Finite-temperature fractional quantum Hall effect

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(Received 28 June 1994)

We investigate the fractional quantum Hall effect at finite temperature using a fermion Chern-Simons field-theoretical approach. In the absence of impurity scattering, the essential aspects of the fractional quantum Hall effect, such as the quantization of Hall conductance, as well as quasiparticle charge and statistics, are not renormalized by thermal fluctuations. On the other hand, we find that the low-energy excitation spectrum at finite T may undergo some qualitative changes as the temperature is raised. Interesting new features include a splitting of the low-energy collective modes and a redshift of the magnetoroton minimum at finite T. Possible experimental consequences are discussed.

INTRODUCTION

The phenomenon of the fractional quantum Hall effect (FQHE) may be understood theoretically as a manifestation of certain two-dimensional (2D) highly correlated fermion states.[1,2] Most of the theoretical efforts thus far have been directed to the study of zero-temperature properties, where the ground state is known to be incompressible and separated from higher energy states by a gap Δ produced by strong electron-electron interaction. On the other hand, experimental measurements are made, of course, at finite temperature. But, from the above mentioned property of the ground state, which has been firmly established through more than a decade of extensive study, it is reasonable to expect that at finite temperature the effect of thermal fluctuations would not be important, as long as $kT \leq \Delta$.

However, for a quantitative comparison between theory and experiment, there are several issues that need to be addressed concerning the finite- (low-) temperature properties of the FQHE. The first one is the fundamental question about the accuracy of the FQHE at nonzero T. How do thermal fluctuations affect the quantization of the Hall conductance? Another important question is the T dependence of the quasiparticle gap which can be explicitly computed at $T = 0$ in Laughlin's theory[1,2] for the "fundamental states" and in Jain's composite fermion theory[3] for general filling fractions and can be implicitly measured experimentally through the T dependence of the longitudinal resistivity.[2,4] In this paper we study FQHE at finite temperature. In particular, we investigate the T dependence of low-energy collective excitations. At zero T, the existence of a special rotonlike excitation in the Laughlin states was demonstrated by Girvin *et al.* through the Feynman-Bijl approach[5] and later by Zhang *et al.* using a Chern-Simons Landau-Ginzburg theory.[6] For general filling fraction ν, this problem was investigated within the composite fermion picture aided with the method of the Chern-Simons transformation.[7–10] Our approach is a fermion Chern-Simons field theory formalism, which has been used previously by Lopez and Fradkin[7] to study FQHE at zero T and by Halperin, Lee, and Read in their study of the $\nu = 1/2$ state.[8] Presented in an equivalent quantum many-body language, Simon and co-workers[9,10] have studied this Chern-Simons composite fermion approach in great detail. An important point addressed in Refs. 8–10 is the problem of mass renormalization of composite fermions. Without treating it properly, this fermion Chern-Simons theory approach will set the system at a spurious energy scale $\hbar\tilde{\omega}_c$ (see below), which is on the order of (although less than) the bare cyclotron energy, while the true physical energy scale of the problem is given by electron-electron interactions. However, the question that concerns us here is the *temperature dependence* of the collective excitations rather than an evaluation of the precise value of the energy gap. Thus, while the issue of mass renormalization is important and its phenomenological Fermi-liquid treatment (discussed in Refs. 8–10) may be straightforwardly generalized to the finite T case, it will not be considered here. This issue, along with some details of the present work, will be discussed somewhere else.[11]

APPROACH

In the spirit of the composite fermion approach, we describe 2D electrons of band mass m_b in a magnetic field at temperature $kT = 1/\beta$ by a coherent functional integral with action (in the unit $\hbar = 1$)

$$S = \int_0^\beta d\tau \int d^2r \left\{ \psi^\dagger(\partial_\tau - ia_0 - \mu)\psi + \frac{1}{2m_b}\left|\left(-i\vec{\nabla} + \frac{e}{c}\vec{A} - \vec{a}\right)\psi\right|^2 + \frac{i}{4\pi\tilde{\phi}}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \right\} + \frac{1}{2}\int_0^\beta d\tau \int d^2r \int d^2r' \psi^\dagger\psi(r)v(\vec{r}-\vec{r}\,')\psi^\dagger\psi(r') \quad , \tag{1}$$

where we attached even flux quanta per fermion through the well-known Chern-Simons term with statistical gauge field a_μ. In the above expression, $\tilde{\phi} = 2p$ and p is an integer. $\vec{A}$ is the vector potential for the magnetic field and the chemical potential μ fixes the Landau level (LL) filling fraction at ν. $v(\vec{r})$ is a two-body interaction potential. For the Coulomb interaction, $v(\vec{r}) = e^2/\epsilon r$, where ϵ is the dielectric constant. A similar Euclidean action has been studied in Ref. 13 in the context of anyon superconductivity.

While this action is equivalent to the usual one without the statistical gauge field, it provides us a convenient starting point for our approximations. Consider the homogeneous *liquid* saddle point solution for the Chern-Simons field $a_\mu = \bar{a}_\mu$ such that

$$\left|\vec{\nabla} \times \vec{\bar{a}}\right| = 2\pi\tilde{\phi}\bar{\rho} \ , \qquad \bar{a}_0 = 0 \ , \tag{2}$$

where $\bar{\rho}$ is the average particle density. In this mean-field theory, an electron experiences an effective magnetic field

$$B_{\rm eff} \equiv \left|\vec{\nabla} \times \vec{A}_{\rm eff}\right| \equiv \left|\vec{\nabla} \times \left(\vec{A} - \frac{c}{e}\vec{\bar{a}}\right)\right| \ . \tag{3}$$

For the filling fraction ν such that $1/(1/\nu - 2p) = n$ (where n is an integer), the mean-field theory possesses a ground state of n filled LL, which is thus incompressible and stable against weak perturbations. The filling fraction ν that satisfies this condition is precisely the value where FQHE arises. This correspondence between the fractional QHE and the integer QHE is precisely the basic idea of Jain's composite fermion theory,[3] which underlines the present approach. Fluctuation corrections may be systematically built around this (stable) saddle point solution.[7]

At finite T, we shall also take such a homogeneous saddle point solution as our starting place and consider only fluctuation corrections on the one-loop level. The implicit assumption is that at thermal equilibrium the composite fermion picture remains a good description; hence effects of the *thermal disintegration* of composite fermions, i.e., the unbinding of particles and (even number of) vortices, are not important for the temperature range under consideration. Experimentally it has been found[12] that the $\nu = 1/2$ anomaly persists in some temperature range in which the FQHE's at other filling fractions are smeared out by thermal fluctuations, which suggests strongly[12] that composite fermions[8] are rather robust against thermal fluctuations, and the disintegration temperature T_d of composite fermions lies well above the (zero T) energy gap of the incompressible FQH states.

With Eqs. (2) and (3), the mean-field action is given by

$$S_0 = \int_0^\beta d\tau \int d^2r \Big\{ \psi^\dagger(\partial_\tau - \mu)\psi + \frac{1}{2m_b}\left|\left(-i\vec{\nabla} + \frac{e}{c}\vec{A}_{\rm eff}\right)\psi\right|^2 \Big\} \ . \tag{4}$$

This action describes a system of noninteracting fermions with magnetic field $B_{\rm eff}$, in which the energy spectrum is given by the effective LL $\epsilon_l = \tilde{\omega}_c(l + \frac{1}{2})$, where $\tilde{\omega}_c = eB_{\rm eff}/m_b c$ is the effective cyclotron frequency. The chemical potential μ is determined by the condition[13]

$$n = \sum_l f(\epsilon_l - \mu) \ , \qquad f(\epsilon_l - \mu) = \frac{1}{e^{\beta(\epsilon_l - \mu)} + 1} \ . \tag{5}$$

To study the Gaussian fluctuations in the Chern-Simons gauge field, we adopt the approach of Refs. 8 and 9 by taking a transverse gauge such that $\vec{a}_T \parallel \hat{y}$ and $\vec{A}_T \parallel \hat{y}$ and we choose $\vec{q} \parallel \hat{x}$. In this case, $j_x(q,\omega)$ is simply given by $\frac{\omega}{q}\rho(q,\omega)$. Shifting a_μ by $\bar{a}_\mu$, i.e., $a_\mu \to \bar{a}_\mu + a_\mu$, we can express the kernel of the Gaussian action for the statistical field a_μ in terms of a 2×2 matrix. At finite frequency, it is given by

$$D^{-1} = \frac{1}{2\pi\tilde{\phi}}\begin{pmatrix} \tilde{n}q^2/\tilde{\omega}_c\Sigma_0 & iq(1-\tilde{n}\Sigma_1) \\ -iq(1-\tilde{n}\Sigma_1) & \tilde{\omega}_c[\frac{\mu_s}{2p}\tilde{l}_0 q + \tilde{n}(\Sigma_2+1)] \end{pmatrix} , \tag{6}$$

with $\tilde{n} = 2pn$ and

$$\Sigma_j = \frac{e^{-x}}{n}\sum_{m<l}\frac{l-m}{(\frac{i\omega_n}{\tilde{\omega}_c})^2 - (l-m)^2}\frac{m!}{l!}x^{l-m-1} \times \{f(\epsilon_m - \mu) - f(\epsilon_l - \mu)\}\left[L_m^{l-m}(x)\right]^{2-j} \times \left[(l-m-x)L_m^{l-m}(x) + 2x\frac{dL_m^{l-m}(x)}{dx}\right]^j \ . \tag{7}$$

In the above equation, $i\omega_n$ is the Matsubara frequency, L_m^l is the Laguerre polynomial, $x = (\tilde{l}_0 q)^2/2$, and $\tilde{l}_0 = \sqrt{c/eB_{\rm eff}}$ is the effective magnetic length. μ_s is the ratio of $\tilde{l}_0$ to the Bohr radius $a_0 = \epsilon/m_b e^2$. To obtain the electromagnetic response, we consider a fluctuation in A_μ such that $A_\mu \to A_\mu + \delta A_\mu$. Integrating out the statistical field a_μ, we arrive at the effective action

$$\tilde{S}_{\rm Gaussian}(\delta A_\mu) = \frac{1}{2}\sum_{q,i\omega_n}\begin{pmatrix} i\delta A_0(-q,-i\omega_n) & \frac{e}{c}\delta A(-q,-i\omega_n) \end{pmatrix} \times K(q,i\omega_n)\begin{pmatrix} i\delta A_0(q,i\omega_n) \\ \frac{e}{c}\delta A(q,i\omega_n) \end{pmatrix} , \tag{8}$$

with the kernel ($i\omega_n \neq 0$)

$$K(q,i\omega_n) = \frac{n}{2\pi(d)}\begin{pmatrix} \frac{q^2}{\tilde{\omega}_c}\Sigma_0 & -iq\Sigma_s \\ iq\Sigma_s & \tilde{\omega}_c\Sigma_r \end{pmatrix} \ . \tag{9}$$

In the above equation, $\Sigma_s = \Sigma_1(1-\tilde{n}\Sigma_1) + \tilde{n}\Sigma_0(\Sigma_2+1)$, $\Sigma_r = 1 + \Sigma_2 + n\mu_s\tilde{l}_0 q[\Sigma_1^2 - \Sigma_0(\Sigma_2+1)]$, and $d = (1-\tilde{n}\Sigma_1)^2 - \Sigma_0[n\mu_s\tilde{l}_0 q + (\tilde{n})^2(\Sigma_2+1)]$. The electromagnetic response is obtained by the usual procedure of substituting the Matsubara frequency $i\omega_n$ by $\omega - i\eta$. The above results differ from the zero T calculations[7,9] by the appearance of the fermion distribution $f(\epsilon_l - \mu)$, hence the necessity of summing up the additional LL's in the expression for Σ_j.

THE FQHE AT FINITE TEMPERATURE

To answer the fundamental question concerning the accuracy of the FQHE at finite temperature, one has to consider dissipative processes such as phonon or impurity scattering. In the absence of them one should expect the same transport properties at finite temperature as those at $T = 0$. In our calculation, this may be seen directly from the zero q response obtained from

$$\Sigma_j(q=0,\omega) = \frac{1}{n}\sum_{m=0}^{\infty}\frac{m+1}{(\frac{\omega}{\tilde{\omega}_c})^2 - 1}\{f(\epsilon_m - \mu) - f(\epsilon_{m+1} - \mu)\} = \frac{1}{(\frac{\omega}{\tilde{\omega}_c})^2 - 1} \ , \tag{10}$$

where in the last equality we have used Eq. (5). The zero frequency response needs to be calculated separately. Straightforward calculation shows that at finite T the compressibility κ acquires an exponential correction.[11] Now we consider the charge and statistics of the quasiparticle [which is defined by excitations from the thermal equilibrium state at a given T (Ref. 14)] at finite temperature. Following Laughlin's gauge argument,[2] which can be readily generalized to the finite T case if one assumes that states in each (pseudo) LL are *uniformly* occupied with probability $f(\epsilon_l - \mu)$, it is reasonable to expect the same fractionalization of quasiparticle charge at finite temperature. Indeed, if one lets δA be a thread of unit flux passing through the origin, one finds that total charge thus induced is νe, independent of T. For $\nu = 1/(2p+1)$, we equate this change to the charge of a quasiparticle, which is in agreement with Laughlin's gedanken experiment[1,2] at zero T. In the general case, the charge of a quasiparticle may be obtained by examining the gauge invariant (finite-temperature) one-particle Green function. Following an argument similar to that of Fradkin,[15] one obtains $eB_{\text{eff}} = e^*B$, which gives the quasiparticle charge $e^* = e/(1+2pn)$. This is the same as the zero-temperature result.[3,15] Statistics of quasiparticles may be obtained from the gradient expansion of Π^0.[15] In the small q limit, the effective Chern-Simons coupling is given by $1/\tilde{\phi}_{\text{eff}} = 1/\tilde{\phi} + n$. Counting the original statistical phase of bare fermion, we find that the statistical phase (relative to the boson) of quasiparticles is $\pi[1-2p/(1+2pn)]$. All these results stated here, which involve only the physics at large length scale, are T independent and are given by those obtained at $T=0$. This is a consequence of the fact that at the long wavelength limit, the temperature appears in our formalism only through the sum $\sum_l f(\epsilon_l - \mu)$, which is constant due to (5). Also as a result of this, the asymptotic behavior of the quasiparticle amplitude distribution remains the same as that at zero T, which can be readily demonstrated by computing responses of the system to a pointlike external charge and an external flux thread.[7,16,17] This result is significant since the statistics of quasiparticles described by the effective Chern-Simons term obtained above corresponds to the phase gain from the adiabatic interchange of two particles separated *infinitely* apart. Changes in the quasiparticle amplitude profile at finite T will then lead to the corrections to the statistics of quasiparticles[16] due to their overlap at finite distance.

COLLECTIVE EXCITATIONS

Collective modes are obtained from the poles in $K(q,\omega)$, which describes the electromagnetic response of the system at a given T. At $q=0$, the cyclotron mode $\omega_c = \frac{n}{\nu}\tilde{\omega}_c$ saturates the f-sum rule, in accordance with Kohn's theorem,[18] which can be readily generalized to the finite T case. As pointed out in the Introduction, the approach adopted here sets a spurious energy scale $\hbar\tilde{\omega}_c$ in the problem. Furthermore, once the saddle point solution Eq. (2) is taken, the interaction $v(\vec{r})$ plays only a nominal role in the perturbation expansion used here. Since FQHE occurs as a result of strong electron-electron correlations, this artifact of our approach seems quite disturbing. One can use the following rationale to justify the fermion Chern-Simons field-theory approach.[3] Although one needs an interaction to create a composite fermion (i.e., the bound state of a bare fermion and $2p$ vortices), hence FQHE, once it is formed, the residual interaction among the composite fermions becomes unimportant. Since the presence of $v(\vec{r})$ does not change the qualitative physics and is deceptive about the role of interaction in this formalism, we shall set μ_s to zero hereafter.

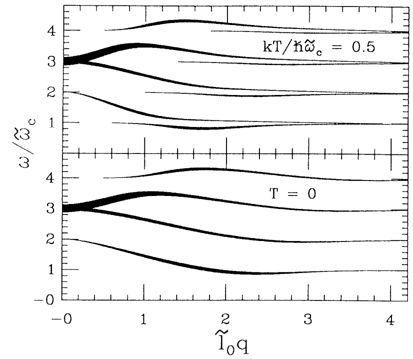

FIG. 1. Lowest few collective modes at $\nu = 1/3$. The width of the curve is proportional to q^2 times the weight of the pole in K_{00}. As T is raised fron zero, each mode splits into two.

Figure 1 shows the evolution of the lowest few branches of the collective modes in the $\nu = 1/3$ case as T is raised from zero. At finite T, each branch of the zero-temperature modes splits into two, where the upper one of the two retains all the weight at small q values. Inspecting the lowest mode, we see that the energy of the roton minimum ω_{min} is rather insensitive to T, while its position q_{min} has a stronger temperature dependence. This situation is depicted in Fig. 2. In general, increasing T causes a redshift of q_{min}. A recent optical experiment measured directly the long wavelength ($q=0$) collective modes.[19] The present calculation suggests that this experiment has actually not detected the

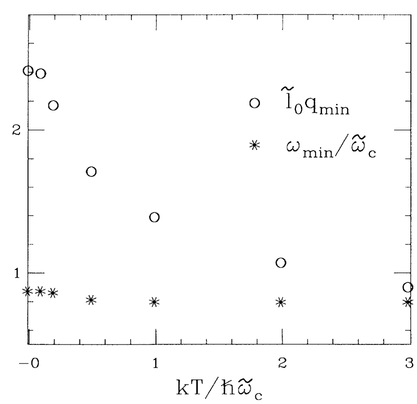

FIG. 2. Energy and position of the roton minimum for the $\nu = 1/3$ state as a function of T. Initially q_{min} is roughly unchanged as T is increased from zero and then decreases faster as T is further increased. ω_{min}, on the other hand, is quite insensitive to T for the whole range of temperature considered.

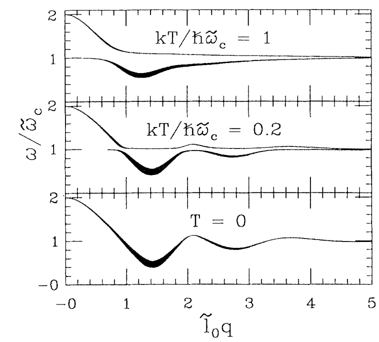

FIG. 3. Evolution of the lowest collective mode in the case of $\nu = 3/7$. As T is increased from zero, this lowest mode also splits into two. The number of roton minima decreases as T is increased, so that for sufficiently large T, only one minimum remains.

lowest branch of the collective mode, which only exists at finite T and vanishes at small q. On the other hand, this lowest mode can in principle be measured through the recently suggested experiment using evanescent field Raman scattering.[20] Since the spectrum at the $q \to \infty$ limit is independent of T, our work also provides a justification for the fitting of σ_{yy} data with a constant activation energy Δ.[2,4]

For FQH states other than the fundamental one, the zero T calculation shows that there are more than one roton minimum and the number of minima corresponds to the number of filled LL's in the composite fermion picture.[9] In Fig. 3 we show the lowest branch of the collective mode in the $\nu = 3/7$ state. At zero T, there are three roton minima. As T is raised, the weakest one, located at a large wave vector ($\approx 4.75\tilde{l}_0 q$), is smeared out first by thermal fluctuations. The second one disappears subsequently at higher T, leaving only one roton minimum at sufficiently high temperature. This is a general feature for all the ν values we have examined. While the position (and the number) of roton minima has not yet been subjected to direct experimental measurement, it does have physical implications according to one of the proposed scenarios of FQH state–Wigner crystal transition.[2,5] In this picture, such a transition is directly related to the softening of the roton minimum and the position of the (softened) roton minimum corresponds to the reciprocal wave vector of the Wigner crystal near the transition. Such a scenario is not supported by the present calculation since the lattice constant of Wigner crystal should be determined by the electron density alone and hence should be independent of T.

CONCLUDING REMARKS

While there are important differences, its similarity to superfluidity has been a useful guide to our understanding of the FQHE.[5,6] It is interesting to compare our results for collective excitations in FQH states at finite temperature with those in the superfluid liquid ^{4}He in which two branches of collective excitations, i.e., the single-phonon and multiphonon modes, were observed.[21] It is tempting to interpret the upper banch of the split lowest zero T mode obtained within our one-loop theory as an indication of the presence of a similar multiphonon collective excitation in FQH states at finite temperature. However, to confirm this picture calculations beyond the one-loop level are needed. Finite-temperature numerical studies are also highly desirable.

ACKNOWLEDGMENTS

The author thanks G.S. Canright, X. Chen, J.K. Jain, and J. Quinn for useful discussions and G.S. Canright and X. Chen for critical readings of the manuscript. Valuable comments from N.E. Bonesteel, D.H. Lee, M. Ma, F.C. Zhang, and S.C. Zhang are also gratefully acknowledged. This work was supported by the National Science Foundation under Grant Nos. DMR-9101542 and DMR-9413057 and by the U.S. Department of Energy through Contract No. DE-AC05-84OR21400, administered by Martin Marietta Energy Systems Inc.

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