Vortex lattice structure in layered superconductors

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We show within the framework of the Lawrence-Doniach theory the possibility of the existence of a combined vortex lattice (one vortex lattice perpendicular and the other parallel to the conducting layers) for a field applied at an arbitrary angle with respect to the crystal axes of a layered superconductor. The phase diagrams are calculated to show the domains where different vortex lattice structures are stable.

I. INTRODUCTION

To treat certain equilibrium properties of the vortex lattice (VL), such as elastic properties, pinning forces, and thermal fluctuations, it is essential to understand the equilibrium VL structure. Given the high anisotropy of the high-T_c materials, finding the configuration of vortices, for a given magnetic induction $\mathbf{B}$ at an angle with respect to the principal axes of the crystal, which minimizes the free energy, has proven to be a nontrivial task. There has been a number of experimental and theoretical results that question the usual type of vortex-lattice structure as worked out theoretically by Campbell *et al.*[1] within the anisotropic London theory. Forgan *et al.*[2] have obtained both sixfold and fourfold symmetry patterns from neutron scattering on twinned Y-Ba-Cu-O single crystals, depending on the orientation of the magnetic field with respect to the symmetry axes. Bolle *et al.*[3] found a mixture of an isotropic vortex lattice embedded in a vortex "chain" structure for single crystals of Bi-Sr-Ca-Cu-O when the applied field is at an angle with respect to the c axis. Similar patterns have recently been observed using decoration on Al-doped Y-Ba-Cu-O single crystals.[4] On the theoretical side, Sardella and Moore,[5] using London theory, have found under certain conditions that instabilities in the VL are possible for highly anisotropic materials. Also, Benkraouda and Clem[6] have found that, for magnetically coupled two-dimensional (2D) vortices,[7] a vortex line (a straight stack of 2D vortices), in the limit of small inductions, becomes unstable when tilted beyond a certain angle with respect to the crystal c axis. A number of recent works have proposed a new vortex-lattice structure different from the standard one with all vortices parallel to the magnetic induction vector $\mathbf{B}$.[8–12] Huse[9] argues that the lattice that is parallel to the superconducting planes pins the vortex lines that are parallel to the c axis, thereby providing a possible explanation for the "chain" structure observed by Bolle *et al.*[3] Both Daemen *et al.*[11] and Sudbo *et al.*[12] using London theory, have shown that the coexistence of two types of vortex lattices where one of the lattices is oriented along one of the crystal principal axes can be stable for a range of the anisotropy ratios $\gamma \equiv \lambda_c/\lambda_{ab}$, where λ_c and λ_{ab} are the magnetic field penetration lengths with the screening currents flowing along the c crystal direction and in the ab planes, respectively. Preosti and Muzikar,[10] also using London theory, have found that a "combined lattice" of two types of vortices, one parallel to the c axis and the other parallel to the ab planes, gives a stable structure for large anisotropy ratio and low inductions.

The common approach to investigate different vortex-lattice structures has been the anisotropic Ginzburg-Landau theory and, in particular, its limiting case, London theory, which is valid for large $\kappa \equiv \lambda/\xi$ far from the upper critical field. Even though London theory works well for homogeneous extreme type-II superconductors, it falls short for highly anisotropic materials such as high-temperature superconductors. For example, as demonstrated by Bulaevskii,[13] one needs to go beyond London theory to explain the unusual angular dependence of torque in Y-Ba-Cu-O. Also, the requirement of high anisotropy is necessary for the coexistence of two types of vortex lattices,[10] thereby invalidating the use of the London theory; see Ref. 14. The theoretical approach that adequately treats highly anisotropic superconductors taking into account their layered nature was proposed by Lawrence and Doniach.[15] The aim of this paper is to explore the vortex-lattice structures in layered superconductors within the framework of Lawrence-Doniach (LD) theory. We will show that for a magnetic field applied at an angle with respect to the c axis, a *combined* lattice consisting of a *perpendicular* lattice (perpendicular to the ab planes) and a *parallel* lattice (along the ab planes) can have lower free energy than the *tilted* structure consisting only of vortices parallel to the vector $\mathbf{B}$.

II. FREE ENERGY OF DIFFERENT VORTEX-LATTICE STRUCTURES

As shown by Bulaevskii *et al.*,[8] the main difference between the vortex structures in LD and anisotropic London models lies in the structure of an individual vortex—a continuous line representing the normal core of a vortex is replaced in the LD model with a stack of pancakes (two-dimensional vortices),[7] located in conducting layers, and interacting through both the magnetic field and the Josephson currents which flow between the layers. We will consider only the limit $B \ll \phi_0/s^2\gamma$ (B is magnetic induction, and s is the spacing between the layers in c direction). In this case, as shown by Bulaevskii *et al.*,[8] the interaction between vortex stacks can still be considered within the framework of the anisotropic London model[1] as the distances between nonlinear Josephson regions (the regions where the profiles of fields and currents are substantially modified because of the Josephson nature of the interplane currents) are large compared to their dimensions. The Josephson nature of the interplane coupling is then manifested in the line energy only.

A. Tilted vortex lattice

The self-energy of vortices (line energy times the density of vortices) for the tilted vortex lattice within LD theory is given by[8]

$$F_{\rm self}^{\rm tilt} = \frac{\phi_0 B\cos\theta}{32\pi^3}\int_{k^2\le\xi_{ab}^2} d^2\mathbf{k}\frac{k_x^2[1+\lambda_c^2(k^2+Q^2)]+Q^2[1+\lambda_{ab}^2(k^2+Q^2)]}{k_x^2[1+\lambda_{ab}^2Q^2+\lambda_c^2k^2][1+\lambda_{ab}^2(k^2+Q^2)]}\,, \tag{1}$$

where $\mathbf{k}=(k_x,k_y)$, $Q^2=2(1-\cos k_x a)/s^2$, $a=s\tan\theta$, θ is the angle between the c axis of the crystal and vector $\mathbf{B}$, and ξ_{ab} is the in-plane coherence length.

The interaction energy in the tilted vortex lattice can be expressed as[16]

$$F_{\rm int}^{\rm tilt} = \frac{B^2}{8\pi}\sum_{\mathbf{G}}\frac{1+\lambda_{zz}^2G^2}{(1+\lambda_{zz}^2G_x^2+\lambda_c^2G_y^2)(1+\lambda_{ab}^2G^2)} - \frac{\phi_0 B}{8\pi}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{1+\lambda_{zz}^2k^2}{(1+\lambda_{zz}^2k_x^2+\lambda_c^2k_y^2)(1+\lambda_{ab}^2k^2)}\,, \tag{2}$$

where $\lambda_{zz}^2 = \lambda_{ab}^2\sin^2\theta + \lambda_c^2\cos^2\theta$. The vectors of the inverse vortex lattice, $\mathbf{G}=(G_x,G_y)$, are $G_x = G_0/\eta m\sqrt{3}/2$ and $G_y = G_0\eta(n-m/2)$, corresponding to a triangular lattice (m and n are integers). Here, $G_0=\sqrt{8\pi^2B/\phi_0\sqrt{3}}$ and $\eta=\rho(\lambda_{zz}/\lambda_c)^{1/2}$, with ρ being the lattice parameter with respect to which the minimization of the expression (2) ought to be performed. For relatively high fields, $B\gg H_{c1}$, $\rho\to1$.[1] Notice that both the sum and the integral in Eq. (2) are divergent but their difference is convergent.

The total free energy of the tilted VL is

$$F^{\rm tilt} = F_{\rm int}^{\rm tilt} + F_{\rm self}^{\rm tilt}\,. \tag{3}$$

B. Combined vortex lattice

Let us now suppose that the same magnetic induction $\mathbf{B}$ is created in the following way: Its perpendicular (to the a-b plane) component ($B\cos\theta$) is due to a perpendicular VL, whereas its parallel (to the a-b plane) component ($B\sin\theta$) is due to a parallel VL.

For the free energy of the perpendicular lattice one can use the London expression,[17] since supercurrents flow only inside superconducting layers and Josephson currents are absent:

$$F^{\rm perp} = \frac{B^2\cos^2\theta}{8\pi}\sum_{\mathbf{G_p}}\frac{1}{1+\lambda_{ab}^2G_p^2}\,, \tag{4}$$

where $\mathbf{G}_p=(G_{px},G_{py})$, $G_{px}=mG_{0p}\sqrt{3}/2$, and $G_{py}=G_{0p}(n-m/2)$. $G_{0p}=\sqrt{8\pi^2B\cos\theta/\phi_0\sqrt{3}}$.

The free energy for the parallel part is again expressed as a sum of the self-energy and the interaction energy:

$$F^{\rm paral} = F_{\rm int}^{\rm paral} + F_{\rm self}^{\rm paral}\,. \tag{5}$$

The interaction energy is obtained upon setting $\theta=\pi/2$ and *then* replacing B with $B\sin\theta$ in Eq. (2):

$$F_{\rm int}^{\rm paral} = \frac{B^2\sin^2\theta}{8\pi}\sum_{\mathbf{G_l}}\frac{1}{1+\lambda_{ab}^2G_{lx}^2+\lambda_c^2G_{ly}^2} - \frac{\phi_0B\sin\theta}{8\pi}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\frac{1}{(1+\lambda_{ab}^2k_x^2+\lambda_c^2k_y^2)}\,, \tag{6}$$

where $G_{lx}=mG_{0l}\sqrt{3}/2\eta_l$ and $G_{ly}=G_{0l}\eta_l(n-m/2)$. $G_{0l}=\sqrt{8\pi^2B\sin\theta/\phi_0\sqrt{3}}$ and $\eta_l=1/\sqrt{\gamma}$. The self-energy in the LD model is given by[8]

$$F_{\rm self}^{\rm paral} = \frac{\phi_0B\sin\theta}{32\pi^3 s}\int_{-\infty}^{+\infty}dk_y\int_{-\pi}^{+\pi}dq\frac{1}{1+\lambda_{ab}^2Q^2+\lambda_c^2k_y^2}\,, \tag{7}$$

where $Q^2=2(1-\cos q)/s^2$. Two mutually perpendicular vortex systems do not interact with each other.[18] Then, the total energy of the combined lattice is given as

$$F^{\rm comb} = F^{\rm perp} + F^{\rm paral}\,. \tag{8}$$

III. RESULTS

We compare the free energy densities for the two vortex-lattice configurations:

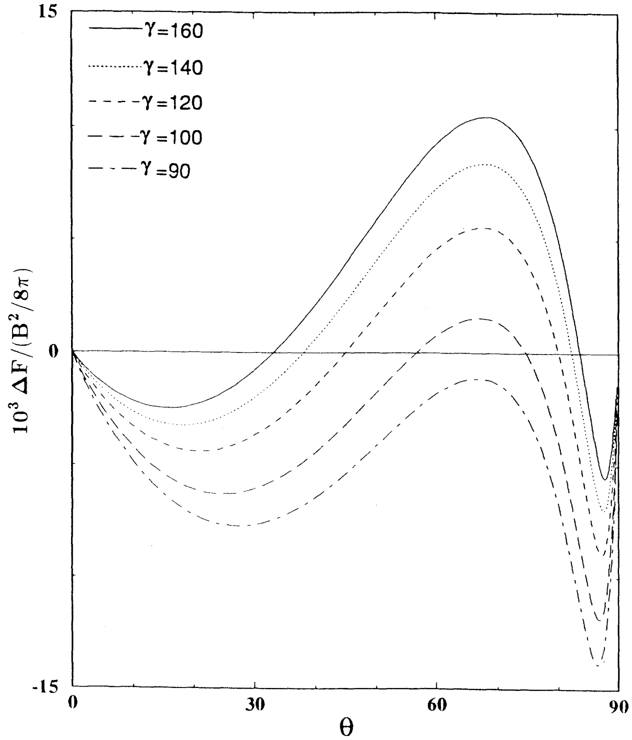

FIG. 1. The difference ΔF in free energy densities of tilted and combined vortex-lattice structures as a function of the angle θ in degrees between the vector of the magnetic induction **B** and the c axis of the crystal for $\xi_{ab}/s = 2$, $\lambda_{ab}/\xi_{ab} = 70$, $l_B/\lambda_{ab} = 2$, and for several values of the anisotropy ratio γ.

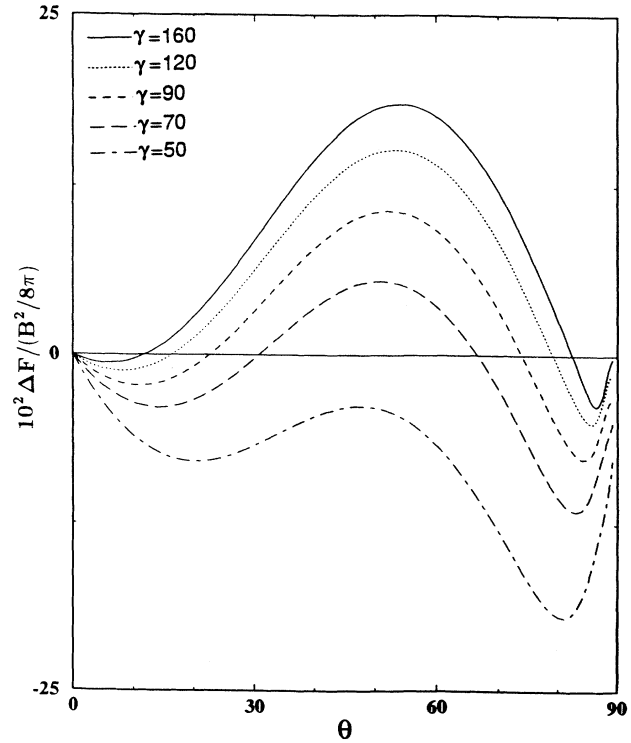

FIG. 2. The same as in Fig. 1, except that $l_B/\lambda_{ab} = 5$.

$$\Delta F = F^{\text{tilt}} - F^{\text{comb}}\,. \tag{9}$$

Throughout our calculations we assume $\xi_{ab}/s = 2$ and $\lambda_{ab}/\xi_{ab} = 70$, which is appropriate for high-temperature superconductors. Typical dependences of the energy difference ΔF on the angle θ for several values of the anisotropy ratio γ and two values of the magnetic induction parameter $l_B = \sqrt{\phi_0/B}$ are shown in Figs. 1 and 2. The conspicuous feature of the curves is that for large enough anisotropies and for a range of angles the energy difference ΔF becomes positive, therefore showing that the combined lattice is stable. There are two "crossing angles" between which the energy difference is positive. Large effective mass anisotropy favors combined structure, as seen in the increase of the region between the two crossing angles. When anisotropy decreases, the region where the combined lattice is stable shrinks and eventually disappears, making the tilted lattice stable for all angles. This agrees with the notion that unconventional vortex-lattice structures can occur only in layered materials for which the Josephson nature of the coupling between the layers (in addition to the magnetic one) is crucial.

The phase diagram anisotropy vs angle is shown in Fig. 3, summarizing the features of the previous two figures. One can see that for the combined structure to be stable it is important to have relatively small inductions; see also Fig. 4. This is because the layered nature of

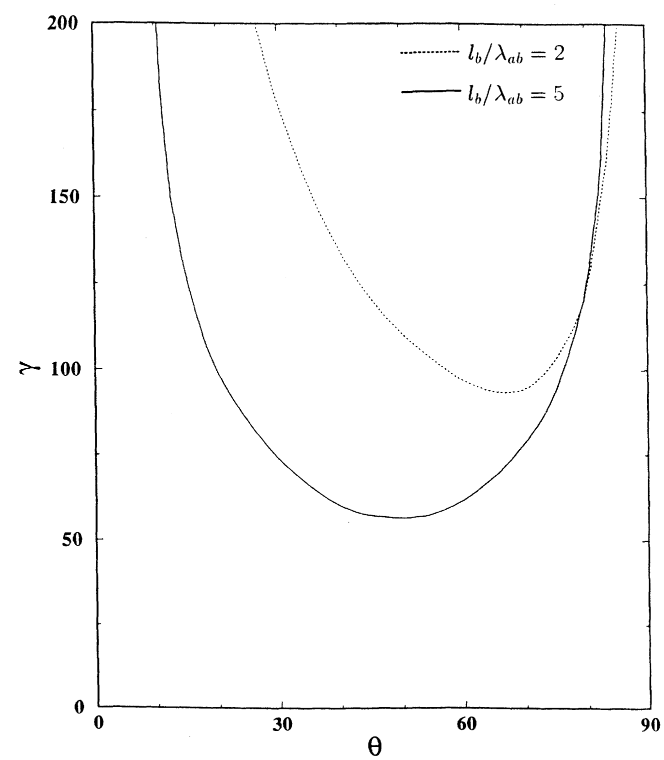

FIG. 3. The phase diagram γ vs θ for $\xi_{ab}/s = 2$, $\lambda_{ab}/\xi_{ab} = 70$, and two different values of l_B/λ_{ab}: 2 and 5. The region above each of the curves is where the combined lattice is stable.

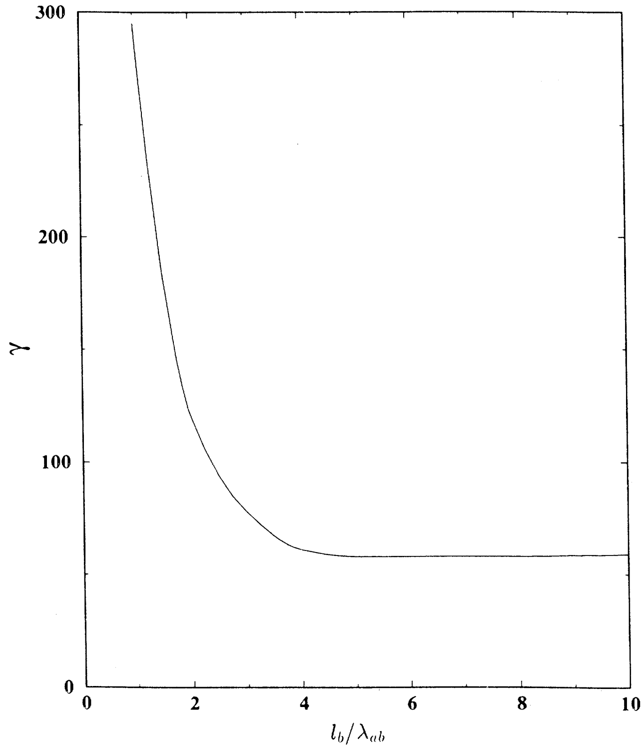

FIG. 4. The phase diagram γ vs l_B/λ_{ab} for $\xi_{ab}/s = 2$, $\lambda_{ab}/\xi_{ab} = 70$, and $\theta = 45°$. The region above the curve is where the combined lattice is stable.

these materials is manifested only in the line energy of vortices (as discussed at the beginning of the previous section), so that only if the line energy is not too small compared to the interaction between vortices (which happens when vortices are relatively far apart) can one see the effect. One then also expects that when the ratio l_B/λ_{ab} becomes very large (interactions negligible), the line separating the two phases should saturate. That is what happens; see Fig. 4.

IV. CONCLUSION

Using Lawrence-Doniach theory, which is appropriate for Josephson-coupled layered systems, we have shown that under certain conditions a vortex structure consisting of two mutually perpendicular vortex lattices—one parallel and the other perpendicular to the *ab* planes—has lower free energy than the usual vortex-lattice with all vortices parallel to the magnetic induction vector **B**. The mass anisotropy parameter γ needs to be large ($\gtrsim 60$) for this new vortex-lattice structure to be stable. If we look at the high-temperature superconductors, we see that such a large γ is present in Bi and Tl families of compounds. In Y-Ba-Cu-O ($\gamma < 10$), on the other hand, only the standard vortex lattice is likely to occur.

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