Universal statistics of transport in disordered conductors

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We study electron counting statistics of a disordered conductor in the low-temperature limit. We derive an expression for the distribution of charge transmitted over a finite time interval by using a result from the random-matrix theory. In the metallic regime, the peak of the distribution is Gaussian and shows negligible sample-to-sample variations. On the contrary, the tails of the distribution are neither Gaussian nor Poisson-like and exhibit strong sample-to-sample variations.

The physics of current fluctuations at low temperature presents an interesting quantum mechanical problem. The classical Johnson-Nyquist noise formula[1] gives a good description of current fluctuations due to thermal fluctuations. However, at low temperature thermal fluctuations are small and another type of noise becomes important. At low temperature, the quantum nature of the current and the discreteness of electron charge is the main source of current fluctuations and for these reasons this noise is called quantum shot noise.

Many of the low-temperature current-fluctuation studies deal with a disordered conductor because it has a simple and well established mathematical description based on Landauer's approach.[2] Lesovik[3] and Yurke and Kochanski[4] studied quantum shot noise in a two-terminal conductor using this approach and found an expression for noise power which is a factor $1-T$ off the classical shot noise, where T is a transmission coefficient. This analysis was generalized to a multiterminal conductor by Büttiker[5], and he also found a reduction of the noise. Physically, the noise reduction is due to the Fermi statistics which leads to correlation of transmission events.

In Landauer's approach, details of current transport are determined by transmission coefficients and there have been many works on the distribution of the coefficients. In the past decade, the random-matrix theory of disordered conductors, pioneered by Dorokhov,[6] has been developed,[7] motivated by theoretical discovery and experimental observation of the universal conductance fluctuations. The theory succeeded in providing a complete characterization of the distribution and it also succeeded in providing insight into the origin of the universal conductance fluctuations. One of the fundamental results in random-matrix theory is the universality of the distribution in the metallic regime and the universality provides a link between the microscopically calculated transmission coefficients and the macroscopically measurable conductance.

Application of the universal distribution to current fluctuations also provides insights into the current fluctuations of a disordered conductor. Beenakker and Büttiker[8] calculated the sample averaged noise power using the universal distribution and found that it depends only on the conductance and that it is one-third of the classical value.

Noise power is a good measure of noise magnitude and its study revealed the reduction of noise due to Fermi correlation. However, compared to Johnson-Nyquist noise, our understanding of quantum shot noise is limited and not many things are known besides the noise magnitude. Our goal in this paper is to explore the physics of low-temperature current fluctuations beyond the noise power. For our purpose, it is useful to look at the behavior of current fluctuations in the time domain, which brings one to the notion of counting statistics of charge transmitted in a conductor over fixed time. A previous study of the counting statistics for a single-channel conductor revealed that the attempts to transmit electrons are highly correlated and almost periodic in time, which leads to binomial statistics.[9]

In the time domain study of current fluctuations, the charge $Q(t)$ (measured in units of e) transmitted over a time interval t is the quantity of interest and the probability distribution $P(Q(t))$ tells everything about the current fluctations. Even at zero temperature, $P(Q(t))$ has finite peak width due to the quantum nature of the To study $P(Q(t))$, it is useful to introduce the characteristic function $\chi(\lambda)$,

$$\chi(\lambda) = \sum_{\text{integer } Q} e^{iQ\lambda} P(Q) \quad \text{for } -\pi < \lambda < \pi\,, \tag{1}$$

because in many cases, $\chi(\lambda)$ is easier to calculate than $P(Q(t))$ itself. $\chi(\lambda)$ is a Fourier transform of $P(Q(t))$ and so once $\chi(\lambda)$ is known we can either take an inverse Fourier transform of it to get an explicit expression for $P(Q(t))$, or expand its logarithm to get cumulants of the distribution:

$$\ln \chi(\lambda) = \sum_{k=1}^{\infty} \frac{(i\lambda)^k}{k!} \langle\langle Q^k \rangle\rangle. \tag{2}$$

In the linear transport regime, we derive a general expression for $\chi(\lambda)$ in terms of transmission coefficients and by combining it with the transmission coefficient distribution for quasi-one-dimensional conductors, we show that

$$\overline{\ln\chi(\lambda)} = \frac{GVt}{e}\text{arcsinh}^2\sqrt{e^{i\lambda}-1}\ , \tag{3}$$

where V is the dc voltage, $G = g(e^2/h) = (Nl/L)(e^2/h)$ is the average conductance, and the bar on the left hand side represents the sample average. Cumulant expansion of Eq. (3) implies that, on average, for $GVt/e \gg 1$, $P(Q(t))$ has a Gaussian peak at GVt/e with $\langle\langle Q^2(t)\rangle\rangle = GVt/3e$. It also implies that even though the peak is Gaussian, the tails show deviation from both Gaussian and Poisson distributions. We estimate sample-to-sample variations of $P(Q(t))$ by studying variances of various quantities and find that for $GVt/e \gg 1$ sample-to-sample variations of $P(Q(t))$ appear only in the tails of $P(Q(t))$ and that around the peak $P(Q(t))$ is universal.

Before we present the derivation of the above result, let us stress that there are two kinds of averages involved. To avoid confusion, we will use an overbar $(\overline{\cdots})$ for an ensemble average, or an average over samples, and an angular bracket $(\langle\cdots\rangle)$ for a quantum average, or a quantum expectation value. Also we reserve a double bracket $(\langle\langle\cdots\rangle\rangle)$ for a cumulant of a quantum expectation value and "var" [var$(\cdots)$] for $\overline{\cdots^2} - \overline{\cdots}^2$.

Now, let us derive Eq. (3). Following Landauer's approach,[2] we consider a conductor sandwiched between two perfect leads. In the linear transport regime, the scattering properties of a conductor are described by a unitary scattering matrix $\hat{S}$ that relates incoming and outgoing amplitudes, $I_{L(R)}$ and $O_{L(R)}$:

$$\hat{S}\begin{pmatrix} I_L \\ I_R \end{pmatrix} = \begin{pmatrix} O_L \\ O_R \end{pmatrix}, \tag{4}$$

where the subscripts L and R stand for the left and the right leads.

The unitarity of $\hat{S}$ is due to the current conservation, and it allows a system to be decomposed into independent channels.[10] Then the decomposition motivates one to study single-channel transport first, where a transmission coefficient T determines the transport. Recently, the counting statistics of single-channel transport was studied.[9] In the low-temperature limit ($k_B\mathcal{T} \ll eV$), the characteristic function $\chi_1(\lambda)$ of a single-channel system becomes

$$\chi_1(\lambda) = (pe^{i\lambda} + q)^M, \tag{5}$$

where $p = T$, $q = 1 - T$, $M = eVt/h$,[11] and $M \gg 1$ is assumed. The inverse Fourier transform of Eq. (5) gives the binomial distribution, which implies that the intervals between subsequent attempts to transmit electrons are quite regular. This regularity is due to the Pauli exclusion principle.

Having the characteristic function of a single channel, we write the total characteristic function $\chi(\lambda)$ as a product,

$$\chi(\lambda) = \prod_j (T_j e^{i\lambda} + 1 - T_j)^M, \tag{6}$$

where T_j is a transmission coefficient of channel j. The product form Eq. (6) follows from the mutual independence of channels. By taking the logarithm of Eq. (6), we get

$$\ln\chi(\lambda) = M\sum_j \ln(T_j e^{i\lambda} + 1 - T_j), \tag{7}$$

and by expanding Eq. (7) in terms of λ we find

$$\langle\langle Q^k(t)\rangle\rangle = M\sum_j \left(T(1-T)\frac{d}{dT}\right)^{k-1} T\Bigg|_{T=T_j}. \tag{8}$$

We note that both $\ln\chi(\lambda)$ and $\langle\langle Q^k(t)\rangle\rangle$ are linear statistics of the T_j's.

Current fluctuations are determined by the distribution of transmission coefficients and the distribution varies from sample to sample even though the samples have the same macroscopic parameters. Therefore, in principle, each sample exhibits distinctive current fluctuations. However, according to the random-matrix theory of disordered conductors, in the metallic regime ($1 \ll g \ll N$) where N is the number of channels, the distribution approaches a universal one.[7] This result provides a motivation to approximate the sample-dependent distribution by the universal one. To exploit the universal distribution, we introduce new variables ν_j and the density function $D(\nu)$ defined by $T_j = 1/\cosh^2\nu_j$ and $D(\nu)d\nu = D(T)dT$, where $D(T)$ is the density function of T_j. According to Ref. 7, $D(\nu)$ is uniform over a wide range of ν,

$$D(\nu) = g \quad \text{for } \nu < \nu_c. \tag{9}$$

We combine Eq. (7) with the universal distribution to obtain

$$\overline{\ln\chi(\lambda)} = Q_0\int_0^\infty d\nu \ln\left(\frac{e^{i\lambda}-1}{\cosh^2\nu} + 1\right), \tag{10}$$

where $Q_0 = gM$. In Eq. (10) the upper limit ν_c is replaced by infinity, which is valid in the metallic regime because for large ν the integrand is exponentially small. The evaluation of the integral then leads to Eq. (3). We note that because $\ln\chi(\lambda)$ is a linear statistic, the universal distribution approximation is equivalent to taking an average over samples.

Cumulants are useful in understanding features of the probability distribution. By using the formula Eq. (2), we obtain the sample averaged cumulants

$$\overline{\langle\langle Q(t)\rangle\rangle} = Q_0, \quad \overline{\langle\langle Q^2(t)\rangle\rangle} = \tfrac{1}{3}Q_0,$$
$$\overline{\langle\langle Q^3(t)\rangle\rangle} = \tfrac{1}{15}Q_0, \quad \overline{\langle\langle Q^4(t)\rangle\rangle} = -\tfrac{1}{105}Q_0,$$
$$\overline{\langle\langle Q^5(t)\rangle\rangle} = -\tfrac{1}{105}Q_0, \quad \overline{\langle\langle Q^6(t)\rangle\rangle} = \tfrac{1}{231}Q_0,$$
$$\overline{\langle\langle Q^7(t)\rangle\rangle} = \tfrac{27}{5005}Q_0, \quad \overline{\langle\langle Q^8(t)\rangle\rangle} = -\tfrac{3}{715}Q_0,$$
$$\overline{\langle\langle Q^9(t)\rangle\rangle} = -\tfrac{233}{36465}Q_0, \quad \overline{\langle\langle Q^{10}(t)\rangle\rangle} = \tfrac{6823}{969969}Q_0, \quad \cdots . \tag{11}$$

The first cumulant is trivial. It is just a definition of G and it shows where the peak of $P(Q(t))$ is. The second cumulant measures the square width of the peak. It is also directly related to the noise power $P = \int dt\, \langle\langle I(0)I(t)\rangle\rangle$, a widely used measure of noise magnitude, by $\langle\langle Q^2(t)\rangle\rangle = tP$ for large t, and its ensemble average is one-third of the classical value Q_0, as pointed out by Beenakker and Büttiker.[8] The third and the fourth cumulants are measures of skewness and sharpness of the peak, respectively, and they are related to three- and four-point current-current correlation functions by similar relations. We note that all cumulants are proportional to Q_0 and that for $Q_0 \gg 1$, $\overline{\langle\langle Q(t)\rangle\rangle}^k \gg \overline{\langle\langle Q^k(t)\rangle\rangle}$. Therefore the peak of the distribution $P(Q(t))$ is Gaussian for the large conductance limit or the long time limit. This result is quite expected from the central limit theorem. Now to see the tails of $P(Q(t))$ we study higher-order cumulants. From Eq. (10), we obtain a general formula for the ensemble averaged kth-order cumulants

$$\overline{\langle\langle Q^k(t)\rangle\rangle} = -i^k \frac{Q_0}{4}\int_{-\infty}^{\infty} dx \frac{1}{\sqrt{1+e^{-x}}} \times \int_{-\infty}^{\infty} dq\, e^{-iqx}\frac{q^{k-1}}{\sinh(\pi q - i0+)}, \tag{12}$$

and by using the steepest descent method twice (see Appendix A), we obtain the asymptotics for large k,

$$\overline{\langle\langle Q^k(t)\rangle\rangle} \sim \frac{Q_0}{(2\pi)^{k-1}}\frac{(k-1)!}{\sqrt{k}} \times \begin{cases} (-1)^{\frac{k+2}{2}} & \text{for even } k, \\ (-1)^{\frac{k+1}{2}} & \text{for odd } k. \end{cases} \tag{13}$$

The high-order cumulants diverge as factorials, which suggests that at the tails $P(Q(t))$ is different from both the Gaussian distribution and the Poisson distribution which describes the classical current fluctuations. In comparison, $\overline{\langle\langle Q^k(t)\rangle\rangle} = 0$ for $k \geq 2$ for a Gaussian distribution and $\overline{\langle\langle Q^k(t)\rangle\rangle} = Q_0$ for $k \geq 1$ for a Poisson distribution.

It is known that in the presence of time reversal symmetry there are order-M corrections to $\overline{\langle\langle Q(t)\rangle\rangle}$ and $\overline{\langle\langle Q^2(t)\rangle\rangle}$ due to weak localization,[12] and it is natural to expect the same kind of corrections to higher-order cumulants. However, because we are interested in the metallic regime, these corrections are small by a factor g and we will ignore them.

A proper next step is to estimate the magnitude of sample-to-sample variations of $P(Q(t))$. Here instead of $\ln\chi(\lambda)$, we examine the variance of $\langle\langle Q^k(t)\rangle\rangle$ to see the variations of $P(Q(t))$. $\langle\langle Q^k(t)\rangle\rangle$ is a linear statistic and the general formula for the variance of a linear statistic $A = \sum_j a(T_j)$ was obtained recently by Beenakker and Rejaei[13] and Chalker and Macêdo[14] as

$$\text{var}\,(A) = \frac{1}{\beta\pi^2}\int_0^{\infty} dk \frac{k\tilde{a}^2(k)}{1+\coth(\frac{1}{2}\pi k)}, \tag{14}$$

$$\tilde{a}(k) = 2\int_0^{\infty} d\nu\, a\left(\frac{1}{\cosh^2\nu}\right)\cos k\nu, \tag{15}$$

where β is a symmetry constant, 1, 2, or 4 depending on the symmetry. We use this formula to obtain

$$\begin{aligned} \text{var}\,[\langle\langle Q(t)\rangle\rangle] &= \frac{2}{15\beta}M^2, \\ \text{var}\,[\langle\langle Q^2(t)\rangle\rangle] &= \frac{46}{2835\beta}M^2, \\ \text{var}\,[\langle\langle Q^3(t)\rangle\rangle] &= \frac{11366}{1447875\beta}M^2, \quad \cdots . \end{aligned} \tag{16}$$

We note that for low-order cumulants $\overline{\langle\langle Q^k(t)\rangle\rangle}^2$ is larger than var $[\langle\langle Q^k(t)\rangle\rangle]$ by at least a factor of g^2, which is large in the metallic regime. Low-order cumulants decide the shape of $P(Q(t))$ around the peak and therefore the small variance of low-order cumulants implies that the peak shape shows little sample-to-sample variation, that is, it is almost universal.

To see the behavior of higher-order cumulants, we obtain an asymptotic form of the variance (see Appendix B) from an approximate variance formula in Ref. 15,

$$\text{var}\ [\langle\langle Q^k(t)\rangle\rangle] \sim \frac{4(2k-1)!}{(2\pi)^{2k}\beta}M^2. \tag{17}$$

According to Eq. (17), for high-order cumulants, var $[\langle\langle Q^k(t)\rangle\rangle]$ becomes larger than $\overline{\langle\langle Q^k(t)\rangle\rangle}^2$ due to its rapidly growing factorial factor, which suggests that the tails of $P(Q(t))$ show large sample-to-sample variations. We argue that this rapid growth of var $[\langle\langle Q^k(t)\rangle\rangle]$ is not an artifact of the approximate variance formula used above because it assumes stronger spectral rigidity than the formula Eqs. (14), and (15) and it has a tendency to slightly underestimate variances. Therefore the large sample-to-sample variation at the tails of $P(Q(t))$ obtained above is not an artifact of the approximation.

In the above, we derived the shape of $\overline{P(Q(t))}$ by examining $\overline{\ln\chi(\lambda)}$ and its cumulant expansion instead of $\ln\overline{\chi(\lambda)}$, which might be an intuitively more appropriate ensemble average because it is directly related to $\overline{P(Q(t))}$. However, we argue that in contrast to intuition, $\overline{\ln\chi(\lambda)}$ is an appropriate ensemble average for the study of current fluctuations. One reason is that, as we remarked earlier, a k-point current-current correlation function is linearly related to $\langle\langle Q^k(t)\rangle\rangle$, whose ensemble average can be obtained from $\overline{\ln\chi(\lambda)}$ by a simple expansion. Another reason is that, as we show later, $\ln\overline{\chi(\lambda)}$ either becomes identical to $\overline{\ln\chi(\lambda)}$ at the short time limit, or is dominated by the conductance fluctuations instead of the current fluctuations.

Calculation of $\overline{\chi(\lambda)}$ is not simple because $\chi(\lambda)$ is not a linear statistic. Muttalib and Chen[16] did this calculation recently for a linear confining potential by the large N limit continuum approximation and showed that at the long time limit $\ln\overline{\chi(\lambda)}$ becomes quite different from $\overline{\ln\chi(\lambda)}$. Here we present a calculation by a perturbation method for an exact confining potential and we believe that our calculation clarifies the reason why the two averages become so different at the long time limit.

Because $\chi(\lambda)$ is not a linear statistic, we need a joint probability distribution of transmission coefficients to average it over ensembles. After the standard variable change $T = 1/(1+x)$, the joint probability distribution $P(\{x\})$ is

$$P(\{x\}) = \exp\left(\beta\sum_{a<b} V(x_a, x_b) + \beta\sum_{a} U(x_a)\right). \quad (18)$$

We choose

$$V(x,y) = (1/2)\ln(x-y) + (1/2)\ln(\operatorname{arcsinh}^2\sqrt{x} - \operatorname{arcsinh}^2\sqrt{y}), \quad (19)$$

$$U(x) = g\operatorname{arcsinh}^2(\sqrt{x})$$

based on the exact calculation of the joint probability distribution function for $\beta = 2$ by Beenakker and Rejaei.[13] Then,

$$\overline{\chi(\lambda)} = \frac{Z_M}{Z_0},$$

$$Z_M = \int \prod_a dx_a \exp\left(\beta\sum_{a<b} V(x_a,x_b) + \beta\sum_a U(x_a) + \sum_a M\ln\frac{x_a + e^{i\lambda}}{x_a+1}\right),$$

$$Z_0 = \int \prod_a dx_a \exp\left(\beta\sum_{a<b} V(x_a,x_b) + \beta\sum_a U(x_a)\right). \quad (20)$$

By expanding $\ln\overline{\chi(\lambda)}$ in terms of M, we find

$$\ln\overline{\chi(\lambda)} = \overline{\ln\chi(\lambda)} + \operatorname{var}[\ln\chi(\lambda)] + O(M^3), \quad (21)$$

and from the formula Eqs. (14), and (15), we obtain

$$\ln\overline{\chi(\lambda)} = gM\operatorname{arcsinh}^2\sqrt{e^{i\lambda}-1} - \frac{M^2}{\beta}\left(3\ln\frac{\operatorname{arcsinh}\sqrt{e^{i\lambda}-1}}{\sqrt{e^{i\lambda}-1}} + \frac{1}{2}i\lambda\right) + O(M^3). \quad (22)$$

Note that for $M \ll g$ (short time limit), $\ln\overline{\chi(\lambda)}$ reduces to $\overline{\ln\chi(\lambda)}$. We expand $\ln\overline{\chi(\lambda)}$ in terms of λ to see features of $\overline{P(Q(t))}$:

$$\ln\overline{\chi(\lambda)} = gM(i\lambda) + \left(\frac{g}{3}M + \frac{2}{15\beta}M^2\right)\frac{(i\lambda)^2}{2!} + \left(\frac{g}{15}M + \frac{2}{315\beta}M^2 + O(M^3)\right)\frac{(i\lambda)^3}{3!} + \cdots. \quad (23)$$

The first expansion coefficient shows that $\overline{\langle Q(t)\rangle} = Q_0 = gM$, which is trivial. The second expansion coefficient, $\overline{\langle Q^2(t)\rangle} - \overline{\langle Q(t)\rangle}^2 = \overline{\langle\langle Q^2(t)\rangle\rangle} + \operatorname{var}[\langle Q(t)\rangle] = Q_0/3 + (2/15\beta)M^2$, indicates that the peak width of $\overline{P(Q(t))}$ has two contributions. The first contribution is related to the noise power, and the second one to the universal conductance fluctuations because $\operatorname{var}[\langle Q(t)\rangle]$ is proportional to the variance of the conductance. (The factor $2/15\beta$ is precisely the variance of the dimensionless conductance.) Note that as $t \to \infty$ the second contribution becomes dominant over the first one. It can be shown that the kth-order expansion coefficient contains k different contributions and at the long time limit the most dominant contribution, which is proportional to M^k, is related to the kth cumulant of the conductance fluctuations. From this analysis we see that the behavior of $\overline{\chi(\lambda)}$ for large t is governed by the conductance fluctuations instead of the current fluctuations.

In summary, we examine the counting statistics of charge to study the current fluctuations at low temperature. By calculating the characteristic function of the probability distribution $P(Q(t))$, we find that $P(Q(t))$ has a Gaussian peak at Q_0 with $\langle\langle Q(t)\rangle\rangle = Q_0/3$ and we also find that the tails of $P(Q(t))$ are different from the tails of Gaussian and classical Poisson distributions. By studying the variances of the cumulants, we establish that, even though the peak location of $P(Q(t))$ varies from sample to sample due to universal conductance fluctuations, the peak shape of $P(Q(t))$ is universal in the metallic regime, and that the sample-to-sample variations show up only at the tails of $P(Q(t))$.

APPENDIX A: ASYMPTOTIC EXPRESSION FOR $\langle\langle Q^K\rangle\rangle$

For the evaluation of q integration, we note that due to its symmetric (or antisymmetry), it is enough to integrate over q from 0 to ∞. For example, for odd k,

$$\int_{-\infty}^{\infty} dq e^{-iqx}\frac{q^{k-1}}{\sinh(\pi q - i0+)} = 2\mathrm{Re}I, \quad (A1)$$

where

$$I = \int_0^\infty dq e^{-iqx} q^{k-1}\frac{2e^{-\pi q}}{1-e^{-2\pi q}}. \quad (A2)$$

Similar relation holds for even k with Re replaced by iIm. Then we change the variable from q to $p = sq$ with $s = k-1$

$$I = s\int_0^\infty dp\, e^{-ispx}(sp)^s\frac{2e^{-s\pi p}}{1-e^{-2s\pi p}} \quad (A3)$$

$$= 2s^{s+1}\int_0^\infty dp\exp[sf_s(p)], \quad (A4)$$

$$f_s(p) \equiv \ln p - ipx - \pi p - \frac{\ln(1-e^{-2s\pi p})}{s}. \quad (A5)$$

The above expression is the standard form for which the steepest descent method gives asymptotic expression for large s, except that $f_s(p)$ depends on s. However, its dependence is exponentially weak and therefore it is safe to ignore the dependence just by dropping the last term of $f_s(p)$. Then following the standard procedure of the steepest descent method, we find

$$I \sim \frac{2(k-1)!}{(\pi+ix)^k} \text{ for large } k \ . \tag{A6}$$

To perform the next integration in Eq. (12) over x, we introduce a variable ν by $e^x = \sinh^2\nu$,

$$J \equiv \int_{-\infty}^{\infty} dx \frac{1}{\sqrt{1+e^{-x}}} \frac{1}{(\pi+ix)^k} \tag{A7}$$

$$= 2\int_0^{\infty} d\nu \exp[kg(\nu)] \ , \tag{A8}$$

$$g(\nu) = -\ln(\pi + i\ln\sinh^2\nu) \ . \tag{A9}$$

Even though it requires many branch cuts, the above expression is still the standard form of the steepest descent methods. Then following the standard procedure of the steepest descent method, we obtain

$$J \sim \frac{1}{\sqrt{k}} \frac{1-i}{(2\pi)^{k-1}} \text{ for large } k \ , \tag{A10}$$

and putting everything together, we finally get

$$\overline{\langle\langle Q^k(t)\rangle\rangle} \sim \frac{Q_0}{(2\pi)^{k-1}} \frac{(k-1)!}{\sqrt{k}} \times \begin{cases} (-1)^{\frac{k+2}{2}} & \text{for even } k \\ (-1)^{\frac{k+1}{2}} & \text{for odd } k \ . \end{cases} \tag{A11}$$

APPENDIX B: VARIANCE OF $\langle\langle Q^K \rangle\rangle$

Beenakker derived a general formula for the variance of linear statistic $A = \sum_j a(T_j)$ assuming the interaction between eigenvalues is exactly logarithmic,[15]

$$\mathrm{var}(A) = \frac{1}{\beta\pi^2}\int_0^{\infty} dq\, \tilde{a}(q)\tilde{a}(-q) q \tanh\pi q \ , \tag{B1}$$

$$\tilde{a}(q) = \int_{-\infty}^{\infty} dx\, e^{iqx} a\left(\frac{1}{1+e^x}\right) \ . \tag{B2}$$

Instead of Fourier transforming each $\langle\langle Q^k(t)\rangle\rangle$, we first take Fourier transform of $\ln\chi(\lambda)$,

$$\tilde{a}(q) = \int_{-\infty}^{\infty} dx e^{ikx} \ln\left(\frac{e^{i\lambda}}{1+e^x} + \frac{e^x}{1+e^x}\right) \tag{B3}$$

$$= \frac{\pi}{k\sinh\pi k}(1-e^{-k\lambda}) \text{ for } -\pi < \lambda < \pi \ , \tag{B4}$$

and expand it in λ to get Fourier tranform $\tilde{a}_k(q)$ of the kth order cumulant,

$$\tilde{a}(q) = \sum_{k=1}^{\infty} \frac{(i\lambda)^k}{k!} \tilde{a}_k(k) \ , \tag{B5}$$

$$\tilde{a}_k(q) = -M\frac{\pi}{q\sinh\pi q}(iq)^k \ . \tag{B6}$$

Then we use Eqs. (B1) and (B2) to obtain

$$\mathrm{var}(\langle\langle Q^k(t)\rangle\rangle) = \frac{1}{\beta k^2}\int_0^{\infty} dq \frac{M^2\pi^2}{\sinh^2\pi q} q^{2k-1}\tanh\pi q \tag{B7}$$

$$= \frac{2M^2}{\beta}\int_0^{\infty} dq \frac{q^{2k-1}}{\sinh 2\pi q} \tag{B8}$$

$$= \frac{M^2}{\beta}\frac{2^{2k}-1}{2^{2k}}\frac{|B_{2k}|}{k} \ , \tag{B9}$$

where B_{2k} is the $2k$th Bernoulli number. And by using an asymptotic expression for Bernoulli number, we finally get

$$\mathrm{var}(\langle\langle Q^k(t)\rangle\rangle) \sim \frac{4(2k-1)!}{(2\pi)^{2k}\beta} M^2 \ . \tag{B10}$$

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