Hall effect in type-II superconductors

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The Hall conductivity for dirty superconductors is found for a large range of the depairing factor Γ. We estimate that in the range $\Delta < \Gamma < \pi T_c$, the change of the distribution functions connected with vortex motion does not have any effect in the Hall conductivity. The quasiclassical approximation is adapted to the treatment of the Hall effect.

I. INTRODUCTION

In recent years a large number of theoretical and experimental papers have appeared that are devoted to the study of the Hall effect in superconductors. In the main quasiclassical approximation in superconductors there exists symmetry between particles and holes and the Hall effect is missing. There are two physical reasons leading to the violation of this symmetry and to the appearance of the Hall effect. One is the same as in normal metal. It is connected with the curvature of the electron trajectory in the magnetic field. This phenomenon gives a main contribution to the Hall effect in superconductors with a large value of the electron mean free path at low temperatures. Quantitatively, this phenomenon was studied in Ref. 1. In the present paper we study the second reason for the Hall effect. Really the Cooper pair is charged. In the BCS approximation the charge is connected with the energy dependence of the density of states near the Fermi surface. This energy dependence also violates charge-hole symmetry. This phenomenon is essential for superconductors with a small value of the electron mean free path. Theoretical results have been obtained in the approximation of the time-dependent Ginzburg-Landau equation in Refs. 2 and 3. However, this model has a very narrow application region. The spin-flip relaxation time Γ^{-1} should be short enough $\Gamma \gg T_c$ (T_c is the transition temperature).

The usual conductivity depends strongly on the concentration of paramagnetic impurities. Near T_c the main contribution to the conductivity is connected with the change of the distribution function of normal excitations. Below we shall prove that the Hall effect is not so strong and is sensitive to the concentration of paramagnetic impurities. In the limit $\Gamma \gg T_c$ our results coincide in the main in the λ^{-1} approximation (λ is the effective constant of the electron-electron interaction) with that of Refs. 2 and 3. In the opposite limit, the Hall effect is independent from the distribution function and hence from the spin-flip relaxation time.

II. *I-V* CHARACTERISTIC OF SUPERCONDUCTORS

The *I-V* characteristic of superconductors in the vortex state (Shubnikov phase) can be found as the linear response[4] to a weak alternating electromagnetic field. Such an approach enables us to derive the expression for the current in the superconductor, averaged over an elementary cell. Technically our method simplifies significantly the calculation of the current. It reduces the problem to a straightforward determination of the linear in frequency part of the equations of motion for the order parameter Δ, vector potential A, and scalar potential φ. The dissipative and Hall components of the current differ not only in the absolute values, but have also quite different origin and structure. In fact, in a weak magnetic field the Hall component of the current is smaller than the dissipative part by the factor T_c/ϵ_F, where ϵ_F is the Fermi energy.

To find the *I-V* characteristic of a superconductor in the vortex state we shall use the Green's function method in the framework of BCS theory. It can be used as equations in real time,[5] as well as in imaginary time.[4] The latter is probably simpler. It will be used below.

The transition to the real time equations is straightforward. Equations for the Green's function in a superconductor are[5]

$$\Big\{ -\frac{\partial}{\partial \tau}\tau_z + \frac{1}{2m}\Big(\frac{\partial}{\partial \mathbf{r}} - ie\mathbf{A}\tau_z\Big)^2 + \hat{\Delta} - e\varphi + \mu - \hat{\Sigma}\Big\}\hat{G}(\mathbf{r},\mathbf{r}',\tau,\tau') = \delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau'), \tag{1}$$

where $\mathbf{A}$ is the vector potential, φ the scalar potential, μ the chemical potential, and

$$\hat{\Delta} = \begin{pmatrix} 0 & \Delta_1 \\ -\Delta_2 & 0 \end{pmatrix}, \quad \tau_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

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As usual, $\hat{G}$ is represented in the form

$$\hat{G} = \begin{pmatrix} G_1 & F_1 \\ -F_2 & G_2 \end{pmatrix}. \quad (3)$$

The self-energy $\hat{\Sigma}$ takes into account the interaction of electrons with impurities (usual and paramagnetic) and phonons. The explicit expression for $\hat{\Sigma}$ is given in Ref. 5.

The charge density ρ, current density j, and order parameters $\Delta_{1,2}$ expressed in terms of the Green's function are

$$\rho = -e\Big\{2e\nu\varphi - T\Sigma_\omega \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\, \hat{G}\Big\},$$
$$j = \frac{e}{m}\int \frac{d^3p}{(2\pi)^3} T\Sigma_\omega \mathrm{Tr}\, \tau_z \mathbf{p}\hat{G},$$
$$\frac{1}{\mid\lambda\mid}\Delta_{1,2} + T\Sigma_\omega \int \frac{d^3p}{(2\pi)^3} F_{1,2} = 0, \quad (4)$$

where $\nu = mp_0/2\pi^2$ is the density of states on the Fermi level and λ is a constant of electron-electron interaction. Below we shall set as usual

$$\lambda_{\text{eff}} = \nu \mid \lambda \mid . \quad (5)$$

The system of the equations (1), (4) should be taken together with the two Maxwell equations for the vector potential $\mathbf{A}$ and scalar potential φ,

$$\frac{1}{4\pi}\text{curl curl}\,\mathbf{A} = j - \frac{i}{4\pi}\frac{\partial}{\partial\tau}\Big(\frac{\partial\varphi}{\partial\mathbf{r}} + i\frac{\partial\mathbf{A}}{\partial\tau}\Big),$$
$$-\frac{1}{4\pi}\Big(-\frac{\partial^2}{\partial\mathbf{r}^2} + 8\pi e^2\nu\Big)\varphi = \frac{-i}{4\pi}\frac{\partial}{\partial\tau}\text{div}\,\mathbf{A} - eT\Sigma_\omega \times \int \frac{d^3p}{(2\pi)^3}\mathrm{Tr}\,\hat{G}. \quad (6)$$

In homogeneous superconductors, the system of equations (1), (4), (6) possess translational symmetry. This means that corrections to the quantities $\Delta_{1,2}, \mathbf{A}, \varphi$ in a weak external field to zeroth order in the frequency ω_0 are equal to[4]

$$\Delta_1^{(1)} = \mathbf{b}\cdot\partial_-\Delta, \quad \Delta_2^{(1)} = \mathbf{b}\cdot\partial_+\Delta^*,$$
$$\varphi_1 = \mathbf{b}\cdot\frac{\partial\varphi_0}{\partial\mathbf{r}}, \quad \mathbf{A}_1 = \mathbf{H}\times\mathbf{b}, \quad (7)$$

where we define the $\partial_\pm = \frac{\partial}{\partial\mathbf{r}} \pm 2ie\mathbf{A}$, $\mathbf{b}$ tanslation vector,

$$\Delta_1 = \Delta_0 + \Delta_1^{(1)}\exp(-i\omega_0\tau),$$
$$\Delta_2 = \Delta_0^* + \Delta_2^{(1)}\exp(-i\omega_0\tau),$$
$$\mathbf{A} = \mathbf{A}_0 + \mathbf{A}_1\exp(-i\omega_0\tau), \quad (8)$$
$$\varphi = \varphi_0 + \varphi_1\exp(-i\omega_0\tau),$$
$$\langle\mathbf{E}\rangle = -\omega_0(\mathbf{B}\times\mathbf{b})\exp(-i\omega_0\tau).$$

The scalar potential φ is not zero even in the static case. Below we shall find its value φ_0. In the general case, Eqs. (1), (4), (6) for the quantities, $\Delta_{1,2}^{(1)}$, $\mathbf{A}_1$, φ_1 (the first-order correction terms in ω_0) have the form[5]

$$\hat{L}\begin{pmatrix}\Delta_1^{(1)}\\ \Delta_2^{(1)}\\ \mathbf{A}_1\\ \varphi_1\end{pmatrix} = -\begin{pmatrix}0\\0\\ \langle j_1\rangle\\ 0\end{pmatrix} + \frac{\omega_0}{4\pi}\begin{pmatrix}0\\0\\ -\frac{\partial}{\partial\mathbf{r}}(\mathbf{b}\cdot\frac{\partial\varphi_0}{\partial\mathbf{r}})\\ -\text{div}\,(\mathbf{H}\times\mathbf{b})\end{pmatrix} + \omega_0\hat{K}\begin{pmatrix}\mathbf{b}\cdot\partial_-\Delta\\ \mathbf{b}\cdot\partial_+\Delta^*\\ \mathbf{H}\times\mathbf{b}\\ \mathbf{b}\cdot\frac{\partial\varphi_0}{\partial\mathbf{r}}\end{pmatrix}, \quad (9)$$

where the operator $\hat{K}$ is defined as

$$\omega_0\hat{K}\begin{pmatrix}\mathbf{b}\cdot\partial_-\Delta\\ \mathbf{b}\cdot\partial_+\Delta^*\\ \mathbf{H}\times\mathbf{b}\\ \mathbf{b}\cdot\frac{\partial\varphi_0}{\partial\mathbf{r}}\end{pmatrix} = -T\Sigma_\omega\int\frac{d^3p}{(2\pi)^3}\begin{pmatrix}F_1^{(1)}\\ F_2^{(1)}\\ -e\mathbf{v}(G_1^{(1)} - G_2^{(1)})\\ e(G_1^{(1)} + G_2^{(1)})\end{pmatrix}. \quad (10)$$

On the right-hand side we keep only terms proportional to the frequency ω_0.

The operator $\hat{L}$ is defined in Eqs. (4) for Δ and (6) for A, φ at $\omega_0 = 0$. The operator $\hat{L}$ has two zero eigenvalues corresponding to the eigenfunction (7) which are simple shifts. This property of the operator $\hat{L}$ enables us to find the I-V characteristics of a superconductor in the vortex state,

$$B^2\hat{\sigma}(\mathbf{B}\times\mathbf{b}) = -\Big\langle\Big(\mathbf{B}\times\partial_+\Delta^*\,,\ \mathbf{B}\times\partial_-\Delta\,,\ \mathbf{B}\cdot\mathbf{H}, \mathbf{B}\times\frac{\partial\varphi_0}{\partial\mathbf{r}}\Big)\hat{K}\begin{pmatrix}\mathbf{b}\cdot\partial_-\Delta\\ \mathbf{b}\cdot\partial_+\Delta^*\\ \mathbf{B}\times\mathbf{b}\\ \mathbf{b}\cdot\frac{\partial\varphi_0}{\partial\mathbf{r}}\end{pmatrix}\Big\rangle$$
$$+\frac{1}{4\pi}\Big\langle\Big(\mathbf{B}\times\frac{\partial\varphi_0}{\partial\mathbf{r}}\Big)\text{div}\,(\mathbf{H}\times\mathbf{b}) - \Big(\mathbf{b}\cdot\frac{\partial\varphi_0}{\partial\mathbf{r}}\Big)\frac{\partial}{\partial\mathbf{r}}(\mathbf{H}\cdot\mathbf{B})\Big\rangle, \quad (11)$$

where $\partial_\pm = \frac{\partial}{\partial \mathbf{r}} \pm 2ie\mathbf{A}$, $\hat{\sigma}$ is the conductivity tensor. The last term in Equation (11) is always small and below we shall omit it. Equation (11) is a general one and it reduces the problem of the calculation of the conductivity to the determination of the linear in ω_0 part of Eqs. (4) for Δ and (6) for $\mathbf{A}, \varphi$. We shall use Eq. (11) to calculate the Hall conductivity in superconductors.

III. QUASICLASSICAL APPROXIMATION

In this section we shall obtain a quasiclassical approximation adapted to the treatment of the Hall effect. For this purpose we shall integrate the Green's function $\hat{G}$ over energy ξ counted from the Fermi level,

$$\hat{g} = \frac{i}{\pi}\int d\xi \hat{G}. \tag{12}$$

The problem that we should take into account in the Hall effect is that the density of states is not a constant near the Fermi level. As a result, we obtain from Eqs. (1), (12)

$$T\Sigma_\omega \int \frac{d^3p}{(2\pi)^3}\hat{G} = -i\nu\pi T\Sigma_\omega \int \frac{d\Omega_p}{4\pi}\left(1 + \frac{\hat{C}_{(\tau)}}{2\gamma}\right)\hat{g},$$
$$T\Sigma_\omega \int \frac{d^3p}{(2\pi)^3}\tau_z \mathbf{p}\hat{G} = -i\nu\pi T\Sigma_\omega \int \frac{d\Omega_p}{4\pi}\tau_z\mathbf{p}\left(1 + \frac{\hat{C}_{(\tau)}}{\gamma}\right)\hat{g}, \tag{13}$$

where

$$\frac{1}{2\gamma} = \frac{1}{\nu}\frac{\partial \nu}{\partial \xi} \tag{14}$$

and the operator $\hat{C}$ is equal to

$$\hat{C}_{(\tau)} = -\hat{\tau}_z\frac{\partial}{\partial\tau} + \frac{i}{2}\mathbf{v}\cdot\left(\frac{\partial}{\partial\mathbf{r}} - 2ie\mathbf{A}\tau_z\right) + \hat{\Delta} - e\varphi - \hat{\Sigma}. \tag{15}$$

Here $\mathbf{v}$ is the velocity at the Fermi level. In the approximation of the free electron gas, we have

$$\gamma = \epsilon_F. \tag{16}$$

Hence, to find the conductivity (including the Hall effect), we should find the Green's function $\hat{g}$. An equation on the Green's function $\hat{g}$ with corrections of order of T/ϵ_F, Δ/ϵ_F can be obtained using the same method as the equation of the quasiclassical Green's function without correction terms.[5–7]

However, it is more convenient to derive equations for the Green's function $\tilde{g}$, where $\tilde{g}$ is defined by the equation

$$\hat{g} = \left(1 + \frac{\hat{\mathcal{D}}_{(\tau)}}{2}\right)\tilde{g}\left(1 + \frac{\hat{\mathcal{D}}^*_{(\tau')}}{2}\right), \tag{17}$$

where the operators $\hat{\mathcal{D}}, \hat{\mathcal{D}}^*$ are equal to

$$\hat{\mathcal{D}}_{(\tau)} = \frac{i}{4\gamma}\left[\mathbf{v}\cdot\left(\frac{\partial}{\partial\mathbf{r}} - 2ie\mathbf{A}_{(\tau)}\tau_z\right)\right],$$
$$\hat{\mathcal{D}}^*_{(\tau')} = -\frac{i}{4\gamma}\left[\mathbf{v}\cdot\left(\frac{\partial}{\partial\mathbf{r}} + 2ie\mathbf{A}(\tau')\tau_z\right)\right], \tag{18}$$

and the operator $\hat{\mathcal{D}}^*$ acts from the right to left. The equation for the Green's function $\tilde{g}$ is

$$-\hat{\tau}_z\frac{\partial\tilde{g}}{\partial\tau} - \frac{\partial\tilde{g}}{\partial\tau'}\hat{\tau}_z + i\left(\mathbf{v}\frac{\partial}{\partial\mathbf{r}}\right)\tilde{g} + i[\hat{H}(\tau)\tilde{g} - \tilde{g}\hat{H}(\tau')] + \frac{1}{2}(\hat{\mathcal{D}}\hat{C} + \hat{C}\hat{\mathcal{D}})\tilde{g} - \frac{1}{2}\tilde{g}(\hat{\mathcal{D}}^*\hat{C}^* + \hat{C}^*\hat{\mathcal{D}}^*)$$
$$+\frac{1}{4\gamma}\left[\frac{\partial\hat{H}}{\partial\mathbf{r}}\tilde{g} + \tilde{g}\frac{\partial\hat{H}}{\partial\mathbf{r}} - ie\left(\mathbf{v}\frac{\partial[\mathbf{v}\cdot\mathbf{A}(\tau)]}{\partial\mathbf{r}}\right)\hat{\tau}_z\tilde{g} - ie\tilde{g}\left(\mathbf{v}\frac{\partial[\mathbf{v}\cdot\mathbf{A}(\tau')]}{\partial\mathbf{r}}\right)\hat{\tau}_z\right] - \frac{1}{2}\frac{\partial\hat{H}}{\partial\mathbf{r}}\frac{\partial\tilde{g}}{\partial\mathbf{p}_\perp} - \frac{1}{2}\frac{\partial\tilde{g}}{\partial\mathbf{p}_\perp}\frac{\partial\hat{H}}{\partial\mathbf{r}}$$
$$-\frac{e^2}{2m}[\mathbf{A}^2(\tau)\tilde{g} - \tilde{g}\mathbf{A}^2(\tau')] - \frac{ie}{2m}\left(\mathbf{A}(\tau)\tau_z\frac{\partial\tilde{g}}{\partial\mathbf{r}} + \frac{\partial\tilde{g}}{\partial\mathbf{r}}\mathbf{A}(\tau')\tau_z\right) = 0, \tag{19}$$

where the operators $\hat{H}, \hat{C}^*$ are equal to

$$\hat{H}(\tau) = -i\left\{\hat{\Delta}(\tau) - e\varphi(\tau) - \hat{\Sigma} + e[\mathbf{v}\cdot\mathbf{A}(\tau)]\hat{\tau}_z\right\},$$
$$\hat{C}^* = \frac{\partial}{\partial\tau'}\tau_z - \frac{i}{2}\left[\mathbf{v}\cdot\left(\frac{\partial}{\partial\mathbf{r}} + 2ie[\mathbf{v}\cdot\mathbf{A}(\tau')\hat{\tau}_z]\right)\right] + \hat{\Delta} - e\varphi - \hat{\Sigma}. \tag{20}$$

The transverse derivative $\partial/\partial\mathbf{p}_\perp$ is defined as

$$\frac{\partial}{\partial\mathbf{p}_\perp} = \frac{\partial}{\partial\mathbf{p}} - \frac{\partial\epsilon(\mathbf{p})}{\partial\mathbf{p}}\frac{\partial}{\partial\epsilon(p)}. \tag{21}$$

The Green's function $\tilde{g}$ satisfies the normalization condition

$$\tilde{g}\tilde{g} \equiv \int d\tau_1\tilde{g}(\tau,\tau_1)\tilde{g}(\tau_1\tau') = \delta(\tau - \tau'). \tag{22}$$

Equation (19) can be rewritten for real time.

To do this, we should extend[6] the Green's function $\tilde{g}$ and matrix $\hat{\tau}_z, \hat{\Delta}$ to the size (4×4) in the following way:

$$\hat{\Delta} \to \check{\Delta} = \begin{pmatrix}\hat{\Delta} & 0\\ 0 & \hat{\Delta}\end{pmatrix}, \quad \tau_z \to \check{\tau}_z = \begin{pmatrix}\hat{\tau}_z & 0\\ 0 & \hat{\tau}_z\end{pmatrix},$$
$$\tilde{g} \to \check{g} = \begin{pmatrix}\tilde{g}^R & \tilde{g}^K\\ 0 & \tilde{g}^A\end{pmatrix}, \tag{23}$$

and change $-\frac{\partial}{\partial\tau} \to i\frac{\partial}{\partial t}$.

The self-energy part $\hat{\Sigma}$ should be modified also. For example, the impurity part of $\hat{\Sigma}_{\text{imp}}$ is

$$\hat{\Sigma}_{\text{imp}} = -\frac{inv}{2}\int d\Omega_{p1}\left[\sigma_{pp_1}\left(1+\frac{\hat{C}}{2\gamma}\right)+\frac{\partial\sigma_{pp1}}{\partial\xi}\hat{C}\right]\hat{g}_{p_1},$$
$$-\frac{i}{2}\int\frac{d\Omega p_1}{4\pi}\hat{\tau}_z\left[\Gamma\left(1+\frac{C}{2\gamma}\right)+\nu\frac{\partial(\Gamma/\nu)}{\partial\xi}\hat{C}\right]\hat{g}_{p_1}\tau_z, \tag{24}$$

where Γ is the inverse spin-flip relaxation time.

IV. CONDUCTIVITY OF SUPERCONDUCTORS IN THE DIRTY LIMIT

Now we shall consider the conductivity in superconductors with small value of electron mean free path and large enough concentration of paramagnetic impurities, so that the condition

$$\Gamma \gg \Delta \tag{25}$$

is fulfilled.

Under these restrictions, we can take into account only corrections that arise in Eqs. (13), (19), (24), from the energy dependence of density at states on the Fermi level. In Eq. (19) we shall neglect all correction terms with derivatives in $\mathbf{r}$. These terms are small in the parameter $|\Delta|^2/(\Gamma+\pi T)^2$.

In the linear in the electrical field approximation, the Green's function $\hat{g}$ is equal to

$$\hat{g}(\tau,\tau') = \hat{g}_0(\tau-\tau') + e^{-i\omega_0\tau}g_1(\tau-\tau'), \tag{26}$$

where[4]

$$\hat{g}_0(\omega) = \begin{pmatrix}\alpha & -i\beta\\ i\beta^* & -\alpha\end{pmatrix},$$
$$\beta = \frac{\Delta}{|\omega|+\Gamma}\left(1+\frac{e\varphi}{2\gamma}\frac{\Gamma}{(|\omega|+\Gamma)^2}\right), \tag{27}$$
$$\alpha = \left(1-\frac{|\Delta|^2}{2(|\omega|+\Gamma)^2}\right)\text{sgn}\omega.$$

So, in the statical case, correction terms in Eq. (24) lead only to the nonessential change of the Green's function β. The value of the scalar potential φ in equilibrium shall be given below.

The Green's function $\hat{g}_1$ can be presented in the form[5]

$$\hat{g}_1(\omega,\omega_+) = \begin{pmatrix} g_1 & f_1\\ -f_2 & g_2\end{pmatrix}\ ,\quad \omega_+ = \omega+\omega_0. \tag{28}$$

In the normal region in ω ($\text{sgn}\omega = \text{sgn}\omega_+$) we obtain

$$f_1 = -\frac{i(\alpha_++\alpha)\Delta_1}{\Gamma(\alpha_++\alpha)+\omega_++\omega} - \frac{e\Delta\varphi_1}{\Gamma(\alpha_++\alpha)+\omega_++\omega}\left(\frac{1}{|\omega|+\Gamma}-\frac{1}{|\omega_+|+\Gamma}\right) - R_1 f_1\ ,$$
$$f_2 = -\frac{i(\alpha_++\alpha)\Delta_2}{\Gamma(\alpha_++\alpha)+\omega_++\omega} + \frac{e\Delta^*\varphi_1}{\Gamma(\alpha_++\alpha)+\omega_++\omega}\left(\frac{1}{|\omega|+\Gamma}-\frac{1}{|\omega_+|+\Gamma}\right) + R_1 f_2. \tag{29}$$

Here $\Delta_{1,2}$ and φ_1 are linear in the electrical field corrections to the order parameter and scalar potential; quantity R_1 is equal to

$$R_1 = \frac{i\Gamma}{4\gamma}\frac{\omega_+(\alpha_++\alpha)+(\omega_+\alpha_+-\omega\alpha)}{\omega_++\omega+\Gamma(\alpha_++\alpha)}. \tag{30}$$

We keep in Eq. (29) only essential corrections, leading to the Hall conductivity.

The functions $g_{1,2}$ in the normal region can be found from the normalization conditions and are equal to

$$g_1 = -\frac{i}{\alpha_++\alpha}(\beta_+f_2+\beta^*f_1),$$
$$g_2 = \frac{i}{\alpha_++\alpha}(\beta f_2+\beta_+^*f_1). \tag{31}$$

In the anomalous region in ω ($\text{sgn}\omega = -\ \text{sgn}\omega_+$) we obtain from the normalization condition

$$f_1 = \frac{i}{\alpha_+-\alpha}(\beta_+g_2+\beta g_1)\ ,$$
$$f_2 = -\frac{i}{\alpha_+-\alpha}(\beta_+^*g_1+\beta^*g_2). \tag{32}$$

For the Green's functions $g_{1,2}$ in the anomalous region, we have[4]

$$g_1-g_2 = -\left(-\mathcal{D}\frac{\partial^2}{\partial r^2}\right)^{-1}\frac{2\omega}{\Gamma^2-\omega^2}(\Delta\Delta_2+\Delta^*\Delta_1)$$
$$-\frac{i}{2\gamma}\left(\omega+\frac{\Gamma}{2}\right)(g_1+g_2)\ ,$$
$$(g_1+g_2) = \hat{L}_1^{-1}\left\{(\Delta\Delta_2-\Delta^*\Delta_1)+\frac{2ie\varphi_1}{\Gamma}(\Gamma^2-\omega^2)\right.$$
$$\left.+\frac{i}{2\gamma}\left(\Gamma+\frac{\omega}{2}\right)(\Delta_1\Delta^*+\Delta\Delta_2)\right\}, \tag{33}$$

where the operator $\hat{L}_1$ is equal to

$$\hat{L}_1 = \left(|\Delta|^2-\mathcal{D}\frac{\Gamma^2-\omega^2}{2\Gamma}\frac{\partial^2}{\partial\mathbf{r}^2}\right) \tag{34}$$

and $\mathcal{D} = vl_{\text{tr}/3}$ is a diffusion coefficient.

Below we shall consider superconductors with a large value of the Ginzburg-Landau parameter x. In the main approximation in this parameter the contribution of the vector potential $\mathbf{A}_1$ to the tensor conductivity can be replaced by the tensor conductivity σ_N in the normal metal. All other essential terms arise from $\Delta_{1,2};\varphi_1$. The kernel $\hat{K}$ in the expression (11) for the conductivity can be presented in the form

$$\hat{K}=\hat{K}^{\rm qcl}+\hat{K}^{\rm corr}, \tag{35}$$

where $\hat{K}^{\rm qcl}$ is the quasiclassical approximation for kernel $\hat{K}$ and $\hat{K}^{\rm corr}$ is the result of the correction terms in Eq. (13). From Eqs. (10), (29), (33) we obtain

$$\begin{aligned}
\hat{K}^{\rm qcl}_{11}\Delta_1+\hat{K}^{\rm qcl}_{12}\Delta_2+\hat{K}^{\rm qcl}_{14}\varphi_1 &= i\nu\Bigg\{\frac{i\Delta_1}{4\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)+\frac{e\varphi_1\Delta}{4(2\pi T)^2}\psi''\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\\
&\quad+\frac{i\Delta}{16T}\int_{-\infty}^{\infty}\frac{d\epsilon}{ch^2\frac{\epsilon}{2T}}\Big[\frac{\Gamma}{\Gamma^2+\epsilon^2}\hat{L}_1^{-1}(\Delta\Delta_2-\Delta^*\Delta_1)+\frac{2\epsilon^2}{(\Gamma^2+\epsilon^2)^2}\hat{L}_2^{-1}(\Delta^*\Delta_1+\Delta\Delta_2)\\
&\quad+2ie\hat{L}_1^{-1}\varphi_1\Big]\Bigg\},\\
\hat{K}^{\rm qcl}_{21}\Delta_1+\hat{K}^{\rm qcl}_{22}\Delta_2+\hat{K}^{\rm qcl}_{24}\varphi_1 &= i\nu\Bigg\{\frac{i\Delta_2}{4\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)-\frac{e\varphi_1\Delta^*}{4(2\pi T)^2}\psi''\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\\
&\quad-\frac{i\Delta^*}{16T}\int_{-\infty}^{\infty}\frac{d\epsilon}{ch^2\frac{\epsilon}{2T}}\Big[\frac{\Gamma}{\Gamma^2+\epsilon^2}\hat{L}_1^{-1}(\Delta\Delta_2-\Delta^*\Delta_1)-\frac{2\epsilon^2}{(\Gamma^2+\epsilon^2)^2}\hat{L}_2^{-1}(\Delta^*\Delta_1+\Delta\Delta_2)\\
&\quad+2ie\hat{L}_1^{-1}\varphi_1\Big]\Bigg\},\\
\hat{K}^{\rm qcl}_{41}\Delta_1+\hat{K}^{\rm qcl}_{42}\Delta_2+\hat{K}^{\rm qcl}_{44}\varphi_1 &= -ie\nu\Bigg\{\frac{\Delta\Delta_2-\Delta^*\Delta_1}{4(2\pi T)^2}\psi''\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\\
&\quad-\frac{1}{8T}\int_{-\infty}^{\infty}\frac{d\epsilon}{ch^2\frac{\epsilon}{2T}}\Big[\hat{L}_1^{-1}(\Delta\Delta_2-\Delta^*\Delta_1)+\frac{2ie(\Gamma^2+\epsilon^2)}{\Gamma}\hat{L}_1^{-1}\varphi_1\Big]\Bigg\},
\end{aligned}\tag{36}$$

where the operator $\hat{L}_1$ given by Eq. (34) in which ω is replaced by $-i\epsilon$, and the operator $\hat{L}_2$ is equal to

$$\hat{L}_2=-\mathcal{D}\frac{\partial^2}{\partial\mathbf{r}^2}. \tag{37}$$

The operator $\hat{L}$ in Eq. (9) is the second variation of the effective action $S_{\rm eff}$ by $\Delta,\Delta^*,\mathbf{A},\varphi$. The statical part of the effective action $S^{\rm st}_{\rm eff}$ can be found with the help of Eqs. (4) and (6) and is given by the expression

$$S^{\rm st}_{\rm eff}=\mathcal{F}_{\rm GL}-\nu\left\{e\varphi-\frac{|\Delta|^2}{4\gamma}\left[\frac{1}{\lambda_{\rm eff}}-\frac{\Gamma}{2\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right]\right\}^2, \tag{38}$$

where $\mathcal{F}_{\rm GL}$ is equal to

$$\begin{aligned}
\mathcal{F}_{\rm GL} &= -\nu\Bigg(|\Delta|^2\left\{\ln\left(\frac{T_c}{T}\right)-\left[\psi\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)-\psi\left(\frac{1}{2}+\frac{\Gamma}{2\pi T_c}\right)\right]\right\}-\frac{\mathcal{D}}{4\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)|\partial_-\Delta|^2\\
&\quad+\frac{|\Delta|^4}{8(2\pi T)^2}\left[\psi''\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)+\frac{\Gamma}{6\pi T}\psi'''\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right]\Bigg).
\end{aligned}\tag{39}$$

In the quasiclassical approximation in the equilibrium state, a scalar potential φ is equal to zero. The correction term in Eq. (13) leads to the appearance of a nonzero value of φ. From Eq. (38) we obtain

$$e\varphi=\frac{|\Delta|^2}{4\gamma}\left[\frac{1}{\lambda_{\rm eff}}-\frac{\Gamma}{2\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right]. \tag{40}$$

So, in first approximation in Δ/ϵ_F, the vortex carries a distributed charge.

Note that the following relation holds:

$$\langle(\mathbf{B}\times\mathbf{e}_1)\cdot(\mathbf{b}\cdot\mathbf{e}_2)\rangle=\frac{1}{2}\mathbf{B}\times\mathbf{b}\langle(\mathbf{e}_1\cdot\mathbf{e}_2)\rangle+\frac{1}{2}\mathbf{b}\langle(\mathbf{B}\times\mathbf{e}_1)\cdot\mathbf{e}_2\rangle, \tag{41}$$

where $\mathbf{e}_{1,2}$ are vectors having rotational symmetry as $\partial\,|\Delta|^2/\partial\mathbf{r}$ [or $(\Delta^*\partial_-\Delta-\Delta\partial_+\Delta^*)$].

The expression (36) enables us to find the quasiclassical part $\hat{\sigma}_{\rm qcl}$ of the conductivity tensor. The matrix elements $\hat{K}^{\rm qcl}_{i\gamma}$ $(i,j=1,2,3)$ contribute to the dissipative part of the conductivity tensor. The Hall component of the conductivity arises only from the terms $\hat{K}^{\rm qcl}_{i4}, \hat{K}^{\rm qcl}_{4i}$ $(i = 1, 2, 3)$.

Using Eqs. (11), (35), and (41) we obtain

$$\hat{\sigma}^{\rm qcl} = \hat{\sigma}^{\rm qcl}_H + \hat{\sigma}^{\rm qcl}_d, \tag{42}$$

where

$$B^2\sigma^{\rm qcl}_d = \frac{\nu}{4\pi T}\psi'\left(\frac{1}{2} + \frac{\Gamma}{2\pi T}\right)\langle(\partial_+\Delta^*)(\partial_-\Delta)\rangle + \frac{\nu}{32T}\int_{-\infty}^{\infty}\frac{d\epsilon}{ch^2(\frac{\epsilon}{2T})}\left\{\frac{\Gamma}{\Gamma^2+\epsilon^2}\langle(\Delta\partial_+\Delta^* - \Delta^*\partial_-\Delta)\hat{L}_1^{-1}(\Delta\partial_+\Delta^* - \Delta^*\partial_-\Delta)\rangle + \frac{2\epsilon^2}{\mathcal{D}(\Gamma^2+\epsilon^2)^2}\langle(|\Delta|^2 - \langle|\Delta|^2\rangle)^2\rangle\right\} \tag{43}$$

and

$$\hat{\sigma}^{\rm qcl}_H = \frac{i\nu e}{B^4}(\mathbf{B}\times\cdots)\left\{\frac{1}{4(2\pi T)^2}\psi''\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\left\langle\left([\mathbf{B}\times(\Delta\partial_+\Delta^* - \Delta^*\partial_-\Delta)]\cdot\frac{\partial\varphi}{\partial\mathbf{r}}\right)\right\rangle - \frac{1}{8T}\left\langle\left([\mathbf{B}\times(\Delta\partial_+\Delta^* - \Delta^*\partial_-\Delta)]\int_{-\infty}^{\infty}\frac{d\epsilon}{ch^2\frac{\epsilon}{2T}}\hat{L}_1^{-1}\frac{\partial\varphi}{\partial\mathbf{r}}\right)\right\rangle\right\} \tag{44}$$

here $(\mathbf{B}\times\cdots)$ stands for the vector product.

The expression (43) for the conductivity $\hat{\sigma}^{\rm qcl}_d$ coincides with the result of Ref. 4. In the region $\Delta \ll \Gamma \ll T$ the last term is the largest. This term arises from the Green's function $g_1 - g_2$ (odd distribution function) and is missing in the approximation of the time-dependent Ginzburg-Landau (TDGL) equation. As a result, the expression (43) coincides with the results in Refs. 8–10 only in the narrow range of large values of the depairing factor $\Gamma \gg \pi T_c$.

V. HALL CONDUCTIVITY

To complete our calculations, we shall find the kernel $\hat{K}^{\rm corr}$. From Eqs. (10), (13), (29), (32), (33) we obtain

$$\hat{K}^{\rm corr}_{11}\Delta_1 + \hat{K}^{\rm corr}_{12}\Delta_2 = \frac{i\nu}{4\gamma}\left\{\frac{1+\lambda_{\rm eff}}{\lambda_{\rm eff}}\Delta_1 - \frac{\Delta}{4T}\int_{-\infty}^{\infty}\frac{d\epsilon}{ch^2\frac{\epsilon}{2T}}\frac{\Gamma^2}{\Gamma^2+\epsilon^2}\hat{L}_1^{-1}(\Delta^*\Delta_1) - \frac{\Gamma\Delta_1}{2\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right\},$$

$$\hat{K}^{\rm corr}_{21}\Delta_1 + \hat{K}^{\rm corr}_{22}\Delta_2 = \frac{i\nu}{4\gamma}\left\{-\frac{1+\lambda_{\rm eff}}{\lambda_{\rm eff}}\Delta_2 + \frac{\Delta^*}{4T}\int_{-\infty}^{\infty}\frac{d\epsilon}{ch^2\frac{\epsilon}{2T}}\frac{\Gamma^2}{\Gamma^2+\epsilon^2}\hat{L}_1^{-1}(\Delta\Delta_2) + \frac{\Gamma\Delta_2}{2\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right\}. \tag{45}$$

In expression (45) we keep only terms leading to the Hall conductivity. With help of Eq. (41) we obtain

$$\hat{\sigma}^{\rm corr}_H = -\frac{i\nu}{8\gamma B^4}\left\{\left[\frac{1+\lambda_{\rm eff}}{\lambda_{\rm eff}} - \frac{\Gamma}{2\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right]\left\langle\frac{\left[(\mathbf{B}\times\frac{\partial|\Delta|^2}{\partial\mathbf{r}})\cdot(\Delta\partial_+\Delta^* - \Delta^*\partial_-\Delta)\right]}{|\Delta|^2}\right\rangle - \frac{1}{4T}\int_{-\infty}^{\infty}\frac{d\epsilon}{ch^2\frac{\epsilon}{2T}}\frac{\Gamma^2}{\Gamma^2+\epsilon^2}\left\langle\left[\left(\mathbf{B}\times\frac{\partial|\Delta|^2}{\partial\mathbf{r}}\right)\cdot\hat{L}_1^{-1}(\Delta\partial_+\Delta^* - \Delta^*\partial_-\Delta)\right]\right\rangle\right\}(\mathbf{B}\times\cdots). \tag{46}$$

Finally, the expression for the conductivity tensor $\hat{\sigma}$ can be presented in the form

$$\hat{\sigma} = \hat{\sigma}_N + \hat{\sigma}^{\rm qcl}_d + \hat{\sigma}^{\rm qcl}_H + \hat{\sigma}^{\rm corr}_H, \tag{47}$$

where $\hat{\sigma}_N$ is the tensor conductivity in the normal metal and $\hat{\sigma}^{\rm qcl}_d$, $\hat{\sigma}^{\rm qcl}_H$, and $\hat{\sigma}^{\rm corr}_H$ are given by Eqs. (43), (44), and (46).

For single vortex, the following relation holds:

$$\Delta\partial_+\Delta^* - \Delta^*\partial_-\Delta = -\frac{2i}{\rho}|\Delta|^2\,\mathbf{e}_\varphi\ \mathrm{sgn}(eB),$$
$$\mathbf{B}\times\mathbf{e}_\rho = \mathbf{e}_\varphi\cdot B, \tag{48}$$

where ρ is the distance from the vortex core and $\mathbf{e}_\rho$, $\mathbf{e}_\varphi$ are unity vectors along $\mathrm{grad}\rho, \mathrm{grad}\varphi$.

In the weak magnetic field ($B \ll H_{c2}$) we obtain from Eqs. (44), (46), and (48)

$$\hat{\sigma}_H^{\rm corr}+\hat{\sigma}_H^{\rm qcl}=\frac{\pi\nu{\rm sgn}(e)}{2\gamma B^2\phi_0}(\mathbf{B}\times\cdots)\Bigg[-\left[\frac{1+\lambda_{\rm eff}}{\lambda_{\rm eff}}-\frac{\Gamma}{2\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right]\mid\Delta_\infty\mid^2$$
$$+\frac{1}{4T}\int_{-\infty}^{\infty}\frac{d\epsilon}{ch^2\frac{\epsilon}{2T}}\int_0^\infty d\rho\rho\frac{\partial\mid\Delta\mid^2}{\partial\rho}\left(\mid\Delta\mid^2-\mathcal{D}\frac{\Gamma^2+\epsilon^2}{2\Gamma}\left\{\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)\right]-\frac{1}{\rho^2}\right\}\right)^{-1}\frac{\mid\Delta\mid^2}{\rho}$$
$$\times\left[\frac{1}{\lambda_{\rm eff}}-\frac{\Gamma}{2\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)+\frac{\Gamma^2}{\Gamma^2+\epsilon^2}\right]\Bigg] \qquad (49)$$

where $\phi_0=\pi/\mid e\mid$ is the flux quantum. In the range $\Gamma\gg\pi T_c$ expressions (44), (46), and (49) in the main in $1/\lambda_{\rm eff}$ approximation coincide with results in Refs. 2 and 3, obtained with help of the time-dependent Ginzburg-Landau equation. In the region $\Gamma\ll\pi T_c$ the last terms in Eqs. (44) and (46) are small. These terms are connected with the change of both distribution functions [even (g_1+g_2) and odd (g_1-g_2)]. As a result, for the Hall component of conductivity in the range $\Gamma\ll\pi T_c$, we have the simple expression

$$\hat{\sigma}_H^{\rm corr}+\hat{\sigma}_H^{\rm qcl}=-\frac{i\nu}{8\gamma B^4}(\mathbf{B}\times\cdots)\frac{1+\lambda_{\rm eff}}{\lambda_{\rm eff}}\times\left\langle\frac{\left(\mathbf{B}\times\frac{\partial|\Delta|^2}{\partial\mathbf{r}}\right)\cdot(\Delta\partial_+\Delta^*-\Delta^*\partial_-\Delta)}{\mid\Delta\mid^2}\right\rangle. \qquad (50)$$

In the weak magnetic field, it reduces to the expression

$$\hat{\sigma}_H^{\rm corr}+\hat{\sigma}_H^{\rm qcl}=-\frac{\pi\nu{\rm sgn}(e)}{2\gamma B^2\phi_0}\frac{1+\lambda_{\rm eff}}{\lambda_{\rm eff}}\Delta_\infty^2(\mathbf{B}\times\cdots). \qquad (51)$$

The order parameter value Δ_∞ without a magnetic field is connected with the critical magnetic field H_{c2} as

$$\frac{eH_{c2}\mathcal{D}}{2\pi T}\psi'\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)=-\frac{\Delta_\infty^2}{4(2\pi T)^2}\left[\psi''\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)+\frac{\Gamma}{6\pi T}\psi'''\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)\right]. \qquad (52)$$

In the weak magnetic field ($B\ll H_{c2}$), the last three terms in Eq. (47) contain large parameters H_{c2}/B relative to the first one.

In superconductors with a large value of the Ginzburg-Landau parameter κ, the order parameter Δ can easily be found in the approximation of the round cell. Near the transition temperature from Eq. (39) we obtain

$$\mid\Delta\mid=\frac{2\sqrt{2}\pi T}{\sqrt{7\zeta(3)}}\sqrt{1-T/T_c}f(\rho/\xi)\ , \qquad \xi=\sqrt{\frac{\pi\mathcal{D}}{8(T_c-T)}}, \qquad (53)$$

where the function f is a solution of the equation

$$\left\{1+\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial}{\partial x}\right)-\frac{1}{x^2}\right\}f=f^3, \qquad (54)$$

with boundary conditions

$$f(0)=0,\qquad \left.\frac{\partial f}{\partial x}\right|_{(\xi\sqrt{eB})^{-1}}=0. \qquad (55)$$

According to Eq. (46) in this approximation the Hall conductivity is equal to

$$\hat{\sigma}_H=-\frac{\pi\nu{\rm sgn}(e)}{\gamma B^2\phi_0\lambda_{\rm eff}}\frac{(2\pi T)^2}{7\zeta(3)}\times(1-T/T_c)\tilde{f}(B/H_{c2})(\mathbf{B}\times\cdots),$$
$$\tilde{f}\left(\frac{B}{H_{c2}}\right)=f^2\left(x_0\sqrt{\frac{H_{c2}}{B}}\right). \qquad (56)$$

The critical magnetic field H_{c2} in the round cell approximation is

$$eH_{c2}\mathcal{D}=\frac{8(T_c-T)}{\pi x_0^2}, \qquad (57)$$

where x_0 is the first zero of the equation

$$J_1(x_0)=0\ ,\quad x_0=1.841\ 18. \qquad (58)$$

In Eq. (58), $J_1(x)$ is the Bessel function. The exact value of H_{c2} is equal to

$$eH_{c2}\mathcal{D}=\frac{4}{\pi}(T_c-T). \qquad (59)$$

Near H_{c2} we have

$$\tilde{f}\left(\frac{B}{H_{c2}}\right)=\left(1-\frac{B}{H_{c2}}\right)J_1^2(x_0)\int_0^{x_0}dxxJ_1^2(x)\Big/\int_0^{x_0}dxxJ_1^4(x)=1.2143\left(1-\frac{B}{H_{c2}}\right). \qquad (60)$$

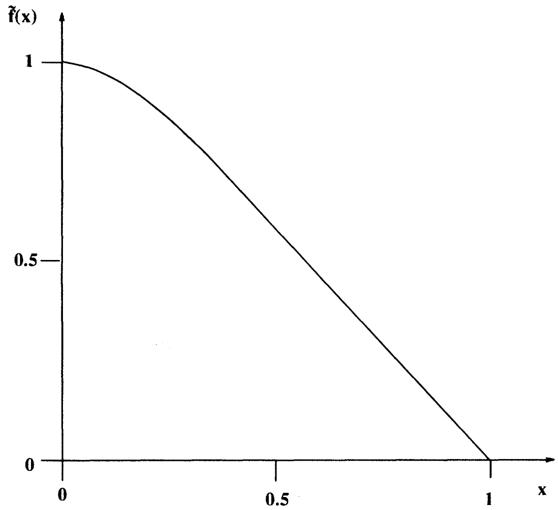

FIG. 1. The magnetic field dependence of the function $\tilde{f}(x)$ in Eq. (56).

And for small values of x

$$\tilde{f}(x) = 1 \ , \ x \ll 1. \tag{61}$$

In Fig. 1 we plot the function $\tilde{f}(x)$ in Eq. (56).

VI. CONCLUSION

We have found the Hall conductivity in dirty superconductors with a large enough depairing factor $\Gamma \gg \Delta$ in the entire range of temperatures and magnetic fields. For a small value of the magnetic field $B \ll H_{c2}$, the Hall conductivity depends on the quantity $1/\gamma$ that is proportional to the derivative of the density of states with respect to the energy. With increasing of the electron mean free path, the corrections to the quasiclassical Green's function become essential and the nature of the Hall conductivity will be changed.

In the range $\Delta < T < \pi T_c$, the Hall conductivity does not depend on the distribution function.

In real superconductors, the pinning phenomenon is very essential for the Hall conductivity, especially in the region of weak magnetic fields $B \ll H_{c2}$.[11,12]

Note added in proof. At low temperatures in the clean limit the Ohmic and Hall conductivities were studied by N. B. Kopkin (Pis'ma Zh. Eksp. Teor. Phys. **60**, 123 (1994) [JETP Lett. **60**, 130 (1994)]).

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