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Quantum Hall effect in a vortex liquid

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A two-dimensional superconducting film in a magnetic field is considered in the adiabatic limit when vortex dynamics is dominated by the Magnus force. By mapping a vortex liquid state into that of electrons in an effective magnetic field, I find that the Hall conductance is quantized as $\sigma_{xy}=(4e^2/h)[1+n(2m-1)]/n$ with $n,m\geqslant 1$ integers, predicting a fractional quantum Hall effect in a boson system. This phenomenon can also account for observed steps in the magnetization curve of $YBa_2Cu_3O_{7-\delta}$. The feasibility of a vortex liquid, i.e., melting of a flux lattice at $T=0$ by quantum fluctuations, is studied.

There is a considerable recent interest in the melting transition of flux lattices in superconductors.[1] In particular quantum fluctuations at low temperatures have been studied,[2–5] with the possibility of inducing melting of the flux lattice at high magnetic fields. It was proposed[4] that high-resistance thin films are good candidates for observing a quantum melting transition at $T=0$. A key issue in these studies is whether the melted phase is a vortex liquid, i.e., whether the superconducting amplitude is large and the dominant fluctuations are due to the superconducting phase.

Recent data on $YBa_2Cu_3O_{7-\delta}$ crystals at temperatures below 1 K and above a threshold field show remarkable steps in the magnetic induction versus field curve.[6] These steps appear in concert with vanishing of the magnetization relaxation rate, indicating a vortex liquid phase. In the present work I show that a vortex liquid has quantized Hall conductance related to the stability of certain vortex densities. The measured steps[6] can therefore be related to the induction field corresponding to these vortex densities.

Another type of experimental data[7] on InO_x thin films has shown a remarkable phase diagram at $T\rightarrow 0$: (a) At low fields vortices are pinned and the system is superconducting; i.e., both longitudinal R_{xx} and Hall R_{xy} resistances vanish. This corresponds to a vortex glass phase.[8] (b) At intermediate fields $R_{xx}\rightarrow\infty$ while R_{xy} is finite. (c) At high fields a normal insulator with $R_{xx},R_{xy}\rightarrow\infty$. The field range for the intermediate phase (b) increases considerably with disorder. Note that since the condensate is destroyed and the system is normal only in phase (c), the dominant fluctuations in phase (b) [not too close to the transition to (c)] are those of the superconducting phase; i.e., phase (b) is a vortex liquid.

The problem of vortex dynamics is closely related to that of electrons in a magnetic field.[9] In particular, the Magnus force, which corresponds to an effective magnetic field, leads to a Hall effect for vortices and has been used to evaluate vortex tunneling.[10–12]

The Magnus force has been derived either by general arguments[9] or by microscopic derivation of the vortex equation of motion.[13–15] The latter shows that the magnitude of the Magnus force depends on the scattering relaxation time τ. Vortex dynamics is dominated by the Magnus force over that of friction if the level spacing $\hbar\omega_0$ of the quantized states in the vortex core satisfies $\omega_0\tau\gg 1$. This pure limit is feasible in clean superconductors with a short coherence length ξ and low normal state resistivity ρ (extrapolated to $T=0$) such as in high-T_c superconductors (HTSC's). Using $\hbar\omega_0\approx\hbar^2/m_e\xi^2$, $\xi\approx 15$ Å, and $\rho=m_e/(n_e e^2\tau)$, where m_e is an electron mass and $n_e\approx 10^{22}$ cm^{-3} is the electron density, the condition $\omega_0\tau\gg 1$ becomes a feasible $\rho\ll 10^{-5}$ Ω cm. In fact a recent experiment[16] has demonstrated the existence of the pure limit by showing a large Hall angle in oxygen-reduced $YBa_2Cu_3O_x$.

The possibility of quantum Hall phenomena has been independently considered in recent studies on Josephson junction arrays.[17–19] These systems involve finite capacitance terms and pinning by the periodic array and therefore are physically distinct from the present superconducting film problem.

In the present work I consider a vortex liquid and study the conditions for vortex adiabatic dynamics, i.e., Magnus force dynamics in the presence of a random potential and vortex-vortex interactions; the pure limit is *not* assumed. I show that the vortex liquid at $T=0$ has a quantized Hall conductance, given by Eq. (5). The melting condition is then examined, showing that formation of a vortex liquid is a feasible scenario, and is supported by the data[7] on InO_x films. Finally I propose that the steps in the magnetization data of $YBa_2Cu_3O_{7-\delta}$ (Ref. 6) can be related to the quantum Hall effect.

Consider an effective Lagrangian[11] for vortices $i=1,\ldots,N_v$ at positions $\mathbf{r}_i=(x_i,y_i)$ in the presence of a random pinning potential $U(\mathbf{r}_i)$,

$$\mathscr{L}_{\text{eff}}=\sum_i\left[\tfrac{1}{2}h\rho_s d(\dot{x}_i-v_s)y_i+U(\mathbf{r}_i)\right]+\sum_{i<j}V(\mathbf{r}_i-\mathbf{r}_j). \tag{1}$$

The vortex-vortex interaction $V(\mathbf{r}_i-\mathbf{r}_j)$ is essentially a repulsive logarithmic one [see Eq. (6) below] while $j_x=e\rho_s v_s$ is an external current in the x direction, ρ_s is the condensate density, and d is the film thickness.

Equation (1) does not necessarily assume a pure limit $\omega_0\tau\gg 1$ since elastic scattering of vortices is incoporated in (1) by the random potential $U(\mathbf{r}_i)$. Equation (1) neglects, however, inelastic scattering in which the composite nature of the vortex is probed, i.e., tunneling of electrons between vortex core levels. This tunneling is weak if the potential fluctuation $\tilde{U}(\mathbf{r})$ as seen by an electron within the core size

ξ is slowly varying, i.e., $|\nabla \tilde{U}(\mathbf{r})|\xi \lesssim \hbar\omega_0$. This corresponds to adiabatic dynamics—the vortex inner structure is unaffected by vortex motion and dynamics can be described by the vortex collective coordinates $\mathbf{r}_i$. Assuming charged impurities, the condition is that the mean spacing between impurities be large compared with $\xi(\xi/\bar{a})^{1/2}$ ($\bar{a}$ is an effective Bohr radius) which is easy to satisfy in HTSC's.

Note that for adiabatic dynamics the difference between Refs. 9 and 13–15 is resolved—the coefficient of the dynamic term in Eq. (1) is a pure Berry phase independent of disorder as in Ref. 9 while the random potential $U(\mathbf{r}_i)$ affects the actual Hall coefficient and can lead to dissipation as in Refs. 13–15. The effective Lagrangian, Eq. (1), is therefore independent of the controversial aspect of the Magnus term.

The properties of the Lagrangian, Eq. (1), can be studied by mapping it into a system of fermions with charge e in a strong magnetic field $B^{\rm eff}$ where $B_{\rm eff}=(hc/2e)\rho_s d$. The strong field condition is due to the absence of a mass term; allowing for a finite vortex mass,[11] m_v, removes this formal condition.

Electrons in a strong magnetic field with repulsive interactions and a random potential have been extensively studied[20–23] in the context of the fractional quantum Hall effect (FQHE). A general class of repulsive interactions, including a logarithmic interaction,[20] is known to yield incompressible states which then imply a fractional Hall conductance.

The energy levels of Eq (1), for $V(\mathbf{r}_i-\mathbf{r}_j)=0$ and $j_x=0$, are Landau bands separated by energies $\hbar eB^{\rm eff}/m_v c$. The degeneracy of a Landau band is $AeB^{\rm eff}/hc$ where A is the area of the system. The vortex density is $N_v/A=B/\phi_0 \equiv 1/a_0^2$, where B is the (real) magnetic field and $\phi_0=hc/2e$ is the flux quantum for a superconductor. Vortices are considered as bosons with a filling fraction of a Landau level,

$$\nu_b=hc/(eB^{\rm eff}a_0^2)=2/(\rho_s d a_0^2). \tag{2}$$

This places a constraint on ν_b since $a_0 \gtrsim \xi$; i.e., vortices should not overlap and B should be below the upper critical field H_{c2}. E.g., for the above HTSC parameters and film thickness of one unit cell (12 Å), $\nu_b \lesssim 0.1$ is necessary. Clearly, lower ρ_s with shorter ξ (i.e., a strong coupling superconductor) will increase this upper bound. When B approaches H_{c2}, ρ_s decreases as $\sim(H_{c2}-B)$ and the upper limit on ν_b seems to increase. In this case, however, superconducting amplitude fluctuations must also be considered; i.e., Eq. (1) is not a complete description.

To describe the vortex ground state I assume that a flux lattice (or a vortex glass) has melted and that a vortex liquid has formed. The effect of the repulsive vortex-vortex interactions is described by a Laughlin-type[20] wave function

$$\Psi\{z_i\}=\prod_{i<j}(z_i-z_j)^{2m}\exp\left[-\sum_i |z_i|^2/4\right], \tag{3}$$

where $z_i=(x_i+iy_i)/\ell^{\rm eff}$, $\ell^{\rm eff}=(\hbar c/eB^{\rm eff})^{1/2}$ is the magnetic length, and m is an integer so that Bose symmetry is satisfied. The wave function (3) minimizes the repulsive energy for larger m, but m is limited by the filling fraction so that $\nu_b<1/2m$. Thus at $\nu_b=1/2m$ there is a discontinuity which implies an incompressible state. This has been confirmed by numerical studies[24,25] which in fact show larger cusps for the energy $E(\nu_b)$ then those of the fermion system.

The corresponding Laughlin ansatz for electrons with filling fraction $1/(2m+1)$ is well established.[20–23] A longer-range interaction, such as a logarithmic one, tends to reduce the stability of a Laughlin state[21] relative to that of a lattice phase; however, within a flux liquid phase $E(\nu)$ has cusps at $\nu_b=1/2m$ even for logarithmic repulsion.[20] Logarithmic interactions, however, affect excitation charges and lead to other peculiarities.[19]

Following Jain[23] one can extend the set of incompressible states by mapping bosons to n filled Landau levels of fermions. The possibility that one occupies states with high energy when the vortex mass m_v is small (i.e., $\hbar eB^{\rm eff}/m_v c$ is large) is irrelevant since incompressibility survives the limit of zero mass.[26] The procedure is to attach $(2m-1)$ flux tubes with flux $\phi_0'=hc/e$ to each fermion so that the original flux per fermion ϕ_0'/n is increased by $(2m-1)\phi_0'$. Permuting two particles results now in an extra Aharonov-Bohm phase of π transforming fermions with charge e into bosons with charge e. (Note that in fact e is arbitrary here—it is only the product $eB^{\rm eff}$ which is defined by the mapping). The procedure is then to assume that the incompressibility of the original n filled Landau levels will persist when the flux tubes are smoothed out to form a uniform magnetic field. The new flux/particle ϕ_0'/ν_b thus defines a boson state with a filling fraction of ν_b, i.e., $\phi_0'/n+\phi_0'(2m-1)=\phi_0'/\nu_b$, so that

$$\nu_b=\frac{n}{1+n(2m-1)}. \tag{4}$$

Since $n\geqslant 1$ and $\nu_b>0$, these states have $m\geqslant 1$ and $\nu_b<1$. In fact, the constraint $a_0\gtrsim\xi$ may imply, via Eq. (2), $\nu_b \ll 1$, and then the relevant states have $m \gg 1$.

Use of charge conjugation leads to a larger set of filling fractions[25] $\nu_f=\nu_b+1$ with ν_f the fermion set of filling fractions for incompressible states. Charge conjugation, however, involves the presence of additional zeroes in the wave function,[25] leading to a larger overlap of vortices and thereby limiting the validity of Eq. (1). Thus Eq. (4) is the more favorable set for ν_b, at least for ν_b not too small.

Incompressibility at the filling fractions of Eq. (4) implies a quantized Hall conductance for the vortex liquid. To see this, note that for $j_x\neq 0$ an incompressible vortex liquid moves with velocity $\dot{x}=v_s$ [see Eq. (1)] and the moving flux generates an electric field in the y direction, $E_y=B\dot{x}/c$. The Hall conductance is therefore $\sigma_{xy}=j_x d/E_y=4e^2/h\nu_b$. Equation (4) then yields the central result of the present work,

$$\sigma_{xy}=\frac{4e^2}{h}\frac{1+n(2m-1)}{n}. \tag{5}$$

I proceed now to examine the melting transition of a vortex lattice at $T=0$. I use the Lindeman criterion[3,5] so that melting occurs when the zero-point motion of a vortex lattice is a certain fraction of the vortex spacing. The vortex-vortex interaction in a thin film has for its Fourier transform[27]

$$V(q)=\frac{\phi_0^2}{2\pi q(1+2\lambda_e q)}, \tag{6}$$

where $\lambda_e=\lambda^2/d$ with λ the London penetration length. Equation (6) is a repulsive potential which is $\sim \ln r$ for $r \ll \lambda_e$ and is $\sim 1/r$ for $r \gg \lambda_e$. Standard methods[28] then yield the elastic constants

$$c_{11}(q)=\frac{B^2}{2\pi q(1+2\lambda_e q)}-\frac{B\phi_0}{(8\pi)^2\lambda_e},$$

$$c_{66}=\frac{B\phi_0}{(8\pi)^2\lambda_e}. \tag{7}$$

The effective Lagrangian in terms of small displacements $\mathbf{u}=(u_x,u_y)$ from an equilibrium lattice is in Fourier space

$$\mathscr{L}=\int [\alpha \dot{u}_x u_y - \tfrac{1}{2}c_{11}(q)q^2u_L^2(q) - \tfrac{1}{2}c_{66}q^2u_T^2(q)]\, dt\, d^2q/(2\pi)^2, \tag{8}$$

where $\alpha=h\rho_s d/2a_0^2$ and u_L, and u_T are the longitudinal and transverse components of $\mathbf{u}$, respectively. Using $\int \dot{u}_x u_y\, dt = -i\int \omega u_L(\omega)u_T(-\omega)\, d\omega/2\pi$ as a frequency integral, the Gaussian integral for the fluctuations in $\mathbf{u}$ becomes

$$\langle \mathbf{u}^2(\mathbf{r},t)\rangle = -i\hbar \int \frac{d\omega\, d^2q}{(2\pi)^3}\frac{c_{11}(q)q^2+c_{66}q^2}{c_{11}c_{66}q^4-\alpha^2(\omega-i\omega\delta)^2}, \tag{9}$$

where $\delta\to+0$ and $|q|<(4\pi)^{1/2}/a_0$ defines the Brillouin zone. For $\lambda_e/a_0 \gg 1$ the result is independent of B,

$$\langle \mathbf{u}^2(\mathbf{r},t)\rangle \approx 2/(\pi\rho_s d). \tag{10}$$

Using a Lindeman melting criterion $\langle \mathbf{u}^2(\mathbf{r},t)\rangle = c_L^2a_0^2$ with[4] $c_L\approx 0.16$ leads with Eq. (2) to a critical filling fraction of $\nu_b^c=\pi c_L^2\approx 0.08$. For $\rho_s d\approx 5\times 10^{14}$ cm^{-2}, melting occurs at $a_0\approx 30$ Å so that the vortex spacing is comparable to ξ of HTSC compounds. To enhance the feasibility of a vortex liquid a candidate compound should have a lower density ρ_s and a shorter coherence length ξ. Note that the melting condition, involving short-range fluctuations, is assumed to be independent of disorder. Disorder, however, turns the vortex lattice into a vortex glass[8] whose melting field into a vortex liquid is in fact sensitive to disorder.[7]

I proceed now to discuss experimental data. The magnetization relaxation data of bulk $YBa_2Cu_3O_{7-\delta}$ is found to vanish above fields of order 50 kG and below 0.3 K.[6] This is consistent with a vortex liquid phase; i.e., the absence of shear modulus allows rapid relaxation to an equilibrium state. Furthermore, steps with constant magnetic induction $B(H)$ appear, spaced by $\Delta B=0.5$–1 kG. I propose the occurrence of a decoupling transition[29] in which the Josephson coupling between CuO_2 superconducting bilayers vanishes. The point vortices on each layer interact then with a $\sim \ln r$ interaction within a layer and with $\sim (d/\lambda)\exp(-nd/\lambda)\ln r$ between layers;[30,31] r is the vortex-vortex separation parallel to the layers, $n-1$ is the number of separating layers, and $d\approx 12$ Å is now the layer separation.

Assume first weak interlayer coupling for which quantization appears as for independent layers.[32] For the main sequence with $n=0$ the steps should be at $B=\phi_0\rho_s d/4m$, corresponding to the prefered stability of integer m states. Changing $m\to m+\Delta m$ leads to $\Delta B/B\approx \Delta m/m$; if $\Delta m=1$, the data imply $m\approx 100$ and $\rho_s d\approx 10^{14}$ cm^{-2}. This is, however, a factor of 5–10 too low as compared with muon spin relaxation data,[33] implying $\Delta m=5$–10. A possible resolution of this issue is by allowing for interlayer effects. Although $d/\lambda\approx 10^{-2}$ implies a weak interlayer coupling, it further decays only beyond a distance of $\lambda/d\approx 100$ layers. Thus the combined effect of all layers can be significant. A possible modification of Eq. (3) for vortices at $z_{i,n}$ on layer n is[32]

$$\Psi_L\{z_{i,n}\}=\prod_{i,j}\prod_{k=1}(z_{i,n}-z_{j,n+k})^{m_k}\Psi\{z_{i,n}\}, \tag{11}$$

with filling fraction (per layer)

$$\nu_b=\left[2m+2\sum_{k=1} m_k\right]^{-1}. \tag{12}$$

The integers m_k measure the strength of the interlayer interaction and depend on the magnetic length.[32] Since many layers are involved, $1\leq k \lesssim 100$, Σm_k can be a large number which is sensitive to the external field; its jump by ≈ 10 as the field is varied can account for the large observed steps. Clearly, a direct measurement of σ_{xy} can resolve this issue.

Consider next the experimental data on InO_x films[7] which demonstrate that a vortex liquid state is indeed feasible and that disorder increases its range of existence. The InO_x system with $d=100$ Å and $B\approx 5$ T implies [Eq. (2)] $\nu_b \ll 1$; in fact the measured[7] Hall resistance of ≈ 10 Ω, as compared with $h/4e^2=6450$ Ω, yields $\nu_b\approx 10^{-3}$. The corresponding large m values [Eq. (4)] imply that the Hall plateaus are dense and disorder can easily destroy them. The vortex liquid is then analogous to an electron Hall insulator with finite R_{xy} while $R_{xx}\to\infty$. Note that presence of dissipative inelastic scattering [which is neglected in Eq. (1)] would delocalize the vortices, leading to a finite R_{xx}. The experimental observation of a phase with finite R_{xy} and $R_{xx}\to\infty$ is therefore strong support to the proposed effective Lagrangian, Eq. (1), and its mapping into fermions even in a system with strong disorder.

The InO_x data[7] show that thinner films and higher fields, which are feasible, can yield higher ν_b values where quantum Hall plateaus are more likely to be observed. In fact the data[16] on 20-μm-thick $YBa_2Cu_3O_x$ shows $\sigma_{xy}\approx -10^4/\Omega$ cm at 13 K, so that a one-unit-cell-thick sample would have (from $\sigma_{xy}=4e^2/h\nu_b$) $\nu_b\approx 0.2$.

The type and strength of disorder are significant from a number of aspects: (i) Inelastic scattering reduces the width of a Hall plateau, analogous to temperature effects in the electron system. Thus, adiabatic dynamics with slowly varying potentials is required. (ii) Disorder helps in inducing melting and allowing for a larger range of fields for a vortex liquid.[7] (iii) The disorder amplitude, however, should not be too large so that the gaps in the Hall plateaus are not destroyed.

In conclusion, I propose that a quantized Hall conductance is possible in a vortex liquid. The required material parameters are (i) thin films of width comparable to a unit cell (or achieving a decoupling transition in a layered superconductor), (ii) a strong coupling superconductor with low condensate density and short coherence length, such as the HTSC compounds, and (iii) random pinning potentials which are slowly varying. The quantization, in fact, may have been indirectly observed in $YBa_2Cu_3O_{7-\delta}$.[6] The possible observation of a FQHE in a vortex liquid would be the first FQHE in a boson system and would demonstrate the significance of the Magnus force.

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[1] Recent reviews in Proceedings of the 4th International Conference on M^{2}M-HTSC, Physica C (to be published).
[2] D. R. Nelson, Phys. Rev. Lett. **60**, 1973 (1988).
[3] G. Blatter and B. Ivlev, Phys. Rev. Lett. **70**, 2621 (1993).
[4] G. Blatter, B. Ivlev Y. Kagan, M. Theunissen, Y. Volokitin, and P. Kes (unpublished).
[5] G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994).
[6] G. T. Seidler, C. S. Carrillo, T. F. Rosenbaum, U. Welp, G. W. Crabtree, and V. M. Vinokur, Phys. Rev. Lett. **70**, 2814 (1993).
[7] M. A. Paalanen, A. F. Hebard, and R. R. Ruel, Phys. Rev. Lett. **69**, 1604 (1992).
[8] D. S. Fischer, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1991).
[9] P. Ao and D. J. Thouless, Phys. Rev. Lett. **70**, 2158 (1993).
[10] M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and S. Levit, Pis'ma Zh. Eksp. Teor. Fiz. **57**, 699 (1993) [JETP Lett. **57**, 711 (1993)].
[11] P. Ao and D. J. Thouless, Phys. Rev. Lett. **72**, 132 (1994).
[12] E. B. Sonin and B. Horovitz, Phys. Rev. B (to be published).
[13] N. B. Kopnin and V. E. Kravtsov, Zh. Eksp. Teor. Fiz. **71**, 1644 (1976) [Sov. Phys. JETP **44**, 861 (1976)].
[14] N. B. Kopnin and V. E. Kravtsov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 631 (1976) [JETP Lett. **23**, 578 (1976)].
[15] N. B. Kopnin and M. M. Salomaa, Phys. Rev. B **44**, 9667 (1991).
[16] J. M. Harris, P. Matl, N. P. Ong, Y. F. Yan, T. Kimura, and K. Kitazawa, Physica C (to be published); J. M. Harris, Y. F. Yan, O. K. C. Tsui, Y. Matsuda, and N. P. Ong, Phys. Rev. Lett. **73**, 1711 (1994).
[17] Yu. V. Nazarov and A. A. Odintsov, Physica B **194**, 1737 (1994).
[18] M. Y. Choi, Phys. Rev. B **50**, 10 088 (1994).
[19] A. Stern, Phys. Rev. B **50**, 10 092 (1994).
[20] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983); in *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer, New York, 1987), p. 223.
[21] F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983); in *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer, New York, 1987), p. 303.
[22] B. I. Halperin, Phys. Rev. Lett. **52**, 1583 (1984).
[23] J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989); Adv. Phys. **41**, 105 (1992).
[24] D. Yoshioka, Phys. Rev. B **29**, 6833 (1984).
[25] X. C. Xie, Song He, and S. Das Sarma, Phys. Rev. Lett. **66**, 389 (1991).
[26] E. H. Rezai and A. H. MacDonald, Phys. Rev. B **44**, 8395 (1991).
[27] J. Pearl, Appl. Phys. Lett. **5**, 65 (1964).
[28] E. H. Brandt, J. Low Temp. Phys. **26**, 735 (1977).
[29] L. I. Glazman and A. E. Koshelev, Phys. Rev. B **43**, 2835 (1991).
[30] K. B. Efetov, Zh. Eksp. Teor. Fiz. **76**, 1781 (1979) [Sov. Phys. JETP **49**, 905 (1979)].
[31] B. Horovitz, Phys. Rev. B **47**, 5947 (1993).
[32] For a review see A. H. MacDonald, Surf. Sci. **229**, 1 (1990).
[33] Y. J. Uemura, Physica B **169**, 99 (1991).