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Atomic- and electron-beam spectroscopy of rarefied semiquantum media

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The quantum-mechanical collective effects accompanying the propagation of a low-energy atomic or electron beam through a rarefied medium are discussed. An unusual dissipation mechanism due to the inelastic scattering of a test particle with thermal collective modes is considered. The phenomenon in question comes into effect even in a thin target at distances less than the mean free path calculated for direct atomic collisions. This effect provides the possibility to study the collective many-body properties of the medium by means of traditional atomic (electron) spectroscopy. The differential cross sections for the inelastic scattering of a probe particle with fluctuations of density, macroscopic magnetization, and entropy in the target are expressed in terms of the corresponding structure functions and calculated both in the hydrodynamic and collisionless regimes. The spin-flip transitions as well as the nonflip scattering are considered in detail. Possible applications of the theory to semiquantum systems such as gaseous $H\downarrow$, $D\downarrow$ and dilute ^{3}He-^{4}He mixtures as well as some aspects of neutron optics and atomic (electron) spectroscopy are briefly discussed.

I. INTRODUCTION

The highly nontrivial collective properties of semiquantum media have attracted a great deal of attention and caused a lot of theoretical and experimental efforts throughout the world. A semiquantum medium possesses a high degree of "classicality" but at the same time displays collective effects of pure quantum-mechanical origin. Quite a big surprise was the fact that even the most "classical" matter in physics—a rarefied gas at rather high temperature, whose particles obey the classical Boltzmann-Maxwell statistics and are on average very far away from each other—under certain conditions exhibits nevertheless fundamentally quantum-mechanical macroscopic properties and collective phenomena.[1,2] An important point is that such a quantal behavior requires neither a quantum degeneracy nor high density. In the case of a rarefied gas of particles with a short-range interaction between them one only needs to sufficiently lower the temperature in order to create the conditions in question:

$$\epsilon_d \ll T \ll \hbar^2/mr_0^2\ , \quad Nr_0^3 \ll 1\ , \tag{1}$$

where $\epsilon_d \sim \hbar^2 N^{2/3}/m$ is the quantum degeneracy temperature, m the mass of a particle, r_0 the interaction range (of the order of an atomic size), and N the atomic density. This criterion may be rewritten in a form of the following hierarchy of the characteristic lengths in the system:

$$r_0 \ll \Lambda_T \ll N^{-1/3}\ , \tag{2}$$

where $\Lambda_T = \hbar/(mT)^{1/2}$ is the average de Broglie wavelength. Thus we can easily see that a gas, being still very far beyond quantum-statistical degeneracy, is at the same time under ultraquantal conditions because the de Broglie wavelength of a particle is much larger than the size of the particle itself. That is why in this limiting case one can expect a medium to possess quantum-mechanical properties.

One of the most exciting features of such a semiquantum medium is the appearance of long-range correlations and a great variety of associated collective effects.[1–3] In a classical gas all correlations fall off very rapidly as an exponential function, $\exp(-r/r_0)$, $r_0 \ll N^{-1/3}$. Therefore particles on average do not "know" of each other and can change their states only at the instant of collision. In a semiquantum gas, despite the fact that the density is also low and the bare interaction is also a short-range one, quantum mechanics comes into effect and provides an effective self-consistent field (similar to the one in the theory of a Fermi liquid) acting on each particle of the gas. As a result a rarefied semiquantum medium may possess a quite unusual spectrum of collective excitation like weakly damped spin waves, the existence of which was demonstrated both in theory and experiment.[4–8] An extremely important feature of the phenomenon in question is that an effective self-consistent field is created, even at distances smaller than the mean free path (but indeed larger than the average interparticle distance $N^{-1/3}$). This gives the possibility to experimentally observe the effect even in the case where direct elastic collisions between particles do not occur, e.g., when detecting spin waves in a polarized semiquantum gas in the Knudsen regime.[9]

The existence of this kind of molecular mean field, which depends on the macroscopic variables like density N and magnetization $\mathbf{M}$, provides a very efficient mechanism of inelastic interaction between a particle in the medium and collective fluctuations of thermodynamical

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quantities.[10] Microscopically such an interaction may be interpreted as the inelastic scattering of paramagnetic particles with thermally excited phonons (density fluctuations), spin modes (fluctuations of magnetization), and entropy fluctuations in a semiquantum gas. Under certain conditions the magnitude of the effect is not small at all, and the scattering mechanism under consideration may significantly affect the transport properties of a medium. For instance, the exchange interaction between an atom and fluctuations of macroscopic magnetization strongly influences the thermal conductivity and viscosity of a spin-polarized semiquantum gas.[11]

Thus an extra channel of inelastic scattering of particles, due to the quantum-mechanical collective properties, is one more typical feature pertaining to a semiquantum media. This fact provides us with the idea to use a cooled atomic beam as a tool to study the spectrum of collective excitations and correlation functions in a rarefied semiquantum system. The probing particles of a beam propagating through a semiquantum target will experience not only direct elastic collisions with the scattering centers of the target but also inelastic scattering with collective modes. It means that by letting a test beam pass through a semiquantum gas as a target and then measuring the intensity of scattered particles at different scattering angles one can infer complete information about the correlation properties and spectrum of collective excitations of the target (similar to what takes place in neutron optics). Moreover, as emphasized above, one can even choose a target in the collisionless regime, e.g., a target with a geometrical thickness less than the mean free path of a testing particle. In the latter case direct atomic collisions practically do not exhibit themselves at all, and everything which is measured pertains to inelastic scattering with collective fluctuations. So a cold atomic beam may be a very good instrument for direct collective mode spectroscopy in a semiquantum media.

The physical pattern of the effect is very simple. We know from optics that once a light wave enters a medium the speed of light changes due to the phenomenon of refraction. The refraction coefficient of a medium may fluctuate and then a light wave experiences scattering with these fluctuations. For the case in question we in fact consider quantum-mechanical refraction of the wave function of an incident particle propagating through a semiquantum medium. The wave function of a probe particle at distances less than the mean free path (for instance, in a thin enough target) is a plane wave. However, the phase of this plane wave is quite different from the one which describes the wave function (also a plane wave) of the same particle propagating in vacuum before it enters the target. Such a difference can be interpreted in terms of the effective refraction coefficient n_{eff} (Ref. 12) related to the ψ function of a probe particle propagating through a target.

In case criterion (1) is fulfilled the contribution of an interaction between the test particle and the medium to the real part of the effective refraction coefficient turns out to be much larger than the imaginary part of n_{eff}. In other words under the conditions in question when the temperature meets criterion (1) one can take into account the quantum-mechanical refraction of the wave function but neglect the dissipation due to the finite mean free path.[1,2] This actually means that the real correction to the self-energy of an incident particle, associated with the effect of quantum-mechanical refraction, considerably exceeds the attenuation (the finite lifetime of a single-particle excitation) due to scattering at distances of the order of the mean free path.

In optics fluctuating macroscopic characteristics cause fluctuations of the refraction coefficient. As a result a light scattering comes along, e.g., Rayleigh scattering from density fluctuations. A similar effect should certainly occur in the case in question. Fluctuations of the quantum-mechanical refraction coefficient n_{eff} can be expressed in terms of fluctuations of the macroscopic variables (the pressure or density, entropy or temperature, and magnetization) describing the thermodynamic state of a medium. Then the influence of the fluctuating refraction coefficient n_{eff} on a test particle reduces to the interaction between the particle and fluctuations of N, T, and $\mathbf{M}$. It is the effect that causes an extra inelastic scattering of particles of a testing beam with collective modes in a semiquantum target.

The typical rarefied semiquantum media which are studied most actively in experiment and theory are indeed spin-polarized atomic hydrogen H$\downarrow$ (Refs. 13–15) and deuterium D$\downarrow$.[16] In this case criterion (1) can be fulfilled very easily, and both systems exhibit a whole variety of macroscopic quantum phenomena. An important point is that the available experimental facilities allow the experiment to be run in the hydrodynamic regime[17,18,6] and in the collisionless Knudsen limit as well.[9] Atomic H$\downarrow$ in a magnetic trap[19] also provides an exciting opportunity to observe quantum-mechanical collective effects in the Knudsen regime. In some cases the developed theory can be directly applied to describe the ensemble of quasiparticles (not real atoms) in condensed media. The most typical example here is a quantum ^{3}He-^{4}He solution where the ^{3}He atoms dissolved in the superfluid background of ^{4}He form a semiquantum gas.[20]

Of course criterion (1) is supposed to be valid for the energy of a testing beam as well. However, it does not seem to be a serious obstacle to run an experiment. Experimentally focusing such a low-energy atomic hydrogen beam was carried out a few years ago.[21] An interesting point is that in experiments[21] a strong scattering of hydrogen atoms within the beam was detected under the conditions when no scattering at all should have occurred (the geometrical size of the beam was less than the corresponding mean free path). Perhaps it was experimental evidence of the interaction between the hydrogen atoms and collective modes in a semiquantum beam. Currently methods of laser cooling enable an experimentalist to cool the beam down to the temperatures $T \sim 0.2$–$0.4\ \mu$K,[22] which is significantly below the upper bound of inequality (1). An experimental setup to study a hydrogen atomic beam propagating through a semiquantum gaseous target of H$\downarrow$ is being built at Harvard (the authors are grateful to I. Silvera for keeping them informed of this project).

If the temperature is too high and criterion (1) is violated, the effectiveness of the phenomenon of quantum-mechanical refraction falls off and the magnitude of the effect in question decreases. However, if one considers a beam of light particles, say, of electrons, the upper temperature limit for the electron-particle interaction in the target will be extremely high, $(\hbar^2/m_e r_0^2) \sim 10^5$ K (here m_e is the bare electron mass, $m_e \ll m$). At such a high temperature all quantum-mechanical macroscopic effects in the target (which itself is not semiquantum anymore) will indeed be suppressed. Nevertheless, the interaction between the electron test beam and the target is still of the ultraquantum origin; i.e., the quantum-mechanical refraction of the electron beam in the target still takes place. Therefore the extra scattering of electrons with collective modes in the target comes into effect despite the fact that the collective fluctuations themselves are absolutely classical. This means that when passing through a medium, a low-energy electron beam is scattered not only by direct collisions with the atoms of the target but also by the interaction with sound waves, spin diffusion modes, and entropy fluctuations in the medium. Electron beams with low enough energy, $E_e \sim 0.1$–0.3 eV, are widely used in various experiments.[23]

In this paper inelastic scattering of slow paramagnetic particles with collective modes in a rarefied spin-polarized (an external magnetic field or dynamically induced polarization) medium is considered. The scattering process where spin of the probe particle is conserved and the spin-flip transition are discussed. The corresponding scattering probabilities considerably differ from each other both qualitatively and quantitatively. Spin polarization strongly affects the picture of scattering and provides a whole number of the additional correlation functions entering the cross sections of the processes under consideration. The differential cross sections of the inelastic scattering of a probe particle with the thermal fluctuations of density and magnetization in a medium are calculated both in the hydrodynamic limit and collisionless regime. Preliminary results were published in Ref. 24.

II. SCATTERING PROBABILITIES

In order to be specific let us consider a rarefied semi-quantum system of particles with spin 1/2 as a target. The spin of the test particle in an incident beam is also assumed to be equal to 1/2. In this case the Hamiltonian of the inelastic interaction between a particle and a macroscopic fluctuation field in the target takes the form[10,11,25,26]

$$\mathcal{H}(\mathbf{r},t) = g_1 N(\mathbf{r},t) + \frac{1}{\beta} g_2 \boldsymbol{\sigma}\cdot\mathbf{M}(\mathbf{r},t) - \beta_0 \boldsymbol{\sigma}\cdot\mathbf{B}(\mathbf{r},t), \tag{3}$$

where $N(\mathbf{r},t)$, $\mathbf{M}(\mathbf{r},t)$, and $\mathbf{B}(\mathbf{r},t)$ are the fluctuating macroscopic variables, namely, the atomic density, magnetization, and magnetic field; $\boldsymbol{\sigma}$ are the Pauli matrices; and β and β_0 are the magnetic moments of a particle of the target and of a testing particle, respectively. The two first terms in Eq. (3) are of purely exchange origin. The latter Zeemann term, of course, contributes essentially less than the exchange ones but it results in a very typical anisotropy of the differential cross section, which can be observed in experiment. The coupling constants g_1 and g_2 can be expressed in terms of the scattering amplitudes. If both a beam and a target consist of identical particles, the quantities g_1 and g_2 have the simple form

$$g_1 = -g_2 = \frac{2\pi\hbar^2}{m} a, \tag{4}$$

where a is the s-wave scattering length. If particles of a beam and of a target are distinguishable, the coupling constants may be calculated as

$$g_1 = \frac{\pi\hbar^2}{M}(3a_+ + a_-)\ , \quad g_2 = \frac{\pi\hbar^2}{M}(a_+ - a_-). \tag{5}$$

Here M is the reduced mass and a_+ and a_- are the s-wave scattering lengths for triplet and singlet scattering, respectively.

The fluctuating magnetic moment $\mathbf{M}(\mathbf{r},t)$ induces the fluctuations of a magnetic field $\mathbf{B}(\mathbf{r},t)$ which are determined by the Maxwell equations in the magnetostatic limit:

$$\mathrm{curl}\,\mathbf{h} = 0\ , \quad \mathrm{div}\,\mathbf{B} = 0\ , \quad \mathbf{B} = \mathbf{h} + 4\pi\mathbf{M}. \tag{6}$$

The solution of Eqs. (6) can be easily found by taking the Fourier transforms

$$B_i(\mathbf{k},\omega) = 4\pi\left[M_i(\mathbf{k},\omega) - \frac{k_i k_l}{k^2} M_l(\mathbf{k},\omega)\right] \tag{7}$$

(summation over dummy indices is used here and throughout). The z axis is chosen along the vector of the spin polarization of the target. In the case of equilibrium in an external magnetic field $\mathbf{H}$, the direction of the z axis indeed coincides with $\mathbf{H}$.

The probability of inelastic scattering of a testing particle for the transition between an initial state $|\mathbf{p},\gamma\rangle$ and a final state $\langle\mathbf{p}\,',\delta|$ is given by the well-known formula of quantum mechanics[12]

$$dw = \frac{1}{\hbar^2}\left|\int_{-\infty}^{\infty}\langle\mathbf{p}\,',\delta|\mathcal{H}|\mathbf{p},\gamma\rangle e^{-i\omega t}dt\right|^2 \frac{d^3p'}{(2\pi\hbar)^3}. \tag{8}$$

Here the Greek indices numerate spin states of a particle (the z projection of spin) and the transition frequency is given by the usual relationship

$$\hbar\omega = \frac{p^2 - p'^2}{2m_0} - \beta_0 H(\sigma^z_{\gamma\gamma} - \sigma^z_{\delta\delta}), \tag{9}$$

where m_0 is the mass of a scattered particle of a beam. Inasmuch as we are interested in inelastic scattering of particles with the thermal fluctuations, expression (8) should also be averaged over fluctuations. After substituting the Hamiltonian, Eq. (3), into Eq. (8) and subsequently averaging, the scattering probability (8) will contain all possible correlation functions of the form

$$\langle A(\mathbf{r}_1\ ,\ t_1)B(\mathbf{r}_2\ ,\ t_2)\rangle \equiv \overline{(AB)}_{\mathbf{r}\ ,\ t}\ , \tag{10}$$

where A and B denote any of the macroscopic variables N, M_i, and B_i ($i = x, y, z$). All pair correlation functions depend in fact only on $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $t = t_1 - t_2$ because of the isotropy of the medium. When calculating the matrix elements in Eq. (8) with plane waves as the wave functions of the initial and final states one can express the scattering probablity in terms of the corresponding dynamic structure factors, i.e., in terms of the Fourier components

$$\overline{(AB)}_{\mathbf{k}\,,\,\omega} = \int \overline{(AB)}_{\mathbf{r}\,,\,t} e^{i\omega t - i\mathbf{k}\cdot\mathbf{r}} dt d\mathbf{r} \;, \tag{11}$$

rather than of the time-coordinate correlators. The specific form of the cross section describing the inelastic scattering of a test particle depends significantly on the spin structure of the matrix elements in Eq. (8). Therefore we will consider the nonflip scattering and spin-flip transition separately.

In order to obtain the differential cross section per one atom of the target one has to normalize the initial wave function of an incident particle to a unit flux density and to divide the whole expression over N. Assuming first that the spin state of the probe particle does not change in the scattering process, $\gamma = \delta \equiv \nu$, and following the standard procedure one can easily find from Eq. (8) that

$$\begin{aligned} d\sigma_{\nu\nu} = \frac{1}{\hbar^2}\frac{m_0}{pN}\frac{d^3p'}{(2\pi\hbar)^3}\Bigg\{ & g_1^2\overline{(\delta N\,\delta N)}_{\mathbf{q},\omega} + \frac{g_2^2}{\beta^2}\overline{(\delta M_z\,\delta M_z)}_{\mathbf{q},\omega} + \beta_0^2\overline{(\delta B_z\,\delta B_z)}_{\mathbf{q},\omega} \\ & -g_2\frac{\beta_0}{\beta}\left[\overline{(\delta B_z\,\delta M_z)}_{\mathbf{q},\omega} + \overline{(\delta M_z\,\delta B_z)}_{\mathbf{q},\omega}\right] \\ & +g_1 g_2\frac{1}{\beta}\sigma^z_{\nu\nu}\left[\overline{(\delta N\,\delta M_z)}_{\mathbf{q},\omega} + \overline{(\delta M_z\,\delta N)}_{\mathbf{q},\omega}\right] - g_1\beta_0\sigma^z_{\nu\nu}\left[\overline{(\delta N\,\delta B_z)}_{\mathbf{q},\omega} + \overline{(\delta B_z\,\delta N)}_{\mathbf{q},\omega}\right]\Bigg\}, \end{aligned} \tag{12}$$

where $\hbar\mathbf{q} = \mathbf{p} - \mathbf{p}'$. The last two terms in Eq. (12) containing $\sigma^z_{\nu\nu}$ explicitly show that the magnitude of the cross section $d\sigma_{\nu\nu}$ depends essentially on whether the spin of an incident particle is parallel to the vector of polarization in the target or opposite to the magnetization. In fact these terms determine the difference between the quantities $\sigma_{\uparrow\uparrow}$ and $\sigma_{\downarrow\downarrow}$.

It is convenient to express all correlators in Eq. (12) in terms of fluctuations of the thermodynamic parameters δM_i and δN. To simplify the final formulas let us introduce the following notations:

$$\begin{gathered} \overline{(\delta M_i \delta M_k)}_{\mathbf{q},\omega} = S_{ik}(\mathbf{q},\omega)\;,\quad \overline{(\delta N \delta N)}_{\mathbf{q},\omega} = G_{00}(\mathbf{q},\omega)\;, \\ \overline{(\delta M_i \delta N)}_{\mathbf{q},\omega} = G_{i0}(\mathbf{q},\omega)\;,\quad \overline{(\delta N \delta M_i)}_{\mathbf{q},\omega} = G_{0i}(\mathbf{q},\omega). \end{gathered} \tag{13}$$

One can easily convince oneself that in the case under consideration the components of dynamic structure function $S_{ik}(\mathbf{q},\omega)$ meet the following criteria:

$$\begin{gathered} S_{xy}(\mathbf{q},\omega) = -S_{yx}(\mathbf{q},\omega)\;,\quad S_{xx}(\mathbf{q},\omega) = S_{yy}(\mathbf{q},\omega)\;, \\ S_{xz} = S_{zx} = S_{yz} = S_{zy} = 0. \end{gathered} \tag{14}$$

Using Eqs. (7) and (14) and the well-known relation of the theory of fluctuations,

$$A(\mathbf{q}\,,\,\omega)B(-\mathbf{q}\,,\,-\omega) = (2\pi)^4\overline{(AB)}_{\mathbf{q},\omega}\;, \tag{15}$$

after simple algebraic transformations we obtain

$$\begin{aligned} d\sigma_{\nu\nu} = \frac{1}{2\pi^2\hbar^5}\frac{m_0^2}{N}\frac{p'}{p}\Bigg\{ & g_1^2 G_{00}(\mathbf{q},\omega) + (4\pi\beta_0)^2\frac{\sin^2 2\phi}{4}S_{xx}(\mathbf{q},\omega) + \left(\frac{g_2}{\beta} - 4\pi\beta_0\sin^2\phi\right)^2 S_{zz}(\mathbf{q},\omega) \\ & +g_1\sigma^z_{\nu\nu}\left(\frac{g_2}{\beta} - 4\pi\beta_0\sin^2\phi\right)\left[G_{0z}(\mathbf{q},\omega) + G_{z0}(\mathbf{q},\omega)\right]\Bigg\} d\epsilon'\frac{do'}{4\pi}, \end{aligned} \tag{16}$$

where $d\epsilon' = p'dp'/m_0$ and ϕ is the angle between the transferred momentum $\hbar\mathbf{q} = \mathbf{p} - \mathbf{p}'$ and the vector of spin polarization so that $q_z = q\cos\phi$.

Expressions (13), (16) yield the cross section of the process whereby the spin of an incident particle is conserved. Assuming that the spin state of a scattered test particle changes—i.e., the spin indices γ and δ in matrix element (8) are not the same any more—$\gamma \neq \delta$, one can obtain the cross section of the spin-flip transition. Similar calculations give rise to the result

$$\begin{aligned} d\sigma_{\mu\nu} = \frac{1}{2\pi^2\hbar^5}\frac{m_0^2}{N}\frac{p'}{p}\Bigg\{ & \left[\left(\frac{g_2}{\beta} - 4\pi\beta_0\right)^2 + \left(\frac{g_2}{\beta} - 4\pi\beta_0\cos^2\phi\right)^2\right] S_{xx}(\mathbf{q},\omega) \\ & -2\sigma^y_{\mu\nu}\left(\frac{g_2}{\beta} - 4\pi\beta_0\right)\left(\frac{g_2}{\beta} - 4\pi\beta_0\cos^2\phi\right) S_{yx}(\mathbf{q},\omega) + (4\pi\beta_0)^2\frac{\sin^2 2\phi}{4}S_{zz}(\mathbf{q},\omega)\Bigg\} d\epsilon'\frac{do'}{4\pi}\;, \end{aligned} \tag{17}$$

where $\mu \neq \nu$. Thus the problem of quantitatively calculating the differential cross section $d\sigma_{\gamma\delta}$ reduces to the problem of finding the appropriate dynamic structure factors G_{00}, G_{0z}, G_{z0}, and S_{ik}, $i,k = x,y,z$. The method to run such a calculation and final results depends considerably on which regime, hydrodynamic or collisionless, takes place in the target under the given scattering conditions.

III. STRUCTURE FUNCTIONS: HYDRODYNAMIC REGIME

The hydrodynamic limit of a small momentum transfer, $ql \ll 1$, where l is the mean free path, is indeed the most interesting case. Normally the criterion $\omega\tau \ll 1$ where $\tau \sim (N\bar{v}r_0^2)^{-1}$ is the "gas-kinetic" relaxation time, is fulfilled as well for the characteristic frequencies at which the intensity of the inelastic scattering in question is particularly high. In this situation one deals with purely thermodynamic fluctuations of the macroscopic variables M_i and N and all correlation functions can be obtained by means of the hydrodynamic equations only.

While we restrict ourselves to considering the fluctuations of N and M_z only, we may describe the target as sort of a two-component mixture of particles with spins up (along the vector of polarization) and down (opposite to the direction of magnetization). Strictly speaking such a description is valid only for frequencies $\omega\tau_d \gg 1$ where τ_d is the dipole-dipole magnetic relaxation time. Under these conditions the probability of a spin-flip transition due to the weak relativistic interaction is negligibly small and may be ignored. In most cases, particularly when fluctuations of the nuclear magnetization are considered, the relaxation time τ_d is very large and therefore there are no serious restrictions on using such an approach.

In addition to the usual thermodynamic variables the hydrodynamic equations of a two-component solution also contain the concentration of the mixture.[27] In the case under consideration instead of the concentration it is more natural and convenient to introduce the degree of polarization, α, defined as follows:

$$N_+ - N_- = N\alpha \ , \quad N_+ + N_- = N \ , \tag{18}$$

where N_+ and N_- are the numbers of particles per unit volume in a target with spin (1/2) and (−1/2), respectively. The macroscopic magnetization M_z can obviously be calculated as $M_z = \beta N\alpha$. Using these variables one can present the linearized equations of hydrodynamics[27] in the form

$$\frac{\partial\alpha}{\partial t} = D\Delta\alpha + \frac{2k_T}{T}\Delta T + \frac{2k_P}{P}\Delta P \ ,$$

$$\frac{\partial T}{\partial t} + \frac{T}{c_P}\left(\frac{\partial S}{\partial P}\right)_{T,\alpha}\frac{\partial P}{\partial t} - \frac{k_T}{c_P}\left(\frac{\partial\mu}{\partial\alpha}\right)_{P,T}\frac{\partial\alpha}{\partial t} = \lambda\Delta T \ , \tag{19}$$

$$\frac{\partial\rho}{\partial t} + \rho\,\mathrm{div}\,\mathbf{v} = 0 \ ,$$

$$\rho\frac{\partial\mathbf{v}}{\partial t} = -\nabla P + \left(\zeta + \frac{4\eta}{3}\right)\nabla\,\mathrm{div}\,\mathbf{v} \ ,$$

where the pressure P, entropy S, and density ρ (all quantities are normalized to the unit mass) are introduced; $\mathbf{v}$ is the macroscopic velocity; η and ζ are the transport coefficients of the first and the second viscosity respectively; λ is the thermometric conductivity; and D is the spin diffusion coefficient. The quantities k_T and k_P being in fact similar to the thermo- and barodiffusion coefficients in a two-component mixture, play the role of the thermomagnetic and baromagnetic diffusion coefficients, respectively. The notation c_P in Eqs. (19) is used for the specific heat of the target. The chemical potential μ, the physical meaning of which is given by the thermodynamic identity for the energy per unit mass,

$$dE = TdS + \frac{P}{\rho^2}d\rho + \frac{1}{2}\mu d\alpha \ , \tag{20}$$

can be easily expressed in terms of the chemical potentials of different spin components:

$$\mu = \frac{1}{m}(\mu_+ - \mu_-). \tag{21}$$

In the hydrodynamic regime the correlation functions of fluctuations, $\langle A(\mathbf{r},t)B(0,0)\rangle$, for any macroscopic variables $A,B = \alpha,T,\rho,\mathbf{v}$ or $A,B = \alpha,S,P,\mathbf{v}$ are determined by the "equations of motion" for $A(\mathbf{r},t)$.[28] The simultaneous correlation functions

$$\langle A(\mathbf{r},t)B(0,0)\rangle|_{t=+0} = \langle A(\mathbf{r})B(0)\rangle \tag{22}$$

are taken as the initial conditions for the hydrodynamic equations. Let us apply this method in order to calculate the sought-after structure functions. To do that it is convenient to solve Eqs. (19) with respect to the Fourier transformants of the correlation functions from the very beginning. It is also helpful to carry out a slightly modified Fourier transformation

$$\langle AB\rangle^+_{\mathbf{q},\omega} = \int_0^\infty dt\int_{-\infty}^\infty e^{i\omega t - i\mathbf{q}\cdot\mathbf{r}}\langle A(\mathbf{r},t)B(0,0)\rangle. \tag{23}$$

This allows us to explicitly take into account the initial conditions (22) at the first stage of the calculations. Finally the dynamic structure factors can be expressed in terms of functions (23) as follows:

$$\langle AB\rangle_{\mathbf{q},\omega} = \langle AB\rangle^+_{\mathbf{q},\omega} + \langle AB\rangle^+_{\mathbf{q},-\omega}. \tag{24}$$

There exists one more important circumstance which significantly facilitates the calculation of dynamic form factors. Inasmuch as fluctuations of pressure P and velocity $\mathbf{v}$ propagate through a fluid with the speed of sound, u, and perturbations of magnetization, α, and temperature T spread in accordance with the equations of thermal conductivity and spin diffusion, one can assume with very good accuracy that at frequencies $\omega \sim Dq^2 \sim \lambda q^2 \ll qu$ only isobaric fluctuations of α and T occur. (Here it is convenient to choose α, T, P as the hydrodynamic variables since the fluctuations of α are independent of the fluctuations of pressure and temperature, i.e., $\langle\delta T\delta\alpha\rangle = \langle\delta\alpha\delta P\rangle = 0$.) Then neglecting the fluctuations of pressure and using the first two equations from (19) as well as the transformation (23) and initial conditions (22) we get

$$(Dq^2 - \imath\omega)\overline{(\delta\alpha\,\delta\alpha)}^{+}_{\mathbf{q},\omega} + \frac{2k_T}{T}q^2\overline{(\delta T\,\delta\alpha)}^{+}_{\mathbf{q},\omega} = \overline{(\delta\alpha\,\delta\alpha)}_{\mathbf{q}}\,,$$
$$\imath\omega\frac{k_T}{c_P}\frac{\partial\mu}{\partial\alpha}\overline{(\delta\alpha\,\delta\alpha)}^{+}_{\mathbf{q},\omega} + (\lambda q^2 - \imath\omega)\overline{(\delta T\,\delta\alpha)}^{+}_{\mathbf{q},\omega} = -\frac{k_T}{c_P}\frac{\partial\mu}{\partial\alpha}\overline{(\delta\alpha\,\delta\alpha)}_{\mathbf{q}}\,, \tag{25}$$

Substituting the obvious initial conditions

$$\langle\delta\alpha(\mathbf{r}_1)\delta\alpha(\mathbf{r}_2)\rangle = \frac{(1-\alpha)T}{N(\partial\mu_+/\partial\alpha)_{P,T}}\delta(\mathbf{r}_1-\mathbf{r}_2) \equiv \Gamma_\alpha\delta(\mathbf{r}_1-\mathbf{r}_2) \tag{26}$$

in Eqs. (25) and using relationship (24) we obtain

$$\overline{(\delta\alpha\,\delta\alpha)}_{\mathbf{q},\omega} = \frac{2\Gamma_\alpha}{\Delta(\mathbf{q},\omega)}Dq^2[\omega^2 + \lambda q^4(\lambda+\gamma)]\,,$$
$$\Delta(\mathbf{q},\omega) = (\omega^2 - D\lambda q^4)^2 + \omega^2 q^4(D+\lambda+\gamma)^2\,, \tag{27}$$

where the parameter γ is defined as follows:

$$\gamma = \frac{2k_T^2}{c_P T}\left(\frac{\partial\mu}{\partial\alpha}\right)_{P,T}. \tag{28}$$

Starting again with the first two equations of (19) and following a similar procedure one can find the equations for the correlation functions of the temperature fluctuations, which are valid at $\omega \sim \lambda q^2 \sim Dq^2$:

$$\frac{2k_T}{T}q^2\overline{(\delta T\,\delta T)}^{+}_{\mathbf{q},\omega} + (Dq^2 - \imath\omega)\overline{(\delta\alpha\,\delta T)}^{+}_{\mathbf{q},\omega} = 0\,,$$
$$(\lambda q^2 - \imath\omega)\overline{(\delta T\,\delta T)}^{+}_{\mathbf{q},\omega} + \imath\omega\frac{k_T}{c_P}\frac{\partial\mu}{\partial\alpha}\overline{(\delta\alpha\,\delta T)}^{+}_{\mathbf{q},\omega} = \overline{(\delta T\,\delta T)}_{\mathbf{q}}. \tag{29}$$

Taking account of the initial conditions

$$\langle\delta T(\mathbf{r}_1)\delta T(\mathbf{r}_2)\rangle = \frac{T^2}{\rho c_V}\delta(\mathbf{r}_1-\mathbf{r}_2)\,, \tag{30}$$

we immediately obtain

$$\overline{(\delta T\,\delta T)}_{\mathbf{q},\omega} = \frac{2T^2}{\rho c_V\Delta(\mathbf{q},\omega)}[\lambda q^2(\omega^2 + D^2q^4) + \gamma\omega^2q^2]\,. \tag{31}$$

Equations (25)–(30) together with relationship (24) also yield

$$\overline{(\delta\alpha\,\delta T)}_{\mathbf{q},\omega} + \overline{(\delta T\,\delta\alpha)}_{\mathbf{q},\omega} = \frac{2k_Tq^2}{\Delta}(\omega^2 - D\lambda q^4) \times\left(\frac{2k_T}{\rho c_V} + \frac{D}{c_P}\frac{\partial\mu}{\partial\alpha}\Gamma_\alpha\right). \tag{32}$$

Expressions (26), (30), and (32) provide us with all the necessary information to find the form factors G_{00}, S_{zz}, G_{0z}, and G_{z0} determining the cross section $d\sigma_{\nu\nu}$ from Eq. (16):

$$G_{00}(\mathbf{q},\omega) = \left(\frac{\partial N}{\partial\alpha}\right)^2_{P,T}\overline{(\delta\alpha\delta\alpha)}_{\mathbf{q},\omega} + \left(\frac{\partial N}{\partial T}\right)^2_{P,\alpha}\overline{(\delta T\delta T)}_{\mathbf{q},\omega} + \left(\frac{\partial N}{\partial\alpha}\right)_{P,T}\left(\frac{\partial N}{\partial T}\right)_{P,\alpha}\left[\overline{(\delta\alpha\delta T)}_{\mathbf{q},\omega} + \overline{(\delta T\delta\alpha)}_{\mathbf{q},\omega}\right]\,,$$
$$G_{0z}(\mathbf{q},\omega) + G_{z0}(\mathbf{q},\omega) = 2\beta N\left(\frac{\partial N}{\partial\alpha}\right)_{P,T}\overline{(\delta\alpha\delta\alpha)}_{\mathbf{q},\omega} + \beta N\left(\frac{\partial N}{\partial T}\right)_{P,\alpha}\left[\overline{(\delta T\delta\alpha)}_{\mathbf{q},\omega} + \overline{(\delta\alpha\delta T)}_{\mathbf{q},\omega}\right]\,,$$
$$S_{zz}(\mathbf{q},\omega) = \beta^2N^2\overline{(\delta\alpha\delta\alpha)}_{\mathbf{q},\omega}. \tag{33}$$

When deriving Eqs. (33) we took into account that the interaction Hamiltonian in the form of Eq. (3) as well as hydrodynamic equations (19) imply that fluctuations of the longitudinal magnetization M_z are due to perturbations of the degree of polarization, α, rather than of the density N.

Formulas (26) and (31)–(33) can in fact be applied to any paramagnetic fluid. In the case of a rarefied gas at low enough temperatures (1) all transport coefficients, which are functions of the degree of polarization, temperature, and density, can be calculated exactly in terms of the two-particle scattering amplitude.[5,29,30] The thermodynamic derivatives can easily be found from the equation of state[31]

$$P = NT\left\{1 + \frac{1}{(18\pi)^{1/2}}\left(\frac{\epsilon_d}{T}\right)^{3/2}(1+\alpha^2) + \pi(Na^3)(1-\alpha^2)\left(\frac{\hbar^2}{ma^2T}+1\right)\right\}. \tag{34}$$

In the leading approximation of a perfect gas where $P \approx NT$ the structure functions G_{0z} and G_{z0} vanish, and the form factor G_{00} reduces to the correlation function of temperature fluctuations. The equations are also drastically simplified in the limiting cases $\alpha \to 0$ and $\alpha \to 1$. Inasmuch as the thermomagnetic coefficient k_T is proportional to $1-\alpha$, at high degrees of polarization, $\alpha \to 1$, the parameter γ becomes very small, $\gamma \to 0$, and Eqs. (26), (31), and (32) reduce to the ones typical for a one-component system (all particles with spin up):

$$\overline{(\delta\alpha\delta\alpha)}_{\mathbf{q},\omega} \approx \frac{T}{N}\left(\frac{\partial\alpha}{\partial\mu_+}\right)_{P,T}(1-\alpha)\frac{2Dq^2}{\omega^2+D^2q^4} ,$$
$$\overline{(\delta T\delta T)}_{\mathbf{q},\omega} \approx \frac{2T^2}{\rho c_V}\frac{\lambda q^2}{\omega^2+\lambda^2q^4} , \quad (35)$$
$$\overline{(\delta\alpha\delta T)}_{\mathbf{q},\omega} \approx \overline{(\delta T\delta\alpha)}_{\mathbf{q},\omega} \approx 0.$$

When $\alpha \ll 1$ the magnitude of γ is small as well, $\gamma \propto \alpha$. Therefore the structure functions $\overline{(\delta\alpha\delta\alpha)}_{\mathbf{q},\omega}$ and $\overline{(\delta T\delta T)}_{\mathbf{q},\omega}$ are still given by the first two equations of (35) whereas the crossing correlation function $\overline{(\delta\alpha\delta T)}_{\mathbf{q},\omega} + \overline{(\delta T\delta\alpha)}_{\mathbf{q},\omega}$ has a finite and not a small value at all.

At high frequencies, $\omega \sim qu \gg \lambda q^2 \sim Dq^2$ (where u is the velocity of sound), there appear two more scattering peaks due to fluctuations of pressure (and velocity). In this case it is convenient to choose P, S, and α as the hydrodynamic variables because fluctuations of pressure and entropy are statistically independent. In terms of these variables the second equation from (19) can be rewritten in the form

$$\frac{\partial S}{\partial t} = \frac{\kappa}{\rho T}\Delta T + \frac{1}{2T}\frac{\partial\alpha}{\partial t}F_\alpha ,$$
$$F_\alpha = 2k_T\left(\frac{\partial\mu}{\partial\alpha}\right)_{P,T} - T\left(\frac{\partial\mu}{\partial T}\right)_{P,\alpha} , \quad (36)$$

where $\kappa = \rho c_P\lambda$ is the thermal conductivity. The quantity ΔT in Eq. (36) as well as in Eq. (19) for $\partial\alpha/\partial t$ should be expressed through the chosen set of variables:

$$\Delta T = \frac{T}{c_P}\Delta S + \left(\frac{\partial T}{\partial P}\right)_{S,\alpha}\Delta P + \left(\frac{\partial T}{\partial\alpha}\right)_{S,P}\Delta\alpha. \quad (37)$$

Taking into account that $\lambda q^2 \sim Dq^2 \ll \omega$, which actually enables us to restrict ourselves to considering the fluctuations of pressure and velocity only, we can transform Eqs. (19) and (36)–(37) and obtain

$$\frac{1}{u^2}\frac{\partial P}{\partial t} + \left[C_\alpha - \frac{\rho\kappa}{T}\left(\frac{\partial T}{\partial P}\right)^2_{S,\alpha}\right]\Delta P + \rho\,\mathrm{div}\,\mathbf{v} = 0 , \quad (38)$$

where the quantities C_α and u are defined as follows:

$$C_\alpha = 2\left[\frac{k_T}{T}\left(\frac{\partial T}{\partial P}\right)_{S,\alpha} + \frac{k_P}{P}\right] \times\left[\left(\frac{\partial\rho}{\partial\alpha}\right)_{P,S} - \rho^2\left(\frac{\partial T}{\partial P}\right)_{S,\alpha}\frac{F_\alpha}{2T}\right] ,$$
$$u^2 = \left(\frac{\partial P}{\partial\rho}\right)_{S,\alpha}. \quad (39)$$

In order to obtain a complete system of equations one should also add the last of the hydrodynamic equations (19) to Eq. (38). Comparing Eq. (38) with the analogous one for fluctuations of pressure P and velocity $\mathbf{v}$ in a one-component fluid[28] one can easily see that the appropriate structure function is described by similar expressions containing, however, the renormalized sound absorption, δ:

$$\overline{(\delta P\delta P)}_{\mathbf{q},\omega} = \frac{\rho T u^3\delta}{(\omega\mp qu)^2+u^2\delta^2} , \quad |\omega\mp qu| \sim u\delta,$$
$$\delta = \frac{q^2}{2\rho u}\left[\zeta + \frac{4\eta}{3} + \frac{\kappa\rho^2u^2}{T}\left(\frac{\partial T}{\partial P}\right)_{S,\alpha} - \rho u^2C_\alpha\right]. \quad (40)$$

Then the main contribution to the cross section $d\sigma_{\nu\nu}$ at frequencies $\omega \sim qu$ is determined with good accuracy by the density-density structure function

$$G_{00}(\mathbf{q},\omega) = \frac{1}{m^2u^4}\overline{(\delta P\delta P)}_{\mathbf{q},\omega}. \quad (41)$$

Let us emphasize once again that in this paper we do not consider any processes related to the slow dipole relaxation of the longitudinal magnetization or connected with the "viscous" renormalization of the hydrodynamic coefficients. The latter comes into effect when the second spatial derivatives of velocity $\mathbf{v}$ are taken into account in the hydrodynamic fluxes[32] and can be important in the case of restricted geometry.

The inelastic scattering of paramagnetic particles of a test beam with collective fluctuations of transverse magnetization is described in terms of the magnetic form factors $S_{ik}(\mathbf{q},\omega)$, $i,k = x,y$, in Eqs. (16), (17) for $d\sigma_{\mu\nu}$. It is this kind of scattering process that contributes the most significantly to the cross section of the spin-flip transition. Scattering on transverse spin modes results in additional peaks in the cross section $d\sigma_{\nu\nu}$ of the nonflip transition as well. Those peaks in $d\sigma_{\nu\nu}$ are indeed not the most prominent as they result from a weak relativistic interaction, but they certainly can be observed in experiment. The structure functions $S_{ik}(\mathbf{q},\omega)$ describing the processes in question can be calculated in the framework of the quasihydrodynamic approach for the transverse magnetization[33,5] in a way similar to the one considered above. On the other hand the same results can be obtained directly from the Boltzmann transport equation with the aid of the small-gradient expansion. The corresponding calculations were performed in Refs. 31, 1, and 10 and the results can be presented in the form

$$S_{xx}(\mathbf{q},\omega) = S_{yy}(\mathbf{q},\omega) = \frac{2\beta^2N\alpha}{1-e^{-\hbar\omega/T}}\left[L(\omega-\omega'_q,\omega''_q) - L(\omega+\omega'_{-q},\omega''_{-q})\right] ,$$
$$S_{yx}(\mathbf{q},\omega) = -S_{xy}(\mathbf{q},\omega) = -i\frac{2\beta^2N\alpha}{1-e^{-\hbar\omega/T}}\left[L(\omega-\omega'_q,\omega''_q) + L(\omega+\omega'_{-q},\omega''_{-q})\right] , \quad (42)$$
$$L(x,y) = \frac{y}{x^2+y^2} ,$$

where ω'_q and ω''_q are the spectrum and the attenuation of transverse spin fluctuations, respectively,[1,5]

$$\omega'_q = \Omega_H + bq^2 , \quad \omega''_q = \frac{b}{\gamma_{\rm int}} q^2 , \quad \Omega_H = \frac{2\beta H}{\hbar} ,$$
$$b = D_0 \frac{\gamma_{\rm int}}{1+\gamma_{\rm int}^2} , \quad \gamma_{\rm int} = 4\pi\hbar \frac{aN\alpha}{T} D_0. \qquad (43)$$

Here D_0 is the spin diffusion coefficient in the unpolarized target, i.e., when $\alpha = 0$.

Expressions (27), (31)–(33), and (40)–(42) obtained above completely determine the inelastic scattering of slow paramagnetic particles with collective modes in the hydrodynamic regime $ql \ll 1$. If the de Broglie wavelength Λ_T of a test particle is comparable or larger than the mean free path l (for instance, when the energy of a test particle is considerably lower than the temperature of the target or when $m_0 \ll m$), then the hydrodynamic regime applies even for scattering at large angles. At higher beam energies, when $\Lambda_T \ll l$, all the results obtained above are in fact valid only for small-angle scattering.

IV. STRUCTURE FUNCTIONS: COLLISIONLESS LIMIT

Under certain conditions the equations of hydrodynamics are no longer valid to describe correlation properties of a semiquantum gas. This may occur, for instance, in the case of a large momentum transfer, $ql \gg 1$, or in the Knudsen collisionless regime when the mean free path considerably exceeds the geometric size of a cell. The latter situation takes place in the experiment with spin-polarized atomic hydrogen H$\downarrow$ captured in a magnetic trap.[19] In the high-frequency limit $\omega\tau \gg 1$, the starting point of all calculations is the collisionless transport equation for the density matrix $\hat{n}$ (distribution function), which can be formulated in the common form

$$\frac{\partial \hat{n}}{\partial t} + \frac{1}{2}\left[\frac{\partial \hat{\epsilon}}{\partial \mathbf{p}}\nabla \hat{n} + \nabla \hat{n}\frac{\partial \hat{\epsilon}}{\partial \mathbf{p}}\right] - \frac{1}{2}\left[\frac{\partial \hat{n}}{\partial \mathbf{p}}\nabla \hat{\epsilon} + \nabla \hat{\epsilon}\frac{\partial \hat{n}}{\partial \mathbf{p}}\right] + \frac{\imath}{\hbar}[\hat{\epsilon} , \hat{n}] = 0, \qquad (44)$$

where the self-consistent energy of a particle, $\hat{\epsilon}$, may be expressed as follows:[1,31,34]

$$\hat{\epsilon} = \frac{p^2}{2m} - \beta \boldsymbol{\sigma}\cdot\mathbf{H} + \sum_{\mathbf{p},\sigma} \hat{f}\cdot\hat{n} ,$$
$$\hat{f} = g(1-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}') , \quad g = 2\pi\hbar^2 a/m . \qquad (45)$$

Here a is the s-wave scattering length. The equilibrium value of the density matrix $\hat{n}_0$ has a traditional form

$$\hat{n}_0 = \frac{n_+ + n_-}{2} + \frac{n_+ - n_-}{2}\boldsymbol{\sigma}\cdot\boldsymbol{l}_0 = \eta + \xi\boldsymbol{\sigma}\cdot\boldsymbol{l}_0, \qquad (46)$$

where $\mathbf{l}_0$ is the unit vector in the direction of spin polarization, and n_+ and n_- are the occupation numbers of the states with spin up and spin down, respectively.

In order to find the sought-after correlation functions one should first calculate the linear response of the system when some periodic external field is applied. Let the effective external field have the form

$$\delta\hat{\epsilon} = U(\mathbf{r},t) - \beta\boldsymbol{\sigma}\cdot\mathbf{h}(\mathbf{r},t) , \quad U, \mathbf{h} \propto \exp(\imath\mathbf{q}\cdot\mathbf{r} - \imath\omega t). \qquad (47)$$

Small perturbations of density matrix $\hat{n}$ caused by the external field will also be sought in a similar form

$$\delta\hat{n} = \nu_{\mathbf{p}}(\mathbf{r},t) + \boldsymbol{\lambda}_{\mathbf{p}}(\mathbf{r},t)\boldsymbol{\sigma} , \quad \nu_{\mathbf{p}}, \boldsymbol{\lambda}_{\mathbf{p}} \propto \exp(\imath\mathbf{q}\cdot\mathbf{r} - \imath\omega t). \qquad (48)$$

Substituting Eqs. (46)–(48) into Eq. (45) yields

$$\hat{\epsilon} = \frac{p^2}{2m} - \beta\boldsymbol{\sigma}\cdot\mathbf{H} + gN(1-\alpha\boldsymbol{\sigma}\cdot\boldsymbol{l}_0) + 2g\sum_{\mathbf{p},\sigma}(\nu_{\mathbf{p}} + \boldsymbol{\sigma}\cdot\boldsymbol{\lambda}_{\mathbf{p}}) + U - \beta\boldsymbol{\sigma}\cdot\mathbf{h} . \qquad (49)$$

In this stage we will restrict ourselves to considering all possible correlations of the longitudinal magnetization and density only. It is these correlation functions that provide the most significant contribution to the cross section $d\sigma_{\nu\nu}$. The structure functions involving fluctuations of transverse magnetization will be calculated at the end of this section.

In view of this one can assume that both $\boldsymbol{\lambda}_{\mathbf{p}}$ and $\mathbf{h}$ are parallel to $\boldsymbol{l}_0$, i.e., only $\lambda_{\mathbf{p}\,z} \equiv \lambda_{\mathbf{p}} \neq 0$ and $h_z \equiv h \neq 0$. As a result the spin commutator $[\hat{\epsilon}, \hat{n}]$ in the Boltzmann equation (44) vanishes. Substituting the density matrix $\hat{n}_0 + \delta\hat{n}$ from Eqs. (44) and (48) and the effective self-energy $\hat{\epsilon}$ from Eq. (49) into the transport equation (44), and then calculating the trace over the spin variables, one can find the first starting equation which takes the form

$$(\omega - \mathbf{q}\cdot\mathbf{v})\nu_{\mathbf{p}} + 2g\mathbf{q}\cdot\mathbf{v}\left(\frac{\partial\eta}{\partial\epsilon}\sum_{\mathbf{p}}\nu_{\mathbf{p}} - \frac{\partial\xi}{\partial\epsilon}\sum_{\mathbf{p}}\lambda_{\mathbf{p}}\right) = \mathbf{q}\cdot\mathbf{v}\left(\frac{\partial\xi}{\partial\epsilon}\beta h - \frac{\partial\eta}{\partial\epsilon}U\right) , \qquad (50)$$

where $\epsilon \approx p^2/2m$ and $\mathbf{v} = \partial\epsilon/\partial\mathbf{p} = \mathbf{p}/m$. In order to derive the second starting equation one should multiply the Boltzmann equation (44) by $\boldsymbol{\sigma}$ and take the trace. Such a procedure yields the following:

$$(\omega - \mathbf{q}\cdot\mathbf{v})\lambda_{\mathbf{p}} - 2g\mathbf{q}\cdot\mathbf{v}\left(\frac{\partial\eta}{\partial\epsilon}\sum_{\mathbf{p}}\lambda_{\mathbf{p}} - \frac{\partial\xi}{\partial\epsilon}\sum_{\mathbf{p}}\nu_{\mathbf{p}}\right) = \mathbf{q}\cdot\mathbf{v}\left(\frac{\partial\eta}{\partial\epsilon}\beta h - \frac{\partial\xi}{\partial\epsilon}U\right) . \qquad (51)$$

The two latter equations are sufficient to calculate the generalized dynamic succeptibility and to infer the sought-after correlation functions.

When taking account of the obvious relationships

$$2\sum_{\mathbf{p}}\nu_{\mathbf{p}} = \delta N , \quad 2\sum_{\mathbf{p}}\lambda_{\mathbf{p}} = \delta M_z , \qquad (52)$$

and integrating Eqs. (50) and (51) over momentum space, one can easily obtain the following result:

$$\delta N = \beta h\frac{2R_\xi}{1-g\Delta} + U\frac{\Delta - 2R_\eta}{1-g\Delta} ,$$
$$\delta M_z = \beta^2 h\frac{2R_\eta + \Delta}{1-g\Delta} - \beta U\frac{2R_\xi}{1-g\Delta} . \qquad (53)$$

Here the quantities R_ξ and R_η are defined as follows:

$$\int \frac{\mathbf{q}\cdot\mathbf{v}}{\omega-\mathbf{q}\cdot\mathbf{v}}\frac{\partial\xi}{\partial\epsilon}\frac{d^3p}{(2\pi\hbar)^3}\equiv R_\xi\ ,$$
$$\int \frac{\mathbf{q}\cdot\mathbf{v}}{\omega-\mathbf{q}\cdot\mathbf{v}}\frac{\partial\eta}{\partial\epsilon}\frac{d^3p}{(2\pi\hbar)^3}\equiv R_\eta\ , \tag{54}$$

and $\Delta = 4g(R_\eta^2 - R_\xi^2)$. The derived expressions in fact provide us with the generalized dynamic succeptibility $\hat\chi(\mathbf{q},\omega)$, which describes the linear response of the medium to an applied external field:

$$\begin{pmatrix}\delta N\\ \delta M_z\end{pmatrix} = \hat\chi\begin{pmatrix}-U\\ h\end{pmatrix}. \tag{55}$$

The susceptibility $\hat\chi(\mathbf{q},\omega)$ is obviously a 2×2 matrix, the elements of which are given by Eqs. (53). When following the fluctuation-dissipation theorem[28] one can now find the correlators entering the cross section (16):

$$\begin{aligned}G_{00}(\mathbf{q}\ ,\ \omega) &= \frac{2\hbar}{1-\exp(-\hbar\omega/T)}\mathrm{Im}\frac{2R_\eta-\Delta}{1-g\Delta}\ ,\\ S_{zz}(\mathbf{q}\ ,\ \omega) &= \frac{2\beta^2\hbar}{1-\exp(-\hbar\omega/T)}\mathrm{Im}\frac{2R_\eta+\Delta}{1-g\Delta}\ ,\\ G_{0z}(\mathbf{q}\ ,\ \omega)+G_{z0}(\mathbf{q}\ ,\ \omega) &= 4\beta\hbar\coth\frac{\hbar\omega}{2T}\mathrm{Im}\frac{R_\xi}{1-g\Delta}\ .\end{aligned} \tag{56}$$

Inasmuch as the temperature is assumed to be high enough, $T \gg \epsilon_d$, the occupation numbers n_+ and n_- and hence the functions $\xi(\mathbf{p})$ and $\eta(\mathbf{p})$ from Eq. (46) can be easily expressed in terms of the classic Maxwell-Boltzmann distribution function $n_0(\mathbf{p})$:

$$\begin{aligned}&\eta(\mathbf{p}) = n_0(\mathbf{p})\ ,\ \xi(\mathbf{p}) = \alpha n_0(\mathbf{p})\ ,\\ &n_0(\mathbf{p}) = \frac{N}{2}\left(\frac{2\pi\hbar^2}{mT}\right)^{3/2}\exp\left(-\frac{p^2}{2mT}\right)\ .\end{aligned} \tag{57}$$

Directly integrating Eqs. (54) with the Maxwell-Boltzmann distribution function leads to the result

$$R_\eta = \frac{N}{2T}\left[1+F\left(\frac{\omega}{2^{1/2}qv_T}\right)\right]\ ,\quad R_\xi = \alpha R_\eta\ , \tag{58}$$

where $v_T = (T/m)^{1/2}$ is the average thermal velocity and function $F(y)$ is defined as

$$\begin{aligned}F(y) &= \frac{y}{\pi^{1/2}}\int_{-\infty}^{\infty}\frac{\exp(-z^2)}{z-y-\imath 0}dz\ ,\\ \mathrm{erfc}\ (\imath y^{1/2}) &= \imath(\pi y)^{-1/2}e^{-y}[F(y^{1/2})+F(y^{-1/2})]\ ,\end{aligned} \tag{59}$$

and has the following asymptotic forms:[35]

$$\begin{aligned}F(y) &= -1-\frac{1}{2y^2}-\frac{3}{4y^4}+\cdots+\imath\pi^{1/2}y\exp(-y^2)\ ,\quad y\gg 1\ ,\\ F(y) &= -2y^2+\imath\pi^{1/2}y\ ,\quad y\ll 1\ .\end{aligned} \tag{60}$$

One can easily convince oneself that in a low-density semiquantum system the interaction strength is always smaller than the temperature, i.e., $|g|N \ll T$. Therefore all the obtained expressions can be expanded in a power series of this small parameter. If only the leading term in this expansion is taken into account (which actually corresponds to the case of a perfect gas), the formulas for the correlation functions become simplified:

$$\begin{aligned}G_{00}(\mathbf{q}\ ,\ \omega) &= \frac{4\hbar}{1-\exp(-\hbar\omega/T)}\mathrm{Im}R_\eta\ ,\\ S_{zz}(\mathbf{q}\ ,\ \omega) &= \frac{4\beta^2\hbar}{1-\exp(-\hbar\omega/T)}\mathrm{Im}R_\eta\ ,\\ G_{0z}(\mathbf{q}\ ,\ \omega)+G_{z0}(\mathbf{q}\ ,\ \omega) &= 4\beta\alpha\hbar\coth\frac{\hbar\omega}{2T}\mathrm{Im}R_\eta\ .\end{aligned} \tag{61}$$

Using the initial transport equation (44) implies actually that criteria $\omega\tau \gg 1$ and $q\Lambda_T \ll 1$ are supposed to be fulfilled. At the same time no particular limitations on the quantity ql are superimposed. As a result the behavior of the correlation functions depends significantly on the density of the medium. In the limit of a low enough density,

$$N|a|^3 \ll \frac{|a|}{\Lambda_T} \ll 1\ , \tag{62}$$

there may be the situation in which the momentum transfer is still small, $q\Lambda_T \ll ql \ll 1$, but the system nevertheless exhibits the entirely collisionless behavior $\omega\tau \gg 1$. In a sense this case still corresponds to the hydrodynamic limit because of small spatial gradients, although the medium is in a purely high-frequency regime and cannot be described by the equations of hydrodynamics at all. It is the regime that is described by the high-frequency part of the correlation functions:

$$\frac{\omega}{qv_T} \gg \frac{\omega}{qv_T}ql \gg 1\ . \tag{63}$$

Using the asymptotic forms (66) of function $F(y)$ one can easily obtain an explicit formula for $\mathrm{Im}R_\eta$ under the conditions discussed above:

$$\mathrm{Im}R_\eta = \frac{N}{2T}\left(\frac{\pi}{2}\right)^{1/2}\exp\left(-\frac{\omega^2}{2q^2v_T^2}\right)\ . \tag{64}$$

In the fully nonhydrodynamic limit $q\Lambda_T \ll 1 \ll ql$ at high frequencies $\omega \gg qv_T$, the correlations functions can still be expressed in terms of function $\mathrm{Im}R_\eta$ from Eq. (64). However, when lowering the frequency,

$$ql \gg \frac{\omega}{qv_T}ql \gg 1\ , \tag{65}$$

the exponential dependence (64) of the correlators is replaced by the power law (60):

$$\mathrm{Im}R_\eta = \frac{N}{2T}\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega}{qv_T}\ . \tag{66}$$

At relatively high densities

$$\frac{|a|}{\Lambda_T} \ll N|a|^3 \ll 1\ , \tag{67}$$

the "semihydrodynamic" regime, where $ql \ll q\Lambda_T \ll 1$, always takes place and the correlation functions have the exponential form (64) provided the criterion (63) is fulfilled.

Now we still have to find the remaining structure functions $S_{xx}(\mathbf{q},\omega) = S_{yy}(\mathbf{q},\omega)$ and $S_{xy}(\mathbf{q},\omega) = -S_{yx}(\mathbf{q},\omega)$, determining the cross sections of nonflip and spin-flip transitions [see Eqs. (16) and (17)]. In order to do this one should retain the spin commutator $[\hat{\epsilon},\hat{n}]$ on the left-hand side of the collisionless Boltzmann equation (44), which was omitted in the previous calculations. On the other hand one can easily demonstrate that terms like $\partial_{\mathbf{p}}\hat{n}\nabla\hat{\epsilon}$ may be neglected when calculating the linear response to a transverse external magnetic field.[1,34] Then the problem in question reduces in fact to one of calculating the spectrum of spin modes in a polarized Boltzmann gas.[4,34] Using the results obtained in Refs. 34 and 1 one can represent dynamic form factors $S_{xx}(\mathbf{q},\omega)$ and $S_{xy}(\mathbf{q},\omega)$ in the form of the Lorentz functions equivalent to Eqs. (42) where functions ω'_q and ω''_q are now defined in a different fashion:

$$\omega'_q = \Omega_H - \frac{(qv_T)^2}{\Omega_{\rm int}} \;, \quad \omega''_q = Zg\Omega_{\rm int} \;. \tag{68}$$

Here the following notation is used:

$$Z = \left(\frac{\pi}{2}\right)^{1/2}\frac{N}{T}\left(2\alpha\frac{T}{\hbar\tilde{\omega}} - 1\right)\frac{\tilde{\omega}}{qv_T}\exp\left(-\frac{\tilde{\omega}^2}{2q^2v_T^2}\right) \;,$$
$$\tilde{\omega} = \omega + \Omega_{\rm int} \;, \quad \Omega_{\rm int} = \frac{2gN\alpha}{\hbar} \;. \tag{69}$$

The obtained expressions can be used to describe correlators $S_{xx}(\mathbf{q},\omega)$ and $S_{xy}(\mathbf{q},\omega)$ in the collisionless regime provided the inequalities $\alpha \gg (|a|/\Lambda_T)$ and $qv_T \ll |\Omega_{\rm int}|$ are fulfilled.

V. MAGNITUDE OF THE EFFECT: NUMERICAL ESTIMATES

As mentioned above, considering properties of a low-density medium has the advantage that all calculations can be done *ab initio*, and rigorous results are thus obtained. Therefore no complicated numerical computations (which might be strongly dependent on the chosen model) are necessary to estimate the magnitude of the effect. Strictly speaking an observation of *any* inelastic scattering within the energy range (1.1) would be direct evidence of the predicted phenomena as the atomic or molecular excitation processes require normally much higher energies. The mechanism in question provides then a unique channel of inelastic scattering. However, one could expect that the effective cross section of the scattering caused by collective fluctuations might be very small due to the low density of the system. It appears not to be true.

To qualitatively understand this it is quite meaningful to make a comparison with typical experiments on neutron scattering. One can easily convince oneself that the physical picture of neutron scattering on collective modes (e.g., on phonons) in condensed matter is very similar to and has many common features with the phenomenon in question. For a neutron beam any target is in fact an extremely low-density system because of the smallness of the interaction range (the size of a nucleus), $r_0 \sim 10^{-13}$ cm. In a neutron-scattering experiment one typically has $Nr_0^3 \sim 10^{-17}$–10^{-15}, which is much less than normal values of the gas parameter in common atomic or molecular rarefied media. Nevertheless, such a "low density" of scattering centers does not prevent us at all from observing the inelastic scattering of neutrons with thermal collective fluctuations in experiment.

Indeed, the "degree of inelasticity" may be rather small due to the low density. For that reason directly measuring the change in the energy of a scattered test particle of an atomic or electron beam may be more difficult than inferring the differential cross section (integrated over the energy of the scattered atom) for a given scattering angle from experimental data. In this case it makes sense to compare the contribution of collective-mode-induced scattering with background scattering which results from the usual elastic collisions between test particles and atoms of the target. It turns out that the low "degree of inelasticity" results normally in rather small scattering angles at which the effect under consideration can reliably manifest itself. However, within those scattering angles the magnitude of the effect may be very big.

To be specific we will make some estimates for the particle-phonon contribution to the nonflip scattering and consider the spin-flip transitions due to the inelastic scattering on the fluctuations of transverse magnetization.

In the classical temperature range the criterion of weak inelasticity, $\epsilon \gg \hbar u/l$ (where ϵ is the energy of the test particle) for the scattering on the hydrodynamic fluctuations of pressure (phonons) reduces obviously to the following:

$$\epsilon \gg T\frac{\Lambda_T}{r_0}Nr_0^3 \;. \tag{70}$$

Under these conditions the contribution of long-wavelength hydrodynamic phonons is limited to the small enough scattering angles Θ:

$$\sin\frac{\Theta}{2} \le A\frac{\hbar}{2pl} \;, \quad p = (2m_0\epsilon)^{1/2} \;, \tag{71}$$

where A is a numerical factor of the order of unity. One can easily come to a conclusion that under the certain conditions

$$\frac{m}{m_0}\frac{\Lambda_T}{r_0}Nr_0^3 \ge 1 \;, \tag{72}$$

there exists an energy range of ϵ [above the threshold given by Eq. (70)] where the particle-phonon interaction is deeply inelastic and strongly affects scattering processes for all possible scattering angles (not only small-angle scattering). Such a special condition can certainly be achieved in experiment. However, a common experimental situation corresponds to a weakly inelastic scattering. Therefore we will restrict ourselves to discussing only the latter case.

The differential cross section for the phonon-induced scattering is determined by Eqs. (16), (40), and (41) and can be presented in the form

$$\frac{d\sigma}{d\epsilon' do'} = \frac{1}{8\pi}\left(\frac{m_0}{M}\right)^2 \frac{p'}{p}\frac{T}{mu^2}a_1^2\frac{1}{\hbar}\frac{u\delta}{(\omega \mp qu)^2 + u^2\delta^2} ,$$

$$a_1 = 3a_+ + a_- . \quad (73)$$

Integrating Eq. (73) over the energy of the scattered particle and taking account of the relation

$$\lim_{x\to 0} \frac{x}{z^2 + x^2} = \pi\delta(z) , \quad (74)$$

we end up with the following expression for the differential cross section in the quasielastic limit:

$$\frac{d\sigma}{do'} = \frac{1}{4}\left(\frac{m_0}{M}\right)^2 \frac{T}{mu^2}a_1^2 . \quad (75)$$

Substituting the velocity of sound in a perfect Boltzmann gas, $3mu^2 \approx 5T$, into Eq. (74) we finally obtain

$$\frac{d\sigma}{do'} = \frac{3}{20}\left(1 + \frac{m_0}{m}\right)^2 a_1^2 . \quad (76)$$

Inasmuch as the differential cross section of the direct elastic collision between two spin-unpolarized particles, $d\sigma_{el}/do'$, is equal to $a_1^2 + 3a_2^2$, where $a_2 = a_+ - a_-$ (see Ref. 12), one can see that both cross sections $d\sigma$ and $d\sigma_{el}$ are of the same order of magnitude; i.e., the effect can easily be detected in experiment. For instance, if the test particles and scattering centers in the target are identical, the increase in the differential cross section due to scattering on the collective density fluctuations is equal to 15%.

Of course, as was already emphasized, the enhancement in the scattering probability occurs only within a rather small-scattering-angle range. To get some idea of how wide this range may be, we will discuss a couple of typical examples. Let us consider the propagation of a test electron beam through the air under normal conditions ($T = 300$ K, $N \approx 10^{19}$ cm^{-3}, $r_0 \approx 5$ Å). In this case all correlations in the target are, indeed, classical although the interaction between the cold electron beam and classical fluctuations bears semiquantum features (see the Introduction). If the energy of the beam, ϵ, is about 1 meV the effect in question can be observed for the scattering angles $\theta \leq \theta_{max} \approx 1.3°$. (Compare with the fact that for a neutron beam of about the same energy typical scattering angles measured in experiment may be approximately 10°.) Lowering the energy of the test beam one could detect the phenomena at larger scattering angles. In the ultralow-energy limit $\epsilon \sim 1$ μeV, the process becomes deeply inelastic and influences the scattering pattern for all Θ.

In the case of a supercooled hydrogen atomic beam with energy $\epsilon \sim 1$ mK (this can be achieved by evaporating from the magnetic trap[19]), which propagates through the rarefied hydrogen target at $T \approx 0.1$ K, $N \approx 10^{17}$ cm^{-3} (see Refs. 17, 18), the maximum scattering angle turns out to be much smaller, $\Theta_{max} \sim (1\text{–}5)''$. It means that in an atomic-beam experiment measuring the forward-scattering processes might be a good tool to discover the effect.

Applying similar arguments to the scattering on the fluctuations of transverse magnetization, which are described by Eqs. (17), (42), and (43), one ends up with the following criterion of quasielastic scattering:

$$\epsilon \geq \hbar|\Omega_{int}| , \quad \Omega_{int} = \frac{4\pi\hbar a N\alpha}{m} , \quad (77)$$

provided the degree of polarization is high enough, $\alpha \gg (r_0/\Lambda_T)$. Here a is the s-wave scattering length for the two-particle interaction in the target. Criterion (77) corresponds to the inelastic scattering of a test particle with weakly damped spin waves (magnons).[10,11] At lower degrees of polarization, $\alpha \ll (r_0/\Lambda_T)$, the fluctuations of transverse magnetization propagate according to the common spin-diffusion law. In this case the criterion of "weak elasticity" is similar to Eq. (70).

Magnon-induced scattering manifests itself at sufficiently small scattering angles:

$$\sin\frac{\Theta}{2} \leq A\frac{\hbar|\Omega_{int}|}{2pv_T} , \quad v_T^2 = \frac{T}{m} . \quad (78)$$

In contrast to Eq. (71) for the phonon-induced scattering the "cutoff" criterion (78) contains the extra large factor $|\Omega_{int}|\tau \gg 1$, where $\tau \sim (Na^2 v_T)^{-1}$ is the relaxation time. If the inequality

$$\frac{m_0}{m} \leq \frac{\hbar|\Omega_{int}|}{T} \quad (79)$$

is fulfilled, the scattering on the spin waves is deeply inelastic and exhibits itself at all scattering angles. In the high-temperature limit $\beta H \ll T$ and within the scattering angle range (78) Eqs. (17), (42), and (43) result in a differential cross section of the form[11,26]

$$\frac{d\sigma}{do'} = 4\left(1 + \frac{m_0}{m}\right)^2 a_2^2 \frac{T}{\hbar\Omega_H}\alpha . \quad (80)$$

In equilibrium where $\alpha = \beta H/T$ the cross section (80) is of the order of the differential cross section for the elastic two-particle collision, $d\sigma_{el}$. On the other hand, under the conditions of "gigantic opalescence"[11] (where one creates a high degree of polarization but keeps the external magnetic field low) the value of $d\sigma$ may considerably exceed $d\sigma_{el}$. In gaseous spin-polarized hydrogen under typical experimental conditions [$T = 0.22$ K, $H = 8$ T, and $\alpha = 0.8$ (Refs. 6, 17, 18)] the increase in the differential cross section due to the inelastic scattering of the hydrogen atom with the fluctuations of nuclear (protonic) magnetization turns out to be of the order of 10^2. However, as was pointed out earlier, such an increase manifests itself only for small-angle scattering. For a test beam of hydrogen atoms with energy $\epsilon \approx 1$ mK (see above) the "cutoff" scattering angle Θ_{max} is about 2′, which is 25 times as large as the value of Θ_{max} for the phonon-induced scattering.

Let us emhasize that the "cutoff" angle Θ_{max} plays a very important role when considering the influence of the scattering on collective modes on transport properties. It explicitly comes into effect when integrating collision terms over the angles in the momentum space. It means that the expressions for transport coefficients obtained in Ref. 11 by averaging over all scattering angles can directly be used only if a criterion similar to Eq. (79) is fulfilled.

Otherwise, correction factors containing $\Theta_{\max}$ should be added.

VI. CONCLUDING REMARKS

We considered quantum-mechanical collective effects which can strongly affect a test beam propagating through a rarefied target. It was shown that particles of the incident beam were essentially scattered not only via direct elastic collisions with the atoms of the target, but inelastic scattering with collective thermal fluctuations in the target plays a very significant role as well. It means that when running a common experiment on atomic- or electron-beam spectroscopy one can study not only atomic collisions but also collective many-body properties of the target provided the energy of the incident beam is not too high. If the temperature of the target is not too high either, the very exciting macroscopic quantum-mechanical effects (like a new propagating mode[36]) exhibit themselves and one can infer the characteristics of those phenomena from the atomic- or electron-spectroscopy data. The magnitude of the effect in question appears to be not small at all. Under certain conditions the cross section of inelastic scattering with collective modes is of the same order of magnitude or even much larger than that of direct atomic collisions.[10,11] If one chooses a target with a thickness less than the mean free path, collective-mode-induced scattering will provide the main contribution to the dissipation of a testing beam at any circumstances.

We discussed propagation of an atomic or electron beam through a semiquantum target as just an example of a possible experimental application. There are of course a lot of other possibilities to study this phenomenon. Neutron optics is probably the most exciting one among them. The formulas obtained above completely determine the inelastic scattering of any slow particle with collective modes in a rarefied semiquantum medium. For a beam of cold neutrons any condensed matter is in fact a rarefied semiquantum medium since all the necessary criteria are always fulfilled. The scattering of a cold neutron beam with the thermal fluctuations of magnetization was considered in Ref. 26. The theory developed in this paper can be directly applied to a neutron beam propagating in a disordered fluid. In a target with polarized nuclei a coupling between fluctuations of the longitudinal magnetization and of the density (the "crossing" structure functions) comes along and significantly influences the intensity of scattered neutrons at a given scattering angle. All one should do in order to calculate the scattering probability is just to reformulate the final expressions of this paper in the spirit of the work.[26]

Probe particles of the beam and particles in the target can indeed be of the same sort (indistinquishable particles). In general one does not need to have a two-component system: a beam and target. All physical arguments about the quantum-mechanical refraction of a wave function as well as the corresponding mathematical formalism are certainly valid for any one-component semiquantum medium at distances less than the mean free path. In this case one actually deals with the contribution of quantum-mechanical refraction to the self-energy per particle of the medium. Then the inelastic interaction between a single-particle excitation and collective mode makes the effective mean free path shorter, which lowers the dissipation in a rarefied semiquantum system. To calculate transport coefficients in this case one has to include the collective modes in consideration and to start with the separate Boltzmann transport equation for each branch of elementary excitation. The contribution of the particle-magnon interaction to the viscosity and thermal conductivity of a spin-polarized semiquantum medium was considered in Ref. 11. It was shown that even under the normal conditions this contribution was not small and could never be ignored. The cross sections obtained in this paper provide us with the starting point in order to also study the influence of the particle-phonon interaction as well as of the scattering with entropy fluctuations, on the transport properties of a rarefied semiquantum medium.

We restricted ourselves to considering the properties of nondegenerate semiquantum media. However, it is clear that all statements hold at temperatures lower than the quantum degeneracy temperature ϵ_d as well. When just using the Fermi-Dirac distribution functions rather than the Maxwell-Boltzmann one for the occupation numbers n_+ and n_- one can easily extend the obtained results to the temperature range $T \leq \epsilon_d$.

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[1]E. P. Bashkin, Usp. Fiz. Nauk **148**, 433 (1986) [Sov. Phys. Usp. **29**, 238 (1986)].

[2]E. P. Bashkin, in *Condensed Matter Theories*, edited by J. S. Arponen, R. F. Bishop, and M. Manninen (Plenum Press, New York, 1988), Vol. 3, p. 179.

[3]Yu. Kagan, B. V. Svistunov, and G. V. Shlyapnikov, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 54 (1988) [JETP Lett. **48**, 56 (1988)].

[4]E. P. Bashkin, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 11 (1981) [JETP Lett. **33**, 8 (1981)].

[5]C. Lhuillier and F. Laloe, J. Phys. (Paris) **43**, 197 (1982); **43**, 225 (1982); **43**, 833 (1982).

[6]B. R. Johnson, J. S. Denker, N. Bigelow, L. P. Levi, J. H. Freed, and D. M. Lee, Phys. Rev. **52**, 1508 (1984); **53**, 302 (1984).
[7]P. J. Nacher, G. Tastevin, M. Leduc, S. B. Crampton, and F. Laloë, J. Phys. Lett. (Paris) **45**, L-441 (1984).
[8]L. R. Levi and A. E. Ruckenstein, Phys. Rev. Lett. **52**, 1512 (1984); **53**, 302 (1984).
[9]N. P. Bigelow, J. H. Freed, and D. M. Lee, Phys. Rev. Lett. **63**, 15 (1989).
[10]E. P. Bashkin, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 320 (1989) [JETP Lett. **49**, 363 (1989)].
[11]E. P. Bashkin, Phys. Rev. B **44**, 12440 (1991).
[12]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Pergamon, New York, 1977).
[13]T. J. Greytak and D. Kleppner, in *New Trends in Atomic Physics*, Proceedings of the Les Houches Summer School, edited by G. Grynberg and R. Stora (North-Holland, Amsterdam, 1984), Vol. 2, p. 1225.
[14]I. F. Silvera and J. T. M. Walraven, in *Progress in Low Temperature Physics*, edited by D. F. Brewer (North-Holland, Amsterdam, 1986), Vol. 10, p. 139.
[15]W. N. Hardy, M. Morrow, R. Jochemsen, and A. J. Berlinsky, Physica B **109–110**, 1964 (1982).
[16]M. W. Reynolds, M. E. Hayden, and W. N. Hardy, Physica (Amsterdam) B **165–166**, 741 (1990).
[17]R. Sprik, J. T. M. Walraven, and I. F. Silvera, Phys. Rev. Lett. **51**, 479 (1983); **51**, 942 (1983).
[18]H. F. Hess, D. A. Bell, G. P. Kochanski, D. Kleppner, and T. J. Greytak, Phys. Rev. Lett. **52**, 1520 (1984).
[19]N. Masuhara, J. M. Doyle, J. C. Sandberg, D. Kleppner, and T. J. Greytak, Phys. Rev. Lett. **61**, 935 (1988).
[20]E. P. Bashkin, Zh. Eksp. Teor. Fiz. **73**, 1849 (1977) [Sov. Phys. JETP **46**, 972 (1977)].
[21]A. Hershcovitch, Phys. Rev. Lett. **63**, 750 (1989).
[22]P. D. Lett, R. N. Watts, C. I. Westbrook, W. D. Phillips, P. L. Gould, and H. J. Metcalf, Phys. Rev. Lett. **61**, 169 (1988).
[23]S. Anderson and B. N. J. Persson, Phys. Rev. Lett. **50**, 2028 (1983).
[24]E. P. Bashkin and S. B. Stepaniants, in *Condensed Matter Theories*, edited by A. Proto *et al.* (Plenum Press, New York, 1992), Vol. 7, p. 207.
[25]E. P. Bashkin, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 322 (1986) [JETP Lett. **44**, 414 (1986)].
[26]E. P. Bashkin, Zh. Eksp. Teor. Fiz. **96**, 2018 (1989) [Sov. Phys. JETP **69**, 1139 (1989)].
[27]L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, New York, 1989).
[28]E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics* (Pergamon, New York, 1980), p. 2.
[29]E. P. Bashkin and A. E. Meyerovich, Adv. Phys. **30**, 1 (1981).
[30]A. E. Meyerovich, J. Low Temp. Phys. **47**, 271 (1982).
[31]E. P. Bashkin, Zh. Eksp. Teor. Fiz. **87**, 1948 (1984) [Sov. Phys. JETP **60**, 1122 (1984)].
[32]V. Zhdanov, Yu. Kagan, and A. Sazykin, Zh. Eksp. Teor. Fiz. **42**, 857 (1962) [Sov. Phys. JETP **15**, 596 (1962)].
[33]A. J. Leggett, J. Phys. C **3**, 448 (1970).
[34]E. P. Bashkin, Zh. Eksp. Teor. Fiz. **93**, 856 (1987) [Sov. Phys. JETP **66**, 482 (1987)].
[35]E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, New York, 1981).
[36]E. P. Bashkin and O. L. Puzyrko, Phys. Rev. Lett. **70**, 2309 (1993).