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Coherent phenomena in inelastic backscattering of electrons from disordered media

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The phenomenon of weak localization in an inelastic-scattering channel is considered in the context of the reflection of moderate-energy electrons with fixed energy loss from a disordered medium. The localization features in the backscattering angular spectrum are found to be manifested under the condition of strong interference, $\omega \ll \gamma$ (here $\hbar\omega$ is the energy loss and γ is the frequency of electron collisions). Unlike the elastic-scattering channel, the center of the enhanced peak is shown to be displaced relative to the exactly backward direction under the condition of oblique incidence of the electrons on a disordered sample surface.

I. INTRODUCTION

Weak localization of conduction electrons and backscattering enhancement of classical waves in disordered media have been studied extensively (see, for example, Refs. 1 and 2). It is well known now that both phenomena have the same physical origin and are connected with the constructive interference of random wave fields, scattered in the backward direction. It manifests itself in the enhancement of scattering in a narrow angular cone of width $\Delta\theta_l \sim 1/kl \ll 1$ in the vicinity of the retroreflection direction (here k is the modulus of the photon or electron wave vector and l is the mean free path).

Also of great interest is the problem of the backscattering of external particles (electrons, neutrons, etc.) incident on disordered samples (amorphous and polycrystalline), where analogous coherent effects take place. Such scattering in systems with various types of disorder has been considered in several theoretical works. In Ref. 3 the weak localization of neutrons was analyzed. The authors of Refs. 4–6 studied the localization of external electrons. In Refs. 4 and 5 it was suggested that backscattering enhancement might be observable for electrons of moderate energies (from hundreds to thousands of eV), which differ significantly from conduction-electron energy.

Unlike in the case of light scattering, the probability of inelastic processes is quite large when an electron of moderate energy propagates in a solid. The significance of inelastic scattering is twofold. On the one hand, in the presence of inelastic collisions the quantum coherence is destroyed by phase-breaking processes, which leads to dissipative electron transport. In this sense, inelastic scattering plays a rather negative role, because it suppresses the probability of elastic scattering and decreases the contribution of scattering processes of high multiplicity to the coherent backscattering intensity in the elastic-scattering channel. For conduction electrons this phenomenon was studied in Refs. 7 and 8. On the other hand, inelastic scattering gives rise to reflected electrons with energies that differ from the energy of the incident particles (the so-called *inelastic-scattering channel).* For example, in experiments on backscattering of electrons from polycrystalline samples of transition-metal oxides and perovskite materials $La_{1-x}Sr_xVO_3$ with random distribution of Sr^{2+} and La^{3+}, the low-energy plasmon modes and interband electron excitations ($\hbar\omega \sim 1$ eV in both cases) were recorded using the high-resolution electron-energy-loss spectroscopy technique.[9]

The interference of elastic and inelastic scattering can engender new coherent effects in the inelastic channel, which arise due to the interference of particle wave fields associated with different realizations of scattering processes in which the electron is multiply scattered elastically and once inelastically. This type of quantum interference was considered in Ref. 10 by an approach in which the electron trajectory is determined by a single elastic incoherent scattering process and a single inelastic collision. It has been shown there that the nonsuccessive nature of elastic and inelastic collisions leads (due to interference) to specific features in the angular spectra of electrons backscattered inelastically. The characteristic angular width of these features is of the order of γ/ω, where γ is the frequency of particle collisions and $\hbar\omega$ is the energy lost by a particle. Quite recently, in Ref. 11 an attempt was made to demonstrate that multiple elastic scattering of electrons does not destroy this coherent phenomenon.

The ratio γ/ω can be considered as a parameter of the strength of interference processes in the inelastic-scattering channel. In this sense the results obtained in Ref. 10 are associated with the weak-interference limit. In the opposite case of strong quantum interference ($\gamma/\omega \gg 1$), the coherent features in the angular spectrum of inelastically backscattered electrons, predicted in Ref. 10, should vanish. We note that this limit corresponds to a small value of the parameter $\omega l/v$ (here v is the modulus of the electron velocity before an inelastic collision), which is equal to the phase shift between the arbitrary electron trajectory that includes an act of single inelastic scattering, and its reversed partner. It is apparent that under the condition $\omega l/v \ll 1$, corresponding to strong interference and to a small phase shift, weak localization should manifest itself in the inelastic-scattering channel. It is precisely this situation that will be studied in the present work. Apart from the fact that exploration of this scattering regime (which is opposite to the one considered in Ref. 10) is of general physical interest in the theory of the inelastic quantum transport of particles, it can turn out to be a method for the study of low-energy excitations in disordered systems by means of the inelastic reflection of electrons with a fixed energy loss.

The paper is organized as follows. In Sec. II we derive a diagrammatic representation for the electron density matrix in the inelastic-scattering channel. In Sec. III the diffusion approximation is introduced to describe multiple elastic scattering in a disordered medium in the presence of wave fields having different energies. Section IV is devoted to the derivation of expressions for the angular spectrum of electrons inelastically reflected from a disordered medium, and the physical sense of various Feynman diagrams is discussed in detail. The form of the angular spectrum of electrons in two opposite cases of inelastic scattering is considered in Sec. V. Section VI contains conclusions.

II. DIAGRAMMATIC TECHNIQUE FOR DENSITY MATRIX OF ELECTRONS IN INELASTIC-SCATTERING CHANNEL

Let an external electron be incident on a disordered medium, which occupies the half space $z>0$. We shall be interested in the angular spectrum of electrons reflected by the medium with a fixed energy loss due to a *single* inelastic collision.

The wave function ψ_n of an electron in the nth inelastic channel is governed by the Schrödinger equation

$$\Delta\psi_n(\mathbf{r})+2m\hbar^{-2}[E_n-U(\mathbf{r})+iU_n']\psi_n(\mathbf{r}) =2m\hbar^{-2}T(\mathbf{r},i\rightarrow n)\psi_i(\mathbf{r}) , \quad (1)$$

where m and E_n are the mass and energy of the electron, $U(\mathbf{r})$ is the random potential describing the interaction of the incident particle with the atoms of the medium, and the effective complex potential iU_n' describes the dissipation due to quantum transitions between the nth and other excited states. $T(\mathbf{r},i\rightarrow n)$ is a matrix element of the Hamiltonian describing the inelastic interaction between electron and medium, calculated with the wave functions of the medium subsystem, associated with the corresponding inelastic process. The wave function ψ_i of an electron in the elastic-scattering channel on the right-hand side of Eq. (1) satisfies the equation

$$\Delta\psi_i(\mathbf{r})+2m\hbar^{-2}[E_i-U(\mathbf{r})+iU_i']\psi_i(\mathbf{r})=0 , \quad (2)$$

where E_i is the initial energy of the electron in matter and iU_i' is a complex potential for the elastic-scattering channel.

The Green's function G_m for the scattering problem is determined by the equation $(m=i,n)$

$$\Delta G_m(\mathbf{r},\mathbf{r}')+2m\hbar^{-2}[E_m-U(\mathbf{r})+iU_m']G_m(\mathbf{r},\mathbf{r}') =2m\hbar^{-2}\delta(\mathbf{r}-\mathbf{r}') , \quad (3)$$

which also contains a complex potential connected with the inelastic mean free path of the electron, $U_m'=\hbar v/2l_{\text{inel}}(E_m)$.

Equations (1)–(3) must be supplemented by boundary conditions. In the case of the elastic-scattering channel $(m=i)$ it is necessary to satisfy the requirement that $\psi_i(\mathbf{r})=\exp(i\mathbf{kr})$ at $z=-\infty$, as well as the continuity of the function ψ_i and its normal derivative across the boundary $z=0$. Analogous requirements of continuity must be satisfied by the electron propagator G_i.

The boundary conditions for the electron wave function ψ_n and propagator G_n in the inelastic-scattering channel have a different form. Namely, they depend on whether the electron moves in matter away from or toward the boundary. In the first case, when $(\mathbf{nk}')>0$, the requirement $\psi_n(\mathbf{r})=0$ at $z=0$ must be satisfied if wave reflection from the boundary is neglected (here $\mathbf{k}'$ is the wave vector of the electron that has undergone an inelastic collision, and $\mathbf{n}$ is the internal normal to the surface). When $(\mathbf{nk}')<0$, the function ψ_n and its normal derivative must be continuous at $z=0$. The boundary conditions for G_n are the same. The requirement of vanishing of ψ_n and G_n at the vacuum-medium boundary when $(\mathbf{nk}')>0$ is associated with the fact that at $z=0$ there are no electrons moving inward from the boundary in the inelastic-scattering channel.

We define the operator

$$\mathbf{U}_\alpha^m=\delta UG_{m0}\delta U\cdots G_{m0}\delta U \quad (4)$$

containing the fluctuating part δU of the random potential $U(\mathbf{r})$ α times. G_{m0} is an integral operator corresponding to the Green's function of an electron with energy E_m in the mean field $\langle U(\mathbf{r})\rangle$,

$$U(\mathbf{r})=\langle U(\mathbf{r})\rangle+\delta U(\mathbf{r}) , \quad (5)$$

where the angular brackets $\langle\cdots\rangle$ denote a statistical average with respect to the ensemble of the fluctuating atomic potential δU, which is assumed to be a Gaussian random process. Then the solution of Eq. (1) can be represented as

$$\psi_n(\mathbf{r})=G_{n0}\left[\mathbf{I}+\sum_{\alpha=1}^{\infty}\mathbf{U}_\alpha^nG_{n0}\right]T(i\rightarrow n) \times\left[\mathbf{I}+\sum_{\beta=1}^{\infty}G_{i0}\mathbf{U}_\beta^i\right]\psi_{i0}(\mathbf{r}) . \quad (6)$$

Here $\mathbf{I}$ is the unit operator and ψ_{i0} denotes the wave function of an electron of energy E_i in the average potential $\langle U\rangle$ in an elastic-scattering channel.

The angular spectrum of inelastically backscattered particles can be defined through the electron density matrix in the nth scattering channel

$$\delta\rho_{nn}(\mathbf{r}_1,\mathbf{r}_2)=\langle\psi_n^*(\mathbf{r}_2)\psi_n(\mathbf{r}_1)\rangle-\langle\psi_n^*(\mathbf{r}_2)\rangle\langle\psi_n(\mathbf{r}_1)\rangle , \quad (7)$$

for which an expansion can be obtained with the aid of Eq. (6):

$$\delta\rho_{nn}(\mathbf{r}_1,\mathbf{r}_2)=\left\{\left[G_{n0}\sum_{\alpha=1}^{\infty}\mathbf{U}_\alpha^n G_{n0}T_{in}\psi_{i0}\right]_1,\left[G_{n0}\sum_{\beta=1}^{\infty}\mathbf{U}_\beta^n G_{n0}T_{in}\psi_{i0}\right]_2\right\}$$
$$+\left\{\left[G_{n0}T_{in}G_{i0}\sum_{\alpha=1}^{\infty}\mathbf{U}_\alpha^i\psi_{i0}\right]_1,\left[G_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}\mathbf{U}_\beta^i\psi_{i0}\right]_2\right\}$$
$$+2\,\mathrm{Re}\left\{\left[G_{n0}\sum_{\alpha=1}^{\infty}\mathbf{U}_\alpha^n G_{n0}T_{in}\psi_{i0}\right]_1,\left[G_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}\mathbf{U}_\beta^i\psi_{i0}\right]_2\right\}+\cdots\ , \tag{8}$$

where subscripts 1 and 2 correspond to radius vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively, and the notation $\{\varphi,\chi\}=\langle\chi^*\varphi\rangle-\langle\chi^*\rangle\langle\varphi\rangle$ was used for short.

It is convenient to average Eq. (8) in a graphical way, as was done for the mutual coherence function of classical waves in an elastic-scattering channel. It has been shown in Ref. 13 that only those connected double-row diagrams contribute to the density matrix that contain an even number of elastic collision vertices δU, joined pairwise by dashed lines in all possible manners. On omitting the diagrams in which at least one dashed line surrounds an inelastic-interaction vertex T_{in} [they are of the order of $1/kl \ll 1$ (Ref. 12)], one can obtain in the first order in Γ_{ni}

$$\delta\rho_{nn}^{(1)}(\mathbf{r},\mathbf{r}')=\int\int d\mathbf{r}_1 d\mathbf{r}_1'\int\int d\mathbf{r}_2 d\mathbf{r}_2'\langle G_n(\mathbf{r},\mathbf{r}_1)\rangle\langle G_n^*(\mathbf{r}',\mathbf{r}_2)\rangle\Gamma_{nn}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$$
$$\times\int\int d\mathbf{r}_3 d\mathbf{r}_3'\langle G_n(\mathbf{r}_1',\mathbf{r}_3)\rangle\langle G_n^*(\mathbf{r}_2',\mathbf{r}_3')\rangle T_{in}(\mathbf{r}_3)T_{ni}(\mathbf{r}_3')\langle\psi_i(\mathbf{r}_3)\rangle\langle\psi_i^*(\mathbf{r}_3')\rangle\ , \tag{9}$$

$$\delta\rho_{nn}^{(2)}(\mathbf{r},\mathbf{r}')=\int\int d\mathbf{r}_1 d\mathbf{r}_1'\int\int d\mathbf{r}_2 d\mathbf{r}_2'\int\int d\mathbf{r}_3 d\mathbf{r}_3'\langle G_n(\mathbf{r},\mathbf{r}_3)\rangle\langle G_n^*(\mathbf{r}',\mathbf{r}_3')\rangle T_{in}(\mathbf{r}_3)T_{ni}(\mathbf{r}_3')\langle G_i(\mathbf{r}_3,\mathbf{r}_1)\rangle$$
$$\times\langle G_i^*(\mathbf{r}_3',\mathbf{r}_2)\rangle\Gamma_{ii}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')\langle\psi_i(\mathbf{r}_1')\rangle\langle\psi_i^*(\mathbf{r}_2')\rangle\ , \tag{10}$$

$$\delta\rho_{nn}^{(3)}(\mathbf{r},\mathbf{r}')=2\,\mathrm{Re}\int\int d\mathbf{r}_1 d\mathbf{r}_1'\int\int d\mathbf{r}_2 d\mathbf{r}_2'\int\int d\mathbf{r}_3 d\mathbf{r}_3'\langle G_n(\mathbf{r},\mathbf{r}_1)\rangle\langle G_n^*(\mathbf{r}',\mathbf{r}_3')\rangle T_{in}(\mathbf{r}_3)T_{ni}(\mathbf{r}_3')\langle G_i^*(\mathbf{r}_3',\mathbf{r}_2)\rangle$$
$$\times\Gamma_{ni}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')\langle G_n(\mathbf{r}_1',\mathbf{r}_3)\rangle\langle\psi_i(\mathbf{r}_3)\rangle\langle\psi_i^*(\mathbf{r}_2')\rangle\ , \tag{11}$$

so that $\delta\rho_{nn}=\delta\rho_{nn}^{(1)}+\delta\rho_{nn}^{(2)}+\delta\rho_{nn}^{(3)}$. The two first terms, given by Eqs. (9) and (10), coincide with those obtained in Ref. 12 in the approximation in which the elastic collisions and the inelastic one were assumed to be strictly successive relative to each other. The third term, given by Eq. (11), was neglected in Ref. 12 and describes the interference of multiple elastic scattering and a single inelastic collision. In Eqs. (9)–(11) $\langle G_m\rangle$ is the average electron propagator in the mth scattering channel, $\langle\psi_i\rangle$ is the average wave function of the electron before the inelastic collision, and the four-point vertex Γ_{ni} describes the evolution of the wave field of the particle in the presence of inelastic collisions (its structure will be considered later).

To calculate the contribution of all inelastic processes we have to sum $\delta\rho_{nn}$ over all the excited n states:

$$\delta\rho_{\mathrm{inel}}(\mathbf{r},\mathbf{r}')=\sum_n\delta\rho_{nn}(\mathbf{r},\mathbf{r}')=\int d\omega\,\Theta(\omega)\delta\rho(\mathbf{r},\mathbf{r}',\omega)\ . \tag{12}$$

Integration over ω in Eq. (12) means integration over the energy lost in an inelastic collision. Therefore $\delta\rho(\mathbf{r},\mathbf{r}',\omega)$ can be considered as a spectral density matrix of the electrons:

$$\delta\rho(\mathbf{r},\mathbf{r}',\omega)=\sum_n\delta\left[\omega-\frac{\varepsilon_n}{\hbar}\right]\delta\rho_{nn}(\mathbf{r},\mathbf{r}')\ . \tag{13}$$

Keeping in mind Eqs. (9)–(11) and Eq. (13), it is convenient to turn to a diagrammatic representation for the spectral density matrix $\delta\rho(\mathbf{r},\mathbf{r}',\omega)$ of the electrons which underwent incoherent elastic scattering and lost the fixed energy $\hbar\omega$ (see Fig. 1). A thin solid line in Fig. 1 corresponds to the average electron Green's function $G(E;\mathbf{r},\mathbf{r}')$ [or $G(E-\hbar\omega;\mathbf{r},\mathbf{r}')$] if this line is to the right (or left) of the wavy line that describes single inelastic scattering (in the following we drop the angular brackets in writing the average propagators). A thick solid line corresponds to the density matrix $\rho_0(\mathbf{r},\mathbf{r}')=\langle\psi_i^*(\mathbf{r}')\rangle\langle\psi_i(\mathbf{r})\rangle$ of the electron with energy E_i that did not experience incoherent elastic scattering. The wavy line denoting an inelastic collision corresponds to the D function

$$D(\mathbf{r},\mathbf{r}',\omega)=\sum_n\delta\left[\omega-\frac{\varepsilon_n}{\hbar}\right]T_{in}(\mathbf{r})T_{ni}(\mathbf{r}')\ . \tag{14}$$

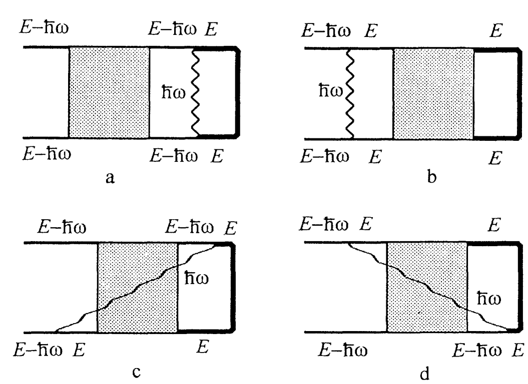

FIG. 1. Diagrammatic representation of the density matrix in an inelastic scattering channel.

Under the assumption of the spatial homogeneity of the inelastic subsystem of the medium $D(\mathbf{r},\mathbf{r}',\omega)=D(\mathbf{r}-\mathbf{r}',\omega)$, and the Fourier transform of the D function is related to the probability $W_{\text{inel}}(\mathbf{q},\omega)$ of inelastic scattering per unit time with momentum transfer $\hbar\mathbf{q}$ and energy loss $\hbar\omega$ by the relation

$$D(\mathbf{q},\omega)=\frac{1}{\nu}\left[\frac{2\pi\hbar^2}{m}\right]^2 W_{\text{inel}}(\mathbf{q},\omega) . \tag{15}$$

The shaded block in Fig. 1 corresponds to the matrix $\Gamma(E,E')$, which describes propagation of the particle with energy E in a random medium, when $E=E'$. If $E\neq E'$ the matrix Γ describes the interference of multiply scattered waves with different energies in the disordered medium. It is equal to the sum of all connected double-row diagrams without incoming and outgoing average Green's functions (see Fig. 2). Two crosses joined by a dashed line are associated with the correlation function $\langle\delta U(\mathbf{r})\delta U(\mathbf{r}')\rangle$ of the fluctuating part of the random potential $U(\mathbf{r})$. The average Green's function can be written in the $\mathbf{p}$ representation as

$$G(E-\hbar\omega,\mathbf{p})=\left[E-\hbar\omega-\frac{\hbar^2\mathbf{p}^2}{2m}+i\frac{k}{l}\right]^{-1} , \tag{16}$$

where l is the electron mean free path, $l^{-1}=l_{\text{el}}^{-1}+l_{\text{inel}}^{-1}$. Thus the pole of the average Green's function contains an imaginary part depending on both elastic and inelastic processes. The contribution connected with the inelastic mean free path is the manifestation of the dissipation of electron waves in matter.

Note that the diagrammatic representation introduced is valid for an arbitrary type of electron-atom potential. The diagram 1(a) describes the motion of an electron in matter in which the particle first undergoes an inelastic collision and then multiple elastic incoherent scattering. The diagram 1(b) corresponds to the process in which the sequence of scattering events is reversed. The diagrams 1(c) and 1(d) describe the contribution of electrons scattered inelastically to the density matrix due to the nonsuccessive nature of the multiple elastic and single inelastic scatterings relative to each other. In other words, these diagrams are associated with the interference of multiple elastic collisions and a single inelastic collision, and are the source of new coherent phenomena in the angular spectrum of electrons backscattered from a disordered medium with fixed energy losses.

It is interesting to note that the diagrammatic representation presented in Fig. 1 is similar to the diagrams for the correction to the conductivity caused by the interaction of the conduction electrons (see, for example, Ref. 1), but, of course, the sense of the elements entering the diagrams is quite different.

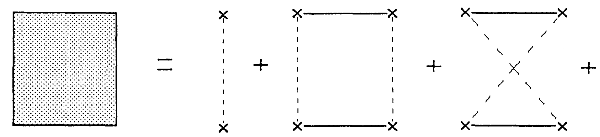

FIG. 2. Graphs contributing to the Γ matrix.

III. Γ matrix in the diffusion approximation

To calculate the Γ matrix it is necessary to sum up all the diagrams shown in Fig. 2. In accordance with general ideas of weak-localization theory, we shall exclude from this graphical series all the diagrams except the ladder and fan-shaped (maximally crossed) ones.

Leaving aside the effects of the anisotropy of the elastic scattering, we write the average and fluctuating parts of the random potential $U(\mathbf{r})$ as follows:

$$\langle U(\mathbf{r})\rangle=-\frac{2\pi\hbar^2}{m}nf , \tag{17}$$

$$\delta U(\mathbf{r})=-\frac{2\pi\hbar^2}{m}f\left[\sum_a \delta(\mathbf{r}-\mathbf{r}_a)-n\right] . \tag{18}$$

Here n is the number of force centers per unit volume, $\mathbf{r}_a$ designates the set of radius vectors of the centers, f is the effective amplitude of electron scattering by an atom, which is connected with some effective elastic scattering path length by the relationship $l_{\text{el}}^*=1/4\pi n|f|^2$. For isotropic elastic scattering l_{el}^* coincides with l_{el} introduced after Eq. (16). In the case of anisotropic scattering it is usually implied that l_{el}^* coincides with the elastic transport mean free path.

By using Eqs. (17) and (18) the contributions of the ladder and fan-shaped diagrams to the Γ matrix can be written as

$$\Gamma_{\text{lad}}(\omega;\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')=\frac{\pi}{l_{\text{el}}^*}\left[\frac{\hbar^2}{m}\right]^2\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1'-\mathbf{r}_2')\times[\delta(\mathbf{r}_1-\mathbf{r}_1')+\Gamma(\omega;\mathbf{r}_1,\mathbf{r}_1')] , \tag{19}$$

$$\Gamma_{\text{fan}}(\omega;\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')=\frac{\pi}{l_{\text{el}}^*}\left[\frac{\hbar^2}{m}\right]^2\delta(\mathbf{r}_1-\mathbf{r}_2')\delta(\mathbf{r}_2-\mathbf{r}_1')\Gamma(\omega;\mathbf{r}_1,\mathbf{r}_1') . \tag{20}$$

Here $\omega=(E-E')/\hbar$, and the new function $\Gamma(\omega;\mathbf{r},\mathbf{r}')$ is the solution of the equation

$$\Gamma(\omega;\mathbf{r},\mathbf{r}')=\frac{1}{4\pi l_{\text{el}}^*}\left\{\frac{\exp[-(1/l+i\omega/v)|\mathbf{r}-\mathbf{r}'|]}{|\mathbf{r}-\mathbf{r}'|^2}+\int d\mathbf{R}\frac{\exp[-(1/l+i\omega/v)|\mathbf{r}-\mathbf{R}|]}{|\mathbf{r}-\mathbf{R}|^2}\Gamma(\omega;\mathbf{R},\mathbf{r}')\right\} , \tag{21}$$

which can be solved analytically in the half space $z>0$. We restrict ourselves, for simplicity, to its solution in the diffusion approximation, which is valid when the condition $\omega l/v\ll 1$ is satisfied. Using the approach of Ref. 14, one can obtain the Fourier transform of the solution of Eq. (21) in the following form:

$$\Gamma(\omega,\mathbf{Q}_\parallel;z,z')=\int d\rho\,\Gamma(\omega,\rho;z,z')\exp[-i\mathbf{Q}_\parallel\rho]$$
$$=\frac{3}{2ll^*_{\rm el}}[\mathbf{Q}_\parallel^2+\beta^2]^{-1/2}\{\exp[-(\mathbf{Q}_\parallel^2+\beta^2)^{1/2}|z-z'|]-\exp[-(\mathbf{Q}_\parallel^2+\beta^2)^{1/2}|z+z'|]\}\ , \tag{22a}$$

where

$$\beta^2=\frac{3}{l^2}\left[1-\frac{l}{l^*_{\rm el}}+i\frac{\omega l}{v}\right] \tag{22b}$$

and $\mathrm{Re}(\mathbf{Q}_\parallel^2+\beta^2)>0$. According to Ref. 15 we require that the function $\Gamma(\omega,\mathbf{Q}_\parallel;z,z')$ vanish on the planes $z=0$ and $z'=0$, but not on the planes $z,z'=-z_0$, where z_0 is the extrapolated length in the Milne problem.[16]

IV. ANGULAR SPECTRUM OF REFLECTED ELECTRONS

The backscattering angular spectrum may be expressed through the diagonal matrix element of the density matrix

$$J_{\rm inel}(\mu_0\to|\mu|,\omega)=\frac{k_\omega^2\mu^2}{(2\pi)^2S}\delta\rho(\mathbf{k}'_\parallel,z=-0;\mathbf{k}'_\parallel,z'=-0;\omega)\ . \tag{23}$$

Here $\mathbf{k}'_\parallel$ is the component parallel to the surface of the wave vector of the emitted electron, $k_\omega^2=k^2-2m\omega/\hbar$, S is the area of the interface, $\mu_0=\cos\theta$, $\mu=\cos\theta'$, and θ and θ' are the incidence and emission angles, respectively. The Fourier component of the density matrix in Eq. (23) is defined in the following way:

$$\delta\rho(\mathbf{k}'_\parallel,z;\mathbf{k}'_\parallel,z';\omega)=\int d\rho\int d\rho'\delta\rho(\rho,z;\rho',z';\omega)\times\exp[-i\mathbf{k}'_\parallel\rho+i\mathbf{k}'_\parallel\rho']\ . \tag{24}$$

According to the rules derived in Sec. II, the contribution of the diagram in Fig. 1(a) to the spectral density matrix can be represented as

$$\delta\rho^{(1)}(\mathbf{r},\mathbf{r}',\omega)=\int d\mathbf{r}_1d\mathbf{r}'_1d\mathbf{r}''_1\int d\mathbf{r}_2d\mathbf{r}'_2d\mathbf{r}''_2G(E-\hbar\omega;\mathbf{r},\mathbf{r}_1)G^*(E-\hbar\omega;\mathbf{r}',\mathbf{r}_2)\Gamma^{\rm el}(\mathbf{r}_1,\mathbf{r}'_1;\mathbf{r}_2,\mathbf{r}'_2)$$
$$\times G(E-\hbar\omega;\mathbf{r}'_1,\mathbf{r}''_1)G^*(E-\hbar\omega;\mathbf{r}'_2,\mathbf{r}''_2)D(\mathbf{r}''_1,\mathbf{r}''_2,\omega)\psi_\mathbf{k}(\mathbf{r}''_1)\psi^*_\mathbf{k}(\mathbf{r}''_2)\ . \tag{25}$$

Here ψ_k denotes the average electron wave field $\langle\psi_i\rangle$ in an elastic-scattering channel, and $\mathbf{k}$ is the wave vector of the electron in the initial state. The quantity $\Gamma^{\rm el}$ is the Γ matrix in the case that the energies of the lower and upper propagation lines are the same,

$$\Gamma^{\rm el}=\Gamma_{\rm lad}(\omega=0)+\Gamma_{\rm fan}(\omega=0)\ .$$

Keeping in mind Eqs. (19) and (20), one can separate the contributions of ladder and fan-shaped parts in Eq. (25).

Let us introduce the Fourier representation for the D function

$$D(\mathbf{r},\mathbf{r}',\omega)=\int\frac{d\mathbf{q}}{(2\pi)^3}D(\mathbf{q},\omega)\exp[i\mathbf{q}(\mathbf{r}-\mathbf{r}')]\ , \tag{26}$$

which is valid in the case when the inelastic collision occurs in the bulk of the matter and the influence of the surface on the properties of the inelastic scattering is not taken into account. We also define the mean wave field of a particle in the elastic scattering channel as

$$\psi_\mathbf{k}(\mathbf{r})=\exp(i\mathbf{k}_\parallel\rho)\psi_\mathbf{k}(z)\ , \tag{27a}$$

$$\psi_\mathbf{k}(z)=\exp[ik_zz-z/2l\mu_0]\ , \tag{27b}$$

and the Fourier representation for the average propagator in the inelastic channel for two types of electron motion in matter (away from and toward the boundary $z=0$) is

$$G(E-\hbar\omega;\mathbf{r},\mathbf{r}')=\int\frac{d\mathbf{p}_\parallel dp_z}{(2\pi)^3}G(E-\hbar\omega;\mathbf{p}_\parallel,p_z)\times\begin{cases}\exp[i\mathbf{p}(\mathbf{r}-\mathbf{r}')] & \text{if } (\mathbf{nk}')<0\ ,\\ \exp[i\mathbf{p}_\parallel(\rho-\rho')]\{\exp[ip_z(z-z')]-\exp[ip_z(z+z')]\} & \text{if } (\mathbf{nk}')>0\ .\end{cases} \tag{28}$$

Then, taking into account the identity[12]

$$G(E-\hbar\omega;\mathbf{k}'_\parallel,z=-0,z')=-\frac{im}{\hbar^2|k'_z|}\psi^*_{\mathbf{k}_\omega^{(-)}}(z')\ , \tag{29}$$

where $\mathbf{k}_\omega^{(-)}=(\mathbf{k}'_\parallel,-|k'_z|)$ and $k'_z=-k_\omega|\mu|$, one can obtain the following expressions for the ladder and fan parts of the contribution of the diagram in Fig. 1(a):

$$\delta\rho_{\rm lad}^{(1)}(\mathbf{k}'_\parallel,z=-0;\mathbf{k}'_\parallel,z'=-0;\omega)=\frac{\pi S}{k_\omega^2\mu^2 l_{\rm el}^*}\int dz\,dz'|\psi_{\mathbf{k}_\omega^{(-)}}(z)|^2[\delta(z-z')+\Gamma^{\rm el}(0;z,z')]$$
$$\times\int\frac{d\mathbf{q}_\parallel dq_z}{(2\pi)^3}D(\mathbf{q}_\parallel,q_z,\omega)|A(E-\hbar\omega;\mathbf{q}_\parallel,q_z;z')|^2\ , \tag{30}$$

$$\delta\rho_{\rm fan}^{(1)}(\mathbf{k}'_\parallel,z=-0;\mathbf{k}'_\parallel,z'=-0;\omega)=\frac{\pi S}{k_\omega^2\mu^2 l_{\rm el}^*}\int dz\,dz'\psi^*_{\mathbf{k}_\omega^{(-)}}(z)\psi_{\mathbf{k}_\omega^{(-)}}(z')$$
$$\times\int\frac{d\mathbf{q}_\parallel dq_z}{(2\pi)^3}D(\mathbf{q}_\parallel,q_z,\omega)\Gamma^{\rm el}(\mathbf{k}_\parallel+\mathbf{k}'_\parallel+\mathbf{q}_\parallel;z,z')$$
$$\times A(E-\hbar\omega;\mathbf{q}_\parallel,q_z;z')A^*(E-\hbar\omega;\mathbf{q}_\parallel,q_z;z)\ . \tag{31}$$

Here the function A denotes the integral

$$A(E-\hbar\omega;\mathbf{q}_\parallel,q_z;z)=-\frac{i}{\pi}\int dp_z G(E-\hbar\omega;\mathbf{k}_\parallel+\mathbf{q}_\parallel,p_z)\frac{p_z\exp(ip_z z)}{p_z^2-[k_z+q_z+i/2l\mu_0]^2}\ , \tag{32}$$

and $\Gamma^{\rm el}(\mathbf{Q}_\parallel;z,z')=\Gamma(\omega=0,\mathbf{Q}_\parallel;z,z')$ [see Eq. (22a)]. In the derivation of these results we assumed that after the inelastic collision the electron continued its motion from the boundary into medium, so that $(\mathbf{nk}')>0$.

The same transformations bring us to the following ladder and fan-shaped contributions of the diagram in Fig. 1(b) to the density matrix:

$$\delta\rho_{\rm lad}^{(2)}(\mathbf{k}'_\parallel,z\doteq-0;\mathbf{k}'_\parallel,z'=-0;\omega)=\frac{\pi S}{k_\omega^2\mu^2 l_{\rm el}^*}\int dz\,dz'|\psi_{\mathbf{k}}(z')|^2[\delta(z-z')+\Gamma^{\rm el}(0;z,z')]$$
$$\times\int\frac{d\mathbf{q}_\parallel dq_z}{(2\pi)^3}D(\mathbf{q}_\parallel,q_z,\omega)|B(E;\mathbf{q}_\parallel,q_z;z)|^2\ , \tag{33}$$

$$\delta\rho_{\rm fan}^{(2)}(\mathbf{k}'_\parallel,z=-0;\mathbf{k}'_\parallel,z'=-0;\omega)=\frac{\pi S}{k_\omega^2\mu^2 l_{\rm el}^*}\int dz\,dz'\psi^*_{\mathbf{k}}(z)\psi_{\mathbf{k}}(z')\int\frac{d\mathbf{q}_\parallel dq_z}{(2\pi)^3}D(\mathbf{q}_\parallel,q_z,\omega)\Gamma^{\rm el}(\mathbf{k}_\parallel+\mathbf{k}'_\parallel-\mathbf{q}_\parallel;z,z')$$
$$\times B(E;\mathbf{q}_\parallel,q_z;z)B^*(E;\mathbf{q}_\parallel,q_z;z')\ , \tag{34}$$

where the function B is defined by the integral

$$B(E;\mathbf{q}_\parallel,q_z;z)=\frac{i}{2\pi}\int dp_z G(E;\mathbf{k}'_\parallel-\mathbf{q}_\parallel,p_z)\frac{\exp(-ip_z z)}{p_z-[k'_z-q_z-i/2l_\omega|\mu|]}\ . \tag{35}$$

In the derivation of Eqs. (33) and (34) we assumed that after the inelastic collision the electron moved from the interior of the medium toward the boundary, $(\mathbf{nk}')<0$.

Let us consider the diagrams in Figs. 1(c) and 1(d). Their contributions are complex conjugates of each other. Keeping this in mind, we can write the ladder contribution in the following manner:

$$\delta\rho_{\rm lad}^{(3)}(\mathbf{k}'_\parallel,z=-0;\mathbf{k}'_\parallel,z'=-0;\omega)=\frac{2\pi S}{k_\omega^2\mu^2 l_{\rm el}^*}{\rm Re}\int dz\,dz'\psi^*_{\mathbf{k}_\omega^{(-)}}(z)\psi^*_{\mathbf{k}}(z')\int\frac{d\mathbf{q}_\parallel dq_z}{(2\pi)^3}D(\mathbf{q}_\parallel,q_z,\omega)[\delta(z-z')+\Gamma^{\rm inel}(\mathbf{q}_\parallel;z,z')]$$
$$\times A(E-\hbar\omega;\mathbf{q}_\parallel,q_z;z')B^*(E;\mathbf{q}_\parallel,q_z;z)\ . \tag{36}$$

The fan contribution is

$$\delta\rho_{\rm fan}^{(3)}(\mathbf{k}'_\parallel,z=-0;\mathbf{k}'_\parallel,z'=-0;\omega)=\frac{2\pi S}{k_\omega^2\mu^2 l_{\rm el}^*}{\rm Re}\int dz\,dz'\psi^*_{\mathbf{k}_\omega^{(-)}}(z)\psi^*_{\mathbf{k}}(z)\Gamma^{\rm inel}(\mathbf{k}_\parallel+\mathbf{k}'_\parallel;z,z')$$
$$\times\int\frac{d\mathbf{q}_\parallel dq_z}{(2\pi)^3}D(\mathbf{q}_\parallel,q_z,\omega)A(E-\hbar\omega;\mathbf{q}_\parallel,q_z;z')B^*(E;\mathbf{q}_\parallel,q_z;z')\ . \tag{37}$$

We note that in Eqs. (36) and (37) $\Gamma^{\rm inel}(\mathbf{Q}_\parallel;z,z')=\Gamma(\omega,\mathbf{Q}_\parallel;z,z')$. By means of Eqs. (15), (23), (24), (30)–(37), and the results of Appendix A one can write the following expression for the angular spectrum of the electrons reflected from the disordered medium that have lost energy $\hbar\omega$ due to a single inelastic collision:

$$J_{\text{inel}}(\mu_0 \to |\mu|,\omega) = \frac{\pi}{v l_{\text{el}}^*} \left[\frac{\hbar^2}{m} \right]^2 \int \frac{d\mathbf{q}_\parallel dq_z}{(2\pi)^3} W_{\text{inel}}(\mathbf{q}_\parallel, q_z, \omega)$$
$$\times \int dz \int dz' ([\delta(z-z') + \Gamma^{\text{el}}(0;z,z')]$$
$$\times \{ |\psi_{\mathbf{k}_\omega^{(-)}}(z)|^2 |\psi_{\mathbf{k}}(z')|^2 [F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v};z',z') + F_{\omega\mathbf{q}}(\mathbf{v}',\mathbf{v}';z,z)] \}$$
$$+ \Gamma^{\text{el}}(\mathbf{k}_\parallel + \mathbf{k}'_\parallel + \mathbf{q}_\parallel; z,z') \psi^*_{\mathbf{k}_\omega^{(-)}}(z) \psi_{\mathbf{k}_\omega^{(-)}}(z') \psi^*_{\mathbf{k}}(z) \psi_{\mathbf{k}}(z') F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v};z,z')$$
$$+ \Gamma^{\text{el}}(\mathbf{k}_\parallel + \mathbf{k}'_\parallel - \mathbf{q}_\parallel; z,z') \psi^*_{\mathbf{k}_\omega^{(-)}}(z) \psi_{\mathbf{k}_\omega^{(-)}}(z') \psi^*_{\mathbf{k}}(z) \psi_{\mathbf{k}}(z') F_{\omega\mathbf{q}}(\mathbf{v}',\mathbf{v}';z',z)$$
$$-2\,\text{Re}\{ [\delta(z-z') + \Gamma^{\text{inel}}(\mathbf{q}_\parallel; z,z')] |\psi_{\mathbf{k}_\omega^{(-)}}(z)|^2 |\psi_{\mathbf{k}}(z')|^2 F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}';z,z') \}$$
$$-2\,\text{Re}[\Gamma^{\text{inel}}(\mathbf{k}_\parallel + \mathbf{k}'_\parallel; z,z') \psi^*_{\mathbf{k}_\omega^{(-)}}(z) \psi_{\mathbf{k}_\omega^{(-)}}(z') \psi^*_{\mathbf{k}}(z) \psi_{\mathbf{k}}(z')$$
$$\times F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}';z',z')]) \; . \tag{38}$$

Here

$$F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}';z,z') = \frac{\exp[-iq_z(z-z')]}{\hbar^2(\omega+\mathbf{v}\mathbf{q})(\omega+\mathbf{v}'\mathbf{q})} \times \left\{ 1 - \exp\left[iz\frac{\omega+\mathbf{v}'\mathbf{q}}{v'_z} \right] \right\} \times \left\{ 1 - \exp\left[-iz'\frac{\omega+\mathbf{v}\mathbf{q}}{v_z} \right] \right\} . \tag{39}$$

In comparison with the result obtained in Ref. 12, the expression given by Eq. (38) takes into account the correct boundary conditions for the electron propagator in the inelastic-scattering channel and the interference between multiple elastic and single inelastic collisions. The detailed calculations of the angular spectrum will be carried out in the following section, whereas some general conclusions can immediately be made from Eq. (38). First, the enhanced peak exists in the inelastic-scattering channel irrespective of the wave vector transferred to the inelastic excitation, if the inequality $\omega l/v \ll 1$ is fulfilled. Second, unlike in pure elastic backscattering the localization peak turns out to be displaced from the exactly backward direction. To see this, let us concentrate on Eq. (38).

The first and second terms in Eq. (38), which contain $F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v};z',z')$ and $F_{\omega\mathbf{q}}(\mathbf{v}',\mathbf{v}';z,z)$, describe the contribution to the spectrum from trajectories in which the elastic and inelastic collisions are strictly successive to each other [see Figs. 3(a) and 3(b)]. These terms introduce no coherent effects in the angular spectrum, and correspond to the incoherent background. Their structure enables one to treat the function

$$g_{\text{inel}}(z) = g_{\text{el}}(z) \int d\mathbf{q}\, W_{\text{inel}}(\mathbf{q},\omega) F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v};z,z) \tag{40}$$

as a distribution of effective radiation sources in the inelastic-scattering channel. Here

$$g_{\text{el}}(z) = |\psi_{\mathbf{k}}(z)|^2 = \exp(-z/2l\mu_0)$$

is the source distribution in the elastic scattering channel.[17] From Eq. (40) we can conclude that the source function g_{inel} differs from g_{el} by an additional integral factor, which depends on the type of inelastic collision. The physical sense of Eq. (40) is transparent enough. The integral factor as a function of the variable z describes the transitions of electrons *into* the coherent inelastic wave field due to the inelastic collisions. At the same time, the exponential function g_{el} describes the ejection of electrons *from* the coherent inelastic wave field due to both elastic and inelastic scattering. Since the source function g_{inel} can differ significantly from that for the elastic-scattering channel, the behavior of the incoherent background in the angular spectrum of inelastically backscattered electrons can differ from the angular background in the elastic channel as well.

The third and fourth terms in Eq. (38), containing $\Gamma^{\text{el}}(\mathbf{k}_\parallel + \mathbf{k}'_\parallel \pm \mathbf{q}_\parallel; z,z')$, are associated with the coherent effects in the inelastic backscattering spectra that arise due to the interference of wave fields before and after the inelastic collision, respectively [Figs. 3(c) and 3(d)]. The appearance of $\mathbf{q}_\parallel$ in the argument of Γ^{el} means that this type of interference depends strongly on the wave vector transferred to the inelastic excitation. In Refs. 12 and 18 it has been shown that this type of coherent feature with an angular width $\Delta\theta_l \sim 1/kl$ occurs only when $q_c l \ll 1$ (here q_c is the characteristic wave vector transferred in the single inelastic collision), and vanishes as the magnitude of $q_c l$ increases. These features become negligible

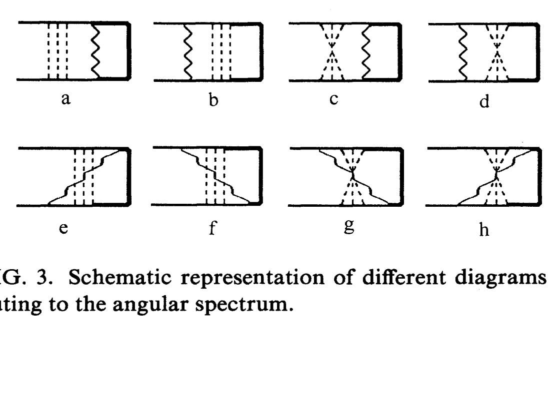

FIG. 3. Schematic representation of different diagrams contributing to the angular spectrum.

when $q_c l > 1$.

The fifth term, containing $F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}';z,z')$, arises from the interference of multiple elastic and single inelastic collisions, when the elastic collisions can be considered as strictly successive [Figs. 3(e) and 3(f)]. This type of coherent effect was studied in Ref. 10, where it has been found that these trajectories lead to the appearance of features in the angular spectra with a width of the order of γ/ω, which is very wide in comparison with the width of the usual weak-localization peak, when the energy loss is much smaller than the electron energy ($\hbar\omega \ll E$). This means that the term in question must be related to the incoherent background in the vicinity of the localization peak.

The last term in Eq. (38) is the most interesting, and it represents the main result of our study. This term is due to the interference of multiple elastic scattering and a single inelastic collision, which are completely nonsuccessive [Figs. 3(g) and 3(h)]. Its integrand contains the factor

$$\Gamma^{\mathrm{inel}}(\mathbf{k}_\parallel+\mathbf{k}'_\parallel;z,z')\psi^*_{\mathbf{k}'^{(-)}_\omega}(z)\psi_{\mathbf{k}'^{(-)}_\omega}(z')\psi^*_{\mathbf{k}}(z)\psi_{\mathbf{k}}(z') ,$$

which in the case of pure elastic backscattering of incident waves with different energies (E and $E-\hbar\omega$) yields, after integration over z and z', an enhanced peak displaced from the retroreflection direction. In our case, when the wave fields are due to the inelastic processes in the medium, this factor is multiplied by the function

$$\int d\mathbf{q}\, W_{\mathrm{inel}}(\mathbf{q},\omega)F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}';z',z') .$$

Since Γ^{inel} does not depend on $\mathbf{q}_\parallel$, the localization peak always exists in the limit $\omega l/v \ll 1$ independently of the wave vector transferred in a single inelastic collision. The shape of the peak is determined by the particular type of inelastic collision.

V. BACKSCATTERING ENHANCEMENT IN INELASTIC-SCATTERING CHANNEL

To forward the analytical calculations, we shall consider below two opposite cases of the single inelastic collision: (i) $q_c l \ll 1$ and (ii) $1 \ll q_c l \ll kl$. The first limit is associated with an inelastic collision in which the characteristic scattering angle $\Delta\theta_s \sim q_c/k$ is much smaller than the range of the weak-localization angles $\Delta\theta_l \sim 1/kl$. The second limit corresponds to the scattering in which $\Delta\theta_s \gg \Delta\theta_l$.

A. The case $q_c l \ll 1$

Under this condition the function $F_{\omega\mathbf{q}}$ in Eq. (38) can be simplified. We note that the exponential factors $\exp(-z/l)$ and $\exp(-z'/l)$ appear in the integrands. This means that the main contribution to the integrals arises from the regions of z and z' that are bounded from above by values of the order of l. Under the conditions $\omega l/v \ll 1$ and $q_c l \ll 1$ this allows us to factorize the function $F_{\omega\mathbf{q}}$ in the integrands of Eq. (38) as follows:

$$F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}';z,z') = \frac{zz'}{\hbar^2 v_z v'_z} . \tag{41}$$

Keeping in mind the discussion in Sec. IV, we can carry out the separation of the angular spectrum, Eq. (38), into a background part and a coherent part in the following way:

$$J^{\mathrm{inc}}_{\mathrm{inel}}(\mu_0\to|\mu|,\omega) = J_0\int_0^\infty d\tau\int_0^\infty d\tau' \exp(-\tau/|\mu|)\exp(-\tau'/\mu_0) \times\{(\tau^2+\tau'^2)[\delta(\tau-\tau')+\Gamma^{\mathrm{el}}(0;\tau,\tau')]+2\tau\tau'[\delta(\tau-\tau')+\mathrm{Re}\Gamma^{\mathrm{inel}}(0;\tau,\tau')]\} , \tag{42}$$

$$J^{\mathrm{coh}}_{\mathrm{inel}}(\mu_0\to|\mu|,\omega) = 2J_0\mathrm{Re}\int_0^\infty d\tau\int_0^\infty d\tau' \exp(-\tau/\tilde{\mu})\exp(-\tau'/\tilde{\mu}^*)\{\tau\tau'\Gamma^{\mathrm{el}}(\mathbf{k}_\parallel+\mathbf{k}'_\parallel;\tau,\tau')+\tau'^2\Gamma^{\mathrm{inel}}(\mathbf{k}_\parallel+\mathbf{k}'_\parallel;\tau,\tau')\} . \tag{43}$$

Here $\Gamma(\mathbf{Q};\tau,\tau') = l\Gamma(\mathbf{Q};z,z')$, $\tau = z/l$, and $\tau' = z'/l$ are normalized coordinates, and

$$(\tilde{\mu})^{-1} = \frac{1}{2}\left[\frac{1}{\mu_0}+\frac{1}{|\mu|}\right] + il(k\mu_0 - k_\omega|\mu|) ,$$

$$J_0 = \frac{\pi}{\mu_0^2 l^*_{\mathrm{el}}}\left[\frac{l}{v}\right]^3 \int\frac{d\mathbf{q}}{(2\pi)^3} W_{\mathrm{inel}}(\mathbf{q},\omega) .$$

Using the diffusion approximation for the Γ matrix derived in Sec. III, we can obtain an analytical result for the enhancement factor $\eta = 1 + J^{\mathrm{coh}}_{\mathrm{inel}}/J^{\mathrm{inc}}_{\mathrm{inel}}$. These calculations lead to a rather unwieldy expression that we do not present.

The enhancement factor η is shown in Fig. 4. The shape of the peak has a pronounced triangular form, and is qualitatively close to that in the elastic-scattering channel, but it is displaced relative to the exactly backward

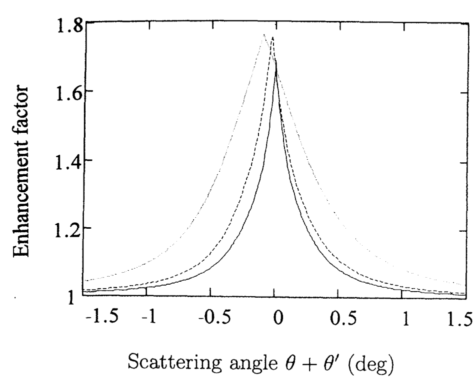

FIG. 4. Enhancement factor η in the case $q_c l \ll 1$. $E = 500$ eV, $\hbar\omega = 1$ eV, $l = 15$ Å, $l/l^*_{\mathrm{el}} = 0.95$. Solid line, incidence angle $\theta = 0°$; dashed line, $\theta = 30°$; dotted line, $\theta = 60°$.

direction in the case of oblique incidence of the electrons. The angular magnitude of the displacement $\Delta\vartheta$ is defined by the equation $|\mathbf{k}_\parallel+\mathbf{k}'_\parallel|=0$, from which for small $\Delta\vartheta$ one obtains

$$\Delta\vartheta=\frac{\hbar\omega}{2E}\tan\theta \; . \qquad (44)$$

The displacement $\Delta\vartheta$ depends on the energy loss $\hbar\omega$, the energy E of incident electrons, and the incidence angle θ. The width of the peak is about $1/kl$ for normal electron incidence, and depends on θ, increasing as $1/\cos\theta$ under oblique incidence.

B. The case $1 \ll q_c l \ll kl$

The growth of the parameter $q_c l$ causes the destruction of the coherent effects in the angular spectrum which are associated with the diagrams shown in Figs. 3(c) and 3(d). This demolition process has been analyzed in Ref. 18. In a mathematical sense the destruction is caused by the diminution of the contribution of the terms containing $\Gamma^{\text{el}}(\mathbf{k}_\parallel+\mathbf{k}'_\parallel\pm\mathbf{q}_\parallel;z,z')$ to the coherent part of the angular spectrum. Namely, the appearance of Γ^{el} with $\mathbf{q}_\parallel$ in the first argument limits the region of integration $\Omega'_{\mathbf{q}}\sim\int d\mathbf{q}$ in $\mathbf{q}$ space, which contributes substantially to the spectrum, to the region $\Omega'_{\mathbf{q}}\sim q_c l^{-2}$. Since in the absence of $\mathbf{q}_\parallel$ in the first argument of Γ^{el} the integration region can be estimated as $\Omega_{\mathbf{q}}\sim q_c^3$, the contributions of diagrams 3(c) and 3(d) are small and of the order of $\Omega'_{\mathbf{q}}/\Omega_{\mathbf{q}}\sim(q_c l)^{-2}$ compared with the contributions of diagrams 3(a) and 3(b). Analogous estimates hold for that part of the contribution from the inelastically crossed diagrams 3(e) and 3(f) that is proportional to $\Gamma^{\text{inel}}(\mathbf{q}_\parallel;z,z')$.

At the same time, the diagrams shown in Figs. 3(g) and 3(h) contribute significantly in the scattering regime $q_c l \gg 1$. This means that coherent angular features will remain in spite of the smallness of diagrams 3(c) and 3(d) already noted above. Therefore we can write the following expression for the background and coherent components of the angular spectrum:

$$J^{\text{inc}}_{\text{inel}}(\mu_0\to|\mu|,\omega)=J_0\left\{\int_0^\infty d\tau\exp\left[-\tau\left[\frac{1}{\mu_0}+\frac{1}{|\mu|}\right]\right][F(\mathbf{v},\mathbf{v},\tau)+F(-\mathbf{v},-\mathbf{v},\tau)-2\,\mathrm{Re}F(-\mathbf{v},\mathbf{v},\tau)]\right.$$
$$\left.+\int_0^\infty d\tau\int_0^\infty d\tau'\exp(-\tau/|\mu|)\exp(-\tau'/\mu_0)\Gamma^{\text{el}}(0;\tau,\tau')[F(\mathbf{v},\mathbf{v},\tau')+F(-\mathbf{v},-\mathbf{v},\tau)]\right\}, \qquad (45)$$

$$J^{\text{coh}}_{\text{inel}}(\mu_0\to|\mu|,\omega)=-2J_0\mathrm{Re}\int_0^\infty d\tau\,d\tau'\exp(-\tau/\tilde{\mu})\exp(-\tau'/\tilde{\mu}^*)\Gamma^{\text{inel}}(\mathbf{k}_\parallel+\mathbf{k}'_\parallel;\tau,\tau')F(\mathbf{v},-\mathbf{v},\tau') \; . \qquad (46)$$

Here $J_0=\pi\hbar^2 l/vm^2 l^*_{\text{el}}$ and function F is defined as

$$F(\mathbf{v},\mathbf{v}';\tau)=\hbar^2\int\frac{d\mathbf{q}}{(2\pi)^3}W_{\text{inel}}(\mathbf{q},\omega)F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}';\tau l,\tau l) \; . \qquad (47)$$

Using the diffusion approximation Eq. (22a), the angular spectrum of inelastically backscattered electrons can be expressed through the Laplace transform F_L of the function F,

$$J^{\text{inc}}_{\text{inel}}(\mu_0\to|\mu|,\omega)=J_0\left\{2F_L\left[\mathbf{v},\mathbf{v};\frac{1}{\mu_0}+\frac{1}{|\mu|}\right]-2\,\mathrm{Re}F_L\left[\mathbf{v},-\mathbf{v};\frac{1}{\mu_0}+\frac{1}{|\mu|}\right]\right.$$
$$\left.+3\left[\mu^{-2}F_L\left[\mathbf{v},\mathbf{v};\frac{1}{\mu_0}\right]+\mu_0^{-2}F_L\left[\mathbf{v},\mathbf{v};\frac{1}{|\mu|}\right]-(\mu^{-2}+\mu_0^{-2})F_L\left[\mathbf{v},\mathbf{v};\frac{1}{\mu_0}+\frac{1}{|\mu|}\right]\right]\right\}, \qquad (48)$$

$$J^{\text{coh}}_{\text{inel}}(\mu_0\to|\mu|,\omega)=6J_0\mathrm{Re}\left\{[\xi^2-(\tilde{\mu})^{-2}]^{-1}\left[F_L(\mathbf{v},-\mathbf{v};(\tilde{\mu}^*)^{-1}+\xi)-F_L\left[\mathbf{v},-\mathbf{v};\frac{1}{\mu_0}+\frac{1}{|\mu|}\right]\right]\right\}, \qquad (49)$$

where

$$\xi=[\beta^2+l^2(\mathbf{k}_\parallel+\mathbf{k}'_\parallel)^2]^{1/2} \; ,$$

and $\mathrm{Re}\xi>0$.

The Laplace transforms of the functions F are calculated in Appendix B. It can be derived from Eqs. (48) and (49) and Eqs. (B4) and (B5) of Appendix B that the enhancement factor η is given by the formula

$$\eta=1+\mu_0^4\frac{\mathrm{Im}\{[1/(1-\xi^2\mu_0^2)][\Lambda(\omega+\gamma\xi''\mu_0,\gamma(1+\mu_0\xi'))/(1+\xi\mu_0)^2-\Lambda(\omega,2\gamma)/4]\}}{\mathrm{Im}\{\Lambda(\omega,\gamma)+[(2\mu_0^2-3)/12]\Lambda(\omega,2\gamma)\}} \; . \qquad (50)$$

Here $\xi' = \mathrm{Re}\xi$, $\xi'' = \mathrm{Im}\xi$, and the function

$$\Lambda(\omega,\gamma) = \int \frac{d\mathbf{q}}{(2\pi)^3} W_{\mathrm{inel}}(\mathbf{q},\omega) \frac{1}{\omega + \mathbf{vq} - i\gamma} \tag{51}$$

is associated with a specific inelastic collision by means of the probability function $W_{\mathrm{inel}}(\mathbf{q},\omega)$; $\gamma = v/l$ is a collision frequency. Equation (50) shows that the shape of η depends on the function W_{inel}, which is different for the different types of inelastic scattering.

For example, in the case of plasmon excitations[19] or excitations of electrons in an atom in the dipole approximation[20] the probability of inelastic collision per unit time $W_{\mathrm{inel}}(\mathbf{q},\omega) \sim q^{-2}\Theta(q_m - q)$. For plasmon excitation q_m is the cutoff wave vector, and for excitation of atomic electrons $q_m \sim 1/a$, where a is a length of the order of the atomic size. In this case $\Lambda(\omega,\gamma)$ can be calculated analytically[21] in the limit $q_m l \gg 1$ and $\omega/\gamma \ll 1$,

$$\Lambda(\omega,\gamma) = \arctan(\omega/\gamma) + i \ln\left[\frac{v q_m}{(\omega^2+\gamma^2)^{1/2}}\right] .$$

The behavior of the enhancement factor under the above-mentioned assumption about W_{inel} is shown in Fig. 5. As can be seen, the magnitude of the enhancement effect is smaller than in the case of $q_c l \ll 1$ (see Fig. 4), but its width is twice as large as that for the case $q_c l \ll 1$, so that the areas under the localization peak are approximately equal in both cases. The peak shape can have a complicated form, with small maxima that are defined by the nature of the single inelastic collision. Note that for oblique incidence the maximum of the enhanced peak is displaced from the exactly backward direction in accordance with Eq. (44).

The additional calculations show that enhancement decreases with the ratio l/l^*_{el}. This means that for the observation of weak-localization effects in the inelastic-scattering channel a weak dissipation ($l_{\mathrm{inel}} \gg l_{\mathrm{el}}$) is needed, which can be achieved by the "freezing" of the disordered system.

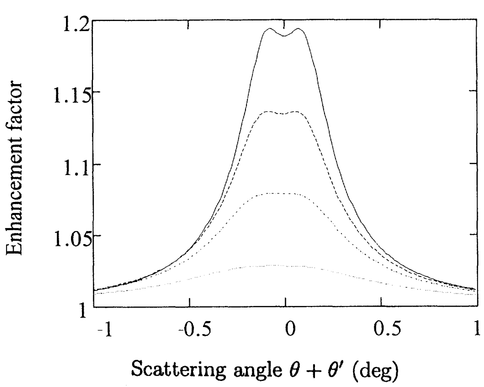

FIG. 5. Enhancement factor η in the case $q_c l \gg 1$. $E = 500$ eV, $\hbar\omega = 0.5$ eV, $l = 20$ Å, $l/l^*_{\mathrm{el}} = 0.99$. Solid line, $\theta = 0°$; dashed line, $\theta = 30°$; dot-dashed line, $\theta = 45°$; dotted line, $\theta = 60°$.

VI. CONCLUSION

The results obtained lead to the conclusion that coherent effects exist in the angular spectrum of moderate-energy electrons ($E = 100-1000$ eV) reflected inelastically from a disordered medium with fixed energy loss. The enhanced peak is shown to exist irrespective of the wave vector transferred in a single inelastic collision. In distinction to the well-studied backscattering enhancement in the elastic channel, the shape of weak-localization features in the angular spectrum of inelastic backscattering is more complicated and depends on the nature of a single inelastic collision. In the case of non-normal incidence of electrons, the center of the enhanced peak is displaced relative to the exactly backward direction. Since the coherent phenomena in an inelastic channel exist independently of the wave vector of the excitation, they can be considered as a possible tool for the experimental study of the wide class of low-energy excitations in disordered media.

APPENDIX A

Let us calculate the function A, defined by Eq. (32). Keeping in mind Eq. (16), we can represent $G(E-\hbar\omega;\mathbf{k}_\parallel+\mathbf{q}_\parallel,p_z)$ in the following way:

$$G(E-\hbar\omega;\mathbf{k}_\parallel+\mathbf{q}_\parallel,p_z) = -\frac{2m}{\hbar^2}[p_z - P^{(+)}(\mathbf{q}_\parallel,\omega)]^{-1} \times [p_z + P^{(+)}(\mathbf{q}_\parallel,\omega)]^{-1} . \tag{A1}$$

$P^{(+)}(\mathbf{q}_\parallel,\omega)$ is that branch of the double-value complex function

$$P(\mathbf{q}_\parallel,\omega) = [k^2 - 2m\omega/\hbar - (\mathbf{k}_\parallel+\mathbf{q}_\parallel)^2 + ik/l]^{-1/2} \tag{A2}$$

for which $\mathrm{Im}P > 0$. In the limit of small-angle inelastic collision ($q \ll k$) $P^{(+)}$ can be represented in the form

$$P^{(+)}(\mathbf{q}_\parallel,\omega) = k_z - \frac{\omega + \mathbf{v}_\parallel \mathbf{q}_\parallel}{v_z} + \frac{i}{2l\mu_0} . \tag{A3}$$

The integral in Eq. (32) is defined by the residue of the integrand at the points $p_z = k_z + q_z + i/2l\mu_0$ and $p_z = P^{(+)}$. The integration contour is closed in the upper half plane of the complex variable p_z due to the finiteness of the function A when $z \to \infty$. As a result we obtain

$$A(E-\hbar\omega;\mathbf{q}_\parallel,q_z;z) = -\frac{\exp[-iq_z z]}{\hbar(\omega+\mathbf{vq})}\left\{1 - \exp\left[-iz\frac{\omega+\mathbf{vq}}{v_z}\right]\right\}\psi_{\mathbf{k}}(z) . \tag{A4}$$

The calculation of the function B defined by Eq. (35) is completely analogous to that of the function A, except that the integration contour must be closed in the lower half plane of the complex variable p_z. Carrying out these calculations, we obtain with the accuracy $\hbar\omega/E \ll 1$

$$B(E;\mathbf{q}_\parallel,q_z;z) = \frac{\exp[iq_z z]}{\hbar(\omega+\mathbf{v}'\mathbf{q})}\left\{1 - \exp\left[-z\frac{\omega+\mathbf{v}'\mathbf{q}}{v'_z}\right]\right\}\psi^*_{\mathbf{k}'^{(-)}_\omega}(z) . \tag{A5}$$

APPENDIX B

Let us calculate the Laplace transform

$$F_L(\mathbf{v},-\mathbf{v};p)=\int_0^\infty d\tau\, F(\mathbf{v},-\mathbf{v};\tau)\exp(-p\tau) \tag{B1}$$

of the function $F(\mathbf{v},-\mathbf{v};\tau)$ using Eq. (47):

$$F_L(\mathbf{v},-\mathbf{v};p)=\int\frac{d\mathbf{q}}{(2\pi)^3}W_{\text{inel}}(\mathbf{q},\omega)\frac{1}{\omega^2-(\mathbf{vq})^2}\left[\frac{1}{p}+\frac{1}{p+2il\omega/v_z}-\frac{1}{p+il(\omega-\mathbf{vq})/v_z}-\frac{1}{p+il(\omega+\mathbf{vq})/v_z}\right]. \tag{B2}$$

If we assume that $W_{\text{inel}}(\mathbf{q},\omega)=W_{\text{inel}}(|\mathbf{q}|,\omega)$, Eq. (B2) can be transformed as follows:

$$F_L(\mathbf{v},-\mathbf{v};p)=2i\frac{l}{v_z}\frac{1}{p(p+2i\omega l/v_z)}\times\int\frac{d\mathbf{q}}{(2\pi)^3}W_{\text{inel}}(\mathbf{q},\omega)\frac{1}{\omega+\mathbf{vq}-ipv_z/l}. \tag{B3}$$

This integral can be rewritten by means of Eq. (51) in the final form

$$F_L(\mathbf{v},-\mathbf{v};p)=2i\frac{l}{v_z}\frac{1}{p(p+2i\omega l/v_z)}\times\Lambda(\omega+p''v_z/l,p'v_z/l), \tag{B4}$$

where $p'=\text{Re}p$ and $p''=\text{Im}p$.

Similar calculations lead to the following expression for the Laplace-transform of the function $F(\mathbf{v},\mathbf{v};\tau)$:

$$F_L(\mathbf{v},\mathbf{v};p)=\frac{il}{v_zp^2}[\Lambda(\omega-p''v_z/l,-p'v_z/l)-\Lambda(\omega+p''v_z/l,p'v_z/l)]. \tag{B5}$$

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