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Electron states in a quasi-one-dimensional charge channel over liquid helium in the presence of a transverse magnetic field

Sviatoslav S. Sokolov

Departamento de Física, Universidade Federal de São Carlos, 13565-905 São Carlos, São Paulo, Brazil and B. I. Verkin Institute for Low Temperature Physics and Engineering, Academy of Sciences of Ukraine, 310164 Kharkov, Ukraine

Nelson Studart

Departamento de Física, Universidade Federal de São Carlos, 13565-905 São Carlos, São Paulo, Brazil

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Calculations are carried out for single-particle energy levels and wave functions for electrons in a quasi-one-dimensional charge channel formed on the surface of liquid helium. The expressions for the spectrum are obtained for two different orientations of the magnetic field perpendicular to the channel. The conditions for the applicability of the formulas derived are established and they can be used in experimental studies of spectroscopic transitions between the confined states and the electron transport of the system.

There has been a lot of interest in recent years in the study of electrical and optical properties of one-dimensional semiconductor nanostructures, mainly due to the great technological progress in crystal growth, lithography, and etching processes. A regime for transport electron was found where the quantization of the conductance is proof of the ballistic one-dimensional transport[1] and the character of the one-dimensional density of states of the electron system was mapped out in tunneling spectroscopy studies.[2] On the other hand, a quasi-one-dimensional system (Q1D) can also be realized by creating a solitary channel of high-mobility electrons on a helium surface strongly distorted by capillary forces due to a substrate formed by two dielectric polymer sheets forming a sharp angle, as shown in Fig. 1.[3,4] The physical realization of this system opened as a possibility of studying different phenomena in the Q1D electron system. Such a system has all the advantages, which are typical for surface electrons (SE) on helium such as the cleanness, homogeneity, and the possibility of a wide variation of the electron density and the holding electric field, in contrast with dirty semiconductor structures where the electron properties are strongly influenced by spatial inhomogeneities, impurity scattering, and so on.

The electrons are free to move along the channel (x axis) but are confined by the potential due to the holding field $E_\perp$ along the z direction and the polarization forces coming from the helium surface and the substrate. In the z direction the confining potential is very well known and generates the two-dimensional subbands. In the y direction the potential, for small deviations of the electron from the bottom of the channel ($y=0$) can be written as

$$V_y(y)=\frac{m\omega_0^2 y^2}{2} \quad , \tag{1}$$

where $\omega_0^2=eE_\perp/mR$, with R the curvature radius of the liquid in the channel ($10^{-4}-10^{-3}$ cm), and m and e are the mass and the charge of the electron, respectively. One can see that only by varying the holding field, one changes significantly the strength of the electron confinement along the y and z directions, and the spectroscopy of the electron states can be successfully determined. Equation (1) is valid when the condition $y \ll R$ is satisfied. The harmonic potential gives the following spectrum:[3,5]

$$E_{n,l}(k_x)=\frac{\hbar^2 k_x^2}{2m}+(n+\tfrac{1}{2})\hbar\omega_0+\Delta_l \quad , \tag{2}$$

and

$$\Psi_{n,l,k_x}=\frac{1}{\sqrt{L_x}}\exp(ik_x x)\chi_n(y)f_l(z) \quad , \tag{3}$$

where k_x is the electron wave vector, L_x is the size of the system in the x direction, $\chi_n(y)$ is the Hermite function, and $n=0,1,2,\ldots$. The energies Δ_l ($l=1,2,3,\ldots$) and wave functions $f_l(z)$ refer to the confinement in the z direction. For $E_\perp \geq 300$ V/cm, the probability of an electron makes

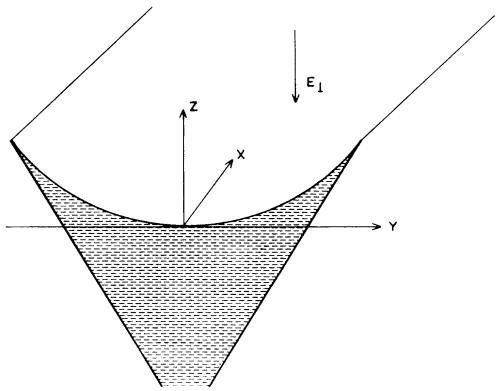

FIG. 1. Schematic view of the channel filled by liquid helium between two dielectric planes. Electrons are free to move along the x axis and the holding field $E_\perp$ is in the z direction.

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transitions from the level $l=1$ to levels $l\geq 2$ negligible. The mean distance of the electron from the bottom of the channel in the z axis for $l=1$ satisfies the condition $\langle z\rangle_1 \ll R$ and we can restrict ourselves to both the energy Δ_1 and wave function $f_1(z)$ obtained for surface electrons over a flat surface of helium.[6]

The aim of the present work is to describe the effect of a transverse magnetic field B on the electron states in the Q1D conducting channel over liquid helium. In the case of SE, we know that the application of a magnetic field perpendicular to the helium surface simply modifies the electron motion in the plane, but does not directly affect the spectroscopic transitions in the z direction. On the other hand, if the field is applied at a certain angle to the surface, then $B_{\|}$ couples the in-plane motion with the spectroscopic transitions while $B_{\perp}$ forces an in-plane periodic motion at the cyclotron frequency $\omega_c = eB/mc$. The Lorentz force due to $B_{\|}$ is equivalent to an oscillating holding field $E_{\perp}$ at frequency ω_c. This modulation of the splitting between the bound states in the z direction produces satellite lines spaced on either side of a transition by integral multiples of ω_c.[7–9] In the Q1D case, due to the confinement of the two degrees of freedom, the application of the magnetic field in a transverse direction couples the spectroscopic transitions in the two directions of confinement. In contrary to the case when B is applied in the direction parallel to the electron motion, by switching on the magnetic field in directions normal to the x axis leads to a mixing of electron states along the direction in which the motion is free and along the direction in which the motion is confined in the absence of the magnetic field. As a result, drastic changes both in the energy spectrum as well as in the electron wave functions take place.

A. Magnetic field applied in the z direction

In this case, by taking the Landau gauge for the vector potential as $\vec{A}=(-By,0,0)$, we can write the electron Hamiltonian as

$$H=\frac{1}{2m}\left[\left(p_x+\frac{eBy}{c}\right)^2+p_y^2+p_z^2\right]+V_y(y)+V_z(z) \quad , \quad (4)$$

where p_i is the operator of the corresponding component of the momentum and $V_z(z)$ is the confining potential in the z direction, which consists of the attractive image potential, a repulsive barrier at the interface and a linear term from the holding electric field. As in the case of SE on helium, the magnetic field does not affect the motion in the z direction. The solution of the Schrödinger equation to the Hamiltonian given by Eq. (4) leads to the following expression for the energy spectrum:

$$E_{n,l}(k_x)=\frac{\hbar^2\omega_0^2k_x^2}{2m\Omega^2}+(n+\tfrac{1}{2})\hbar\Omega+\Delta_l \quad , \quad (5)$$

where the frequency $\Omega=\sqrt{(\omega_c^2+\omega_0^2)}$ indicates the hybrid effect between the geometric and magnetic confinement. The wave function in the y direction is written as

$$\chi_n(y)=\frac{1}{\sqrt{2^n n!\,\pi^{1/2}l_c^*}}\exp\left(-\frac{(y-Y)^2}{2l_c^{*\,2}}\right)H_n\left(\frac{y-Y}{l_c^*}\right) \quad , \quad (6)$$

where $l_c^{*2}=\hbar/m\Omega$ is the renormalized magnetic length, $Y=-\hbar\omega_c k_x/m\Omega^2$ is the y coordinate of the center of the electron orbit, and $H_n(x)$ is the Hermite function. Equations (5) and (6) are very similar to the expressions derived for a two-dimensional electron system in the presence of a uniform positive one-dimensional charge density, which also leads to a parabolic potential.[10] However, in the present case, the potential given by Eq. (1) is valid only for $y\ll R$, and a careful analysis of the applicability of the equations obtained is imperative. Then we first calculate the mean square of the displacement of the electron in the y direction, which can be written as

$$\langle y^2\rangle_n=(n+1/2)\,l_c^{*2}+\frac{\hbar^2\omega_c^2k_x^2}{m^2\Omega^4} \quad . \quad (7)$$

The first term is smaller than the mean square displacement at $B=0$, given by $\langle y_0^2\rangle_n=(n+1/2)y_0^2$, where $y_0=\sqrt{\hbar/m\omega_0}$ is the localization length in the y direction. The second term has a maximum at $\omega_c=\omega_0$ and the major contribution of this term to $\langle y^2\rangle_n$ at $\omega_c=\omega_0$ is approximately the same as $\langle y_0^2\rangle_0$ for temperatures around 1 K and for holding fields $E_{\perp}\leq 3000$ V/cm.[5] So, the inequality $\sqrt{\langle y^2\rangle_n}\ll R$ is satisfied even for large enough n, which gives strong evidence that Eqs. (5) and (6) can be used for describing the electron states in Q1D channels on liquid helium in a wide range of holding fields. In the limit of $\omega_c\ll\omega_0$, we reproduce the results of zero magnetic field given by Eq. (2). In the opposite limit, $\omega_0\to 0$ (for a given $E_{\perp}$, this limit can be achieved as $R\to\infty$), we obtain the familiar result of the Landau quantization of electron motion in the x-y plane. Finally, we observe that in order to have $|Y|<L_y$, where L_y is the characteristic size of the system in the y direction, the possible values of k_x must fulfill the condition $|k_x|<m\Omega^2L_y/\hbar\,\omega_c$.

B. Magnetic field applied in the y direction

Under such orientation of the magnetic field, the most convenient gauge for the vector potential is $\vec{A}=(Bz,0,0)$ which leads to the electron Hamiltonian given by

$$H=\frac{1}{2m}\left[\left(p_x-\frac{eBz}{c}\right)^2+p_y^2+p_z^2\right]+V_y(y)+V_z(z) \quad . \quad (8)$$

In this case the magnetic field does not influence the electron motion along the y direction. As in the previous situation ($B\,\|\,z$) the x component of the electron momentum is conserved and the wave function has the same form as that given in Eq. (3) with $\chi_n(y)$ expressed by the same function as in the case $B=0$.[3] However the wave function $f_l(z)$ satisfies the following equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 f_l(z)}{\partial z^2}+\left[-\frac{\Lambda_0}{z}+eE_{\perp}^*z+\frac{m\omega_c^2z^2}{2}\right]f_l(z)=\epsilon_l f_l(z) \quad , \quad (9)$$

where $E_{\perp}^*=E_{\perp}-\hbar\omega_c k_x/e$ is an effective holding field which incorporates the motion in the x direction. The total

energy spectrum is the same as in Eq. (3) with $\Delta_l=\epsilon_l$. In Eq. (9), we have used the explicit form of the potential $V_z(z)$, given by[11,12]

$$V_z(z)=-\frac{\Lambda_0}{z}+eE_\perp^* z \ , \tag{10}$$

with $\Lambda_0=(e^2/4)(\varepsilon_{\rm He}-1)/(\varepsilon_{\rm He}+1)$, $\varepsilon_{\rm He}$ being the dielectric constant of helium. There is no analytical solution for Eq. (10). However, it is possible to obtain an expression for the energy of the ground level $l=1$, by using the variational method. Using the trial wave function as

$$f_1(z)=2\gamma^{3/2}z\exp(-\gamma z) \ , \tag{11}$$

which takes into account the presence of a potential barrier at the liquid-vapor interface satisfying the boundary condition $f_1(0)=0$, one can write the energy of the ground state as

$$\Delta_1=\frac{\hbar^2\gamma^2}{2m}-\Lambda_0\gamma+\frac{3}{2}\frac{eE_\perp^*}{\gamma}+\frac{3}{2}\frac{m\omega_c^2}{\gamma^2} \ . \tag{12}$$

The parameter γ should be determined from the condition $\partial\Delta_1/\partial\gamma=0$. If we write $\gamma=\gamma_0 x$ with $\gamma_0=m\Lambda_0/\hbar^2$, the variational parameter is found by solving the following quartic equation:

$$x^4-x^3-\left(\frac{\gamma_\perp^*}{\gamma_0}\right)^3 x-\left(\frac{\gamma_c}{\gamma_0}\right)^4=0 \ , \tag{13}$$

where $(\gamma_\perp^*)^3=3eE_\perp^* m/2\hbar^2$ and $(\gamma_c)^4=3m^2\omega_c^2/\hbar^2=3l_c^{-4}$, where l_c is the usual magnetic length. In the limit of $B=0$, the solution of Eq. (13) coincides with the results of Ref. 13. The positive definite conditions that should be imposed on Eq. (13) lead to possible values of k_x in the interval $eE_\perp/\hbar\omega_c< k_x< eE_\perp/\hbar\omega_c+m\omega_c/\hbar\gamma_0$. Even though it is possible to obtain an analytical solution of Eq. (13), the mathematical approach is quite cumbersome and we prefer to solve it numerically. The calculated value, for thermal electron momentum $k_x=\sqrt{mT}/\hbar\simeq 3.55\times10^{-5}$ cm^{-1}, is $x=1.2$ and $x=1.5$ for $B=5$ T and $B=10$ T, respectively, and holding fields up to 400 V/cm. We must emphasize that we are considering the magnetic field in the positive direction of the y axis. In the case of the opposite direction, the possible values of k_x are negative and are connected with $-\omega_c$.

If the magnetic field is weak, i.e., the condition $\hbar\omega_c\ll\Delta_1^{(0)}$ is satisfied with $\Delta_1^{(0)}$ being the energy of the level $l=1$ for $B=0$, the correction to the energy spectrum can be calculated in perturbation theory and leads to the result

$$E_{n,l}(k_x)=\frac{\hbar^2k_x^2}{2m}+(n+\tfrac{1}{2})\hbar\omega_0+\Delta_l^{(0)}+\frac{m\omega_c^2}{2}\int_0^\infty z^2[f_l^{(0)}(z)]^2dz-m\omega_c^2 z_0\int_0^\infty z[f_l^{(0)}(z)]^2dz \ , \tag{14}$$

where $z_0=\hbar k_x/m\omega_c$ and $f_l^{(0)}(z)$ is given by Eq. (11) and γ_1 is defined by Eq. (13) for $B=0$. After straightforward calculation, we obtain for the level $l=1$ the following expression for the energy spectrum:

$$E_{n,1}(k_x)=\frac{\hbar^2k_x^2}{2m}+(n+\tfrac{1}{2})\hbar\omega_0+\Delta_1^{(0)}+\frac{3}{2}\frac{m\omega_c^2}{\gamma_1^2}(1-\gamma_1 z_0) \ . \tag{15}$$

The shift of the energy state is a maximum for the case $E_\perp=0$ when γ_1 is a minimum and $\gamma_1=\gamma_0=m\Lambda_0/\hbar^2=1.31\times10^6$ cm^{-1}. In such conditions $|\Delta_1^{(0)}|=-\hbar^2\gamma_0^2/2m=7.58$ K or 157.9 GHz in frequency units. In this way, we can estimate the quantity $\delta_1=\frac{3}{2}m\omega_c^2/\gamma_1^2(1-\gamma_1 z_0)$ for $k_x<0$ and $|k_x|=\sqrt{mT}/\hbar$ for $T=1$ K as $\delta_1=6.6$ GHz, if $B=5$ T, and $\hbar\omega_c=0.67$ K $\ll\Delta_1^{(0)}$. A similar procedure was used in the calculations of the energy spectrum SE over a flat helium liquid when a magnetic field in the direction parallel to the surface is applied.[7]

In the limit of high enough holding fields, one can neglect the first term in Eq. (10) and Eq. (9) can be solved analytically. The corresponding solutions for the spectrum and the wave functions $f_l(z)$ can be expressed as

$$E_{n,l}(k_x)=\frac{\hbar^2k_x^2}{2m}+(n+\tfrac{1}{2})\hbar\omega_0+\hbar\omega_c\left(l+\frac{1}{2}-\frac{\zeta_l^2}{4}\right) \ , \tag{16}$$

and

$$f_l(z)=CD_l\left(\frac{z}{l_c/\sqrt{2}}-\zeta_l\right) \ , \tag{17}$$

where $D_l(x)$ is the parabolic cylinder function and $-\zeta_l$ are the zeros of $D_l(x)$, $\zeta_l>0$. The possible values for k_x are determined by the conditions imposed to solve Eq. (9) and yields $k_x=eE_\perp/\hbar\omega_c+m\omega_c l_c\zeta_l/\sqrt{2}\hbar$.

In the limit of $l\gg1$, the energy spectrum can be written as

$$E_{n,l}(k_x)=\frac{\hbar^2k_x^2}{2m}+(n+\tfrac{1}{2})\hbar\omega_0+\left(\frac{3\pi}{2}\right)^{2/3}\left[\frac{(\hbar k_x-eE_\perp/\omega_c)^2}{2m}(l\hbar\omega_c)^2\right]^{1/3} \ . \tag{18}$$

This expression agrees with the results of Ref. 14 in which the energy of high states of SE over a flat helium surface subject to a magnetic field in the direction parallel to the

surface were calculated in the quasiclassical approximation.

In conclusion, we have calculated the electronic structure of Q1D electrons localized in the channel filled with liquid helium in the presence of a magnetic field in the transverse direction to the channel axis. The effect of the magnetic field on the spectroscopic transitions of the system was demonstrated and the wave functions and the energy spectrum were evaluated. The results of the present work can be used in the experimental study of the spectroscopic transitions between the confined states and the electron transport properties of the Q1D charge system on liquid helium.

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[1] C. W. J. Beenakker and H. van Houten, *Solid State Physics*, edited by H. Ehrenreich and D. Turnbull (Academic, New York, 1991), Vol. 44, p. 1, and references therein.

[2] C. C. Eugster and J. A. del Alamo, Phys. Rev. Lett. **67**, 3586 (1991); C. C. Eugster, J. A. del Alamo, M. J. Rooks, and M. R. Melloch, Appl. Phys. Lett. **64**, 3157 (1994).

[3] Yu. Z. Kovdrya and Yu. P. Monarkha, Fiz. Nizk. Temp. **12**, 1011 (1986) [Sov. J. Low Temp. Phys. **12**, 571 (1986)].

[4] O. I. Kirichek, Yu. P. Monarkha, Yu. Z. Kovdrya, and V. N. Grigor'ev, Fiz. Nizk. Temp. **19**, 458 (1993) [Low Temp. Phys. **19**, 323 (1993)].

[5] S. S. Sokolov, G. Q. Hai, and N. Studart, Phys. Rev. B (to be published).

[6] V. B. Shikin and Yu. P. Monarkha, J. Low Temp. Phys. **16**, 193 (1974).

[7] C. L. Zipfel, T. R. Brown, and C. C. Grimes, Surf. Sci. **58**, 283 (1976).

[8] C. L. Zipfel, T. R. Brown, and C. C. Grimes, Phys. Rev. Lett. **37**, 1760 (1976).

[9] P. M. Platzman and B. I. Halperin, Phys. Rev. B **18**, 226 (1978).

[10] M. J. Harrison, Phys. Rev. B **36**, 9399 (1987).

[11] M. W. Cole and M. H. Cohen, Phys. Rev. Lett. **23**, 1238 (1969).

[12] V. B. Shikin, Zh. Eksp. Teor. Fiz. **58**, 1748 (1970) [Sov. Phys. JETP **31**, 936 (1970)].

[13] Yu. P. Monarkha, S. S. Sokolov, and V. B. Shikin, Solid State Commun. **38**, 611 (1980).

[14] V. B. Shikin and Yu. P. Monarkha, *Two-dimensional Charge Systems in Helium* (Nauka, Moscow, 1989), p. 31 (in Russian).