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Effect of phase breaking on quantum transport through chaotic cavities

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We investigate the effects of phase-breaking events on electronic transport through ballistic chaotic cavities. We simulate phase breaking by a fictitious lead connecting the cavity to a phase-randomizing reservoir and introduce a statistical description for the total scattering matrix, including the additional lead. For strong phase breaking, the average and variance of the conductance are calculated analytically. Combining these results with those in the absence of phase breaking, we propose an interpolation formula, show that it is an excellent description of random-matrix numerical calculations, and obtain good agreement with several recent experiments.

Recently there has been great interest in the effects of quantum-mechanical interference on electronic transport through ballistic quantum dots.[1] In these microstructures both the phase-coherence length and the elastic mean free path exceed the system dimensions. Thus the leads into the dot can be thought of as electron waveguides and the dot itself as a resonant cavity.

Experimentally[2–9] one observes random (but reproducible) fluctuations in the conductance as the magnetic field,[2,4,9] the Fermi energy,[5] or the shape of the cavity[9] is changed. The sensitivity to small changes in these parameters shows that these fluctuations are caused by quantum interference. In addition to conductance fluctuations, several interference effects which survive averaging over many microstructures are observed;[6–8] in particular, an increase in the average resistance at zero magnetic field called the weak-localization correction (WLC).[4,5,8] The two main features of these interference effects are their *shape*—the characteristic field or energy to which they are sensitive—and their *magnitude*—simply how big these quantum corrections are compared to the classical conductance. Here we concentrate on the magnitude and merely note that a theory for the shape has been extensively developed.[10–13]

The magnitude of quantum interference effects in chaotic cavities has recently been studied by making a statistical ansatz for the S matrix describing the scattering.[12–16] References 15 and 16 developed a random S-matrix theory by assigning to S an "equal *a priori* distribution" once the symmetry requirements were imposed. The results for the average, variance, and probability density of the conductance were in good agreement with numerical calculations.[15] However, the random-matrix predictions for both the weak-localization correction and the variance are larger than the experimental results.[8,9] In addition, the measured probability density is close to a Gaussian distribution[9] when there are two propagating modes per lead ($N=2$), while random-matrix theory predicts a Gaussian distribution only for $N \geqslant 3$.[15]

Inherent in Refs. 15 and 16 is the assumption (among others) that one can neglect processes which destroy the coherence of the wave function. In this paper we show that this assumption is largely responsible for the discrepancy between theory and experiment mentioned above. We make specific predictions for the dependence of the quantum transport corrections on the degree of phase breaking which may be tested by experiments.

To simulate the effects of phase-breaking events we adopt a model suggested by Büttiker:[17] in addition to the physical leads 1,2 attached to reservoirs at chemical potentials μ_1, μ_2, a lead 3 connects the cavity to a phase-randomizing reservoir at μ_3. This model has been discussed extensively for disordered materials[18] and, more recently, for ballistic quantum dots by Marcus *et al.*[3] A similar model has been used for absorption of microwaves in chaotic-scattering from cavities.[19] Requiring the current in lead 3 to vanish determines μ_3; the two-terminal dimensionless conductance is then found to be

$$g \equiv G/(e^2/h) = 2\left[T_{21} + \frac{T_{23}T_{31}}{T_{32}+T_{31}} \right] , \tag{1}$$

where T_{ij} is the transmission coefficient for "spinless electrons" from lead j to lead i. The factor of 2 accounts for spin explicitly. We call N the number of channels in leads 1 and 2, N_ϕ that in lead 3, and $N_T = 2N + N_\phi$. The N_ϕ channels in lead 3 are physically related to the phase-breaking scattering rate γ_ϕ via the relation $N_\phi/2N \approx \gamma_\phi/\gamma_{\rm esc}$ where $\gamma_{\rm esc}$ is the escape rate from the cavity. In deriving Eq. (1) no phase relation is assumed between the N_ϕ channels and the $2N$ original ones, and the various currents are thus added incoherently.

We now make the fundamental assumption of an equal *a priori* distribution for the *total* $N_T \times N_T$ scattering matrix S, once the symmetry requirements have been imposed. S is, of course, unitary, and is symmetric in the absence of a magnetic field because of time-reversal symmetry. We assume that the statistics of the total S matrix are given by the circular ensembles of random matrix theory.[20] For $B=0$ the orthogonal ensemble (denoted $\beta=1$) is appropriate while for nonzero B we use the unitary ensemble ($\beta=2$). In contrast to previous studies of the eigenphases,[10,16] total transmission,[15,16] or individual S-matrix elements,[19] we treat the statistics of g given in Eq. (1). In the rest of the paper, we first derive results valid in the weak and strong phase-breaking limits, then combine these into an interpolation formula which simulations show to be valid, and finally compare with experiments.

We start by recalling the result for the WLC and variance at $N_\phi=0$ given in Ref. 15:

$$\delta g \equiv \langle g\rangle^{(\beta=1)}-\langle g\rangle^{(\beta=2)}=-N/(2N+1)\,, \tag{2a}$$

$$\mathrm{var}g=\begin{cases}\dfrac{4N(N+1)^2}{(2N+1)^2(2N+3)}\,, & \beta=1\\[2ex] \dfrac{N^2}{4N^2-1}\,, & \beta=2\,.\end{cases} \tag{2b}$$

In addition to these results, the probability density of g was calculated for $N=1,2,3$.

The case $N_\phi=1$ and $\beta=1$ is a special one which can be analyzed in detail. From the joint probability distribution of S-matrix elements in one row,[21]

$$P(S_{11},\dots,S_{1N_T})\propto(1-|S_{11}|^2)^{-N}\delta\left(1-\sum_{i=1}^{N_T}|S_{1i}|^2\right)\,, \tag{3}$$

one can show that the joint distribution of the T_{3j}'s is

$$P(\{T_{3j}\})=\frac{N(2N-1)!(T_{31}T_{32})^{N-1}\delta\left(1-\sum T_{3j}\right)}{[(N-1)!]^2(T_{31}+T_{32})^N}\,. \tag{4}$$

Remarkably, the $\langle g\rangle$ that one obtains by integrating over this distribution is identical to that for $N_\phi=0$.

In the limit $N_\phi\gg1$, we obtain results to leading order in $1/N_\phi$. First, expand the conductance of Eq. (1) in powers of δT_{ij} where $T_{ij}=\langle T_{ij}\rangle+\delta T_{ij}$. The average transmission is[15] $\langle T_{ij}\rangle=N_iN_j/(N_T+\delta_{\beta1})$ as in the Hauser-Feshbach formula of nuclear physics.[23] Using Ref. 22, one finds that the correlations among the T_{ij}'s are

$$\begin{aligned}\langle\delta T_{ij}\delta T_{kl}\rangle=A_\beta\{&N_iN_j(N_T+\delta_{\beta1})(N_T+2\delta_{\beta1})\delta_{ik}\delta_{jl}\\ &-N_iN_jN_l(N_T+\delta_{\beta1})\delta_{ik}-N_iN_jN_k(N_T+\delta_{\beta1})\delta_{jl}\\ &+N_iN_jN_kN_l(1+\delta_{\beta1})\}\,,\end{aligned} \tag{5a}$$

$$A_1=[N_T(N_T+1)^2(N_T+3)]^{-1}\,, \tag{5b}$$

$$A_2=[N_T^2(N_T^2-1)]^{-1} \tag{5c}$$

for $i\neq j$, $k\neq l$. (For $\beta=1$, the indices of the T's have been permuted so as to maximize coincidences.) Note that $\langle\delta T_{ij}\delta T_{kl}\rangle$ is at least of order $1/N_\phi^2$. Thus

$$\langle g\rangle=2\left[\langle T_{21}\rangle+\frac{\langle T_{23}\rangle\langle T_{31}\rangle}{\langle T_{32}\rangle+\langle T_{31}\rangle}\right]+O(1/N_\phi^2). \tag{6}$$

The largest contribution to $\langle g\rangle$ is the classical conductance $N_1N_2/(N_1+N_2)$; for $N_\phi=0$ this comes from $\langle T_{21}\rangle$ while for large N_ϕ it comes from the second term in Eq. (6) (the incoherent part). The WLC is then given by

$$\delta g=-N/N_\phi+O(1/N_\phi^2). \tag{7}$$

For the variance one finds from Eq. (5)

$$\mathrm{var}g=\left(\frac{N}{N_\phi}\right)^2\frac{2}{\beta}\left[1+\frac{2-\beta}{2N}\right]+\cdots \tag{8}$$

and, for the ratio of the variances for $\beta=1,2$,

$$\frac{(\mathrm{var}g)^{(\beta=1)}}{(\mathrm{var}g)^{(\beta=2)}}=2\left(1+\frac{1}{2N}\right)+\cdots\,. \tag{9}$$

Of course the magnitude of the quantum corrections decreases as the phase breaking increases. That var$g$ decreases with N_ϕ reflects the fact that each S-matrix element fluctu-

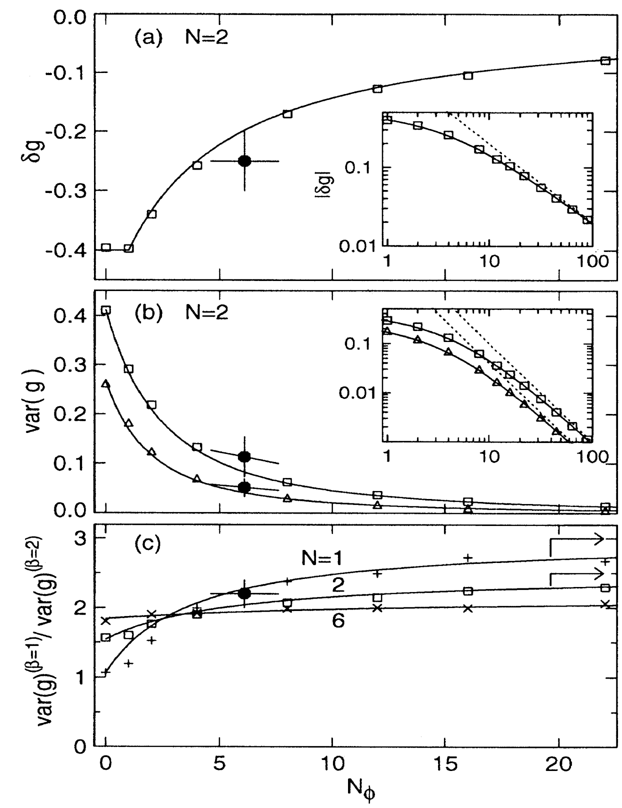

FIG. 1. Magnitude of quantum transport effects as a function of the number of phase-breaking channels, N_ϕ, on linear and log-log (insets) scales. (a) The weak-localization correction ($N=2$). (b) The variance in the orthogonal (squares) and unitary (triangles) cases ($N=2$). (c) The ratio of the variance in the orthogonal case to that in the unitary for $N=1$, 2, and 6; the arrows mark the $N_\phi\to\infty$ limit for $N=1,2$. Open symbols are numerical results (20 000 matrices used, statistical error is the symbol size). Solid lines are interpolation formulas. Dotted lines are asymptotic results. Solid circles are experimental results of Ref. 9 corrected for thermal averaging. The interpolation formulas are excellent except for $N=1$ and small N_ϕ [panel (c)].

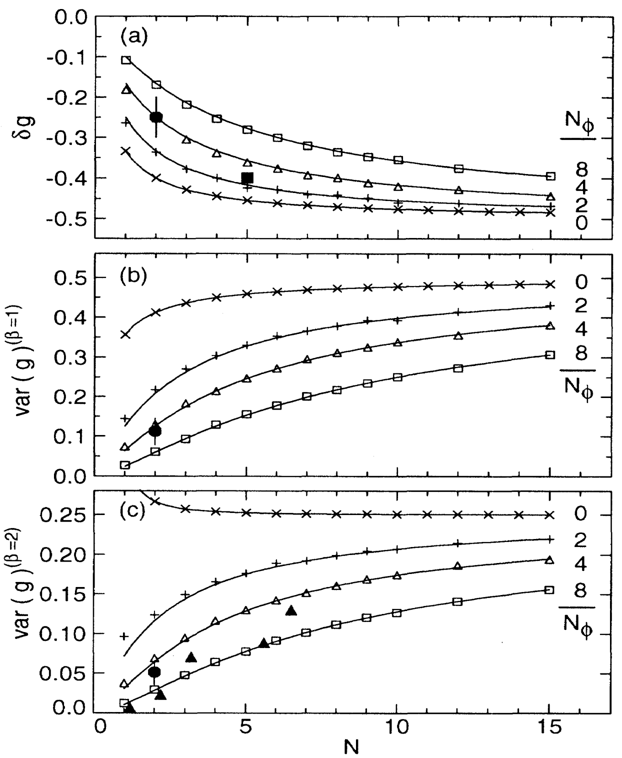

FIG. 2. Magnitude of quantum transport effects as a function of the number of channels in the leads, N, for $N_\phi=0$, 2, 4, and 8. (a) The weak-localization correction. (b) The variance for the orthogonal case ($B=0$). (c) The variance for the unitary case (nonzero B). Open symbols are numerical results (as in Fig. 1). Solid lines are interpolation formulas. Solid symbols are experimental results of Refs. 4 (triangles), 8 (squares), and 9 (circles) corrected for thermal averaging. The introduction of phase breaking decreases the "universality" of the results but leads to good agreement with experiment.

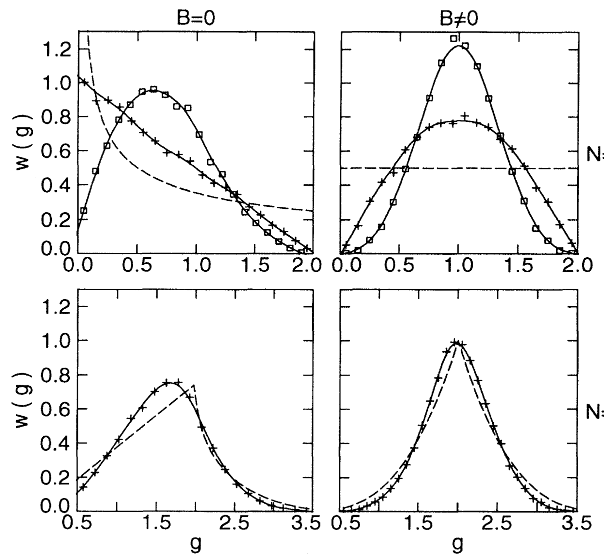

FIG. 3. Probability density of the conductance in the orthogonal (first column) and unitary (second column) cases for $N=1$ (first row) and $N=2$ (second row). Increasing the phase breaking from zero (dashed lines, analytic) to $N_\phi=1$ (plus symbols, numerical) to $N_\phi=2$ (squares, numerical) moves the distribution towards a Gaussian.

ates less as N_T increases. The deviation of the ratio of the variances from 2 is highly unusual; in fact, for $N=1$ the ratio can be as high as 3.

We present in Figs. 1–3 results of simulations of the random-matrix model in order to check the asymptotic behavior and suggest improvements. The points are obtained by generating random $N_T \times N_T$ unitary or orthogonal matrices and computing g from Eq. (1). In Fig. 1, $N=2$ was selected for comparison with the experiment of Ref. 9. The insets in Fig. 1 show that the convergence of the numerical to the asymptotic results [Eqs. (7) and (8)] is rather slow. The curves in the main parts of Fig. 1 are interpolation formulas inferred from the small and large N_ϕ results. For the WLC, Eqs. (2a) and (7) suggest

$$\delta g \simeq -N/(2N+N_\phi) \quad . \tag{10}$$

For the fluctuations of the conductance, we combine square roots of variances,

$$(\mathrm{var} g)^{1/2} = [(\mathrm{var} g)_{N_\phi=0}^{-1/2} + (\mathrm{var} g)_{N_\phi \gg 1}^{-1/2}]^{-1} \quad , \tag{11}$$

because in the numerical simulations the change in var$g$ is clearly linear in N_ϕ, not quadratic. So, for $\beta=2$

$$\mathrm{var} g \simeq N^2/[(4N^2-1)^{1/2}+N_\phi]^2 \quad . \tag{12}$$

For $\beta=1$ a similar, but more complicated, expression holds. These interpolation formulas agree very well with the numerical results; the only significant deviation is for $N=1$ and small N_ϕ [Fig. 1(c)].

The results at fixed N_ϕ (Fig. 2) are relevant to experiments at fixed temperature in which the size of the opening to the cavity is varied. Though δg and var$g$ are nearly independent of N in the perfectly coherent limit—a result known as "universality"—phase-breaking channels cause variation. Thus the universality can be seen only if $N_\phi \ll N$; otherwise, the behavior is approximately linear, as in some experiments.[4,9] Clearly, phase breaking must be included in interpreting the experiments.

In addition tQ the mean and variance, the probability density of the conductance $w(g)$ is experimentally measurable.[9] Figure 3 shows that as N_ϕ increases $w(g)$ tends towards a Gaussian and is therefore fully characterized by its mean and variance. For $N \geq 3$, $w(g)$ is essentially Gaussian[15] even for $N_\phi=0$, so that N_ϕ affects $w(g)$, apart from the mean and variance, only for $N<3$. For $N=1$, the highly non-Gaussian distributions[15,16] at $N_\phi=0$ become approximately Gaussian at $N_\phi=3$; the intermediate distributions, $N_\phi=1,2$, are shown. For $N=2$, the $N_\phi=0$ distribution is closer to a Gaussian than for $N=1$, and hence weaker phase-breaking produces a Gaussian distribution. We show numerical results only for $N_\phi=1$.

Finally, we use our model to interpret the experiments of Refs. 4, 8, and 9. First, Fig. 1 shows the measured N_ϕ,

δg, and var$g$ of Ref. 9 for $N=2$. (N_ϕ is measured through the temperature dependence of the correlation function of the fluctuations.[3]) Before comparison, the variance must be corrected for thermal averaging: convolution over the derivative of the Fermi function produces a reduction of $\sim 0.22-0.38$ for a temperature of $50-100$ mK. We have increased the measured var$g$ by the inverse of this reduction factor; the WLC is not affected since it is already an average effect. The error bars shown result from both the uncertainty in temperature (for the variance) and the experimental fluctuations at small B. [For the moment, we do not assign further physical significance to the oscillations seen in the experiment. The error bar in Fig. 1(b) is sloped because N_ϕ contributes to the total width of the levels and hence to the thermal reduction factor.] Note that we have not fit the theory to the data and yet the agreement is very good. Second, in Fig. 2 we show as solid symbols the data of Refs. 4, 8, and 9. The N_ϕ deduced from comparing to our calculations is in good agreement with the value estimated independently in the experiments: $N_\phi=7-9$ for Ref. 4, $N_\phi=2$ for Ref. 8, and $N_\phi=4-8$ for Ref. 9. Finally, the probability density of the conductance for the estimated $N_\phi=4-8$ of Ref. 9 is Gaussian, consistent with experiment. Observation of the interesting non-Gaussian distribution of g obtained in the theory for $N=1,2$ and $N_\phi=0$ requires greatly reduced phase breaking.

In conclusion, we have presented a random-matrix model that simulates the effects of phase breaking on transport through ballistic chaotic cavities. The analysis of recent experiments indicates that one can find a value of N_ϕ that consistently describes the data. It is not necessary to identify the phase-breaking channels with a particular place in real space; in fact, our random-matrix treatment implies that the phase-breaking processes are distributed throughout the cavity as in the experiments. Further experiments are needed to test quantitatively the dependence on N and N_ϕ that we predict.

Note added. Recently, we received some results by P. W. Brouwer and C. W. J. Beenakker with some overlapping material for $N=1$.

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