Motion of a Josephson vortex under a temperature gradient

A. A. Golubov
Institute of Solid State Physics, Russian Academy of Science, 142432 Chernogolovka, Moscow District, Russia

G. Yu. Logvenov*
Faculty of Science, Kyoto University, Kyoto 606-01, Japan

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A theory of a Josephson vortex motion under a temperature gradient is presented. Microscopic calculations of the temperature dependence of a Josephson vortex transport entropy, S_φ, are done in the whole temperature range. It is shown that $S_\varphi \rightarrow 0$ as $T \rightarrow 0$ and T_c. Results of the theory are compared with our experimental data for superconductor–normal-metal–superconductor (SNS) and superconductor-insulator-superconductor Josephson junctions. The value of a transport entropy obtained from our experiments is in good agreement with theoretical estimates. An additional maximum of the transport entropy has been found experimentally in long SNS junctions with $d_N/\xi_N \gg 1$. This result is explained in the framework of the microscopic model presented in the paper.

I. INTRODUCTION

Thermoelectric and thermomagnetic effects in the mixed state of high-temperature superconductors (HTSC's) have attracted much attention last years (see, for example, Refs. 1–13. In this paper only the thermomagnetic Nernst effect will be at the center of our attention. When a temperature gradient is applied to a sample, a thermal force $F_{\rm th}=S_\varphi \nabla T$ acts on the vortices. As a result, viscous motion of Abrikosov vortices (A vortices) in the mixed state of a superconductor under a temperature gradient induces the transverse Nernst voltage. This effect is connected with an entropy transport in a direction parallel to the temperature gradient, i.e., with a vortex creation at one side and destruction at the other. Thermal energy is absorbed at the side where vortices are generated and it is transferred to the other side where vortices are expelled out of the sample.

Recently we observed the Nernst effect in the mixed state of Josephson superconductor–normal-metal–superconductor (SNS) sandwiches Ta/Cu/Nb.[14,15] It was the first report of the Nernst voltage due to the motion of Josephson vortices (J vortices). The problem of the Nernst effect due to the motion of J vortices was raised also by Coffey[16,17] in a context of HTSC's and multilayered systems. An important difference between J and A vortices is that the latter have a normal core, whereas J vortices do not. However, there is no fundamental reason for a qualitative difference between two thermodynamic phenomena—thermal diffusion of J vortices and that of A vortices.

In this paper a theory of the flux motion under a temperature gradient and experimental results of the observation of this thermomagnetic effect (Nernst effect) in SNS and superconductor-insulator-superconductor (SIS) Josephson junctions are presented. The paper consists of two parts. First, the theory of flux motion of J vortices in a Josephson junction under a temperature gradient is presented. Numerical calculations of a temperature dependence of the transport entropy of a J vortex in the whole temperature range between $T=0$ and T_c are performed. In the second part we estimate the transport entropy of the J vortex using our experimental results for SNS Josephson junctions. The magnitude of the Nernst effect in SIS Josephson junctions and the possibility of its observation are also discussed.

II. THEORY

A theoretical approach to the problem of the Nernst effect in the Josephson junctions was reported by us shortly in Refs. 14 and 15 and independently by Coffey in Refs. 16 and 17. Here a more detailed theory of motion of Josephson vortex under a temperature gradient is presented.

Let us consider a long one-dimensional Josephson junction of length $L \gg \lambda_J$ under a temperature gradient ∇T applied along the junction, and let at an initial moment of time a Josephson vortex is situated at the point x_0. The temperature gradient along the junction leads to a spatially inhomogeneous distribution of the critical current density, $j_c(x)$. As a result, the vortex is subjected to an effective force F. We will show that in linear approximation in ∇T this force can be written in the form

$$F=S_\varphi \nabla T \ , \tag{1}$$

and will calculate the corresponding transport coefficient S_φ.

The energy of a static vortex in a Josephson junction is given by the well-known expression

$$H=\int_{-\infty}^{\infty}[\tfrac{1}{2}(\nabla\varphi)^2+1-\tilde{j}_c(\tilde{x})\cos\varphi(\tilde{x})]d\tilde{x} \ . \tag{2}$$

Here $\tilde{x}=x/\lambda_J$, where λ_J is the Josephson penetration depth,

$$\lambda_J=(c\Phi_0/8\pi^2 d j_{c0})^{1/2} \ ,$$

and the critical current density $\tilde{j}_c = j_c / j_{c0}$, where $j_{c0} = j_c(\tilde{x} = \tilde{x}_0)$ is the critical current density at the initial fluxon position. (Φ_0 is a flux quantum.) It is convenient to introduce the dimensionless function $f(\tilde{x})$ according to

$$\tilde{j}_c(\tilde{x}) = 1 + f(\tilde{x}), \quad f(\tilde{x}_0) = 0 \quad f(\tilde{x}) \ll 1 \ . \tag{3}$$

The function $f(\tilde{x})$ describes the critical current density variation along the junction due to a temperature gradient. It will be estimated below for a number of specific models of a junction. The phase difference $\varphi(\tilde{x})$ obeys the perturbed sine-Gordon equation

$$\varphi_{xx} = [1 + f(\tilde{x})] \sin\varphi(\tilde{x}) \ . \tag{4}$$

In zeroth order to the perturbation $f(\tilde{x})$, Eq. (4) has the well-known solution

$$\varphi_0 = 4 \tan^{-1}[\exp(\tilde{x} - \tilde{x}_0] \ . \tag{5}$$

In this approximation the fluxon energy according to Eq. (2) does not depend on its position $\tilde{x}$; i.e., the fluxon is not subjected to an external force. To calculate a force applied to the vortex in the presence of the temperature gradient one should consider a correction to its energy Eq. (2) in first order to $f(\tilde{x})$. One has from Eqs. (2), (4), and (5)

$$H = 8 - \int_{-\infty}^{+\infty} f(\tilde{x} - \tilde{x}_0) \cos\varphi(\tilde{x} - \tilde{x}_0) d\tilde{x} \ . \tag{6}$$

As a result a force applied to the unit length of a vortex can be written in the form

$$\begin{aligned} F &= -[\partial H / \partial \tilde{x}]_{\tilde{x} = \tilde{x}_0} \\ &= -4 \int_{-\infty}^{+\infty} f(t) \sinh(t) \cosh^{-2}(t) dt \ . \end{aligned} \tag{7}$$

To determine the function $f(\tilde{x})$ let us first consider the simplest case of T close to T_c. For a tunnel junction one can use the expression for the critical current density $\tilde{j}_c$,[18]

$$\tilde{j}_c = \frac{\pi \Delta^2(0)}{4 e R_N T_c} \ . \tag{8}$$

For SNS junctions with small NS boundary transparency and $d_N / \xi_N \gg 1$ the critical current $\tilde{j}_c$ was derived in Refs. 19 and 20,

$$\tilde{j}_c = \frac{\pi \Delta^2(0)}{4 e R_N T_c} \left[2 + \frac{d_N}{\gamma_B \xi_N} \right] \exp(-d_N / \xi_N) \quad \text{for } \gamma_B \gg 1 \ . \tag{9}$$

Here R_N is the normal state resistance of a junction multiplied by the junction area; $\Delta(0)$ is the order parameter in junction electrodes near the weak link region. The parameter γ_B is defined as $\gamma_B = (2/3)(l_N/\xi_N)\langle (1-D)/D \rangle$, where D is the NS boundary transparency, the brackets $\langle \cdots \rangle$ denote angle averaging, d_N, ξ_N, and l_N are the thickness, the coherence length, and the mean free path in the normal layer, respectively.

For a tunnel junction and for a SNS junction with $\sigma_N / \sigma_S \ll \gamma_B \xi_N / \xi_S$ (ξ_S is the coherence length in the superconducting banks) one can use the rigid boundary conditions $\Delta(0) = \Delta_{\rm BCS}(T)$, where $\Delta^2_{\rm BCS}(T) = 8\pi^2 T_c (T_c - T) / 7\zeta(3)$ near T_c. Assuming that ∇T is small, one can write the expression for $f(\tilde{x})$ in the following form:

$$f(\tilde{x}) = \tilde{j}_c(\tilde{x}) / \tilde{j}_{c0} - 1 = \tilde{x} \frac{\partial \tilde{j}_c}{\partial T} \frac{\partial T}{\partial \tilde{x}} \ . \tag{10}$$

Substituting Eq. (10) into Eq. (7) and going from dimensionless units to the physical ones we get

$$\begin{aligned} F &= 4 \frac{\partial \tilde{j}}{\partial T} \frac{\partial T}{\partial \tilde{x}} \int_{-\infty}^{+\infty} t \sinh(t) \cosh^2(t) dt \\ &\equiv \frac{2\Phi_0}{\pi c} k_B \lambda_J \frac{\partial j_c}{\partial T} \frac{\partial T}{\partial x} \ . \end{aligned} \tag{11}$$

Using the explicit dependences $\tilde{j}_c(T)$ [see Eqs. (8), (9)] we can finally write the expression for the vortex entropy S_φ in the following form:

$$\begin{aligned} S_\varphi &= [8\pi^2 / 7\zeta(3)] k_B \lambda_J (R_0 / R_N), \quad \text{for SIS} \\ &= [8\pi^2 / 7\zeta(3)] k_B \lambda_J (R_0 / R_N) [2 + d_N / \gamma_B \xi_N] \\ &\quad \times \exp(-d_N / \xi_N) \quad \text{for SNS} \ , \end{aligned} \tag{12}$$

where

$$R_0 = \frac{\pi \hbar}{2 e^2} \simeq 6.5 \ \mathrm{k}\Omega \ .$$

As is seen from Eqs. (11), (12), the Josephson vortex is subjected to the force which is proportional to the temperature gradient; i.e., the direction of this force is *from the cold to the hot edge of a junction.* The proportionality coefficient S_φ is the transport entropy per unit vortex length. There exists a close analogy with the thermal transport of Abrikosov vortices. The transport entropy of the Abrikosov vortices was widely discussed in the literature theoretically[21–23] and measured both for low-T_c and high-T_c samples.[1–13] The important difference between A and J vortices is that the latter has no normal core. The entropy of bound states in a core is often interpreted in the literature as a source of the transport entropy. Nevertheless, as is shown above, a J vortex moves in a temperature gradient, i.e., also has transport entropy.

Let us discuss some limits in the temperature dependence of S_φ: $T \cong T_c$ and $T \cong 0$. As follows formally from Eq. (12), S_φ diverges as T approaches T_c for SIS junctions and SNS junctions with small NS boundary transparency. For SNS junctions with large NS boundary transparency, when the condition $\sigma_N / \sigma_S \gg \gamma_B \xi_N / \xi_S$ holds, the temperature dependence of the order parameter $\Delta(0)$ is given by $\Delta(0) \propto (T_c - T)$ (see Ref. 24), whereas for SNS junctions with small NS boundary transparency and for SIS junctions $\Delta(0) \propto (T_c - T)^{1/2}$. Then $j_c \propto (T_c - T)^2$ and $S_\varphi = \text{const}$ at $T \to T_c$ for the first case, whereas $j_c \propto (T_c - T)$ and $S_\varphi \to \infty$ at $T \to T_c$ for the second case. However, Eq. (12) is not applicable in this limit. The divergence should be cutted off at a temperature T^* such that $\lambda_J(T > T^*) > L$, where L is a junction length. To show this explicitly, one shall change the limits of integration, $+\infty$, to L and then obtain $S_\varphi \propto (L/\lambda_J)^2 \propto (T_c - T)$ for SIS junctions and $(T_c - T)^2$ for SNS junctions with large NS boundary transparency. On the other hand, Eq. (12) was derived assuming that T is sufficiently close to T_c where Eqs. (8) and (9) for $\tilde{j}_c$ are applicable. Therefore Eq. (12) is not applicable at $T \cong 0$. It is the reason why S_φ according to Eq. (12) does not go to zero as it should according to the third law of thermodynamics.

To show explicitly that S_φ really vanishes in this limit, one should use a microscopic model to calculate the critical current density j_c at arbitrary temperature. Let us consider the case of arbitrary T. In this case the general expression for the critical current density of SNS Josephson junctions can be written in the form[20]

$$j_c = (2\sigma_N/e)\pi T \sum_\omega G_N^2 \Phi_N^* \Phi_N' / \omega^2 \;, \tag{13}$$

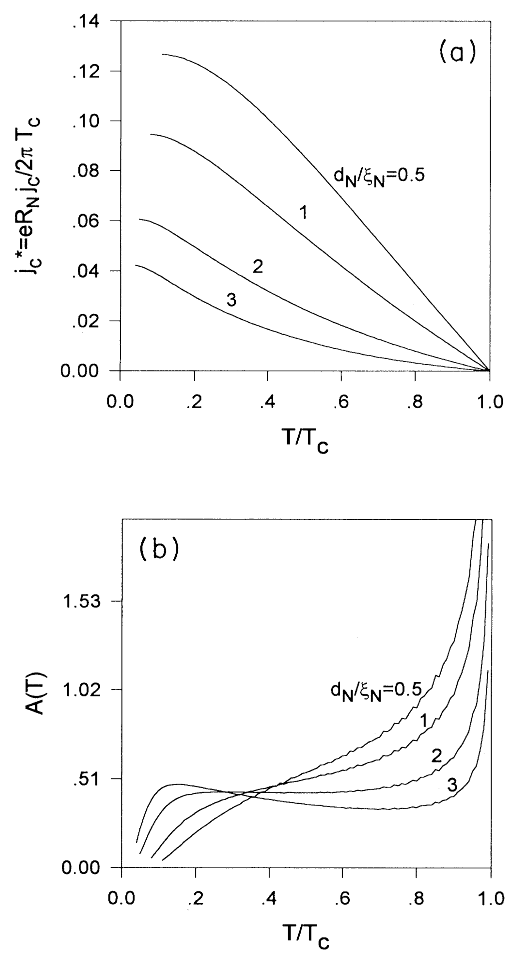

FIG. 1. (a) Temperature dependences of the critical current, $j_c^* = eR_N j_c / 2\pi T_c$, of SNS Josephson junctions for different thicknesses d_N/ξ_N: (0.5,1,2,3) and $\gamma_B = 5$, $\gamma = 1$. (b) Dimensionless function $A(T) = (eR_N T_c/2\pi)^{1/2} j_c^{-1/2} dj_c/dT$, which determines the transport entropy of the Josephson vortex, S_φ, according to Eq. (15).

where $\omega = \pi T(2n+1)$ is the Matsubara frequency, and G_N and Φ_N are the normal and anomalous Green's functions in the N layer. To determine Φ_N and G_N one should solve the microscopic Usadel equations,[25] which are valid in the whole temperature range. We have done the numerical calculations for $\gamma_B = 5$ (which corresponds to the real experimental conditions as discussed below) by the method described in Ref. 26 and various values of $\gamma = \sigma_N \xi_s / \sigma_s \xi_N$ and thickness of a normal layer d_N. The results of calculations are shown in Fig. 1(a). It is seen that positive curvature of the $j_c(T)$ dependence exists at low temperatures for sufficiently large d_N/ξ_N values [it is due to the factor $\exp(-d_N/\xi_N)$ with $\xi_N \propto 1/T$]. However, at $T \cong 0$, $j_c(T)$ levels off and thus $dj_c/dT \propto 0$. Figure 1(b) shows the results for S_φ/T. The dimensionless function $A(T)$ as defined as

$$A(T) = \left[\frac{eR_N T_c}{2\pi}\right]^{1/2} j_c^{-1/2} dj_c/dT \;. \tag{14}$$

The entropy S_φ is expressed through $A(T)$ in the following way:

$$S_\varphi = \left[\frac{\Phi_0^3}{\pi^3 ecd_N R_N T_c}\right]^{1/2} A(T) \;. \tag{15}$$

It is seen from Fig. 1(b) that the temperature dependence of S_φ could be rather complicated; in particular, an additional maximum could have taken place at low T in the case $d_N/\xi_N \gg 1$. However, this calculation demonstrates that S_φ goes to zero at $T \to 0$ in accordance with the third law of thermodynamics. This is also the case in a more general case of vortex thermal motion in a vertical stack of Josephson junctions (multilayered system). For the case of S/N multilayer parallel critical currents along the layers were shown to have the property $dj_c/dT \to 0$ as $T \to 0$ in Ref. 27 (see Fig. 4 of Ref. 27). The same property holds for the perpendicular j_c. The Nernst effect in a multilayered system will be discussed separately.

III. DISCUSSION OF EXPERIMENTAL RESULTS

In this section we present our discussion of the experimental results obtained by us in SNS (Refs. 14 and 15) and SIS (Ref. 28) Josephson junctions. First of all we recall the experimental results of SNS junctions.[14] We omit the details of the experimental procedure and preparation of our samples. For more details see Ref. 14. The Nernst coefficient $E_y/\nabla_x T$ of the asymmetric Nb/Cu/Ta Josephson junction versus temperature at different magnetic fields is shown in Fig. 2. The Nernst coefficient has a maximum near the critical temperature of the Ta plate of our sample and then it tends to zero at $T \to T_c$ and decreases at lower temperatures. This temperature dependence of the Nernst coefficient is similar to that in HTSC's (see for instance Refs. 1–13). At lower temperatures the Nernst voltage in our SNS junction de-

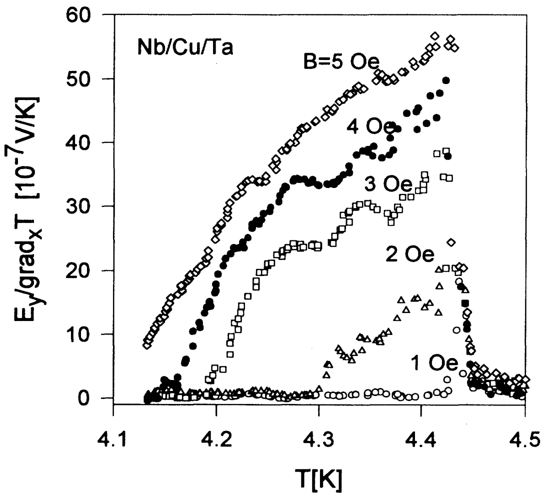

FIG. 2. Nernst coefficient $E_y/\nabla_x T$ of the asymmetric Nb/Cu/Ta Josephson junction versus T at different magnetic fields B (1 Oe, 2 Oe, 3 Oe, 4 Oe, 5 Oe).

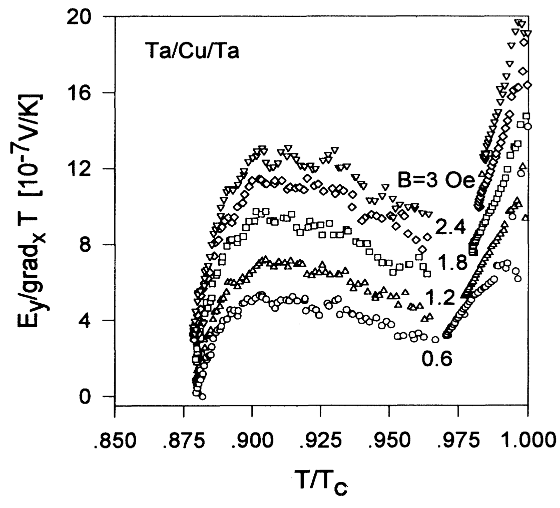

FIG. 3. Nernst coefficient $E_y/\nabla_x T$ of the symmetrical Ta/Cu/Ta Josephson junction with $d_N/\xi_N \cong 10$ versus T/T_c at different magnetic fields B (0.6 Oe, 1.2 Oe, 1.8 Oe, 2.4 Oe, 3 Oe).

creases remarkably and may be due to the pinning of Josephson vortices.

In Ref. 14 we estimated the value of the transport entropy using our data as

$$S_\varphi = 3\times 10^{-12}\,\mathrm{J/mK}$$

near T_c at $T=4.35$ K. Let us estimate a value of the transport entropy S_φ, according to Eq. (12) for the SNS case. Substituting the resistance of our sample $R_N=1.6\times 10^{-9}\ \Omega$, the sample area 0.1 cm^2 and Josephson penetration depth $\lambda_J=50\ \mu$m we have obtained the value of $S_\varphi=2\times 10^{-12}$ J/Km which is in reasonable agreement with our experimental value.

The measurements shown in Fig. 2 were made of asymmetric Nb/Cu/Ta Josephson junctions. Symmetrical SNS junctions Ta/Cu/Ta with large d_N/ξ_N values were also measured. Figure 3 shows the Nernst coefficient versus temperature for a thick Ta/Cu/Ta junction with $d_N/\xi_N \cong 10$. It is seen that an additional maximum of the Nernst coefficient, $E_y/\nabla_x T = S_\varphi \rho_N/\Phi_0$, exists at low temperatures (here E_y is an electric field across the junction due to the flux flow under a temperature gradient, ρ_N is resistivity of normal layer, and Φ_0 is magnetic flux quantum). This observation is in qualitative agreement with the results of calculations presented in Fig. 1(b) for a thick symmetrical SNS junction. Namely, with an increase of d_N/ξ_N an additional maximum of S_φ appears at low temperatures. Results of a quantative comparison between the model and measurements will be presented elsewhere.

We also have measured the Nernst voltage in SIS long Josephson junctions. We used Nb/AlO$_x$/Nb Josephson junction[28] with different dimensions. Typical parameters of the junctions used were following: the critical current I_c (4.2 K) = 20 mA, the Josephson penetration depth $\lambda_J \cong 15\ \mu$m, and the resistance multiplied by the sample area was $0.1\ \Omega\times 2.4\times 10^{-4}$ cm^2. The Nernst voltage was measured by applying a temperature gradient along the junctions in different magnetic fields. However, any Nernst voltage caused by the flux motion in the SIS Josephson junctions was not observed within the resolution of our experiment, $\delta V \sim 10^{-7}$ V. We observed a new thermoeffect in these tunnel Josephson junctions which was caused by a temperature dependence of the viscosity drag coefficient along the Josephson junction.[28] The fact of the absence of the Nernst voltage is not inconsistent with our theoretical estimations according to Eq. (12). Indeed, as follows from Eq. (12) with the parameters of our Josephson junction given above, $S_\varphi = 5\times 10^{-15}$ J/Km. It is about three orders of magnitude lower than that for SNS Josephson junctions estimated above. Using this value of S_φ and the measured resistivity of our SIS junctions at the flux flow step ρ_{FF}, we estimated the Nernst voltage in a SIS junction as $V_N = d\rho_{FF} S_\varphi \nabla T/\Phi_0 \cong 10^{-8}$ V. This value is less than the resolution of our experimental setup.

In summary, a theory of flux motion a J vortex under a temperature gradient has been presented. The microscopic calculations of the temperature dependences of a transport entropy, S_φ, of the J vortex were done. It was shown implicitly that $S_\varphi \to 0$ as $T\to 0$ and as $T \to T_c$. The theoretical results were compared with the experimental data obtained for SNS and SIS Josephson junctions. The magnitude of the transport entropy of a Josephson vortex was estimated from our experimental data. The value obtained is in good agreement with theoretical predictions. An additional maximum of the transport entropy was found experimentally in long SNS junctions with $d_N/\xi_N \gg 1$, which can be explained in the framework of the microscopic model presented in this paper.

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*Permanent address: Institute of Solid State Physics, Russian Academy of Sciences, Chernogolovka, Moscow District 142432, Russia.

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