Sign reversal of the Hall effect in untwinned single-crystal superconducting $YBa_2Cu_3O_{7-\delta}$

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(Received 20 September 1994)

For untwinned single-crystal $YBa_2Cu_3O_{7-\delta}$ in a magnetic field H, we find that the in-plane Hall conductivity σ_{xy} just below the superconducting transition temperature T_c is the sum of two terms, C_1/H and C_2H, where C_1 and C_2 are field independent, in agreement with recent theoretical predictions of Dorsey *et al.* and Kopnin *et al.* C_1 and C_2 have opposite signs, as required by the observation that the Hall effect changes sign as the applied field is varied. C_1 is proportional to τ^2, and C_2 is approximately proportional to τ, where $\tau=(T_c-T)/T_c$, so the crossover field H_0, where $\sigma_{xy}=0$, is proportional to $\tau^{1/2}$.

INTRODUCTION

In a type-II superconductor in an applied magnetic field H, quantized magnetic vortices (fluxoids) are formed by supercurrents, forming the mixed state, for H larger than the lower critical field H_{c1} and less than the upper critical field H_{c2}. The Hall effect in the mixed state has never been successfully analyzed to account for all of the salient features of the observations, even in the classical superconductors.[1] The high-temperature superconductors have also exhibited puzzling Hall effect phenomena in the mixed state.[2] One of the most puzzling of these features has been a reversal of the sign of the Hall effect in the neighborhood of the superconducting transition temperature T_c as the magnetic field is varied.

This sign reversal of the Hall effect has been observed for $YBa_2Cu_3O_{7-\delta}$,[3–9] $Bi_2Sr_2CaCu_2O_8$,[4] and $Tl_2Ba_2CaCu_2O_8$,[10,11] (YBCO, BSCCO, and TBCCO). Indeed, the sign reversal had already been seen in some of the classical superconductors, such as the elements Nb (Ref. 12) and V (Ref. 13) and In-Pb alloys.[14] The Hall effect's sign reversal for $YBa_2Cu_3O_{7-\delta}$ (YBCO) has been observed for single untwinned crystals.[8] Those crystals were very pure and had extraordinarily low resistivity, indicating correspondingly high sample quality; they were made by a method known to produce untwinned crystals with very low flux pinning.[15] It therefore seems useful to further analyze the resistivity and Hall-effect data of Ref. 8 in light of recent theoretical developments.

ANALYSIS

The resistivity and Hall-effect data of Ref. 8 were obtained simultaneously on the same sample, with the applied magnetic field H oriented perpendicular to the copper-oxygen planes; we will call that the z direction. The current density J in the sample was in the x direction, and the Hall electric field E was in the y direction, indicating positively charged current carriers (holes). For this geometry, the Hall effect can be described in terms of the Hall coefficient R_H, the Hall angle θ_H, the off-diagonal matrix element of the resistivity ρ_{yx}, or the off-diagonal matrix element of the conductivity σ_{xy}. These quantities are related by the equations

$$R_H=\frac{E_y}{J_xH_z}=\frac{\rho_{yx}}{H_z}\ , \tag{1}$$

$$\tan\theta_H=\frac{\rho_{yx}}{\rho_{xx}}\ , \tag{2}$$

$$\sigma_{xy}=\frac{\rho_{yx}}{\rho_0}\ , \tag{3}$$

where

$$\rho_0=\rho_{xx}\rho_{yy}-\rho_{xy}\rho_{yx}\ . \tag{4}$$

For the superconductors under discussion here, the resistivity and conductivity tensors are antisymmetric, and their off-diagonal elements are odd functions of H_z. (The data are plotted in this paper for $H_z>0$, reversed from the direction used in the plots of Ref. 8 to conform with the notation of Ong and others.) In the normal state, $R_H>0$ for hole conduction (as seen in YBCO) and $\rho_0>0$, so ρ_{yx} and σ_{xy} are also positive. Although Hall-effect results for superconductors have traditionally been expressed in terms of R_H, θ_H, or ρ_{yx}, it is of particular interest to discuss such data in terms of σ_{yx}, because Vinokur *et al.*[16] pointed out that this parameter should be independent of disorder in the sample. According to theoretical results of Dorsey and co-workers[17–19] and of Kopnin, Ivlev, and Kalatsky,[20] the Hall conductivity σ_{xy} in the superconducting state should be the sum of two terms:

$$\sigma_{xy}=\sigma^s_{xy}+\sigma^n_{xy}\ , \tag{5}$$

where σ^s_{xy}, arising from the motion of the magnetic vortices, should be proportional to $1/H$. Because σ^n_{xy} arises from the motion of quasiparticles in the regions of the vortex cores, we may expect it to be proportional to H. Accordingly, σ_{xy} should be of the form

$$\sigma_{xy}=C_1/H+C_2H\ , \tag{6}$$

where the coefficients C_1 and C_2 are independent of H, but are expected to depend on the temperature T. At low

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fields, σ_{xy} should therefore be proportional to $1/H$, and such a field dependence has been observed by Samoilov, Ivanov, and Johansson in TBCCO films.[11] Their results indicate that the C_2H term in Eq. (6) should be replaced by a field-independent term, however; this behavior is different from the YBCO crystal results we focus on here. Ong's group has recently obtained experimental results on YBCO which obey Eq. (6) at high fields.[21] The angular dependence of Ong's data fit the theory of Geshkenbein and Larkin,[22] which, like the work of Dorsey and co-workers and of Kopnin, Ivlev, and Kalatsky, is based on the time-dependent Ginzburg-Landau theory.

To test the validity of Eq. (6), we fit our data to the relation

$$\sigma_{xy}H = C_1 + C_2H^2 \; . \tag{7}$$

In calculating σ_{xy}, one uses Eq. (4); this requires that one know ρ_{yy}. This we find from the known, reproducible[23] ratio of ρ_{xx} to ρ_{yy}. If C_1 and C_2 have opposite signs, then the Hall effect can change sign as H is varied, as seen in the experiments. We will find that Eq. (7) describes the YBCO data of Ref. 8 very well in the neighborhood of the sign reversal of the Hall effect.

The temperature dependences of the coefficients C_1 and C_2 are the new results that we are reporting here. We will not attempt to relate these temperature dependences to theory, because theory indicates that would require one to take into account details of the electronic structure, such as the shape of the Fermi surface and the energy dependence and anisotropy of the electron density of states, the superconducting pairing potential, and various relaxation times.[17–20]

RESULTS

We will now determine how closely Eq. (7) fits the data on the sample,[8] which had a value of $T_c = 94.5$ K, as determined from the midpoint of its resistive transition in zero field. (In all of our graphs, the scatter of the points is much greater than the systematic experimental uncertainties, except for a field-independent and temperature-independent scale factor arising from a small uncertainty in the sample size.[8]) Figures 1 and 2 show $\sigma_{xy}H$ vs H^2. Also shown there are straight-line fits. (All of our fits are least-squares fits.) In making these fits, we have ignored some of the points at the lowest values of H^2, since they curve away from such a linear fit and/or have a large amount of scatter, possibly because of flux-pinning effects. The points ignored in the fits are indicated by the open symbols in these two figures. It is evident that a straight line provides an excellent fit to the data points in the neighborhood of the reversal of the Hall effect's sign, i.e., near the crossover field H_0, where σ_{xy} passes through 0.

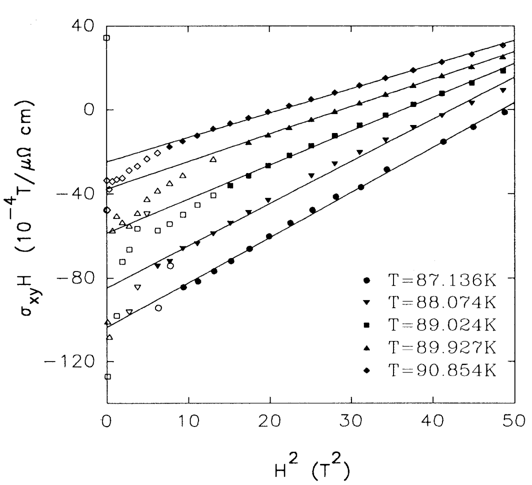

FIG. 1. $\sigma_{xy}H$ vs H^2 for an untwinned single crystal of YBCO at five temperatures T just below $T_c = 94.5$ K. The data points indicated by open symbols were not used in making the linear fits that are shown.

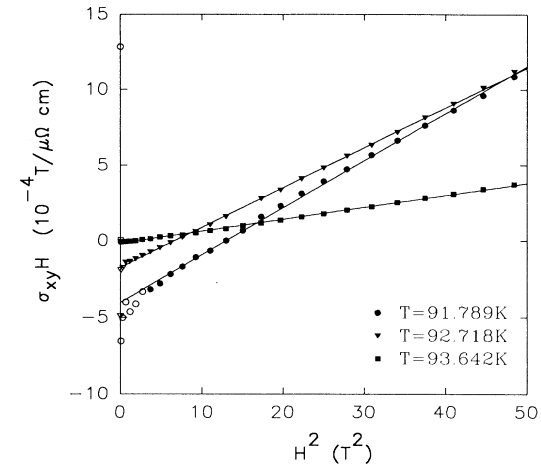

FIG. 2. $\sigma_{xy}H$ vs H^2 for an untwinned single crystal of YBCO at three temperatures T just below $T_c = 94.5$ K. The data points indicated by open symbols were not used in making the linear fits that are shown.

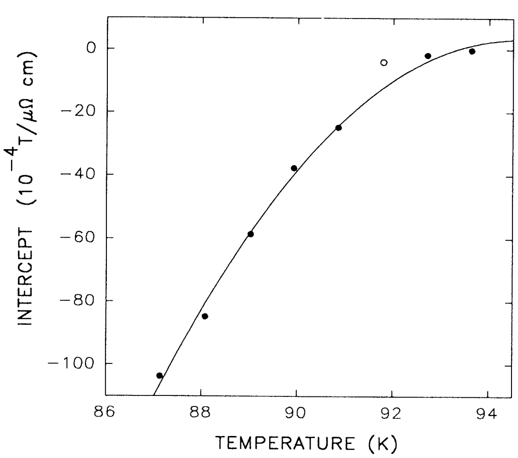

FIG. 3. C_1 vs T. C_1 is the intercept of the straight-line fits shown in Figs. 1 and 2. The curve is a fit to the power law $C_1 = A_1\tau^2$, where $\tau = (T_c - T)/T_c$. The data point indicated by the open symbol was not used in making the fit that is shown. A_1 and all of the other fitting parameters are listed in Table I.

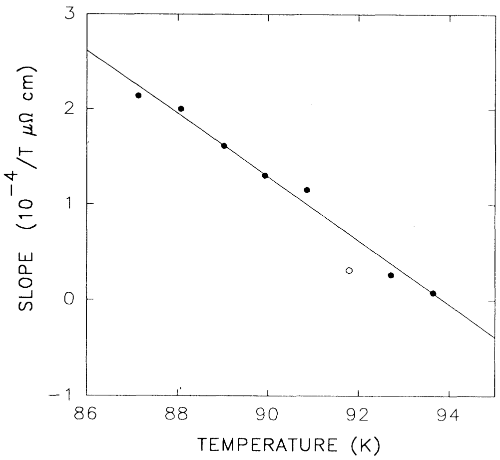

FIG. 4. C_2 vs T. C_2 is the slope of the straight-line fits shown in Figs. 1 and 2. The data point indicated by the open symbol was not used in making the fit that is shown.

Figures 3 and 4 show plots of the intercept C_1 and the slope C_2, respectively, of the straight-line fits shown in Figs. 1 and 2. We have fitted C_1 to a power law:

$$C_1 = A_1 \tau^n , \tag{8}$$

where $\tau = (T_c - T)/T_c$. We plot $\ln C_1$ vs $\ln \tau$, and the slope should be n. As shown in Fig. 5, this power law provides a good fit to the temperature dependence of C_1. In determining n, we ignore the data point at 91.789 K, since Figs. 3 and 4 show that it deviates from the behavior of the other points. The values of all of our fitting parameters are listed in Table I. The best fit for n is 2.00 ± 0.11. Perhaps this approximate integer value can be derived from a simple scaling argument. The fit with $n = 2$ is shown in Fig. 3.

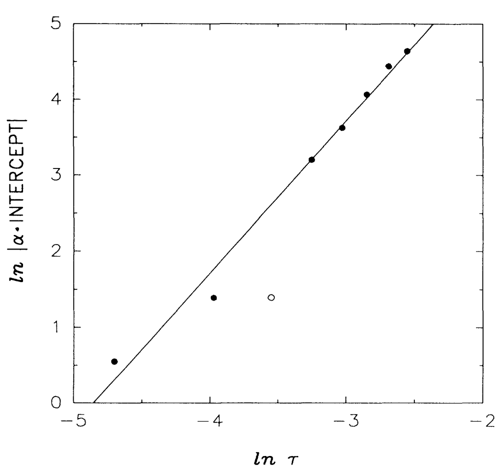

FIG. 5. $\ln|\alpha C_1|$ vs $\ln \tau$, where $\alpha = 10^4 \mu\Omega\,\text{cm/T}$. The data point indicated by the open symbol was not used in making the fit that is shown. The straight line is a fit to the power law $C_1 = A_1 \tau^n$. We find the exponent $n = 2.00 \pm 0.11$. See the fit to $n = 2$ in Fig. 3.

TABLE I. Fitting parameters and uncertainties (one standard deviation).

Quantity	Value	Units
A_1	-1.68 ± 0.64	$\text{T}/\mu\Omega\,\text{cm}$
A_2	-0.214 ± 0.085	$10^{-4}/\text{T}\,\mu\Omega\,\text{cm}$
B_2	-31.6 ± 1.7	$10^{-4}/\text{T}\,\mu\Omega\,\text{cm}$
D	26.4 ± 7.6	T
n	2.00 ± 0.11	

Figure 4 shows that C_2 is a linear function of the temperature T,

$$C_2 = A_2 + B_2 \tau , \tag{9}$$

again ignoring the data point at 91.789 K. A straight-line fit going through $C_2 = 0$ at $T = T_c$ (i.e., at $\tau = 0$) would have fit almost as well. According to Eq. (6), Eq. (8) with $n = 2$, and Eq. (9), the crossover field H_0 would then be given by

$$H_0 = \left[-\frac{C_1}{C_2} \right]^{1/2} = D\tau^{1/2} . \tag{10}$$

Figure 6 shows H_0^2 vs τ, with a straight-line fit.

CONCLUSIONS

We have analyzed Hall-effect data which were obtained on a sample of $YBa_2Cu_3O_{7-\delta}$ with high quality (shown by low resistivity) and made by a method known to yield pure crystals with low resistivity and low flux pinning.[8] The results are well fitted by Eq. (7), and therefore by Eq. (6), in the field neighborhood of the sign reversal in the Hall conductivity σ_{xy}. The points deviate from the fit at lower field values, possibly because of flux pinning. The coefficients C_1 and C_2 in Eq. (7) have been fitted to a power law and a linear relation, given in Eqs. (8) and (9), respectively. The fitted power-law exponent is found to be $n = 2.00 \pm 0.11$. The crossover field H_0, where σ_{xy} passes through 0, is proportional to $\tau^{1/2}$. The success of

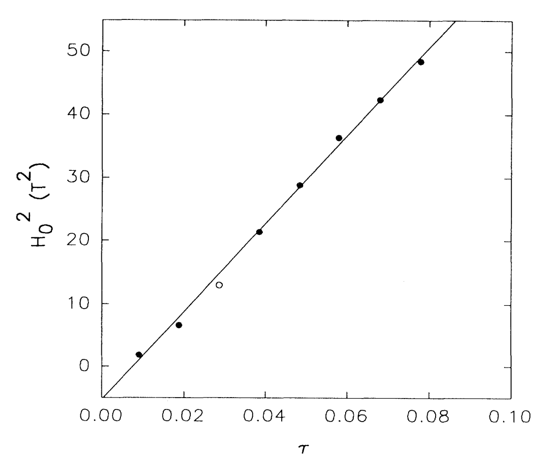

FIG. 6. The square of the crossover field H_0 vs τ, with a straight-line fit to the relation $H_0 = D\tau^{1/2}$. The data point indicated by the open symbol was not used in making the fit that is shown.

the fit to Eq. (6) verifies theoretical predictions of Dorsey and co-workers[17–19] and of Kopnin, Ivlev, and Kalatsky.[20] We hope that the theory can be extended to yield the temperature dependences of the coefficients C_1 and C_2.

ACKNOWLEDGMENTS

This research was supported in part by the National Science Foundation under Grant No. DMR-9318740.

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