Extraordinary Hall effect in magnetic multilayers

Shufeng Zhang
Physics Department, New York University, 4 Washington Place, New York, New York 10003
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The extraordinary Hall resistivity of magnetic multilayers is derived by using the Kubo formalism. It is found that the commonly used scaling relation between the extraordinary Hall resistivity and the ordinary resistivity is *not* valid. Hall voltages in the plane of the layers and perpendicular to the plane of the layers are quite different, just as the ordinary resistivities are different for these two geometries. Recent experimental results are discussed within this formulation.

I. INTRODUCTION

Recently, much attention has been focused on the electrical conductivity and magnetoresistance of magnetic multilayered structures and granular solids. Transport properties are controlled by two factors indigenous to these magnetically inhomogeneous solids: (1) spin-dependent scattering and spin-dependent potentials, and (2) spatially varying scattering rates. The first factor is the origin of giant magnetoresistance (GMR) while the second controls the magnitude of GMR in terms of layer thicknesses and mean free paths. These two ingredients make an analysis of transport in magnetic multilayered structures complex because one has to take into account both spatial and spin inhomogeneities. Theories based on the Boltzmann equation[1,2] and the Kubo formalism[3–5] have been developed to properly include this inhomogeneous nature of the transport phenomena. Up until now, these theories addressed only the electrical conductivities and magnetoresistance. It is not clear how other transport properties in magnetic materials, e.g., the extraordinary Hall effect (EHE), should be modified for multilayered structures and granular solids.

In explaining recent experimental data on the extraordinary Hall effect in layered structures[6–9] and granular solids,[10] one has relied on the theoretical description[11] of the EHE developed for *homogeneous magnetic materials* because there has been no theoretical attempt for the EHE in layered structures. In this paper, we give the first derivation of the EHE in these structures; we show that the behavior of the EHE in magnetic multilayered structures is *different* from that of homogeneous magnetic materials; in particular, the scaling relation between the Hall resistivity and ordinary resistivity is *not* valid for structures under consideration.

The origin of the EHE is well understood; it is from spin-orbital coupling. The spin-orbital coupling results in the breaking of the time symmetry. As a consequence, the scattering matrix contains an asymmetric term with respect to incident and scattered wave vectors (skew scattering).[12] The spin-orbit coupling also causes an anomalous velocity[13] due to noncommutivity between the position operator and spin-orbit Hamiltonian. The contribution to the EHE from the anomalous velocity is known as a side jump.[11,14,15] For homogeneous magnetic materials or alloys, the extraordinary Hall resistivity from skew scattering is proportional to the ordinary resistivity while that from the side jump is proportional to the square of the resistivity. However, it is not clear whether these scaling relations are held for magnetic multilayers.

II. EXTRAORDINARY HALL RESISTIVITY

We consider a layered structure consisting of alternately magnetic and nonmagnetic layers. The growth direction (perpendicular to the interfaces) is denoted as z. There are three geometries to discuss the Hall effect: (1) For a magnetic field applied in the z direction, the driving current and Hall voltage are in the plane of the layers; this is the conventional geometry for which experiments have been carried out. (2) The driving current is in the z direction while the magnetic field and Hall voltage are in the plane of layers. (3) The Hall voltage is measured along the z direction while the magnetic field and the current are in the plane of the layers. As it is known that the resistivity for the current in the plane (CIP) of layers is very different from that for the current perpendicular to the plane (CPP) of the layers,[16,17] one expects that the Hall effects are different for these three cases. We give the detailed derivation of the EHE for the first case; for the other two cases, the same derivation procedure follows and we only write down the final results.

Let us denote the driving current as the x direction, and the Hall voltage is measured along the y direction. The magnetic field is along the z direction (same as growth direction). For layered structures, translation invariance in the plane of the layer is preserved; therefore the electric fields in the plane of the layers are constant and the current density varies only with the coordinate z. The linear response for each spin channel s is written as

$$j_x^s(z) = \int \sigma_{xx}^s(z,z')E_x dz' + \int \sigma_{xy}^s(z,z')E_y dz', \quad (2.1)$$

$$j_y^s(z) = \int \sigma_{yx}^s(z,z')E_x dz' + \int \sigma_{yy}^s(z,z')E_y dz'. \quad (2.2)$$

To obtain the Hall field E_y, it is necessary to constrain the current density $j_y^s(z)$. We note that condition $j_y^s(z) = 0$ is inappropriate because it cannot be fulfilled at *all* positions z. There are always *internal* charge as well as spin currents in the y direction. Nevertheless, one can define the Hall electric field by imposing an external boundary condition such that the total current density along the y direction is zero, i.e., $\sum_s \int j_y^s(z)dz = 0$. Thus, from Eqs. (2.1) and (2.2), the Hall resistivity is

$$\rho_{yx} \equiv \frac{-E_y}{\sum_s \int j_x^s(z)dz} = \rho_{\rm CIP}^2 \sigma_{yx}, \tag{2.3}$$

where we have dropped the second term in Eq. (2.1) because the Hall fields are very small compared to the driving field, and have defined

$$\rho_{\rm CIP} = \left(\frac{1}{L} \sum_s \int \sigma_{xx}^s(z,z')dzdz' \right)^{-1} \tag{2.4}$$

for the CIP resistivity and

$$\sigma_{yx} = \frac{1}{L} \sum_s \int \sigma_{yx}^s(z,z')dzdz' \tag{2.5}$$

for the off-diagonal conductivity (Hall conductivity). The ordinary resistivity $\rho_{\rm CIP}$ has been studied for layered structures by several authors.[1–5] The main objective here is to derive the σ_{yx} from the side jump mechanism.

III. MODEL CALCULATION

We model the scattering from impurities within the layers as well as at interfaces by a scattering potential which includes spin-orbit coupling; for a single impurity (scattering center) it is written as

$$H' = V(\mathbf{r}) + \lambda_{\rm s.o.}\mathbf{M} \cdot [\nabla V(\mathbf{r}) \times \mathbf{p}], \tag{3.1}$$

where $\mathbf{M}$ stands for the magnetization of ferromagnetic layers and $\lambda_{\rm s.o.}$ is an effective spin-orbit coupling between conduction electrons and the impurity. The spin-orbit scattering produces the Hall resistivity in two ways. First it results in the *noncommutivity* between the coordinate operator and scattering potential Eq. (3.1), and therefore produces an anomalous velocity term[14,15]

$$\omega_{\mathbf{k}} = \frac{1}{i\hbar} \langle \mathbf{k} | [\mathbf{r}, H'] | \mathbf{k} \rangle_{\rm sc}, \tag{3.2}$$

where $|\mathbf{k}\rangle_{\rm sc}$ is a scattering conduction electron state. Second, the scattering matrix element of Eq. (3.1),

$$H'_{\mathbf{k},\mathbf{k}'} = \langle \mathbf{k} | H' | \mathbf{k}' \rangle = V_{\mathbf{k},\mathbf{k}'}(1 + i\lambda_{\rm s.o.}\mathbf{M} \cdot \mathbf{k} \times \mathbf{k}'), \tag{3.3}$$

is asymmetric (second term) with respect to the $\mathbf{k}$ and $\mathbf{k}'$ which gives rise to the skew scattering. To study these two effects in the Kubo formalism, one needs to redefine the velocity operator for the side jump and to include vertex corrections for skew scattering. For multilayered structures, the resistivities are usually much larger than that of individual bulk materials; the side jump may be the main source for the EHE.[11] Therefore, we concentrate our discussion on the side jump.

The anomalous velocity, Eq. (3.2), may be calculated in a perturbative way.[14] The scattering state for the Hamiltonian, Eq. (3.1), is expressed as

$$|\mathbf{k}\rangle_{\rm sc} = |\mathbf{k}\rangle + G_0 H' |\mathbf{k}\rangle_{\rm sc}, \tag{3.4}$$

where $G_0 = (\epsilon_{\mathbf{k}\sigma} - H_0)^{-1}$ is the unperturbed Green's function, and $|\mathbf{k}\rangle$ is the plane wave state. By placing the above scattering state into Eq. (3.2), we find that the anomalous velocity is, to second order of H',

$$\omega_{\mathbf{k}} = \frac{\hbar\lambda_{\rm s.o.}}{\tau^s(z)} \mathbf{k} \times \mathbf{M}, \tag{3.5}$$

where $\tau^s(z)$ is the *local* relaxation time in the *absence* of spin-orbit coupling. Since the local relaxation time varies from one layer to the next, it may be more convenient to express the anomalous velocity, Eq. (3.5), in real space, i.e.,

$$\omega(\mathbf{r}) = \frac{i\hbar\lambda_{\rm s.o.}}{\tau^s(z)} \mathbf{M} \times \boldsymbol{\nabla} . \tag{3.6}$$

The current density also gains a term proportional to the anomalous velocity

$$\begin{aligned} \mathbf{j}^s(\mathbf{r}) = & \frac{e\hbar}{2mi} \Psi_s^\dagger(\mathbf{r}) \overleftrightarrow{\boldsymbol{\nabla}}_{\mathbf{r}} \Psi_s(\mathbf{r}) \\ & + \frac{e\hbar\lambda_{\rm s.o.}}{2i\tau^s(z)} \Psi_s^\dagger(\mathbf{r}) \mathbf{M} \times \overleftrightarrow{\boldsymbol{\nabla}}_{\mathbf{r}} \Psi_s(\mathbf{r}), \end{aligned} \tag{3.7}$$

where Ψ_s is the wave function and $\overleftrightarrow{\boldsymbol{\nabla}}_{\mathbf{r}} = (\overrightarrow{\boldsymbol{\nabla}}_{\mathbf{r}} - \overleftarrow{\boldsymbol{\nabla}}_{\mathbf{r}})/2$ is the antisymmetric gradient operator. The first term is simply the conventional current density. The conductivity in the Kubo formalism is expressed in terms of the current-current correlation function. For our case the current consists of the normal part and the anomalous part, Eq. (3.7). While the correlation between normal current densities gives an ordinary conductivity, the correlation between the anomalous current and the normal current gives rise to the two-point local Hall conductivity

$$\begin{aligned} \sigma_{yx}^s(z,z') = & C \sum_s \int G_s^r(\mathbf{r},\mathbf{r}') \frac{m\lambda_{\rm s.o.}M_z}{\tau^s(z)} \overleftrightarrow{\partial}_x \overleftrightarrow{\partial}_{x'} G_s^a(\mathbf{r},\mathbf{r}') \\ & \times d(x-x')d(y-y'), \end{aligned} \tag{3.8}$$

where $C = e^2\hbar^3/2\pi m$ and $G^{r(a)}$ is the impurity averaged retarded (advanced) Green's function. Within the local relaxation time approximation (or local self-energy approximation), the Green's function satisfies Dyson's equation[4,5]

$$\left[\epsilon_F + \frac{\hbar^2}{2m} \nabla_{\mathbf{r}} - i \frac{\hbar}{\tau^s(z)} \right] G_s^r(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}'). \tag{3.9}$$

In the limit of weak scattering, $\epsilon_F \gg \hbar/\tau^s(z)$, the above equation has an explicit solution

$$G_s^r(\mathbf{r},\mathbf{r}') = -\frac{\hbar^2}{2m} \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|} \exp\left[-\frac{1}{2} \int_{\mathbf{r}}^{\mathbf{r}'} dr''/\lambda^s(z) \right], \tag{3.10}$$

where the integral in the exponent is along the straight line from point $\mathbf{r}$ to $\mathbf{r}'$ and we have introduced the local mean free path $\lambda^s(z) = v_F\tau^z(z)$. Upon substituting Eq. (3.10) into Eq. (3.8), and by integrating the coordinates in the plane of the layers x-x' and y-y', we find that for $k_F^{-1} \ll \lambda^s(z)$ (quasiclassical limit),

$$\sigma^s_{yx}(z,z') = \frac{m\lambda_{\text{s.o.}}M_z}{\tau^s(z)}\sigma^s_{\text{CIP}}(z,z'), \tag{3.11}$$

where

$$\sigma^s_{\text{CIP}}(z,z') = \frac{e^2k_F^2}{8\pi^2\hbar}\int_1^\infty dt(t^{-1}-t^{-3}) \times \exp\left[-t\int_{z_<}^{z_>} dz''/\lambda^s(z'')\right] \tag{3.12}$$

is the CIP two-point conductivity[5] and $z_<$ ($z_>$) is the smaller (larger) of z and z'. Finally, the Hall conductivity, Eq. (2.5), is simply the double integrations of the local Hall conductivity, Eq. (3.11), with respect to z and z', and summed over the spin variables.

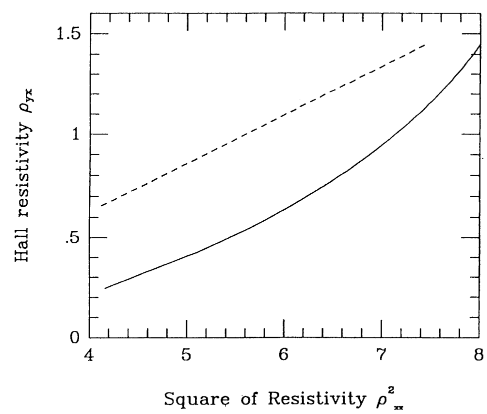

FIG. 1. The Hall resistivity as a function of the square of the ordinary resistivity (arbitrary units) in the limit of large mean free paths. The ratio of relaxation time for spin up to spin down electrons in magnetic layers is taken to be 10, $\tau^\uparrow_m/\tau^\downarrow_m = 10$. The relaxation time in nonmagnetic layers is $\tau_{nm} = \tau^\uparrow_m$ for the solid line and $\tau_{nm}{}' = 2\tau^\uparrow_m$ for the dashed line.

IV. LIMITING CASES

Our discussions of the extraordinary Hall effect are based on Eq. (3.11). When one applies it to a homogeneous magnetic material, the local conductivity $\int dz'\sigma^s_{\text{CIP}}(z,z')$ is proportional to the relaxation time which is canceled by the prefactor in Eq. (3.11). One concludes that the Hall conductivity, Eq. (3.11), is independent of the scattering potential or the relaxation time and the Hall resistivity, Eq. (2.3), is simply proportional to the square of ordinary resistivity. This is precisely the result first given by Luttinger[13] and emphasized by Berger.[11] However, for layered structures which consist of magnetic and nonmagnetic layers, the prefactor $\lambda_{\text{s.o.}}M_z/\tau^s(z)$ in Eq. (3.11) is governed by scattering in magnetic layers and interfaces while $\int dz'\sigma^s_{\text{CIP}}(z,z')$, Eq. (3.12), depends on, in general, the scattering in magnetic layers and at interfaces as well as in nonmagnetic layers. Therefore, the Hall conductivity σ_{yx} *depends* on the scattering potentials. The layered structures *invalidate* the simple scaling law between Hall resistivity and ordinary resistivity. To further illustrate this statement, we first examine two limiting cases where simple expressions for σ_{yx} can be derived. In the next section, we show numerical results for various realistic situations.

(1) Local limit. It is defined that the mean free path is much less than the layer thickness. Then we can view the multilayer as a resistor network that each layer has a well-defined resistance. The local CIP conductivity is therefore inversely proportional to local relaxation time,

$$\sigma^s_{\text{CIP}}(z,z') \propto \tau^s(z)\delta(z-z'). \tag{4.1}$$

By placing this equation into Eq. (3.11), we immediately see that σ^s_{yx} is independent of scattering potentials. Therefore, the scaling law is reestablished in the local limit.

(2) Long mean free path limit. Another extreme case is that the mean free path is much larger than the layer thickness. In this limit, electrons sample many layers before they are scattered. Therefore the ordinary two-point conductivity can be calculated in terms of average scattering in the structure:[3]

$$\int dz'\sigma^s_{\text{CIP}}(z,z') = \frac{A(t_m+t_{nm})}{m}\left(\frac{t_{nm}}{\tau_{nm}}+\frac{t_m}{\tau^s_m}\right)^{-1}, \tag{4.2}$$

where $A = e^2\hbar^3k_F^3/6\pi^2$, and t_{nm} and t_m are the thicknesses of nonmagnetic and magnetic layers, respectively. For simplicity, we have not included interface scattering. By placing the above equation into Eq. (3.11), we find that the Hall conductivity is

$$\sigma_{yx} = \lambda_{so}M_zAt_m\sum_s\left(t_m+t_{nm}\frac{\tau^s_m}{\tau_{nm}}\right)^{-1}. \tag{4.3}$$

Clearly, the Hall conductivity depends on the ratio of relaxation times in magnetic layers and nonmagnetic layers. This feature is quite different from the homogeneous magnetic materials where σ_{yx} is proportional to the side jump Δy (Ref. 11) which is independent of the scattering potential or the relaxation time. In Fig. 1, we show the relation between the Hall resistivity and the ordinary resistivity when the relative layer thickness between magnetic layers and nonmagnetic layers is varied. It is clear that as long as the local mean free path is not the same for different layers, the Hall resistivity is not simply proportional to the square of the resistivity.

V. NUMERICAL RESULTS

Experimentally, the extraordinary Hall effects have been studied for magnetic multilayers[6–9] and granular solids.[10] In these structures, the mean free path is usually larger than layer thicknesses or the size of granules. Therefore, it is *not* in the local limit and the simple scaling behavior ($\rho_{xy} \propto \rho_{xx}^2$) is not expected. In interpreting experimental data for various scaling behaviors, one anticipates quite different systems compared to the bulk materials. Without knowing detail scattering parameters, these scaling laws have no specific meaning. A quantitative comparison between various experimental data and our formulation, Eq. (3.11), relies on the knowledge of sets of parameters: local mean free paths for each layer, scattering parameters at interfaces, and spin-orbital coupling. Since the variation of the resistivity is measured experimentally through varying temperature from 4.2 to 300 K, one also requires to know the temperature dependence of these parameters. Unfortunately, little has been determined. Nevertheless, to get a possible realistic estimation, we assume a range of parameters which characterize most interesting multilayers, e.g, Co/Cu and Fe/Cr. Particularly, we consider three possible cases to illustrate the scaling behavior (interface scattering is neglected for simplicity; the typical layer thickness of 20 Å is chosen for magnetic and nonmagnetic layers): (1) Only the mean free path of magnetic layers depends on temperature, Fig. 2(a), (2) only the mean free path of nonmagnetic layers depends on temperature, Fig. 2(b), and (3) both mean free paths depend on temperature with *fixed* ratio, Fig. 2(c). These results show that the power n in the scaling law $\rho_{xy} \propto \rho_{xx}^2$ can be smaller [Fig. 2(a)], greater [Fig. 2(b)], or equal to [Fig. 2(c)] 2. This prediction is consistent with experimental observations that different experimental groups have reported rather different exponents of the scaling.[6–10]

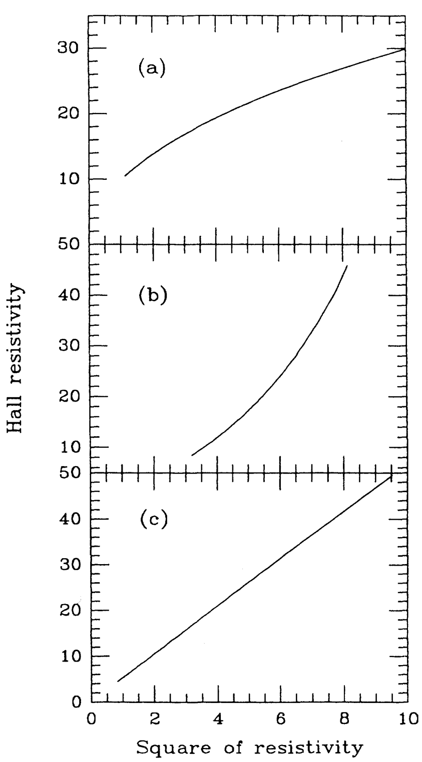

FIG. 2. The Hall resistivity as a function of the square of ordinary resistivity (arb. units) with each layer thickness of 20 Å. (a) The mean free path of spin up channel $\lambda_m^\uparrow$ and down channel $\lambda_m^\downarrow$ in magnetic layers are held at at 75 and 37.5 Å while the mean free path of nonmagnetic layers λ_{nm} varies from 300 to 30 Å. (b) λ_{nm} is held at 100 Å while $(\lambda_m^\uparrow)$ varies from 30 to 200 Å and $(\lambda_m^\downarrow)$ from 15 to 100 Å. (c) All the mean free paths vary with their ratios fixed.

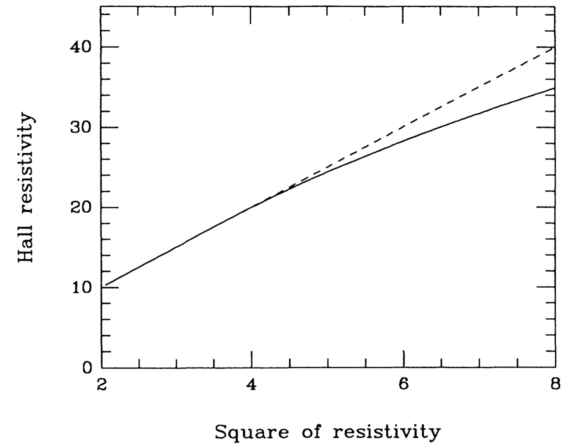

FIG. 3. The Hall resistivity as a function of the square of ordinary resistivity (arb. units) with fixed mean free paths $\lambda_m^\uparrow = 75$ Å, $\lambda_m^\downarrow = 37.5$ Å, and $\lambda_{nm} = 150$ Å. The thicknesses of magnetic and nonmagnetic layers are held equal and vary from 5 to 500 Å. The dashed line is the ordinary scaling law. The two curves start to separate at approximately 100 Å (note that the resistivity increases with the decrease of the layer thickness).

To further establish the evolution of the scaling behavior from the two limiting cases discussed in the Sec. IV, we study the Hall effect in terms of variation of the layer thickness. In Fig. 3 the Hall resistivity (solid curve) is plotted against the square of the resistivity by varying the layer thickness from 5 to 500 Å for fixed mean free paths. As a reference, we also show the dashed curve which is the ordinary scaling law. The deviation between these two curves starts approximately from thickness smaller than 100 Å; i.e., when the layer thickness is larger than 100 Å the standard scaling law is valid (two curves are overlapped). This is precisely the case which can be treated as a local limit. When the layer thickness is comparable or smaller than the mean free paths, a significant deviation between two curves appears. Since the interesting range of the layer thickness for most of the experiments is not on the local limit, we conclude that the ordinary scaling law is not applicable for these systems.

VI. OTHER GEOMETRIES

Up until now, we only calculated the Hall voltage in the plane of the layers. The Hall voltage for the other two cases described in the Introduction can also be obtained. The new ingredient for these two cases is that CPP resistivity is involved. In deriving the Hall resistivities, the electric field in the direction of the growth direction varies with position in space. For the driving current perpendicular to the plane of the layers (case 2), the linear response equations are

$$j_z^s(z) = \int \sigma_{zz}^s(z,z')E_z^s(z')dz' + \int \sigma_{zx}^s(z,z')E_x dz', \tag{6.1}$$

$$j_x^s(z) = \int \sigma_{xz}^s(z,z')E_z^s(z')dz' + \int \sigma_{xx}^s(z,z')E_x dz', \tag{6.2}$$

where $E_z^s(z')$ is the internal spin-dependent electrical field.[18] In the case of no spin mixing, the current density $j_z^s(z)$ for each spin channel is a constant, i.e., $j_z^s(z) = j_z^s$. Since the second term in Eq. (6.1) is much smaller than the first term, one can show that the electrical field has the form $E_z(z) \propto j_z^s/\tau^s(z)$.[18] By inserting this electrical field into Eq. (6.2) and by following the derivation similar to the derivation of Eqs. (2.3) and (3.12), we find

$$\rho_{zx}(z,z') = \rho_{\rm CIP}\rho_{\rm CPP} \sum_s \frac{\sigma_{zx}(z,z')}{\tau^s(z')A^s}, \tag{6.3}$$

where $\rho_{\rm CPP} = \sum_s \int dz dz' \sigma_{\rm CPP}^s(z,z')$ is the CPP conductivity,[5]

$$\sigma_{\rm CPP}^s(z,z') = \frac{e^2k_F^2}{4\pi\hbar}\int_1^\infty \frac{dt}{t^3}\exp\left[-t\int_{z_<}^{z_>} dz''/\lambda^s(z'')\right], \tag{6.4}$$

the off-diagonal conductivity is

$$\sigma_{zx}^s(z,z') = \frac{m\lambda_{\rm s.o.}M_y}{\tau^s(z)}\sigma_{\rm CPP}^s(z,z') \tag{6.5}$$

and the constant A^s is defined as $A^s = \int dz/\tau^s(z)$. For the Hall voltage measured in the z direction (case 3), the same derivation procedure follows. We find

$$\frac{\rho_{xz}}{\rho_{xy}} = \frac{\rho_{zz}}{\rho_{xx}}. \tag{6.6}$$

Although these results for different geometric realization can be tested by experiments in principle, the actual experimental setup is not at all trivial.[19] Currently, two groups are able to measure the CPP conductivity. In order to measure the CPP Hall resistivity one encounters difficulties of avoiding the current shunting if superconductor leads are used[16] or of obtaining a uniform current distribution.[17]

VII. SUMMARY

In summary, we have derived extraordinary Hall resistivities for magnetic multilayered structures; they are expressed in terms microscopic scattering parameters. To obtain a more complete picture of physical scattering processes and parameters in multilayered structures, one should simultaneously examine the magnetoresistivity and the Hall effect. We have found that the simple scaling behavior between the Hall resistivity and ordinary resistivity which has been frequently used to analyze experimental data is *not valid* for these structures. In addition, we have arrived at the correlated relations between the Hall resistivities and ordinary resistivities in CIP and CPP geometries.

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