal strength would be sufficient to produce the backflow in the superfluid part to satisfy the total current constraint.

The dynamics of the temporal gauge fields $\lambda$ and $\beta$ can be determined after integrating out the quadratic $h$, $p$, and $q$ fields in the Lagrangian (1). The propagators $D^\lambda = -\langle\delta\lambda\delta\lambda\rangle$ and $D^\beta = -\langle\delta\beta\delta\beta\rangle$, where $\delta\lambda = \lambda - \mu_p$ and $\delta\beta = \beta - \mu_q$ with $\mu_p$ and $\mu_q$ as the chemical potentials for $p$ and $q$ species, respectively, are then given by

$$D^{\lambda(\beta)}(q,\omega) = -[(\Pi_h^{-1} + \Pi_{q(p)}^{-1})^{-1} + \Pi_{p(q)}]^{-1}. \tag{4}$$

$\Pi_{h(p,q)}$ is the density-density correlation function for $h$ ($p$, $q$) species, which could all be regarded as fermions here.[11] $\Pi_h$ for a 2D gas is typically of order of $1/t_h$, whereas $\Pi_p$ and $\Pi_q \sim \beta$ as can be seen below. Since we are interested in the temperature range of $\beta = \frac{1}{k_B T} \gg \frac{1}{t_h}$, we shall then approximate $D^\lambda \simeq -1/\Pi_p$ and $D^\beta \simeq -1/\Pi_q$ in the rest of the paper.

To get the polarization function $\Pi_p$, we first note that the $p$ species stay in the LLL. Define $\gamma_{s'sq} \equiv \sum_k \langle s'|k+q\rangle\langle k|s\rangle$ where $|s\rangle$ refers to the degenerate states in the LLL and $|k\rangle$ is an eigenstate for a free particle. Then the "bubble" diagram $\Pi_p = \beta^{-1}\sum|\gamma|^2 G^p G^p$ can be written down in real frequency as

$$\Pi_p^R(q,\omega) = \sum_{s's} |\gamma_{s'sq}|^2 \int_{-\infty}^{\infty} \frac{d\omega' d\omega''}{(2\pi)^2} \frac{f(\omega') - f(\omega'')}{\omega + \omega' - \omega'' + i0^+} \rho_p(s,\omega')\rho_p(s',\omega'') \ , \tag{5}$$

where $f(\omega) = 1/(e^{\beta\omega}+1)$ and $\rho_p$ is the spectral weight function of the $p$-particle Green function $G^p$. If there is no broadening in the Landau level, i.e., $\rho_p(s,\omega) = 2\pi\delta(\omega - \omega_0^p)$ ($\omega_0^p = \frac{1}{2}\omega_c^p - \mu_p$, where $\omega_c^p = \frac{H}{m_p}$ is the cyclotron frequency), then (5) would show that $\Pi_p^R(q,w) = 0$ for $\omega \neq 0$. The same also happens to $\Pi_q^R$. According to (4), however, $D^{\lambda,\beta}$ would thus become divergent at $|\omega| > 0$. Coupling to such strong fluctuations of $D^{\lambda,\beta}$ in (1) would force the Landau levels of $p$ and $q$ broadened. This procedure, of course, has to be treated in a self-consistent way as follows. One may assume, at the beginning, a broadening $\Gamma_p(\omega)$ for $\rho_p$:

$$\rho_p(\omega) = \frac{2\Gamma_p(\omega - \omega_0^p)}{\Gamma_p^2(\omega - \omega_0^p) + (\omega - \omega_0^p)^2}. \tag{6}$$

Then the broadening $\Gamma_p(\omega - \omega_0^p) = -\mathrm{Im}\Sigma^p(\omega)$ is determined by (without including the vortex correction):

$$\mathrm{Im}\Sigma_R^p(s,\omega) = -\frac{1}{2}\sum_{qs'} |\gamma_{s'sq}|^2 \int \frac{d\Omega}{2\pi}[n(\Omega) + f(\omega+\Omega)] \times \rho_p(\omega+\Omega)P^\lambda(q,\Omega), \tag{7}$$

in which $n(\Omega) = (e^{\beta\Omega} - 1)^{-1}$ and $P^\lambda = -2\mathrm{Im}D^\lambda \approx$

$2\mathrm{Im}\Pi_p/|\Pi_p|^2$ . The chemical potential in $\omega_0^p$ is decided by an equation $\delta = 1/(2\pi)\sum_s \int d\omega\rho_p(\omega)f(\omega)$. In these self-consistent equations, one can rescale all the frequencies by $\omega - \omega_0^p = \xi\beta^{-1}$ and find the quantities $\beta\Gamma_p(\xi\beta^{-1}) \equiv \eta(\xi)$ and $\beta\omega_0^p$ to be *independent* of both temperature as well as the coupling constants. Then at $\xi = 0$, one gets

$$\Gamma_p(0) = \eta(0)k_BT. \tag{8}$$

The coefficient $\eta(0)$ has a weak doping dependence as shown in Fig. 1. (An ultraviolet cutoff $q_c \sim 1/a_0$ for the momentum $q$ has been used in the numerical calculation.) A typical curve of $\eta(\xi)$ as a function of $\xi$ is also presented in the inset of Fig. 1, which is approximately linear in $\xi$ at $|\xi| \gg 1$, or equivalently, $\Gamma_p(\omega) \propto |\omega|$ at $|\omega| \gg k_BT$. A similar result holds for the $q$ species. Finally we note that in the limit $\delta \to 0$, the condition that $\Pi_{p,q}$ dominates over $\Pi_h$ will be no longer valid and one needs to retain all of the contributions from $\Pi_{p,q,h}$ in (4). In this case, $\Gamma_{p,q}(0)$ could deviate from the linear-$T$ dependence.

The relaxation rates $\hbar/\tau_{p,q}$ for $p$ and $q$ species due to scattering to the gauge fluctuations should be obtained by inserting the vertex correction in the self-energies like (7), which can be reduced to the well-known $1 - \cos\theta$ factor[6] in the quasielastic case. But since $D^{\lambda,\beta}(q,\Omega)$ have very weak $q$ dependence, $\cos\theta$ essentially has no important effect and $\hbar/\tau_{p,q} \simeq 2\Gamma_{p,q}$. As $\tau_q \approx \tau_p$, one can use a single relaxation time $\tau_s$ to represent them later: $\frac{\hbar}{\tau_s} = 2\eta(0)k_BT$. The transport rate $\hbar/\tau_h$ for $h$ species can be also calculated from scattering with $D^\lambda$ and $D^\beta$. Due to the weak $q$ dependence of $P^\lambda$ and $P^\beta$, we get

$$\frac{\hbar}{\tau_h} = -2\mathrm{Im}\Sigma_R^h(k,0)$$
$$\simeq 2D_h \int d\xi[n(\xi) + f(\xi)][P^\lambda(\xi) + P^\beta(\xi)],$$

where the density of states $D_h = m_h a^2/(2\pi)$. A rescaling on the right-hand side of the above equation gives $\hbar/\tau_h = \eta'(k_BT)^2/t_h$. The numerical value of $\eta'$ is found $\sim 8$ with a weak doping dependence.

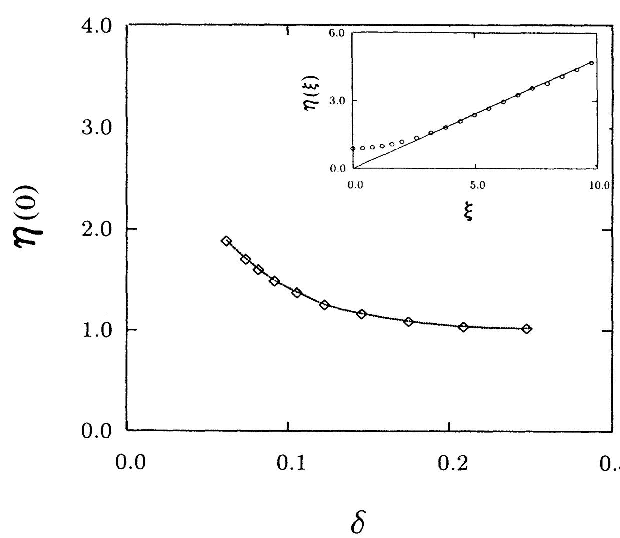


FIG. 1. The coefficient $\eta(0)$ (square) as a function of the doping concentration $\delta$. The inset shows a curve of $\eta(\xi)$ (circle) vs $\xi$ at $\beta\omega_0^p = 0$.

*Resistivity.* Apply the external electric field $\boldsymbol{\varepsilon} = -\nabla \mathbf{a}_0^{\mathrm{ext}}$, with $\mathbf{a}^{\mathrm{ext}} = 0$. $\boldsymbol{\varepsilon}$ will not only act on the holon part because the fields $\lambda$ and $\beta$ will partially transfer the effect to $p$ and $q$ particles: $\lambda \to \lambda + \lambda_{\mathrm{ext}}$, and $\beta \to \beta + \beta_{\mathrm{ext}}$. So the net electric field on $h$ is equal to $\boldsymbol{\varepsilon}_h = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_s$ where $\boldsymbol{\varepsilon}_s = \nabla\lambda_{\mathrm{ext}} + \nabla\beta_{\mathrm{ext}}$. $\boldsymbol{\varepsilon}_h$ and $\boldsymbol{\varepsilon}_s$ will be decided by maintaining the constraint (3). For the $h$ species, the Drude formula of the resistivity is $\rho_h = \frac{m_h}{e^2 n\tau_h}$ where $n$ is the density of hole, and then $\boldsymbol{\varepsilon}_h = \rho_h \mathbf{J}_h^l$. On the other hand, as $p$ and $q$ species are subjected to the fictitious magnetic fields, one would expect the nonzero Hall effect for each of them. However, in terms of the density constraint (2), two wave packets of $p$ and $q$ particles, whose sizes are much smaller than the average distance of holes, have to be bound together during the relaxation time. As each of them sees the fictitious magnetic fields in a different direction in (1), the pure transverse effect will thus be canceled out through the internal force. Using the center-migration theory,[15] one finds that the center coordinate of the wave packets for $p$ and $q$ will be simply accelerated by the external electric field $\boldsymbol{\varepsilon}_s$ added onto them, with an effective mass $m_s = m_p + m_q$. As the relaxation times for $p$ and $q$ are characterized by a single $\tau_s$, the corresponding Drude formula reads $\rho_s = \frac{(m_p+m_q)}{e^2 n\tau_s}$ with $\rho_s$ connected to $\boldsymbol{\varepsilon}_s = \rho_s \mathbf{J}_s^l$, where $\mathbf{J}_s^l = \mathbf{J}_p^l = \mathbf{J}_q^l$. According to the current constraint (3), we find that the electron resistivity satisfies the Ioffe-Larkin-like[6] combination rule $\rho = \rho_h + \rho_s$. Since $\rho_h/\rho_s \simeq (\frac{J}{t})\frac{k_BT}{t} \ll 1$, one therefore obtains the linear-$T$ resistivity

$$\rho \propto k_BT\ . \tag{9}$$

The corresponding relaxation rate is $\hbar/\tau_s \simeq 2k_BT$, in which the coefficient is independent of the coupling strength but with a weak doping dependence (cf. Fig. 1). When $|\omega| \gg k_BT$, one also has $\hbar/\tau_s \propto |\omega|$. All of them agree well with the optical measurements.[1] Generally one expects to see a small $T^2$ upturn in (9) from $\rho_h$ at a sufficiently high temperature.

*Hall effect.* Now apply the external magnetic field $\boldsymbol{a}^{\mathrm{ext}}$. $\boldsymbol{a}^{\mathrm{ext}}$ will solely act on the $h$ species since there is no internal transverse gauge field in (1) to transfer the effect to $p$ and $q$ species. One may use the kinetic equation to write down the off-diagonal coefficient $\rho_{xy}^h$ for $h$ species in the form $\rho_{xx}^h = \rho_h$ and $\rho_{xy}^h = -\rho_h\omega_H\tau_h$ where the cyclotron frequency $\omega_H$ could differ from the bare one $\omega_H^0 = \frac{eB}{cm_h}$ by an enhanced cyclotron mass $m_H$. This is due to the fact that in the presence of the external magnetic field, a wave packet of holon $h$ has to drag the wave packets of the $p$ and $q$ together to go through the cyclotron motion. This drag force cannot make the $p$ and $q$ species performing a coherent cyclotron motion in the external magnetic field but will enhance the effective cyclotron mass of the $h$ species since $p$ and $q$ are confined in the LLL and thus their effective masses are very large. Such a big cyclotron mass is in contrast to the longitudinal transport masses discussed before. Note that the electric field can be directly applied to both the $p$ and $q$ species inside their orbitals which changes the latter into the current-carrying states and, as the consequence, the total longitudinal effects are simply added up in $\rho_s$ as well as in $\rho$.

The total transport coefficients can be determined as follows. The current $\mathbf{J}_h$ are decomposed into the longitudinal and transverse components as $\mathbf{J}_h = \mathbf{J}_h^l + \mathbf{J}_h^t$, with $\mathbf{J}_h^l = \sigma_{xx}^h \boldsymbol{\epsilon}_h$, $\mathbf{J}_h^t = \sigma_{xy}^h(\boldsymbol{\epsilon}_h \times \hat{\mathbf{z}})$. The longitudinal component $\mathbf{J}_h^l$ has to satisfy the current constraint (3), i.e., $\mathbf{J}_h^l = \mathbf{J}_s^l$, which determines the strengths of $\boldsymbol{\epsilon}_h$ and $\boldsymbol{\epsilon}_s$ through $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_h + \boldsymbol{\epsilon}_s$. Then simple algebra will lead to $\rho_{xx} = \rho_h + \rho_s/(1+\cot^{-2}\theta)$, where the cotangent Hall angle $\theta$ defined by $\cot\theta = \rho_{xx}/\rho_{yx}$ is found as

$$\cot\theta = \frac{1}{\omega_H \tau_h} = \alpha T^2 + C, \tag{10}$$

with $\alpha = \eta' k_B^2/(\hbar\omega_H t_h)$ and the constant $C$ originating from the scattering of $h$ species with impurities. $\rho_{xx}$ shows a magneto-resistance effect through $\cot^{-2}\theta$ which is usually small, e.g., $\cot^{-2}\theta = 4 \times 10^{-4}$ for Y-Ba-Cu-O at T=100 K.[4] The $T^2$ law for the cotangent Hall angle in (10) has been a very good fitting[2,4] to the experimental measurements of high-$T_c$ copper-oxide materials. Anderson[3] proposed that a second scattering rate is needed to interpret the experimental data.[4] It has been supported by the Zn impurity effect which causes an extra $T$-independent contribution in $\cot\theta$.[4] The scattering rate $\hbar/\tau_h$ found in the present work appears naturally in (10) to serve as such a second scattering rate. The Hall coefficient $R_H$ can be written as $R_H \simeq \frac{1}{nec}\left(\frac{m_p+m_q}{m_H}\right)\frac{\tau_h}{\tau_s}$. In order to fit the Hall angle data in Y-Ba-Cu-O, it has been found that the cyclotron mass $m_H$ has to be very large, e.g., $m_H \approx 45 m_e$, if $t_h \simeq 8 \times 830$ K.[4] A mechanism for such an enhancement of $m_H$ comes naturally in the present approach.

Finally we briefly discuss the thermopower $\mathcal{S} = \mathcal{S}_h + \mathcal{S}_s$. $\mathcal{S}_h$ for the $h$ species gives a small temperature-dependent contribution but $\mathcal{S}_s$ will be dominant. When the temperature is extrapolated down to zero where the broadening of the Landau level vanishes, $\mathcal{S}_s$ will be simply related to the entropy of $p$ and $q$ species and one finds a Heikes-like formula $\mathcal{S}_s = \frac{k_B}{e}\ln(\frac{1-2\delta}{2\delta})$, which agrees very well with the overall doping dependence in $(La_{1-x}Sr_x)_2CuO_4$ (Ref. 16) (taking $\delta = 2x$) and other materials.[5] The broadening effect is expected to become important with increase of temperature and the corresponding temperature dependence of $S_s$ needs to be further explored. The absence of the spin effect in the thermopower measurements[17] also lend a support to the present charge-spin separation picture.

In conclusion, we have shown that the Lagrangian (1), describing the composite holon excitation in the flux-binding phase, can systematically explain the main features of the anomalous transport properties (the resistivity, the Hall effect, and the thermopower) in the normal state of the high-$T_c$ superconductors. In order to explore other normal-state properties, one needs also to include the spin excitation which has been decoupled from the charge component in the flux-binding phase and will be discussed elsewhere.

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[1] For a review, see D. B. Tanner and T. Timusk, in *Physical Properties of High Temperature Superconductors*, edited by P. M. Ginsberg ( World Scientific, Singapore, 1992 ), Vol. 3, p. 363.

[2] For a review, see N. P. Ong, *Physical Properties of High Temperature Superconductors* (Ref. 1), Vol. 2, p. 459.

[3] P. W. Anderson, Phys. Rev. Lett. **67**, 2092 (1991).

[4] T. R. Chien, Z. Z. Wang, and N. P. Ong, Phys. Rev. Lett. **67**, 2088 (1991).

[5] For a review, see A. B. Kaiser and C. Uher, in *Studies of High Temperature Superconductors*, edited by A. Narlikar (Nova Science, New York, 1991).

[6] N. Nagaosa and P. A. Lee, Phys. Rev. Lett. **64**, 2450 (1990); Phys. Rev. B **46**, 5621 (1992).

[7] L. B. Ioffe and P. B. Wiegmann, Phys. Rev. Lett. **65**, 653 (1990); L. B. Ioffe and G. Kotliar, Phys. Rev. B **42**, 10 348 (1990).

[8] P. W. Anderson, Phys. Rev. Lett. **64**, 1839 (1990); **65**, 2306 (1990).

[9] Z. Y. Weng, D. N. Sheng, and C. S. Ting, Phys. Rev. B **49**, 607 (1994).

[10] P. Lederer *et al.*, Phys. Rev. Lett. **63**, 1519 (1989); Y. Hasegawa *et al.*, J. Phys. Soc. Jpn. **59**, 822 (1990); P. B. Wiegmann, Phys. Rev. Lett. **65**, 2070 (1990); A. M. Tikofsky, *et al. ibid.* **69**, 3670 (1992).

[11] Z. Y. Weng, D. N. Sheng, and C. S. Ting, Phys. Rev. B **50**, 9470 (1994).

[12] Y. Kitazawa and H. Murayama, Phys. Rev. B **41**, 11101 (1990); X. Wilczek, *Fractional Statistics and Anyon Superconductivity* (World Scientific, Singapore, 1990).

[13] R. B. Laughlin, Phys. Rev. Lett. **60**, 2677 (1988); C. B. Hanna, R. B. Laughlin, and A. L. Fetter, Phys. Rev. B **40**, 8745 (1989).

[14] The total local density of spinons and $p$ species is equal to $a^{-1/2}$ in the normal state. The same is true for the eon and $q$ species case.

[15] R. Kubo *et al.*, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press, New York 1965), Vol. 17, p. 270.

[16] J. R. Cooper *et al.*, Phys. Rev. B **35**, 8794 (1987).

[17] R. C. Yu *et al.*, Phys. Rev. B **37**, 7963 (1988).