Tilted magnetic field effect on a double-layer quantum Hall system

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The effect of a parallel magnetic field on the quantum Hall state of a double-layer system at total filling factor $\nu=(4n+1)$, with n being the Landau-level index, is studied. While our result is consistent with previous ones, which indicate that the system at $\nu=1$ undergoes a parallel-magnetic-field-induced phase transition, we find that the phase transition also takes place at higher filling factors. For a certain parameter range, we observe an instability of the quantum Hall state towards probably a charge-density-wave state at a smaller parallel magnetic field before the phase transition takes place.

The competition between the tunneling energy and the Coulomb energy in a double-layer quantum Hall effect (QHE) system has attracted considerable attention.[1–3] Recently, Murphy *et al.* have measured[4] the activation energy of the $\nu=1$ QHE in a tilted magnetic field in the double-layer QHE system. They found that the activation energy drops rapidly by up to an order of magnitude as the parallel magnetic field (PMF) $B_{\parallel}$ increases to a critical point, suggesting that the system undergoes a phase transition to a new quantum Hall state (QHS). The activation energy of this QHS is almost independent of further increase in $B_{\parallel}$. Theoretically, this phase transition was studied in a beautiful paper[5] by Yang *et al.* by mapping the system onto a ferromagnetic model with the layer degree of freedom playing the role of spin. From this magnetic model, they estimated the critical value of the PMF at which the phase transition takes place, and the result is in qualitative agreement with the experiment.

In this paper, by employing a diagrammatic approach developed by Kallin and Halperin[6] and applied to a double-layer system by Fertig[7] and Brey,[3] we study a double-layer OHE system in the presence of a tilted magnetic field at filling factor $\nu=(4n+1)$, with n being the Landau-level index ($n=0$, i.e., $\nu=1$ corresponds to the experiment[4] of Murphy *et al.*). We find that the ground state which optimizes the tunneling energy is unstable with respect to a state which has a good exchange energy as the PMF reaches a critical point. This demonstrates that the system undergoes a PMF-induced phase transition. This result, which has been obtained for $\nu=1$ in Ref. 5, is valid for all filling factors $\nu=4n+1$, although the nature of the ground state after the phase transition may not be the same for different filling factors. We also calculate the energy of the collective mode associated with the symmetric to antisymmetric excitation. We find that there exists a parameter range where the mode goes soft at a finite PMF, resulting in the absence of the QHE before the occurrence of the above-mentioned phase transition.

In the absence of the PMF, we can model the tunneling by the following Hamiltonian:

$$H_t=-t\int d\mathbf{r}\,\Psi_1^{\dagger}(\mathbf{r})\Psi_2(\mathbf{r})+\text{H.c.}\ , \tag{1}$$

where t is the hopping integral between the two layers, and $\Psi_{\sigma}^{\dagger}(\mathbf{r})$, which creates an electron at $\mathbf{r}$ in the σth layer ($\sigma=1$ and 2), can be written as

$$\Psi_{\sigma}^{\dagger}(\mathbf{r})=\sum_{n,k}\phi_{nk}^{*}(\mathbf{r})a_{\sigma nk}^{\dagger}\ , \tag{2}$$

with $\phi_{nk}(\mathbf{r})=e^{iky}\Phi_n(x+kl_0^2)$ being the Landau orbital and $l_0=\sqrt{hc/eB_{\perp}}$ being the magnetic length associated with the perpendicular magnetic field $B_{\perp}$. Now we apply the PMF along the $-x$ direction and choose the corresponding vector potential as $\mathbf{A}_{\parallel}=(0,0,-y)B_{\parallel}$. In the presence of the PMF the tunneling Hamiltonian becomes

$$H_t=-t\int d\mathbf{r}\,e^{-iQy}\Psi_1^{\dagger}(\mathbf{r})\Psi_2(\mathbf{r})+\text{H.c.}\ , \tag{3}$$

with $Q=d(eB_{\parallel}/\hbar c)$ and d being the interlayer separation. By using the equation

$$\begin{aligned}\int d\mathbf{r}\,e^{-iQy}&\phi_{nk}^{*}(\mathbf{r})\phi_{n'k'}(\mathbf{r})\\&=\delta_{k+Q,k'}e^{-Q^2l_0^2/4}\left[\frac{n'!}{n!}\right]^{1/2}\left[\frac{-Ql_0}{\sqrt{2}}\right]^{n-n'}\\&\quad\times L_{n'}^{n-n'}(Q^2l_0^2/2)\ ,\end{aligned} \tag{4}$$

where L_n^m is a Laguerre polynomial, and making the transformation

$$a_{\sigma nk}^{\dagger}=\frac{1}{\sqrt{2}}\sum_{s=\pm1}c_{snk-(\sigma-1)Q}^{\dagger}s^{\sigma-1}\ , \tag{5}$$

where c operators represent electrons in the symmetric ($s=1$) or antisymmetric ($s=-1$) states, the tunneling Hamiltonian can be written as

$$H_t=-\sum_{nk}t_n^{Q}(c_{+1nk}^{\dagger}c_{+1nk}-c_{-1nk}^{\dagger}c_{-1nk})\ , \tag{6}$$

where

$$t_n^{Q}=te^{-Q^2l_0^2/4}L_n(Q^2l_0^2/2)\ . \tag{7}$$

In obtaining Eq. (6), the hopping between different Landau levels is neglected because it is much smaller than the cyclotron energy. On the other hand, the Coulomb interactions

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$$H_{\rm int}=\sum_{\sigma_1\sigma_2}\sum_{n_1n_2n_3n_4}\sum_{k_1k_2k_3k_4}\delta_{k_1+k_2,k_3+k_4}V^{\sigma_1\sigma_2}_{n_1n_2n_3n_4}(k_1-k_4,k_1-k_3)a^{\dagger}_{\sigma_1n_1k_1}a^{\dagger}_{\sigma_2n_2k_2}a_{\sigma_2n_3k_3}a_{\sigma_1n_4k_4}\ , \tag{8}$$

where

$$V^{\sigma_1\sigma_2}_{n_1n_2n_3n_4}(p_1,p_2)=\tfrac{1}{2}\sum_{\mathbf{q}}\frac{2\pi e^2}{\epsilon q}\delta_{p_1,q_y}e^{-iq_xp_2l_0^2}e^{-q^2l_0^2/2}\left[\frac{n_3!n_4!}{n_1!n_2!}\right]^{1/2}(-1)^{n_2-n_3}\left[\frac{q_y-iq_x}{\sqrt{2}}l_0\right]^{n_1+n_2-n_3-n_4}L_{n_4}^{n_1-n_4}(q^2l_0^2/2)$$
$$\times L_{n_3}^{n_2-n_3}(q^2l_0^2/2)e^{-qd(1-\delta_{\sigma_1\sigma_2})} \tag{9}$$

can be rewritten, in terms of the c operators, as

$$H_{\rm int}=\tfrac{1}{2}\sum_{s_1s_2s_3s_4}\sum_{n_1n_2n_3n_4}\sum_{k_1k_2k_3k_4}\delta_{k_1+k_2,k_3+k_4}V^{n_1n_2n_3n_4}_{s_1s_2s_3s_4}(k_1-k_4,k_1-k_3)c^{\dagger}_{s_1n_1k_1}c^{\dagger}_{s_2n_2k_2}c_{s_3n_3k_3}c_{s_4n_4k_4}\ , \tag{10}$$

where

$$V^{n_1n_2n_3n_4}_{s_1s_2s_3s_4}(k_1-k_4,k_1-k_3)=\tfrac{1}{2}[V^1_{n_1n_2n_3n_4}(k_1-k_4,k_1-k_3)(1+s_1s_2s_3s_4)$$
$$+V^2_{n_1n_2n_3n_4}(k_1-k_4,k_1-k_3-Q)s_2s_3+V^2_{n_1n_2n_3n_4}(k_1-k_4,k_1-k_3+Q)s_1s_4] \tag{11}$$

and

$$\begin{aligned}V^1_{n_1n_2n_3n_4}(p_1,p_2)&=V^{11}_{n_1n_2n_3n_4}(p_1,p_2)\\&=V^{22}_{n_1n_2n_3n_4}(p_1,p_2)\ ,\\V^2_{n_1n_2n_3n_4}(p_1,p_2)&=V^{12}_{n_1n_2n_3n_4}(p_1,p_2)\\&=V^{21}_{n_1n_2n_3n_4}(p_1,p_2)\ .\end{aligned} \tag{12}$$

In the double-layer QHE system without PMF, the QHE occurs at integer filling factors. Among them, only odd filling factors are of interest because the energy gap is directly related to the tunneling energy. Moreover, since $\nu=1$ and 3 correspond to the first Landau level, $\nu=5$ and 7 correspond to the second level, etc., we only need to consider $\nu=4n+1$ in general. At this filling factor, because of the tunneling gap, all the symmetric and antisymmetric states with two spin directions in the first $n-1$ Landau levels are filled, and only the symmetric states with one spin direction in the nth Landau level are filled. The exchange self-energy can be obtained as

$$\Sigma_{ms}=-\sum_{m's'}\sum_{q}\int\frac{d\omega}{2\pi}G_{m's'}(\omega)e^{i\omega0^+}V^{m'mm'm}_{s'ss's}(q-k,0)\ , \tag{13}$$

where the Green's function is

$$G_{ms}(\omega)=\frac{1}{\omega-(m+\frac{1}{2})\hbar\omega_c+st_m^Q-\Sigma_{ms}+i\delta_{ms}}\ , \tag{14}$$

with δ_{ms} being 0^+ if a state denoted by ms is occupied, and is 0^- otherwise. From Eq. (10), one can calculate the total exchange energy of the system at $\nu=4n+1$. The Q (or $B_\parallel$) dependent part of the total exchange energy is only contributed by electrons in the nth Landau level, while the Q- (or $B_\parallel$) dependent part of the exchange energy of an electron in the nth Landau level can be written in a simple form,

$$E^n_{\rm exc}(Q)=-\tfrac{1}{2}[F^{(1)}_{n,2}(0,Q)+F^{(1)}_{n,2}(0,-Q)]\ , \tag{15}$$

where

$$\begin{aligned}F^{(1)}_{n,i}(k_x,k_y)&=\sum_q e^{ik_xql_0^2}V^i_{nnnn}(q,k_y)\\&=\frac{e^2}{2\epsilon}\int_0^\infty dp\,e^{-p^2l_0^2/2}[L_n(p^2l_0^2/2)]^2\\&\quad\times e^{-pd(i-1)}J_0(kpl_0^2)\ ,\end{aligned} \tag{16}$$

with J_0 being a Bessel function. The PMF dependence of $E^n_{\rm exc}(Q)$ given by Eq. (15) is numerically calculated and the result is presented in Fig. 1, where the zero point is chosen at its zero PMF value. The result clearly shows that the exchange energy is an increasing function of the PMF.

For $\nu=1$, this conclusion has been obtained by Yang *et al.* (Ref. 5), and the physics involved can be understood as follows: In the absence of a PMF, the ground state can be described by a wave function whose in-plane coordinate part is of the Jastrow form. This state not only optimizes the tunneling energy, but also has a very good interaction energy. When a PMF is applied, in order to optimize the tunneling energy, the Jastrow form of the ground-state wave function has to be distorted resulting in the loss of exchange energy. As can be seen from Fig. 1, the loss of exchange energy continues to rise as the PMF increases. Therefore, as pointed out in Ref. 5, when the PMF reaches a critical point, the system would adopt a ground state which optimizes the interaction energy instead of the tunneling energy, and a phase transition takes place. A good candidate for the ground state after the phase transition is the original ground state in the absence of the PMF. The considerable exchange energy this state gains more than compensates for the complete

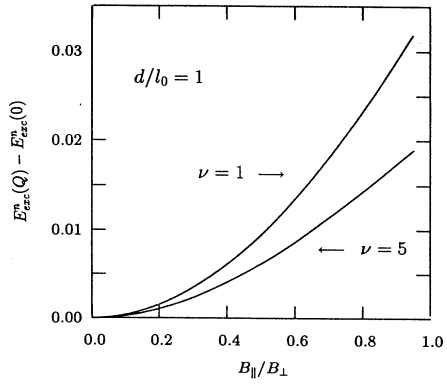

FIG. 1. The PMF dependence of $E^n_{\rm exc}(Q)-E^n_{\rm exc}(0)$ given by Eq. (15) (in units of $e^2/\epsilon l_0$) for $\nu=1$ (upper curve) and 5 (lower curve). The interlayer separation $d/l_0=1$ is chosen in the calculation.

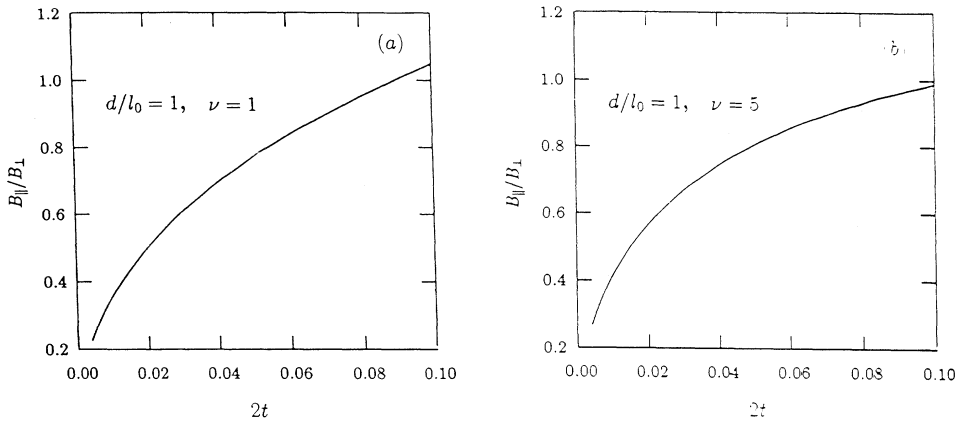

FIG. 2. Variation of the critical value of the PMF $B_\parallel/B_\perp$ as a function of the tunneling energy $2t$ (in units of $e^2/\epsilon l_0$) for $\nu=1$ [Fig. 1(a)] and 5 [Fig. 1(b)]. The interlayer separation $d/l_0=1$ is chosen in the calculation.

loss of the tunneling energy. Although the above analysis can apply only to the $\nu=1$ system due to the lack of a Jastrow wave function in higher Landau levels, the result presented in Fig. 1 clearly shows the statement that the ground state loses its interaction energy in order to optimize the tunneling energy to be valid for higher filling factor systems. Therefore, one can expect the system to experience an instability at a critical value of the PMF at higher filling factors such as $\nu=5$. Whether the state is the QHS or not after the phase transition is not yet clear, and deserves further study (such as a finite-size study). Nevertheless, we can calculate the critical value of the PMF at which the ground state that optimizes the tunneling energy is unstable with respect to a state which is the ground state in the absence of a PMF. This estimate gives an upper bound of the critical value of the PMF for which a phase transition takes place.

According to the above analysis, the critical field can be estimated from the following equation:

$$2t_n^Q+F_{n,2}^{(1)}(0,Q)+F_{n,2}^{(1)}(0,-Q)=2F_{n,2}^{(1)}(0,0)\ , \tag{17}$$

where $F_{n,i}^{(1)}(k_x,k_y)$ is given by Eq. (16). In Fig. 2, the critical value of the PMF as a function of the bare tunneling gap $2t$ for $\nu=1$ and 5 is plotted. The critical value is an increasing function of the tunneling gap for all filling factors. The result for $\nu=1$ is consistent with the experimental observation[4] and with previous theoretical study.[5] We would like to mention here that we did not use the experimental value of the interlayer separation[4] in our calculation, because otherwise the ground state would be in a non-QHE phase[2] even in a zero PMF. This discrepancy can be attributed[2] to the neglect of layer width in our model.

The above discussion is based on the assumption that the system we have studied is always in a QHE state before the phase transition takes place. As we shall see below, this is not always true. In order to address this issue, we have studied the collective mode associated with the symmetric to antisymmetric excitation,[2,3,7] i.e., $s=1\rightarrow s=-1$ with the same n. This collective mode can be obtained by solving the following vertex equation:[6,7]

$$\begin{aligned}\Gamma^n_{s_1,s_2}(q_1,q_2;\mathbf{k},\omega)=&\delta_{s_1,-1}\delta_{s_2,1}\delta_{q_2-q_1,k_y}\tfrac{1}{2}(1+e^{-k_xQl_0^2})e^{-k^2l_0^2/4}e^{-ik_x(q_1+q_2)l_0^2/2}L_n(k^2l_0^2/2)\\&+\sum_{s_3s_4}\sum_{q_3q_4}\delta_{q_2+q_3,q_1+q_4}[V^{nnnn}_{s_2s_3s_4s_1}(q_2-q_1,q_2-q_4)-V^{nnnn}_{s_2s_3s_1s_4}(q_2-q_4,q_2-q_1)]\\&\times\int\frac{d\omega'}{2\pi i}G_{ns_3}(\omega')G_{ns_4}(\omega+\omega')\Gamma^n_{s_3,s_4}(q_3,q_4;\mathbf{k},\omega)\ ,\end{aligned} \tag{18}$$

where the vertex $\Gamma^n_{s_1,s_2}(q_1,q_2;\mathbf{k},\omega)$ is defined through a "spin-density" Green's function.[6,7] We would like to point out that, unlike the zero PMF case, where the ordinary density Green's function has no pole within one Landau level,[7] in the presence of the PMF the ordinary density Green's function has the same pole as that obtained from the spin-density Green's function. As shown in Ref. 6, the vertex equation (18) can be diagonalized as

$$\tilde\Gamma^n_{s_1,s_2}(\mathbf{k},\omega)=\delta_{s_1,-1}\delta_{s_2,1}\tfrac{1}{2}(1+e^{-ik_xQl_0^2})e^{-k^2l_0^2/4}L_n(k^2l_0^2/2)+\sum_{s_3s_4}D^n_{s_3s_4}(\omega)[\tilde V^n_{s_2s_3s_4s_1}(\mathbf{k})-\bar V^n_{s_2s_3s_1s_4}(\mathbf{k})]\tilde\Gamma^n_{s_3,s_4}(\mathbf{k},\omega)\ , \tag{19}$$

where

$$\begin{aligned}\tilde\Gamma^n_{s_1,s_2}(\mathbf{k},\omega)&=\sum_q e^{ik_xql_0^2}\Gamma^n_{s_1,s_2}\left[q+\frac{k_y}{2},q-\frac{k_y}{2};\mathbf{k},\omega\right]\ ,\\ \tilde V^n_{s_1s_2s_3s_4}(\mathbf{k})&=\sum_q e^{ik_xql_0^2}V^{nnnn}_{s_1s_2s_3s_4}(k_y,q)\ ,\\ \bar V^n_{s_1s_2s_3s_4}(\mathbf{k})&=\sum_q e^{ik_xql_0^2}V^{nnnn}_{s_1s_2s_3s_4}(q,k_y)\ ,\\ D^n_{s_1s_2}(\omega)&=\int\frac{d\omega'}{2\pi i}G_{ns_1}(\omega')G_{ns_2}(\omega+\omega')\ .\end{aligned} \tag{20}$$

From Eq. (19), we can obtain the collective excitation energy

$$\begin{aligned}\omega^2=&[\Delta^n_{+-}(Q)-F^{(1)}_{n,2}(k_x,k_y+Q)-F^{(1)}_{n,2}(k_x,k_y-Q)]\\&\times[\Delta^n_{+-}(Q)-2F^{(1)}_{n,1}(\mathbf{k})+2F^{(2)}_{n,1}(\mathbf{k})\\&-2\cos(k_xQl_0^2)F^{(2)}_{n,2}(\mathbf{k})]\ ,\end{aligned} \tag{21}$$

where

$$\Delta^{n}_{+-}(Q)=2t_n^Q+F^{(1)}_{n,2}(0,Q)+F^{(1)}_{n,2}(0,-Q)\ ,$$

$$F^{(2)}_{n,i}(\mathbf{k})=\sum_q e^{ik_x ql_0^2}V^i_{nnnn}(k_y,q)$$
$$=\frac{e^2}{2\epsilon kl_0^2}e^{-k^2l_0^2/2}[L_n(k^2l_0^2/2)]^2e^{-kd(i-1)}\ , \qquad (22)$$

and $F^{(1)}_{n,i}(\mathbf{k})$ is given in Eq. (16). It can easily be seen that Eq. (21) reduces to the zero PMF result[2,3,7] at $Q=0$. When Q is nonzero, although $\omega(\mathbf{k})=\omega(-\mathbf{k})$, this mode depends not only on the magnitude of $\mathbf{k}$ but also on the direction of $\mathbf{k}$. This is the result of the existence of the PMF, which destroys the rotational symmetry along the z direction. To see this more clearly, one can choose a PMF along the y direction with the corresponding vector potential $\mathbf{A}_\parallel=(0,0,-x)B_\parallel$. The resulting tunneling Hamiltonian can be obtained from Eq. (3) with the exponential factor e^{-iQy} replaced by e^{-iQx}. By using the following equation

$$\int d\mathbf{r}\, e^{-iQx}\phi^*_{nk}(\mathbf{r})\phi_{n'k'}(\mathbf{r})=\delta_{k,k'}e^{iQkl_0^2}e^{-Q^2l_0^2/4}\left[\frac{n'!}{n!}\right]^{1/2}\left[\frac{iQl_0}{\sqrt{2}}\right]^{n-n'}L_{n'}^{n-n'}(Q^2l_0^2/2) \qquad (23)$$

corresponding to Eq. (4), and making the transformation $a^\dagger_{\sigma nk}=(1/\sqrt{2})\sum_{s=\pm1}c^\dagger_{snk}(se^{ikQl_0^2})^{\sigma-1}$ instead of Eq. (5), the same tunneling Hamiltonian as Eq. (6) is found, while the interaction Hamiltonian can be obtained from Eqs. (10), (11), and (12) by the following replacement:

$$V^2_{n_1n_2n_3n_4}(k_1-k_4,k_1-k_3\pm Q)$$
$$\rightarrow V^2_{n_1n_2n_3n_4}(k_1-k_4,k_1-k_3)e^{\mp i(k_1-k_4)Ql_0^2}\ . \qquad (24)$$

By repeating the same calculation, one obtains the collective excitation energy which is given by Eq. (21) with k_x and k_y exchanged.

It has been shown in Ref. 2 that there exists a phase boundary which separates OHE states from non-QHE states in a double-layer system with a zero PMF. By numerically calculating the collective-mode energy given by Eq. (21), we find that there exists a parameter range connecting to the phase boundary of Ref. 2 on the QHE side, where the collective mode goes soft at a finite PMF before the above-studied phase transition takes place. For example, for $\nu=1$, $d/l_0=1.3$, and $2t/(e^2/\epsilon l_0)=0.07$, the critical value of PMF which would lead to the above-studied phase transition is $0.8B_\perp$. However, the collective mode given by Eq. (21) will go soft at $B_\parallel=0.63B_\perp$, indicating an instability toward a probably charged density-wave state before the above-studied phase transition takes place. It should be pointed out that this instability is not simply caused by the reduction of the effective tunneling energy. Because, taking the above example again, the effective tunneling energy t_n^Q is only reduced by about 5% from its zero PMF value, which is still considerably larger than the tunneling energy on the upper phase boundary for $d/l_0=1.3$. In fact, this is a combined effect of the tunneling energy and the Coulomb energy brought about by a PMF. A similar result can be obtained for higher filling factors. Therefore, one can add one more phase boundary to the $[d/l_0-2t/(e^2/\epsilon l_0)]$ phase diagram obtained in Ref. 2, where there is only one phase boundary which separates the no-QHE phase from the QHE phase. This additional phase boundary, which is determined by requiring the critical magnetic field obtained from Eq. (17) to be the same as that obtained form Eq. (21) when the collective mode goes soft, is approximately a straight line parallel to the horizontal axis [the $2t/(e^2/\epsilon l_0)$ axis] within the range of $2t/(e^2/\epsilon l_0)<0.1$, and has the same intercept on the vertical axis (d/l_0 axis) as the old phase boundary does. In this new phase diagram, above the upper phase boundary (the old one), is the no-QHE phase. In between the two phase boundaries, the system may undergo a PMF-induced QHS to a charge-density-wave-state phase transition. Below the lower boundary (the new one), the system may undergo the kind of phase transition that is observed experimentally in Ref. 4.

Finally, we would like to make an interesting connection between the phase transition in a double-layer QHE system experimentally observed by Murphy *et al.*,[4] and the magnetic-field-induced phase transition proposed recently by Strong, Clarke, and Anderson[8] in a different context. Although they may differ in details, one can see that the two phase transitions are the same in nature in the sense that in each case the system completely gives up tunneling energy as the phase transition takes place.

In conclusion, we have studied the PMF effect on a double-layer QHE system. We found that the experimentally observed phase transition at $\nu=1$ can also take place at higher filling factors. Moreover, we found that within a certain parameter range, the system may undergo a PMF-induced QHS to a charge-density-wave-state phase transition.

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