# Effect of fractons on the exciton dynamics in dilute magnets

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The effect of fractons on the exciton dynamics in dilute magnets is investigated theoretically. It is predicted that the total intensity of the hot exciton-fracton absorption band varies with temperature as $T^{\bar{d}+2\bar{d}/D}$, where $\bar{d}$ is the fracton dimension and $D$ the fractal dimension. Also, the fracton-exciton interaction results in a quadratic temperature dependence of the exciton intersublattice relaxation rate in dilute antiferromagnets, substantially different from that (cubic temperature dependence) of magnon-exciton scattering. Implication of the results for experimental study of the magnon-fracton crossover in dilute magnets is demonstrated.

## I. INTRODUCTION

The nature of the exciton dynamics in magnets is an interesting subject which has received consideration attentions.[1-13] The creation and movement of these magnetic excitons usually involve low-energy magnetic excitations, such as spin waves. For example, a hot exciton band in dilute magnets is often observed,[3,6] where the optical absorption of an exciton is combined with the annihilation of a thermally crated magnon. In dilute magnets, where the short-range exchange interaction (SREI) becomes increasingly important, the magnetic ions form a percolating system if the SREI is dominant. This percolating network has fractal structure if the bond concentration is not too far from the percolation threshold. The low-energy excitation spectra of such percolating structures have been studied by Alexander and Orbach[14] using a scaling picture which is called the fracton model. Although some important properties of the crossover, such as the fracton (spectral) dimension and the exact density of states still remain to be clarified, their introduction into condensed-matter physics has been shown to be successful in the identification of many physical properties of topologically disordered materials.[15-18] The purpose of the present paper is twofold. First, we study the effect of fractons on the temperature dependence of two important quantities characterizing the exciton dynamics, i.e., the intensity of the hot exciton absorption bands in dilute magnets and the exciton intersublattice relaxation rate in dilute antiferromagnets (AF's). Secondly, we suggest that the study of exciton dynamics in dilute magnets can be a way to investigate the fracton dynamics, whose results should make it possible to perform a careful check of the characteristics of the magnon-fracton crossover experimentally.

This paper is organized in the following way. In Sec. II we calculate the temperature dependence of hot exciton-fracton absorption bands in dilute magnets. We show that the intensity of the hot exciton-fracton band varies with temperature as $T^{\bar{d}+2\bar{d}/D}$, where $D$ is the fractal dimension and $\bar{d}$ the fracton (spectral) dimension. The temperature dependence of the exciton intersublattice relaxation rate in dilute AF's is studied in Sec. III, where we find that the fracton-exciton interaction results in a quadratic temperature dependence of the exciton intersublattice relaxation rate in dilute antiferromagnets, substantially different from the cubic temperature dependence induced by magnon-exciton scattering. In Sec. IV we discuss the implication of the results for experiments. The conclusions of this paper are given in Sec. V.

## II. THE TEMPERATURE DEPENDENCE OF THE INTENSITY OF HOT EXCITON-FRACTON ABSORPTION BANDS IN DILUTE MAGNETS

Hot and cold magnon sidebands have been studied widely in magnets.[3-10] It has been considered[6] a well-established method for investigating the spin correlations of magnetic crystals by studying the optical exciton-magnon transitions. Of particular interest is the temperature variation of hot exciton bands, where the optical absorption of an exciton is combined with the annihilation of a thermally created magnon. The intensity of hot bands is expected to increase with increasing temperature since the transition strength depends on the thermal population of magnons. The temperature dependence of hot exciton-magnon sidebands was calculated numerically by Shinagawa and Tanabe[4] in $MnF_2$. Robbins and Day[9] studied the temperature dependence of the oscillator strengths of exciton-magnon absorption bands in

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metamagnetic transition-metal dihalides and pointed out that the temperature variations of the hot bands are significantly dependent on the spin-wave dispersion. Later Tsuboi and Ahmet[7,8] compared experimental and theoretical work on antiferromagnets and conjectured that the total intensity can be described by $T^d$ at temperatures below $T_N$, where $d$ is the Euclidean dimension.

In this section we study the implication of the magnon-fracton crossover for the temperature dependence of the hot exciton-fracton band intensity. The calculations will be done in a way analogous to Refs. 9 and 10. We first consider a ferromagnet diluted with nonmagnetic ions since in a ferromagnet only hot bands are allowed,[6] which is different from an AF, where both hot bands and cold bands are expected. If the concentration of magnetic ions approaches the percolating threshold, the magnet can be considered to be self-similar and the magnon-fracton crossover effect becomes appreciable. The incident photon creates an exciton of wave vector $k$ and a fracton is annihilated simultaneously. Then the intensity of this transition is expressed as

$$I(\omega,T)=\sum_k I_k(\omega,T) \ , \tag{1}$$

where $I_k(\omega,T)$ is given by

$$I_k(\omega,T)\propto |M_k|^2\langle n_k\rangle\delta(\hbar\omega+\hbar\Omega_k-E_k) \ . \tag{2}$$

Here $M_k$ is the electric-dipole matrix element, $E_k$ is the energy of an exciton with wave vector $k$, $n_k$ is the thermal occupation of fractons with frequency $\Omega_k$,

$$\langle n_k\rangle=\left[\exp\left[\frac{\hbar\Omega_k}{k_BT}\right]-1\right]^{-1} , \tag{3}$$

and the total number of fractons is

$$\sum_k\langle n_k\rangle=\int_{\Omega_c}^{\Omega_f}\rho(\Omega)\langle n(\Omega)\rangle d\Omega \ , \tag{4}$$

where $\Omega_c$ is the magnon-fracton crossover frequency and $\Omega_f$ the fracton cutoff frequency.

We assume that the thermally created fractons have small $k$ (which is true for low temperatures); then only matrix elements $M_k$ having small $k$ need to be taken into account. For a centrosymmetric system, the leading terms in $M_k$ are approximately proportional to $k$.[9] The total intensity is then obtained as

$$I(T)=\int I(\omega,T)d\omega \ . \tag{5}$$

The dispersion[17] of fractons is different from that of magnons,

$$\Omega\sim k^{D/\bar{d}} \ , \tag{6}$$

which leads to the fracton density of states (DOS)

$$\rho(\Omega)\sim\Omega^{\bar{d}-1} \ . \tag{7}$$

Combining these equations, one has

$$I(T)\propto\int_{\Omega_c}^{\Omega_f}\Omega^{\bar{d}+2\bar{d}/D-1}\left[\exp\left[\frac{\hbar\Omega}{k_BT}\right]-1\right]^{-1}d\Omega \ . \tag{8}$$

Putting $\hbar\Omega/k_BT=x$, Eq. (8) is rewritten as

$$I(T)\propto T^{\bar{d}+2\bar{d}/D}\int_{x_c}^{x_f}x^{\bar{d}+2\bar{d}/D-1}[\exp(x)-1]^{-1}dx \ . \tag{9}$$

Since the integral over $x$ is a definite value, one can see that the total intensity $I(T)$ of the hot band varies with temperature as $T^{\bar{d}+2\bar{d}/D}$, giving a prediction about the exciton dynamics in dilute magnets.

The $T^{\bar{d}+2\bar{d}/D}$ dependence is, however, valid only for magnets at the percolation limit, where $\Omega_c\to 0$ and the effect of fractons is significant. For those magnets with smaller doping levels, the effect of magnons must be taken into account. Since the magnons have different dispersion and density of states from those of fractons, the integration over $\Omega$ must then be divided into two terms, one of which gives the contribution of the magnon sideband and the other that of the fracton sideband. For clarity the result will not be presented here. Nevertheless, it is natural to consider that the total intensity on these occasions is sensitive to the details of the magnon-fracton crossover.

In dilute antiferromagnets, where both hot and cold bands exist, the calculation can be done in the same way for the hot bands, and the $T^{\bar{d}+2\bar{d}/D}$ dependence is also expected. The difference is that the anisotropy-associated gap in the spin excitation spectrum may lead to some term modifying the temperature dependence in antiferromagnets, as shown by Tsuboi and Ahmet.[8] Though the temperature variation must be calculated for individual materials, we would like to point out that the $T^{\bar{d}+2\bar{d}/D}$ dependence of hot exciton bands can be achieved by choosing appropriate materials and it can be expected in a series of dilute magnets.

We note with interest that the optical exciton transitions in $Fe_{0.6}Z_{0.4}F_2$ have been studied by Kleemann and Uhlig.[6] One of their observations is that the moment $M_0$ (which is defined as the integrated intensity) of the $E$ absorption line varies with temperature as $T^{1.6}$, considerably different from the variation of the undoped crystal (whose moment $M_0$ increases with temperature as a power law with an exponent of 4.2). We think that this can be regarded as a possible trace of the fracton sideband. Using the Alexander-Orbach value[14] for the spectral dimension ($\bar{d}=\frac{4}{3}$) and the fractal dimension of a three-dimensional percolating network ($D=2.5$), our calculation predicts a $T^{2.4}$ dependence. Considering that the moment $M_0$ is a measure of the total intensity of the $E$ band (which includes pere exciton and hot and cold exciton lines) rather than the hot $M^-$ band, and the fact that the $Fe_{0.6}Zn_{0.4}F_2$ crystal is not a critically doped one, we think that our theory is relevant to their experiment despite the discrepancy, and their result can be considered as possible evidence for hot exciton-fracton transitions.

## III. THE TEMPERATURE DEPENDENCE OF THE EXCITON INTERSUBLATTICE RELAXATION RATE IN DILUTE ANTIFERROMAGNETS

Using the magnon approximation, Ueda and Tanabe[11] have studied the motion of an exciton in an AF. Their study indicted that at low temperatures the exciton moves on one sublattice coherently and at higher temperatures moves diffusively on both sublattices. The nature of exciton motion changes gradually at the temperature where $\Gamma_0+Z\Gamma_1$ is comparable to the exciton bandwidth. Here $\Gamma_0$ represents the fluctuation of the exciton energy, $\Gamma_1$ the transition probability between the nearest-neighbor intersublattice pair, and $Z$ the coordinate number. Ueda and Tanabe showed that within the framework of spin-wave theory both $\Gamma_0$ and $\Gamma_1$ increase approximately with $T^3$. Measurement of the intersublattice transition rate of magnetic excitons was carried out in $MnF_2$ by Holzrichter, Macfarlane, and Schawlow,[12] who used a technique based on the detection of magnetization changes with the help of a pickup coil. By proper adjustment of the polarization of the incident light, excitons are created on one sublattice and magnons on the opposite one. The resultant nonequilibrium populations of free excitons and magnons subsequently relax to their equilibrium distributions by way of intersublattice transitions. More recently, Hollman, Arts, and Wijn[13] measured the same rate as a function of temperature for the excitons arising from the ${}^6A_1\rightarrow{}^4T_1(I)$ transition in $MnF_2$ in both a nominally pure crystal and one slightly doped with $Zn^{2+}$ ions. They found that the rate is dominated by scattering with thermal magnons at temperatures above 10 K, and increases approximately with $T^3$, consistent with the prediction of Ueda and Tanabe.[11]

In dilute AF's, however, the exciton intersublattice relaxation has not been well investigated. One can expect the intersublattice relaxation process in dilute AF's because of the existence of the magnon-fracton crossover. In this section, we shall examine the effect of fractons on the temperature dependence of the exciton intersublattice relaxation rate in dilute AF's. We consider a dilute AF with appreciable intersublattice transfer matrix elements (such as Zn-diluted $MnF_2$). The exciton-magnon (fracton) pairs are created on neighboring magnetic ions and then transitions occur between the two opposite sublattices via off-diagonal exchange. The first problem we encounter when calculating the exciton intersublattice transfer rate due to the fracton-exciton interaction comes from the localization of the fraction wave functions. Since the magnetic excitons are usually localized, this situation is somewhat similar to that faced by Alexander, Entin-Wohlman, and Orbach[19] when studying the relaxation and nonradiative decay of localized electronic states involving fractons. The localization breaks the equivalence between electronic states there. However, that is not the case here. Dominant intersublattice relaxations take place between nearest-neighbor pairs on opposite sublattices. The size of a magnetic exciton is typically of the order of magnitude of the lattice parameter $a$, while the localization length $l_\Omega$ of a fracton depends on its energy:

$$l_\Omega \propto \Omega^{-\bar{d}/D}\ , \quad \Omega > \Omega_c\ , \tag{10}$$

where the crossover frequency $\Omega_c$ is a function of the correlation length $\xi_p$:[15]

$$\Omega_c \propto \xi_p^{-D/\bar{d}}\ , \tag{11}$$

and $\xi_p$ is

$$\xi_p = a\,|\Delta p|^{-\nu}\ . \tag{12}$$

Here $\Delta p = p - p_c$, with $p$ the bond concentration of the network and $p_c$ the percolation threshold. Therefore the localization length of a fracton is within the range $a < l_\Omega < \xi_p$. If most of the excitations with energy $\Omega > \Omega_c$ are low-frequency fractons (which is true for low temperatures), then the localization length of a fracton can be generally rather large. So long as we consider the nearest-neighbor interaction, the localization of the fracton wave function does not suppress the interaction and has no practical import for the calculation of the exciton intersublattice relaxation rate.

We follow the calculation of Ueda and Tanabe[11] and give a short account[13] of their relevant theoretical results first. The dominant exciton-magnon interaction Hamiltonian is given by

$$\begin{aligned} H_I = & -\sum_{jl} 2J_{jl}S[\epsilon_{jl}(A_j^\dagger A_j a_j^\dagger a_j + B_l^\dagger B_l b_l^\dagger b_l) + \phi_{jl}(A_j^\dagger A_j + B_l^\dagger B_l)(a_j^\dagger b_l^\dagger a_j b_l) + \rho_{jl}(A_j^\dagger A_j b_l^\dagger b_l + B_l^\dagger B_l a_j^\dagger a_j)] \\ & + \sum_{jl}\{L_{jl}(A_j^\dagger B_l a_j b_l^\dagger + B_l^\dagger A_j b_l a_j^\dagger) + L'_{jl}[B_l^\dagger A_j(a_j^\dagger a_j^\dagger + b_l b_l) + A_j^\dagger B_l(a_j a_j + b_l^\dagger b_l^\dagger)]\}\ , \end{aligned} \tag{13}$$

where $a_j^\dagger$ $(a_j)$ and $b_l^\dagger$ $(b_l)$ are the usual local-spin-deviation creation (annihilation) operators, $A_j^\dagger$ $(A_j)$, $B_l^\dagger$ $(B_l)$ the exciton creation (annihilation) operators, and $j$ and $l$ denote sites on the spin-up and spin-down sublattices, respectively. Definitions of the coefficients in Eq. (13) can be found in Ref. 13 and they are not repeated here.

The two summations of Eq. (13) describe the modulation of the exciton energy due to the interaction and the exciton intersublattice transition, respectively. In deriving the expression for the exciton intersublattice transition rate, the density matrix formalism is used and the exciton-magnon interaction is treated as a perturbation up to second order. The magnon system is assumed to be thermalized quickly enough. Then one has two terms providing for the exciton damping. One is

$$\Gamma_0=Z(2JS)^2(Z\epsilon^2\gamma^{(1)}+\phi^2\gamma^{(2)}+\rho^2\gamma^{(3)}) , \tag{14}$$

which is a measure of the fluctuation of the exciton energy induced by magnon scattering, and the other,

$$\Gamma_1=Z(2L'^2\gamma^{(3)}+L^2\gamma^{(4)}) , \tag{15}$$

denotes the magnon-assisted intersublattice transition rate. The parameters $\gamma^{(1)}-\gamma^{(4)}$ are correlation functions between spin-deviation operators. In general, $\Gamma_0+Z\Gamma_1$ will be observed as the linewidth if the intersublattice transfer matrix element is appreciable.

Within the magnon approximation, which neglects correlations between sites, $\gamma^{(4)}=\gamma^{(1)}$ and $\gamma^{(2)}=\gamma^{(3)}$ hold. By introducing the magnon density of states

$$\rho(\Omega)=\frac{1}{N}\sum_k\delta(\Omega-\Omega_k) , \tag{16}$$

and the weighted one

$$\rho_c(\Omega)=\frac{1}{N}\sum_k\delta(\Omega-\Omega_k)\cosh2\theta_k , \tag{17}$$

one has

$$\gamma^{(1)}=2\pi\int_0^\infty d\Omega\, n(\Omega)[n(\Omega)+1]\tfrac{1}{2}\{[\rho_c^2(\Omega)+\rho^2(\Omega)]\} \tag{18}$$

and

$$\gamma^{(3)}=2\pi\int_0^\infty d\Omega\, n(\Omega)[n(\Omega)+1][\rho_c^2(\Omega)-\rho^2(\Omega)] , \tag{19}$$

where $\tanh2\theta_k=\gamma_k$,and $n(\Omega)$ is the Bose-Einstein distribution function for magnons with frequency $\Omega$.

The temperature dependence of $\Gamma_1$ is contained in $\gamma^{(1)}$ and $\gamma^{(3)}$. If $\rho(\Omega)$ is proportional to $\Omega^2$, then at low temperatures a $T^3$ dependence of $\gamma^{(1)}$ and $\gamma^{(3)}$, and thus $\Gamma_0$ and $\Gamma_1$ results, as shown by Ueda and Tanabe.[11] This conclusion is regarded to hold for pure crystals and those with very low impurity concentration where the magnon approximation works fairly well. In dilute AF's, where the fractal geometry is expected to play a role, extended magnons cross over to localized fractons at the crossover frequency $\Omega_c$, as noted earlier in this paper. The dispersion of a fracton has been given by Eq. (6) and the fracton cutoff frequency $\Omega_f$ is related to $\Omega_c$ by

$$\Omega_f=\Omega_c(\xi_p/a)^{D/\bar{d}} . \tag{20}$$

After being properly normalized, the fracton DOS is inserted into Eqs. (18) and (19) to replace that of the magnons and then into Eq. (15). We suppose that $ka \ll 1$ holds for the fracton excitations at low temperatures, so that one has for a cubic structure

$$\rho_c(\Omega)=\left[\frac{Z}{2}\right]^{1/2}\left[\frac{\Omega_f}{\Omega}\right]^{\bar{d}/D}\rho(\Omega) . \tag{21}$$

Numerical results, shown in Fig. 1, suggest an increase of $\Gamma_1$ with temperature according to a power law with an exponent of 2 (as is evident from the inset). This quadratic temperature dependence is substantially different from the $T^3$ dependence arising from the magnon-exciton interaction.

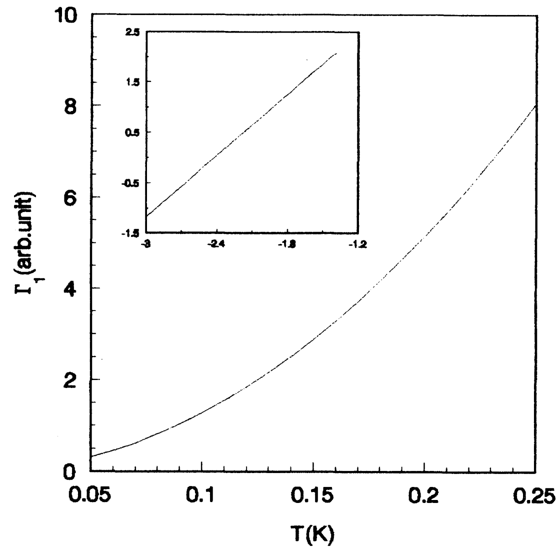


FIG. 1. Exciton intersublattice relaxation rate $\Gamma_1$ due to the fracton-exciton interaction in dilute AF's as a function of temperature. Shown in the inset is the log-log plot, which indicates that $\Gamma_1$ increases with temperature as a power law with an exponent of 2.

The $T^2$ dependence which arises from the fracton-exciton interaction is expected to occur in those dilute crystals with $\Delta p\to0$ (where $\Omega_c\to0$ and the fracton-exciton interaction dominates). Generally, however, the dilute AF has finite $\Delta p$ (and therefore finite $\Omega_c$); then the DOS crosses over from $\Omega^2$ dependence to $\Omega^{\bar{d}-1}$ dependence at $\Omega_c$:

$$\rho(\Omega)=\begin{cases} A\left[\dfrac{\Omega}{\Omega_c}\right]^2 & \text{for } \Omega<\Omega_c, \\ A\left[\dfrac{\Omega}{\Omega_c}\right]^{\bar{d}-1} & \text{for } \Omega>\Omega_c . \end{cases} \tag{22}$$

Here we have assumed continuity of the DOS at $\Omega_c$, as the effective-medium approximation (EMA) calculation[20] and large-scale computer simulation[15] (LSCS) predict. The condition that the total number of modes be equal to that of the completely Euclidean case leads us to

$$\Omega_c=\Omega_m\left[\frac{3}{\bar{d}}\Delta p^{-\nu D}-\left[\frac{3}{\bar{d}}-1\right]\right]^{-1/3} . \tag{23}$$

Here $\Omega_m=2ZJS$ is the maximum magnon energy in the undoped regime. Then $\Omega_f$ is determined by Eq. (20). The prefactor of the DOS (i.e., the DOS at $\Omega_c$), on the other hand, is obtained by using the normalization $\int_0^{\Omega_f}\rho(\Omega)d\Omega=1$; it is

$$A=\frac{1}{\Omega_c[\frac{1}{3}+\frac{3}{4}(\Delta p^{-\nu D}-1)]} . \tag{24}$$

At this stage, calculation of the exciton intersublattice relaxation rate is possible for dilute AF's with different magnetic-ion concentrations. We shall write only the expression for $\gamma^{(1)}$ here:

$$\gamma^{(1)}/\pi = A^2 \int_0^{\Omega_c} n(\Omega)[n(\Omega)+1]\left[1+\left[\frac{\Omega_m}{\Omega}\right]^2\right]\left[\frac{\Omega}{\Omega_c}\right]^4 d\Omega$$

$$+A^2 \int_{\Omega_c}^{\Omega_f} n(\Omega)[n(\Omega)+1]\left[1+\frac{Z}{2}\left[\frac{\Omega_f}{\Omega}\right]^{2\bar{d}/D}\right]\left[\frac{\Omega}{\Omega_c}\right]^{2(\bar{d}-1)} d\Omega . \tag{25}$$

Roughly speaking, the rate $\Gamma_1$ can be regarded as a weighted sum of a $T^3$ term and a $T^2$ term, and the weight is strongly $p$ dependent. For clarity the calculated results will not be presented here; we simply point out that this approach provides a more adequate description of the exciton intersublattice relaxation process in dilute AF's and allows one to compare quantitatively with future experiments.

It is still necessary to elucidate the possibility of observing the quadratic temperature dependence since very-long-lifetime excitons usually become trapped on magnetic ions that are perturbed by neighboring impurity ions, and trapped excitons do not have the same temperature dependence of intersublattice transfer as that of free excitons. Recent experiments by Hollman, Arts, and de Wijn[13] have shown that the trapping effect appears at low temperatures (below 10 K in their measurement) only. Thermal activation by magnons (fractons) or phonons at higher temperatures will cause trapped excitons to scatter back to the free-exciton level, thereby decreasing the effectiveness of the trap and recovering the temperature dependence of the free-exciton intersublattice relaxation rate.

## IV. DISCUSSIONS

Despite much effort devoted to the study of fracton dynamics since the discussion by Alexander and Orbach,[14] some points remain to be clarified. The spectral dimensionality $\bar{d}$, for example, has been conjectured[14] to be $\frac{4}{3}$ for percolating networks in all dimensions $d \geq 2$ and the scaling ansatz[21] supports this argument; while LSCS suggests[15] $\bar{d} \approx 0.97$ in percolating AF's and detailed EMA calculation[20] yields $\bar{d} = \frac{2}{3}$ in randomly dilute AF's. No consensus has been achieved.[22] On the other hand, the exact shape of the DOS in the vicinity of $\Omega_c$ is still unknown. It has been suggested[23] that, because of an extended crossover region where both phonon (magnon) and fracton excitation exist, there should be a "bump" at the crossover frequency $\Omega_c$. The scaling analysis of Aharony *et al.*[21] also suggests that the missing low-frequency states must be recovered at the crossover region and therefore an excess of states may be expected in this region. An EMA calculation[20] of the spin-wave spectrum in a percolating AF, however, shows that a smooth transition in the DOS occurs at $\Omega_c$. A LSCS by Nakayama, Yakubo, and Orbach[15] does not support the suggestion of a bump in the DOS near $\Omega_c$ either.

In the direction of experimental study, there are a great many publications[24] studying the phonon-fraction crossover, which is in sharp contrast to the magnon-fracton crossover, where only a few works have been published. Among them, Uemura and Birgeneau[25] reported the observation of fractons in Zn-diluted $MnF_2$ using the neutron-scattering method. Salamon and Yeshurun[26] fitted the data of magnetization in a series of dilute amorphous magnets to the magnon-fracton crossover regime and showed that the deviations from the Bloch $T^{3/2}$ law are a result of the magnon-fracton crossover. But, to the present author's knowledge, there is not yet a detailed experiment on dilute magnets identifying clearly the fraction dimension and examining carefully the crossover region. This situation, in our opinion, can be partly attributed to the technical difficulties of these experiments. Considering this, we think that the study of exciton dynamics can be a somewhat convenient experiment whose interpretation should provide a fairly sensitive and detailed check of the character of the magnon-fracton crossover in dilute AF's. To some extent, the importance of these experiments is similar to the role the specific-heat measurement[27] of disordered materials played in the study of phonon-fracton crossover, since the measured quantities are both the integration of the DOS over the Bose-Einstein function. Specific-heat measurement has been used[27] to determine many parameters which are important in fracton dynamics, such as the crossover energy, spectral dimensionality, the component of fractons, the density of states over the whole energy range, etc. Exciton dynamics measurement can also provide much information about the magnon-fracton crossover and can be a way to study the fracton dynamics experimentally. Experiments aiming at this have not been reported to our knowledge and research in this way will be worthwhile.

It is noteworthy that the study of hot exciton-fracton bands is superior to that of the exciton intersublattice relaxation rate since the temperature dependence of the hot exciton band intensity is more sensitive to the details of the magnon-fracton crossover, and hot exciton bands can be expected in both dilute ferromagnets and antiferromagnets.

## V. CONCLUSIONS

In this paper we have studied the effect of fractons on the temperature dependence of the hot exciton band intensity in dilute magnets and the exciton intersublattice relaxation rate in dilute antiferromagnets. The intensity of hot exciton-fracton absorption bands in dilute magnets varies with temperature as $T^{\bar{d}+2\bar{d}/D}$. The fracton-exciton interaction leads to a quadratic temperature dependence of the exciton intersublattice relaxation rate, substantially different from that of the magnon-exciton interaction. Therefore the fractons could manifest themselves by influencing the exciton dynamics in dilute magnets. We demonstrated that the results imply that the measurement of the two quantities can be a method for the experimental study of fracton dynamics. Though the fracton effect on exciton dynamics is certainly not restricted to the above two quantities (other properties such as the line shape are also influenced by the presence of fractons), the above two are more direct and have important experimental implications. By choosing appropriate materials, one can perform a careful check on the characteristics of the magnon-fracton crossover through the measurement of the temperature dependence of the hot exciton band intensity in dilute magnets and the exciton intersublattice relaxation rate in dilute antiferromagnets.

## ACKNOWLEDGMENTS

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[22] We have calculated the rate using different (scaling, EMA, LSCS) values of $\bar{d}$. The results do not show significant deviation from $T^2$ dependence. This is also true for the DOS with or without a bump. However, for quantitative comparison with experiments, their effects must be taken into consideration.

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