Coulomb blockade and current-voltage characteristics of ultrasmall double tunnel junctions with external circuits

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The theory on the Coulomb blockade in ultrasmall double junctions with external circuits is proposed, which self-consistently describes tunneling currents and charged states of islands without resorting to the stochastic technique. The island Hamiltonian in the theory contains the chemical potential (charged state) of the island to be determined self-consistently through the current continuity conditions. A general expression for the tunneling current is obtained together with self-consistency equations for the charge state of the island. Current-voltage characteristics are discussed analytically at zero temperature in the limits of high and low impedance environments. Coulomb gaps and the difference in current-voltage characteristics between symmetric and asymmetric double junctions (Coulomb staircases) were also analytically discussed in connection with the concept of neutral and charged islands. At finite temperatures current-voltage characteristics and the number of charges on the island under equilibrium configuration are calculated numerically for various junction parameters and temperatures. The theory describes resonably well the fundamental features of the ultrasmall double junctions with external circuits.

I. INTRODUCTION

Single-electron tunneling in ultrasmall tunnel junctions has attracted a great deal of interest in recent years.[1–4] The controlled transfer of electrons one by one is based on the Coulomb blockade of tunneling, which manifests itself as the charging energy of a single electron becomes larger than the energies of thermal and quantum fluctuations. Various phenomena due to the Coulomb blockade have been extensively studied both theoretically[3,5–11] and experimentally[12–14] and devices exploiting them have been also presented.[15,17–19]

It has also been shown that the Coulomb blockade of tunneling is strongly affected by the external circuits connected with the tunnel junctions (electromagnetic environment effect).[6–9] In a single junction, the Coulomb blockade is severely suppressed unless the impedance of the external circuit is much larger than the resistance quantum $R_q \equiv \pi\hbar/e^2$ (high impedance limit). In this case the change of elementary charge e is almost smeared out by the charge fluctuations on the electrodes directly coupled to the external circuit. As Devoret *et al.*[8] showed, in the single tunnel junction with capacitance C coupled to the external inductance L, whose total impedance is $Z_t(\omega) = iC^{-1}\omega/[\omega_L^2 - (\omega - i\delta)^2]$ $(\delta \to +0)$, the average charge fluctuation induced on the junction electrodes is given as

$$\langle \delta Q^2\rangle = \left(\frac{\hbar C}{e}\right)^2 \int_0^\infty d\omega\, \omega \frac{\mathrm{Re} Z_t(\omega)}{R_q} \coth\left(\frac{1}{2}\beta\hbar\omega\right) = \frac{e^2}{2}\frac{\hbar\omega_L}{E_c}\left(\frac{1}{\exp(\beta\hbar\omega_L) - 1} + \frac{1}{2}\right), \quad (1.1)$$

where $\omega_L = 1/\sqrt{LC}$ is the frequency of the environmental mode, $E_c = e^2/(2C)$ is the charging energy of a single electron, $\beta = 1/kT$, k is the Boltzmann constant, and T is the absolute temperature. If $\hbar\omega_L/E_c \gg 1$, which corresponds to low impedance limit $(L \to 0)$, the charge fluctuation exceeds e even at $T = 0$ since the zero point energy of the environmental mode smears out the charging energy.

The result can be generalized to the case where the total impedance of the electromagnetic environment is expressed as the series of many LC circuits with frequency $\omega_\ell \equiv 1/\sqrt{L_\ell C_\ell}$ and weight w_ℓ $(\ell = 1, 2, \ldots,)$,

$$Z_t(\omega) = \sum_\ell w_\ell \frac{1}{C_\ell} \frac{i\omega}{\omega_\ell^2 - (\omega - i\delta)^2} \quad (1.2)$$

with $\sum_\ell w_\ell = 1$. Since an LC circuit is equivalent to a harmonic oscillator, the electromagnetic environment can be understood as an assembly of the harmonic oscillators which serves the energy reservoir for the tunnel junction.

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This is nothing but the picture of Caldeira and Leggett.[20] Equation (1.1) tells us that the Coulomb blockade can be observed if $\hbar\omega_L/E_c < 1$ at $T = 0$, which is the condition at which a tunneling electron can dissipate its energy E_c by exciting the environmental modes $\hbar\omega_L$. Therefore, the Coulomb blockade can be recognized as the suppression of tunneling due to the energy dissipation. Taking account of the electromagnetic environmental effect as well as the thermal and quantum fluctuations is indispensable for understanding the phenomena due to the Coulomb blockade.

Concerning the double junction system, where the charging effect appears in a different way for Q (total charge) and q (island charge), the theory by Grabert *et al.*[11] was the first along this direction. While Q changes continuously and largely fluctuates unless $\hbar\omega_L/E_c \ll 1$, the change in q only results from the tunneling as far as the junction resistances are much larger than R_q. Therefore, there is no charge fluctuations on the island, and the Coulomb blockade on the island can be observed irrespective of the environmental impedance.

Following this work, but in a more refined way, the localization effect of electrons on the Coulomb blockade was studied.[21] The localization of electrons in the electrodes and leads, which gives rise to the increase in the total impedance of the system, tends to stabilize the Coulomb blockade. This can also be understood as a direct consequence of the effect of energy dissipation on tunneling. In this theory, the charged states of the island were treated explicitly through the commutation relations between charges and their conjugate phases, so the energy change by the tunneling of a single electron from or to the island through the junctions was quite naturally described. Although the theory correctly describes current-voltage (I-V) characteristics at low bias voltages such as Coulomb gaps (I-V characteristics in the neutral island), it cannot explain Coulomb staircases which are the I-V characteristics in charged islands for asymmetric double junctions. It is because the Hamiltonian of the island is of the form proportional to the square of q, which implies that the most stable charged state of the island is always neutral irrespective of the junction parameters. Grabert *et al.*[11] avoided the problem by assuming the presence of charged island states from the beginning. They solved the master equation for the probabilities of the charged states to calculate the tunneling currents. Charged states, however, should be determined self-consistently with corresponding tunneling currents through the current continuity condition.

In the present paper we propose the theory on the Coulomb blockade in ultrasmall double junctions with external circuits, which self-consistently describes tunneling currents and charged states without resorting to the stochastic technique. In Sec. II, we derive the island Hamiltonian which correctly describes charging effects on the island. According to Ref. 21, the general expression for the tunneling current is obtained in Sec. III. In Sec. IV the current-voltage characteristics are discussed. Analytical results for low and high impedance limits and numerical results for realistic cases are also shown here. Section V is apportioned to conclusions and discussions.

II. HAMILTONIAN OF THE SYSTEM

Following a model in Ref. 11, let us consider a voltage-biased double junction system consisting of two tunnel junctions in series coupled to an external circuit with impedance $Z(\omega)$ as depicted in Fig. 1. In the following, for simplicity, we consider only an inductance L as the external circuit, i.e., $Z(\omega) = i\omega L$. The Hamiltonian of the system is then written in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_T , \tag{2.1}$$

where $\mathcal{H}_0$ is the Hamiltonian in the absence of tunneling of electrons and $\mathcal{H}_T$ is the Hamiltonian which describes the transfer of electrons by tunneling through junctions. $\mathcal{H}_0$ consists of $\mathcal{H}_{\rm es}$ and $\mathcal{H}_{\rm em}$ which describe, respectively, the electronic states of three electrodes and the electromagnetic energy of the entire circuit, and thus is written as

$$\mathcal{H}_0 = \mathcal{H}_{\rm es} + \mathcal{H}_{\rm em} . \tag{2.2}$$

Although $\mathcal{H}_{\rm es}$ contains intra-electrode interactions due to the electron-impurity and electron-electron scatterings as well as kinetic terms, we neglect the interactions for the moment,

$$\mathcal{H}_{\rm es} = \sum_{i=1}^{3} \sum_{\boldsymbol{k},\sigma} \epsilon_i(\boldsymbol{k}) a^{(i)\dagger}_{\boldsymbol{k},\sigma} a^{(i)}_{\boldsymbol{k},\sigma} , \tag{2.3}$$

where $a^{(i)\dagger}_{\boldsymbol{k},\sigma}$ ($a^{(i)}_{\boldsymbol{k},\sigma}$) is the creation (annihilation) operator of electrons with wave vector $\boldsymbol{k}$, energy $\epsilon_i(\boldsymbol{k})$, and spin σ in the ith electrode.

In order to describe the dynamics of charge Q_i ($i = 1, 2$) on the junction capacitors under the electromagnetic environment, let us introduce the variable φ_i canonically conjugate to Q_i, which corresponds to the magnetic flux induced in the circuit by self-induction of L. These variables satisfy the following canonical commutation relations:

$$[Q_i \, , \, \varphi_j] \;=\; i\hbar\delta_{i,j} \, , \quad [Q_i \, , \, Q_j] \;=\; [\varphi_i \, , \, \varphi_j] \;=\; 0 \, . \tag{2.4}$$

Using these variables, $\mathcal{H}_{\rm em}$ is of the form

$$\mathcal{H}_{\rm em} = \frac{(\varphi_1 + \varphi_2)^2}{2L} + \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} - Q_1 \frac{\mu_{2,1}}{e} - Q_2 \frac{\mu_{3,2}}{e} , \tag{2.5}$$

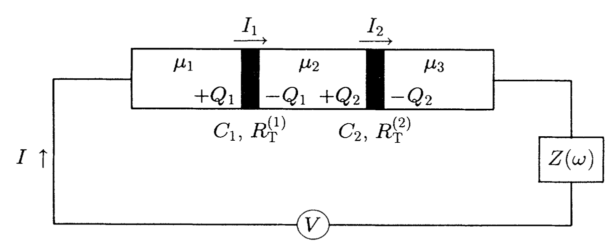

FIG. 1. A voltage-biased double tunnel junction with capacitances C_1 and C_2 and tunnel resistances $R_T^{(1)}$ and $R_T^{(2)}$ coupled to an external circuit with impedance $Z(\omega) = i\omega L$.

which describes the magnetic energy due to the self-induction of the external circuit, the Coulomb charging energy stored in the junctions, and the works done by the external bias voltage V. Here $\mu_{i+1,i}/e = (\mu_{i+1} - \mu_i)/e$, where μ_i is the chemical potential of ith electrode (voltage across the ith junction). Note that $eV = \mu_{3,2} + \mu_{2,1}$. It is convenient to express $\mathcal{H}_{\rm em}$ as the term coupled to the external environment and the other.[11] Let us introduce charges Q and q, and define their canonical phases φ and ψ as

$$Q_i = Q + (1 - \kappa_i)\eta_i q\,, \tag{2.6a}$$

$$\varphi_i = \kappa_i\varphi + \eta_i\psi\,, \tag{2.6b}$$

where $\kappa_i = C/C_i$, $\eta_i = (-1)^i$ $(i = 1, 2)$, $C_\Sigma = C_1 + C_2$, and $C = C_1C_2/C_\Sigma$. The new variables satisfy the following commutation relations:

$$[Q\,,\,\varphi] = [q\,,\,\psi] = i\hbar\,, \tag{2.7a}$$

$$[Q\,,\,q] = [Q\,,\,\psi] = [\varphi\,,\,\psi] = [\varphi\,,\,q] = 0\,. \tag{2.7b}$$

The operator Q expresses the total charge of a double junction with capacitance C carried by the external circuit and q expresses the (excess) charge on the island with capacitance C_Σ. It is quite natural to assume that Q has a continuous eigenvalue. Concerning q, however, it is plausible to assume eigenstates such as

$$q\,|q\rangle = me\,|q\rangle, \tag{2.8}$$

where m is the integer, since the change in q only results from the tunneling as far as the junction resistances $R_T^{(i)}$ are much larger than R_q.

In terms of these variables Eq. (2.5) is rewritten as

$$\mathcal{H}_{\rm em} = \mathcal{H}_{\rm env} + \mathcal{H}_c\,, \tag{2.9}$$

where

$$\mathcal{H}_{\rm env} = \frac{\varphi^2}{2L} + \frac{Q^2}{2C} - QV\,, \tag{2.10a}$$

$$\mathcal{H}_c = (q/e - n_c)^2 U\,, \tag{2.10b}$$

apart from any constant shift in $\mathcal{H}_c$. Note that

$$U \equiv \frac{e^2}{2C_\Sigma} \tag{2.11}$$

is the charging energy of a single electron on the island and

$$n_c = \frac{-\mu_{2,1} + \kappa_1 eV}{2U} \tag{2.12}$$

is the *noninteger* charge offset which will be determined self-consistently together with the tunneling current under a certain bias condition for a given set of junction parameters. $\mathcal{H}_{\rm env}$ is equivalent to the Hamiltonian of the electromagnetic environment in a single junction system with capacitance C embedded in the external circuit with inductance L. It should be noted that n_c self-consistently determined reflects the electromagnetic environment effect. On the other hand, $\mathcal{H}_c$ is the Hamiltonian which specifies the charged state of the island and has not been treated properly in previous works.[11,21] The statistical average of the number of charges on the island with respect to $\mathcal{H}_c$, $\langle q/e\rangle_c$, which is a function of n_c, can be determined self-consistently including the environmental effect and does not always become integral as seen below.

The second term in Eq. (2.1), which specifies the tunneling through two junctions, is of the form

$$\mathcal{H}_T = \sum_{i=1}^{2}\left\{\mathcal{H}_T^{(i)} + \mathcal{H}_T^{(i)\dagger}\right\}\,, \tag{2.13}$$

$$\mathcal{H}_T^{(i)} = \sum_{\boldsymbol{k},\boldsymbol{k}',\sigma} T_{\boldsymbol{k},\boldsymbol{k}'}^{(i)} e^{ie\varphi_i/\hbar} a_{\boldsymbol{k},\sigma}^{(i)\dagger} a_{\boldsymbol{k}',\sigma}^{(i+1)}\,, \tag{2.14}$$

where $T_{\boldsymbol{k},\boldsymbol{k}'}^{(i)}$ is the matrix element for the electron tunneling from $\boldsymbol{k}$ to $\boldsymbol{k}'$ through the ith junction. $\mathcal{H}_T^{(i)}$ describes the tunneling process which annihilates an electron with wave vector $\boldsymbol{k}'$ on an $(i+1)$-th electrode and creates an electron with wave vector $\boldsymbol{k}$ on an ith electrode. An instantaneous change of Q_i accompanied by tunneling is consistently specified by the factor $e^{ie\varphi_i/\hbar}$ in Eq. (2.14). In fact, the commutation relation Eq. (2.4) yields

$$Q_i\mathcal{H}_T^{(i)} = \mathcal{H}_T^{(i)}(Q_i - e)\,, \tag{2.15}$$

which means that the charge on the junction is changed by e due to the tunneling of a single electron through the ith junction.

III. TUNNELING CURRENT

Since the tunneling current of the double junction system can be obtained by the current continuity condition, let us first consider a current flowing through the ith junction I_i. With the aid of Heisenberg's equation of motion for Q_i, the current operator for the ith junction is given as

$$\mathcal{I}_i = \frac{e}{i\hbar}\left(\mathcal{H}_T^{(i)} - \mathcal{H}_T^{(i)\dagger}\right)\,. \tag{3.1}$$

Within the lowest order in $\mathcal{H}_T^{(i)}$, the tunneling current through the ith junction is expressed as

$$I_i = \frac{1}{i\hbar}\int_{-\infty}^{0} dt\,\langle[\mathcal{I}_i\,,\,\mathcal{H}_T(t)]\rangle\,, \tag{3.2}$$

where $O(t) \equiv e^{it\mathcal{H}_0/\hbar}\,O\,e^{-it\mathcal{H}_0/\hbar}$ and

$$\langle\ \rangle \equiv \frac{\mathrm{Tr}\,\{\exp(-\beta\breve{\mathcal{H}}_0)\,\cdots\}}{\mathrm{Tr}\,\exp(-\beta\breve{\mathcal{H}}_0)}\,, \tag{3.3a}$$

$$\breve{\mathcal{H}}_0 = \mathcal{H}_0 - \sum_{i=1}^{3}\sum_{\boldsymbol{k},\sigma}\mu_i a_{\boldsymbol{k},\sigma}^{(i)\,\dagger} a_{\boldsymbol{k},\sigma}^{(i)}\,. \tag{3.3b}$$

Inserting Eqs. (2.13) and (3.1) into Eq. (3.2) and writing the virtual time evolution by $\breve{\mathcal{H}}_0$ as $\breve{O}(t) = e^{it\breve{\mathcal{H}}_0/\hbar}\,O\,e^{-it\breve{\mathcal{H}}_0/\hbar}$, Eq. (3.2) becomes

$$I_i = i\frac{e}{\hbar}\int_{-\infty}^{+\infty} dt \left\langle \frac{1}{i\hbar}\left[\check{\mathcal{H}}_T^{(i)\dagger}(t),\, \check{\mathcal{H}}_T^{(i)}(0)\right]\right\rangle \times \exp\left(i\frac{\mu_{i+1,i}}{\hbar}t\right) . \tag{3.4}$$

In order to calculate Eq. (3.4), it is convenient to use the Matsubara Green's function defined by

$$X_i(\tau) \equiv -\left\langle T_\tau\, \check{\mathcal{H}}_T^{(i)\dagger}(\tau)\check{\mathcal{H}}_T^{(i)}(0)\right\rangle , \tag{3.5}$$

where $\tau \equiv it$ is the imaginary time and T_τ denotes the imaginary time ordered operator. Then, Eq. (3.4) is expressed as

$$I_i = -\frac{2e}{\hbar}\mathrm{Im}\left[\frac{1}{\hbar}\tilde{X}_i\left(i\omega_\ell = \frac{\mu_{i+1,i}}{\hbar} + i\delta\right)\right] , \tag{3.6}$$

where

$$\tilde{X}_i(i\omega_\ell) = \int_0^{\hbar\beta} d\tau\, e^{i\omega_\ell\tau}X_i(\tau) , \tag{3.7}$$

$\omega_\ell = 2\pi\ell/(\hbar\beta)$ ($\ell = 0, \pm1, \cdots, \pm\infty$) and $\delta \to +0$.

Equation (3.5) contains two kinds of phase correlation functions. One of them is the correlation function of φ (Ref. 11) defined by

$$\mathcal{F}_\varphi(\tau, \kappa_i) \equiv \left\langle T_\tau\, e^{-i\kappa_i e\varphi(\tau)/\hbar}e^{i\kappa_i e\varphi(0)/\hbar}\right\rangle_{\mathrm{env}} \equiv \exp\left\{\kappa_i^2 J(\tau)\right\} , \tag{3.8}$$

where $\langle\ \rangle_{\mathrm{env}}$ denotes the ensemble average over eigenstates of $\mathcal{H}_{\mathrm{env}}$ and

$$J(\tau) = \frac{E_c}{\hbar\omega_L}\left\{\coth\frac{\hbar\beta\omega_L}{2}(\cosh\omega_L\tau - 1) - \sinh\omega_L|\tau|\right\} . \tag{3.9}$$

Here

$$E_c = \frac{e^2}{2C} , \tag{3.10a}$$

$$\omega_L = \frac{1}{\sqrt{LC}} , \tag{3.10b}$$

are, respectively, the charging energy of a single electron felt by capacitance C and a frequency of the environmental mode described by $\mathcal{H}_{\mathrm{env}}$. The other is the correlation function of ψ defined by

$$\mathcal{F}_\psi(\tau, \eta_i) \equiv \left\langle T_\tau\, e^{-i\eta_i e\psi(\tau)/\hbar}e^{i\eta_i e\psi(0)/\hbar}\right\rangle_c = \left\langle \exp\left\{-\frac{U}{\hbar}\left[|\tau| - 2\eta_i(q/e - n_c)\tau\right]\right\}\right\rangle_c , \tag{3.11}$$

where $\langle\ \rangle_c$ denotes the ensemble average over eigenstates of $\mathcal{H}_c$. The factor in Eq. (3.11), $U[1 - 2\eta_i(q/e - n_c)]$ $(\tau > 0)$, describes the change in charging energy on the island with the charged state $|q\rangle$ caused by a single-electron transfer through the junctions, assuring the energy conservation relation together with the factor in Eq. (3.8). Grabert *et al.*[11] did not introduce the correlation function of ψ, since they eventually treated the variable q classically. Therefore the energy conservation relation was introduced into the expression of current by hand. Using Eq. (2.8), Eq. (3.11) is expressed as

$$\mathcal{F}_\psi(\tau, \eta_i) = \frac{\displaystyle\sum_{m=-\infty}^{\infty} \exp\left\{-\beta U(m - n_c)^2 - \frac{U}{\hbar}\left[|\tau| - 2\eta_i(m - n_c)\tau\right]\right\}}{\displaystyle\sum_{m=-\infty}^{\infty} \exp\left\{-\beta U(m - n_c)^2\right\}} . \tag{3.12}$$

Using these correlation functions, $X_i(\tau)$ becomes

$$X_i(\tau) = -\hbar^2\mathcal{F}_\varphi(\tau, \kappa_i)\mathcal{F}_\psi(\tau, \eta_i)\alpha_T^{(i)}(\tau) , \tag{3.13}$$

where

$$\alpha_T^{(i)}(\tau) = -\frac{1}{\hbar^2}\sum_{\boldsymbol{k},\boldsymbol{k}',\sigma}\left|T_{\boldsymbol{k},\boldsymbol{k}'}^{(i)}\right|^2 \mathcal{G}_{i,\sigma}(\boldsymbol{k}, \tau)\mathcal{G}_{i+1,\sigma}(\boldsymbol{k}', -\tau) \tag{3.14}$$

reflects details of the electronic states of electrodes on both sides of the ith junction through the Matsubara Green's function of the electron,

$$\mathcal{G}_{i,\sigma}(\boldsymbol{k}, \tau) = -\left\langle T_\tau\, \check{a}_{\boldsymbol{k},\sigma}^{(i)}(\tau)\check{a}_{\boldsymbol{k},\sigma}^{(i)\dagger}(0)\right\rangle = \frac{1}{\hbar\beta}\sum_{\nu=-\infty}^{\infty}\frac{e^{-i\omega_\nu\tau}}{i\omega_\nu - [\epsilon_i(\boldsymbol{k}) - \mu_i]/\hbar} , \tag{3.15}$$

with $\omega_\nu = (2\nu + 1)\pi/(\hbar\beta)$. Assuming that $\left|T_{\boldsymbol{k},\boldsymbol{k}'}^{(i)}\right|^2 = \left|T^{(i)}\right|^2$, Eq. (3.14) reduces to

$$\alpha_T^{(i)}(\tau) = \frac{1}{2\pi^2}\frac{R_q}{R_T^{(i)}}\left(\frac{\pi}{\hbar\beta}\mathrm{cosec}\frac{\pi\tau}{\hbar\beta}\right)^2 , \tag{3.16}$$

where $R_T^{(i)} = R_q \,/\, \left\{4\pi^2 N_i(0)N_{i+1}(0)\, |T^{(i)}|^2\right\}$ and $N_i(0)$ are, respectively, the tunneling resistance of the ith junction and the density of states in the ith electrode. Effects of the electron-electron and electron-impurity interactions within the same electrodes only give rise to the simple T dependence of tunneling resistances as far as only the lowest order in $\mathcal{H}_T^{(i)}$ is taken into account.[21]

Noting that

$$\alpha_T^{(i)}(\tau) = \frac{1}{\hbar\beta}\sum_\nu e^{-i\omega_\nu\tau}\left\{-\frac{1}{2\pi}\frac{R_q}{R_T^{(i)}}|\omega_\nu|\right\} \tag{3.17}$$

(Ohmic dissipation), the tunneling current Eq. (3.6) is expressed as

$$I_i = \frac{1}{eR_T^{(i)}}\mathrm{Im}\left\{\frac{1}{\hbar\beta}\sum_\nu \hbar|\omega_\nu - \omega_\ell|\int_0^{\hbar\beta} d\tau\, e^{i\omega_\ell\tau}\mathcal{F}_\varphi(\tau,\kappa_i)\mathcal{F}_\psi(\tau,\eta_i)\right\}_{i\omega_\ell\to\mu_{i+1,i}/\hbar+i\delta} . \tag{3.18}$$

Since it is not so convenient to handle Eq. (3.18), let us derive the real-time expression for I_i. Noting that $X_i(\tau)$ has no singularity except for poles at $\tau = \ell\hbar\beta$ ($\ell = 0, \pm1, \ldots, \pm\infty$), Eq. (3.7) can be written as

$$\tilde{X}_i(i\omega_\ell) = -i\hbar^2\int_0^\infty dt\, e^{-\omega_\ell t}\tilde{\mathcal{F}}_\psi(it,\eta_i)\left[\tilde{\mathcal{F}}_\varphi(it,\kappa_i)\alpha_T^{(i)}(it+\delta) - \tilde{\mathcal{F}}_\varphi(-it,\kappa_i)\alpha_T^{(i)}(it-\delta)\right] \tag{3.19}$$

for $\omega_\ell > 0$, where we defined

$$\tilde{\mathcal{F}}_\varphi(\tau,\kappa_i) = \mathcal{F}_\varphi(\tau,\kappa_i)\exp(-U\tau/\hbar)\,, \qquad \tilde{\mathcal{F}}_\psi(\tau,\eta_i) = \mathcal{F}_\psi(\tau,\eta_i)\exp(U\tau/\hbar)\,, \tag{3.20}$$

and the following relation was used: $\mathcal{F}_\varphi(\tau+\hbar\beta,\kappa_i)\mathcal{F}_\psi(\tau+\hbar\beta,\eta_i) = \tilde{\mathcal{F}}_\varphi(-\tau,\kappa_i)\tilde{\mathcal{F}}_\psi(\tau,\eta_i)$ for $\tau \geq 0$. Substituting Eq. (3.19) in Eq. (3.6) and using Eqs. (3.8), (3.11), (3.16), and (2.12), we arrive at

$$I_i = \frac{1}{eR_T^{(i)}}\left(\kappa_i eV + 2U\eta_i\left\langle\frac{q}{e}\right\rangle_c - \Phi_i(eV)\right)\,, \tag{3.21}$$

where

$$\Phi_i(eV) = \frac{i\hbar}{\pi}\int_{-\infty}^{+\infty} dt\left(\frac{\pi}{\hbar\beta}\mathrm{cosech}\frac{\pi t}{\hbar\beta}\right)^2\tilde{\mathcal{F}}_\varphi^{(-)}(it,\kappa_i)\left\langle\sin\left\{\frac{t}{\hbar}\left(\kappa_i eV + 2U\eta_i\frac{q}{e}\right)\right\}\right\rangle_c \tag{3.22}$$

with

$$\tilde{\mathcal{F}}_\varphi^{(-)}(it,\kappa_i) = \frac{1}{2}\left\{\tilde{\mathcal{F}}_\varphi(it,\kappa_i) - \tilde{\mathcal{F}}_\varphi(-it,\kappa_i)\right\}\,. \tag{3.23}$$

Let us consider the average number of island charges. It is given as

$$\left\langle\frac{q}{e}\right\rangle_c = \left\lfloor n_c + \frac{1}{2}\right\rfloor + \sum_{j=1}^\infty \frac{\sinh(2\beta U\delta n_c)}{\cosh(2\beta U\delta n_c) + \cosh\{2\beta U(j-1/2)\}}\,, \tag{3.24}$$

where $\lfloor x\rfloor$ is the greatest integer less than or equal to x and

$$\delta n_c = n_c - \lfloor n_c + 1/2\rfloor\,. \tag{3.25}$$

As expected, the average number of island charges is given as a function of n_c for a given set of bias condition, junction parameters, and temperature. In particular, at sufficiently low temperatures relative to U/k, Eq. (3.24) may be written as

$$\left\langle\frac{q}{e}\right\rangle_c = \left\lfloor n_c + \frac{1}{2}\right\rfloor + \frac{1}{2}\left\{\tanh\beta U(1/2+\delta n_c) - \tanh\beta U(1/2-\delta n_c)\right\}\,, \tag{3.26}$$

since the summation in Eq. (3.24) is dominated by the term with $j = 1$. As easily verified, if n_c happens to be an integral or a half-integral value, Eq. (3.24) gives the result, $\langle q/e\rangle_c = n_c$ at any temperature. In Fig. 2, $\langle q/e\rangle_c$ is shown as a function of n_c for $T = 0$ and $U/kT = 10$. We can thereby interpret $\lfloor n_c + 1/2\rfloor$ as the number of charges on the island under equilibrium configuration at sufficiently low temperatures unless n_c is a half-integral value. The point where n_c takes a half-integral value is the transition point around which $\langle q/e\rangle_c$ increases or decreases by one.

In order to determine n_c we have to utilize the current continuity condition

$$I = I_1 = I_2\,, \tag{3.27}$$

which leads to the self-consistency equation with respect to n_c,

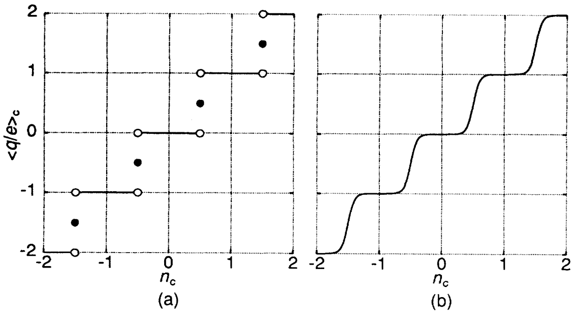

FIG. 2. $\langle q/e\rangle_c$ as a function of n_c for (a) $T = 0$ and (b) $U/kT = 10$.

$$(r_1\kappa_2 - r_2\kappa_1)eV + 2U\left\langle\frac{q}{e}\right\rangle_c - r_1\Phi_2(eV) + r_2\Phi_1(eV) = 0, \quad (3.28)$$

where $r_i = R_T^{(i)}/R_\Sigma$ and $R_\Sigma = R_T^{(1)} + R_T^{(2)}$. Eliminating $\langle q/e\rangle_c$ from Eqs. (3.21) and (3.28), we finally obtain the tunneling current of the double junction system,

$$I = \frac{1}{eR_\Sigma}\{eV - \Phi_1(eV) - \Phi_2(eV)\}, \quad (3.29)$$

where the ensemble average by $\mathcal{H}_c$ involved in $\Phi_1(eV)$ and $\Phi_2(eV)$ must be evaluated using n_c determined by Eq. (3.28).

Noting that the ensemble average by $\mathcal{H}_c$ satisfies the relation $\langle f(q)\rangle_c|_{n_c\to -n_c} = \langle f(-q)\rangle_c$, where $f(q)$ is an arbitrary function of q, we can easily prove that the function $\Phi_i(eV)$ satisfies $\Phi_i(-eV)|_{n_c\to -n_c} = -\Phi_i(eV)$. Therefore, if n_c is a solution of Eq. (3.28) for a given bias voltage V, $-n_c$ is a solution of Eq. (3.28) for $-V$. These results guarantee that I-V curves traced by Eq. (3.29) are symmetrical with respect to the origin ($I = V = 0$).

IV. CURRENT-VOLTAGE CHARACTERISTICS

Let us discuss the tunneling currents obtained above in more detail. First we derive the analytical expressions for the currents in the high and low impedance limits at zero temperatures, and discuss I-V characteristics. Next we show the numerical results at finite temperatures.

A. Zero temperature

As seen from the behavior of $\langle q/c\rangle_c$ at very low temperature, the zero-temperature limit of the ensemble average by $\mathcal{H}_c$ takes different forms whether n_c becomes a half-integral or not. We cannot assume whether n_c becomes a half-integral or not before taking the zero-temperature limit, since n_c is finally fixed to solve the self-consistency equation (3.28). Therefore, the ensemble average by $\mathcal{H}_c$ should be evaluated retaining the same temperature dependence as in Eq. (3.26). Only after that can we obtain the correct expressions at zero temperature.

Within the same order as the low-temperature expression for $\langle q/c\rangle_c$, the ensemble average by $\mathcal{H}_c$ in (3.22) is evaluated as

$$\left\langle\sin\left\{\frac{t}{\hbar}\left(\kappa_i eV + 2U\eta_i\frac{q}{e}\right)\right\}\right\rangle_c = \{1-\gamma_R(n_c)\}\sin\left\{\frac{t}{\hbar}(\kappa_i eV + 2U\eta_i n)\right\} + \sum_{\sigma=\pm1}\frac{\gamma_R(n_c)+\sigma\gamma_I(n_c)}{2}\sin\left\{\frac{t}{\hbar}(\kappa_i eV + 2U\eta_i\{n+\sigma\})\right\}, \quad (4.1)$$

where $n = \lfloor n_c + 1/2\rfloor$ and

$$\gamma_R(n_c) = \frac{1}{2}\{1 - \tanh\beta U(1/2+\delta n_c)\tanh\beta U(1/2-\delta n_c)\}, \quad (4.2a)$$

$$\gamma_I(n_c) = \frac{1}{2}\{\tanh\beta U(1/2+\delta n_c) - \tanh\beta U(1/2-\delta n_c)\}. \quad (4.2b)$$

Applying the short-time expansion to Eq. (3.23) in the high impedance limit we get

$$\tilde{\mathcal{F}}_\varphi^{(-)}(it,\kappa_i) = -i\sin\left(\frac{\kappa_i E_c}{\hbar}t\right). \quad (4.3a)$$

In the low impedance limit, on the other hand, Eq. (3.23) reduces to

$$\tilde{\mathcal{F}}_\varphi^{(-)}(it,\kappa_i) = -i\sin\left(\frac{U}{\hbar}t\right), \quad (4.3b)$$

since $J(it)\sim 0$. Using Eqs. (4.1) and (4.3), Eq. (3.22) becomes

$$\Phi_i(eV) = \kappa_i eV + 2U\eta_i\{n+\gamma_I(n_c)\} - \{1-\gamma_R(n_c)\}W_\beta\left(\kappa_i eV + 2U\eta_i n,\ E_g^{(i)}\right) - \sum_{\sigma=\pm1}\frac{\gamma_R(n_c)+\sigma\gamma_I(n_c)}{2}W_\beta\left(\kappa_i eV + 2U\eta_i\{n+\sigma\},\ E_g^{(i)}\right) \quad (4.4)$$

with

$$E_g^{(i)} = \begin{cases} \kappa_i E_c & \text{for the high impedance limit} \\ U & \text{for the low impedance limit.} \end{cases} \tag{4.5}$$

Here we introduce

$$W_\beta(x,\ y) = \frac{x-y}{1-e^{-\beta(x-y)}} + \frac{x+y}{1-e^{\beta(x+y)}}, \tag{4.6}$$

which reduces to

$$W(x,\ y) = \{x - y\,\mathrm{sgn}(x)\}\Theta(|x| - y) \tag{4.7}$$

at zero temperature, where $\Theta(x)$ is the step function. Inserting Eqs. (3.26) and (4.4) into Eq. (3.21), we obtain

$$I_i = \frac{1}{eR_T^{(i)}}\left[\{1-\gamma_R(n_c)\}W_\beta(\kappa_i eV + 2U\eta_i n,\ E_g^{(i)}) + \sum_{\sigma=\pm1}\frac{\gamma_R(n_c)+\sigma\gamma_I(n_c)}{2}W_\beta(\kappa_i eV + 2U\eta_i\{n+\sigma\},\ E_g^{(i)})\right] \tag{4.8}$$

for the current through the ith junction in the low-temperature limit, giving the tunneling current in the double junction system as

$$I = \frac{1}{eR_\Sigma}\sum_{i=1}^{2}\left[\{1-\gamma_R(n_c)\}\,W_\beta\left(\kappa_i eV + 2U\eta_i n,\ E_g^{(i)}\right) + \sum_{\sigma=\pm1}\frac{\gamma_R(n_c)+\sigma\gamma_I(n_c)}{2}W_\beta\left(\kappa_i eV + 2U\eta_i\{n+\sigma\},\ E_g^{(i)}\right)\right], \tag{4.9}$$

with the current continuity condition

$$\sum_{\sigma=\pm1}\frac{\gamma_R(n_c)+\sigma\gamma_I(n_c)}{2\{1-\gamma_R(n_c)\}}[r_2W_\beta(\kappa_1 eV - 2U\{n+\sigma\},\ E_g^{(1)}) - r_1W_\beta(\kappa_2 eV + 2U\{n+\sigma\},\ E_g^{(2)})] + r_2W_\beta\left(\kappa_1 eV - 2Un,\ E_g^{(1)}\right) - r_1W_\beta\left(\kappa_2 eV + 2Un,\ E_g^{(2)}\right) = 0\,. \tag{4.10}$$

In the following, let us consider I-V characteristics at $T=0$ in the high and low impedance limits separately.

1. *High impedance limit:* $\hbar\omega_L/E_c \ll 1$

At zero temperature, Eq. (4.8) reduces to

$$I_i = \frac{1}{eR_T^{(i)}}\frac{1}{2}\sum_{n'=\pm\lfloor\pm n_c+1/2\rfloor}\kappa_i W(eV + 2(1-\kappa_i)\eta_i E_c n',\ E_c)\,, \tag{4.11}$$

from which we find conditions for the Coulomb blockade of tunneling at each junction:

$$\frac{eV-E_c}{2\kappa_2 E_c} \le n' \le \frac{eV+E_c}{2\kappa_2 E_c} \quad \text{for } i=1,$$

$$-\frac{eV+E_c}{2\kappa_1 E_c} \le n' \le -\frac{eV-E_c}{2\kappa_1 E_c} \quad \text{for } i=2.$$

The tunneling current of the double junction is suppressed when both conditions are satisfied at the same time. This leads to $|eV| \le E_c$ and $n'=0$. The former gives a Coulomb gap,

$$V_c^H = \frac{E_c}{e}\,, \tag{4.12}$$

for the double junction in this limit. The latter means $\lfloor n_c + 1/2\rfloor = 0$, i.e., no charge is stored on the island (neutral island). Note that n_c cannot be uniquely determined only by that equation. When the tunneling is blocked, the voltage across the ith junction is equal to CV/C_i under the neutral island. From this, together with $\mu_{i+1,i} = \kappa_i eV + 2U\eta_i n_c$, we get $n_c = 0$.

Hereafter we examine I-V characteristics only for positive V, since the current-voltage characteristics are symmetric with respect to the origin as mentioned above. As the bias voltage V exceeds V_c^H the tunneling current begins to flow. Note that at $T = 0$, Eq. (4.10) reduces to

$$\sum_{\sigma=0,-1} r_2\kappa_1 W\Big(eV - 2\kappa_2 E_c\{n+\sigma\},\ E_c\Big) = \sum_{\sigma=0,-1} r_1\kappa_2 W\Big(eV + 2\kappa_1 E_c\{n+\sigma\},\ E_c\Big)\,, \quad (4.13a)$$

if n_c is a half-integral, and

$$r_2\kappa_1 W(eV - 2\kappa_2 E_c n,\ E_c) = r_1\kappa_2 W(eV + 2\kappa_1 E_c n,\ E_c)\,, \quad (4.13b)$$

if otherwise.

In the symmetric case, when the characteristic time constants of the junctions are equal, i.e., $r_1\kappa_2 = r_2\kappa_1$, the charged state of the island is always neutral, $n = 0$, irrespective of bias voltage and charging energy. Therefore, there is no Coulomb staircase and the tunneling current in the junction system is simply given by

$$I = \frac{1}{R_\Sigma}\left(V - \frac{E_c}{e}\right) \qquad (V \geq E_c/e)\,. \quad (4.14)$$

In the asymmetric case, when $r_1\kappa_2 \neq r_2\kappa_1$, on the other hand, we obtain from Eq. (4.13) discrete voltage steps

$$\left.\begin{array}{c} V_n^H \\ V_{n-1/2}^H \end{array}\right\} = V_c^H + \frac{eR_\Sigma}{R_T^{(2)}C_2 - R_T^{(1)}C_1} \times \left\{\begin{array}{l} n \\ n - \frac{1}{2} \end{array}\right. \quad (4.15)$$

$(n = 0, \pm 1, \pm 2, \ldots,)$, with

$$\mathrm{sgn}(n) = \mathrm{sgn}\left(R_T^{(2)}C_2 - R_T^{(1)}C_1\right)\,. \quad (4.16)$$

Tunneling current is then given by Eq. (4.14) with these voltage steps and shows steplike structures (Coulomb staircases). Note that V_n^H determines current in the voltage range which corresponds to

$$n - \frac{1}{2} < n_c < n + \frac{1}{2}\,, \quad (4.17)$$

while $V_{n-1/2}^H$ determines current only at this point which corresponds to

$$n_c = n - \frac{1}{2}\,. \quad (4.18)$$

Results obtained are the direct consequence of an incremental charging of the island by unit charge e due to the difference in RC time constants of the junctions.

As will be seen below, the tunneling current at $kT \ll E_c$ continuously varies smoothly linking the current steps obtained at $T = 0$, since $\langle q/e\rangle_c$ varies continuously keeping steplike structures as shown in Fig. 2.

2. Low impedance limit: $\hbar\omega_L/E_c \gg 1$

In this limit, Eq. (4.8) becomes

$$I_i = \frac{1}{eR_T^{(i)}}\frac{1}{2}\sum_{n'=\pm\lfloor \pm n_c + 1/2\rfloor} W\Big(\kappa_i eV + 2\eta_i U n',\ U\Big)\,. \quad (4.19)$$

Conditions for the Coulomb blockade of tunneling at each junction are

$$\kappa_1\frac{eV}{2U} - \frac{1}{2} \leq n' \leq \kappa_1\frac{eV}{2U} + \frac{1}{2} \qquad \text{for } i = 1,$$

$$-\kappa_2\frac{eV}{2U} - \frac{1}{2} \leq n' \leq -\kappa_2\frac{eV}{2U} + \frac{1}{2} \quad \text{for } i = 2,$$

which are satisfied simultaneously if $n' = 0$ and $|eV| \leq \min(U/\kappa_1,\ U/\kappa_2)$. According to the same argument as in the high impedance limit, we obtain $n_c = 0$ and the Coulomb gap

$$V_c^L = \min[e/(2C_1),\ e/(2C_2)] \quad (4.20)$$

for the double junction in this limit, beyond which nonzero tunneling current flows. Noting that Eq. (4.10) reduces to

$$\sum_{\sigma=0,-1}\left[r_2\kappa_1 W\big(eV - 2U\{n+\sigma\}/\kappa_1,\ U/\kappa_1\big) - r_1\kappa_2 W\big(eV + 2U\{n+\sigma\}/\kappa_2,\ U/\kappa_2\big)\right] = 0 \quad (4.21a)$$

if n_c is the half-integral, and

$$r_2\kappa_1 W(eV - 2Un/\kappa_1,\ U/\kappa_1) - r_1\kappa_2 W(eV + 2Un/\kappa_2,\ U/\kappa_2) = 0 \quad (4.21b)$$

if otherwise, we get

$$\left.\begin{array}{c} V_n^L \\ V_{n-1/2}^L \end{array}\right\} = \frac{eR_\Sigma}{R_T^{(2)}C_2 - R_T^{(1)}C_1} \times \left\{\begin{array}{l} n - \dfrac{r_1 - r_2}{2} \\ n - \dfrac{1}{2} - \dfrac{r_1 - r_2}{2} \end{array}\right. \quad (4.22)$$

$(n = 0, \pm 1, \pm 2, \ldots,)$, with

$$\mathrm{sgn}(n) = \mathrm{sgn}\left(R_T^{(2)}C_2 - R_T^{(1)}C_1\right)\,, \quad (4.23)$$

provided that $R_T^{(2)}C_2 \neq R_T^{(1)}C_1$. Tunneling currents at these voltage steps are given by

$$I = \frac{1}{R_\Sigma}\left(V - \frac{e}{C_\Sigma}\right)\,. \quad (4.24)$$

As in the high impedance limit, there is no Coulomb

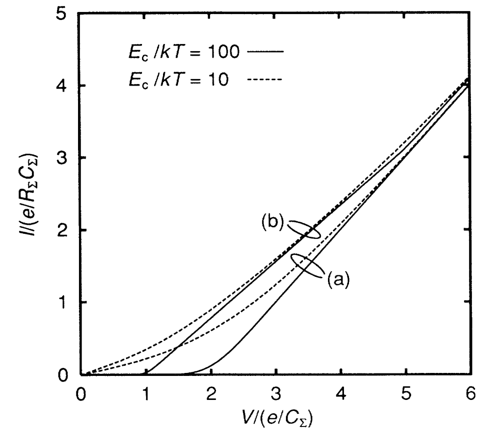

FIG. 3. Current-voltage characteristics of a symmetric double junction with $R_T^{(1)} = R_T^{(2)}$ and $C_1 = C_2$ at temperatures $E_c/kT = 100$ (solid lines) and 10 (dashed lines). Curves (a) and (b) are for the environmental impedances, $\hbar\omega_L/E_c = 0.01$ and 1, respectively.

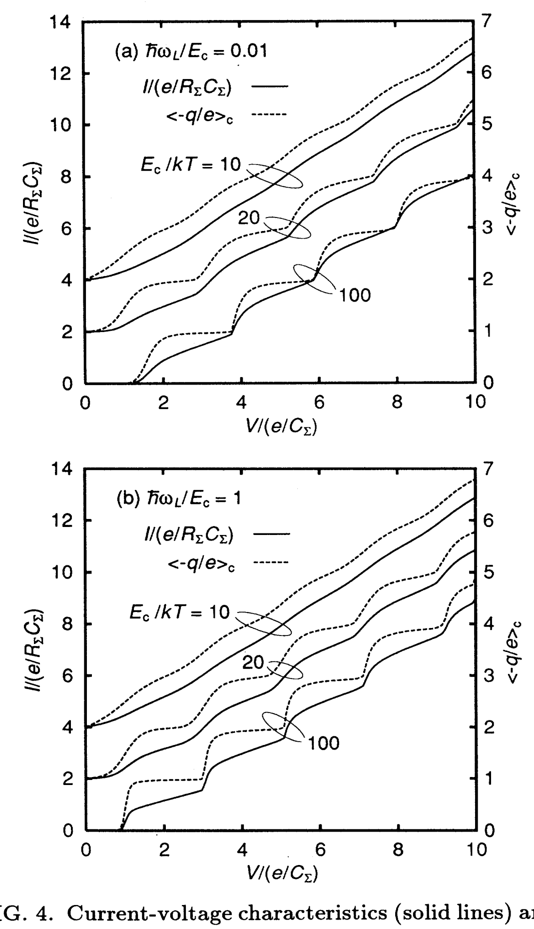

FIG. 4. Current-voltage characteristics (solid lines) and the number of charges on the island (dashed lines) of an asymmetric double junction with $R_T^{(1)}/R_T^{(2)} = 100$ and $C_1 = C_2$ at temperatures $E_c/kT = 100$, 20, and 10. (a) and (b) are for the double junctions coupled to the limits of high ($\hbar\omega_L/E_c = 0.01$) and low ($\hbar\omega_L/E_c = 1$) impedance environments, respectively. Curves for $E_c/kT = 20$ and 10 are offset for clarity ($I = 0$ at $V = 0$).

staircase if $R_T^{(2)}C_2 = R_T^{(1)}C_1$ and then the tunneling current is expressed by Eq. (4.24) for the voltage range $V \geq e/C_\Sigma$.

B. Finite temperatures

At finite temperatures we need numerical calculations even in the two extreme limits. Figure 3 shows the current-voltage characteristics for the symmetric junction with $R_T^{(1)} = R_T^{(2)}$ and $C_1/C_2 = 1$ for $E_c/kT = 100$ and 10. We choose here the impedances of external circuits as (a) $\hbar\omega_L/E_c = 0.01$ and (b) $\hbar\omega_L/E_c = 1$, which nearly correspond to high and low impedance limits, respectively. Coulomb gaps are clearly seen for both (a) and (b) for $E_c/kT = 100$. In (a) and (b), Coulomb gaps are nearly equal to $E_c/e = 2e/C_\Sigma$ and $2U/e = e/C_\Sigma$, re-

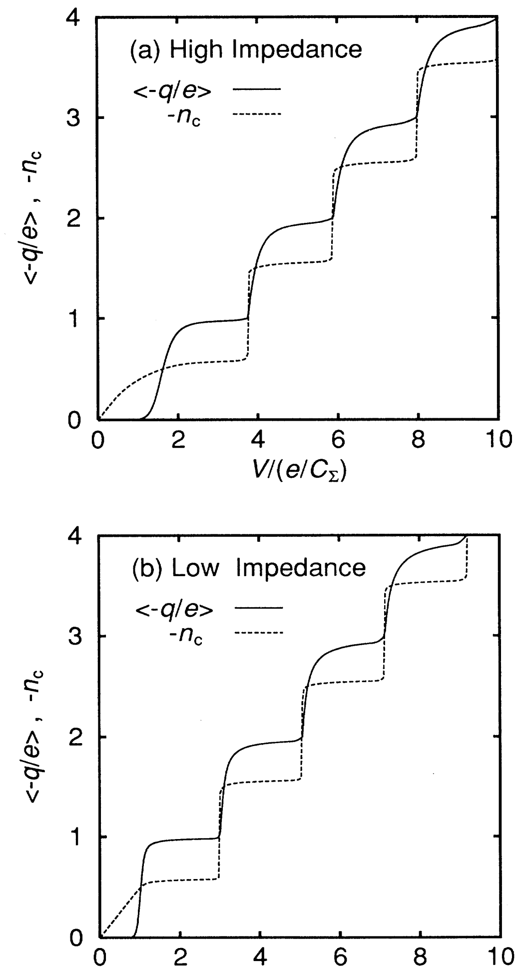

FIG. 5. $\langle -q/e \rangle_c$ (solid line) and $-n_c$ (dashed line) as functions of bias voltage for the asymmetric double junction with the same as junction parameters in Fig. 4 at temperature $E_c/kT = 100$. (a) and (b) are for the limits of high ($\hbar\omega_L/E_c = 0.01$) and low ($\hbar\omega_L/E_c = 1$) impedance environments, respectively.

spectively. Note that the Coulomb blockade of tunneling is rather rapidly lifted as the temperature increases. In fact, neither (a) nor (b) has an apparent Coulomb gap any more when E_c/kT becomes of the order of 10.

Figures 4(a) and 4(b) show the current-voltage characteristics for an asymmetric double junction with $R_T^{(1)}/R_T^{(2)} = 100$ and $C_1/C_2 = 1$ in the high and low impedance environments, respectively. The equilibrium numbers of electrons on the island, $\langle -q/e \rangle_c$, are also shown as a function of V. Coulomb staircases, which are the steplike increases in the currents with increasing bias voltage, are clearly seen in Figs. 4(a) and 4(b). Corresponding steplike variations of $\langle -q/e \rangle_c$ are also seen.

Figure 5 shows $-n_c$ as well as $\langle -q/e \rangle_c$ as functions of bias voltage for the same double junction as in Fig. 4. For that junction, differences in the chemical potential between adjacent electrodes are shown in Fig. 6. The gentle slopes in the I-V curves correspond to the voltage regions where $\langle -q/e \rangle_c$ becomes a certain integer, n, provided that the temperature is low enough as compared with E_c/k. In that voltage region, $-n_c$ varies from $n-1/2$ to $n+1/2$ as seen in Fig. 5. As noted above, when $\langle -q/e \rangle_c$ varies from n to $n+1$, $-n_c$ is nearly equal to $n-1/2$.

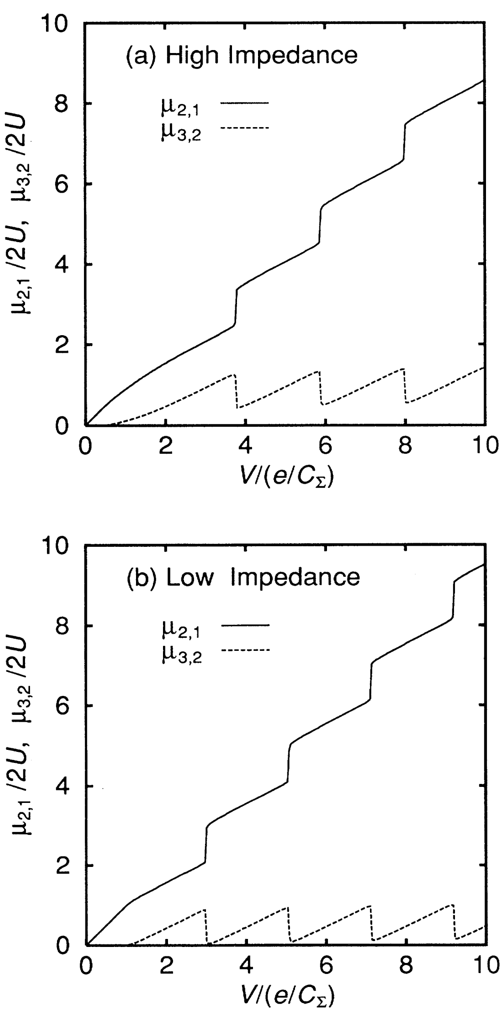

FIG. 6. Differences of chemical potential between adjacent electrodes, $\mu_{2,1}$ and $\mu_{3,2}$, as functions of bias voltage for the same asymmetric double junction as in Fig. 4.

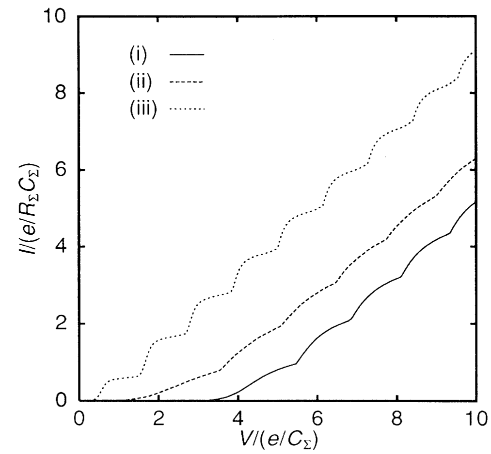

FIG. 7. Current-voltage characteristics of an asymmetric double junction with $R_T^{(1)}/R_T^{(2)} = 100$ and $C_1/C_2 = 10$ at temperature $E_c/kT = 100$ for various environmental impedances; $\hbar\omega_L/E_c = 0.01$ (solid line), 0.1 (dashed line), and 1 (short-dashed line).

Figure 7 shows current-voltage characteristics of an asymmetric double junction with $R_T^{(1)}/R_T^{(2)} = 100$ and $C_1/C_2 = 10$ at temperature $E_c/kT = 100$ for three environmental impedances. As $\hbar\omega_L/E_c$ increases, the Coulomb gap reduces from E_c/e to $e/(2C_1)$ but the steplike increase in current becomes more conspicuous because the electrons can determine the presence of the island more clearly in the weak impedance region.

It is instructive to see that the incremental charging of the island changes its sign depending on the difference in RC time constants of the junctions. I-V characteristics and the corresponding $\langle -q/e \rangle_c$ are shown, respectively, in Figs. 8 and 9 for various RC time constants. A change

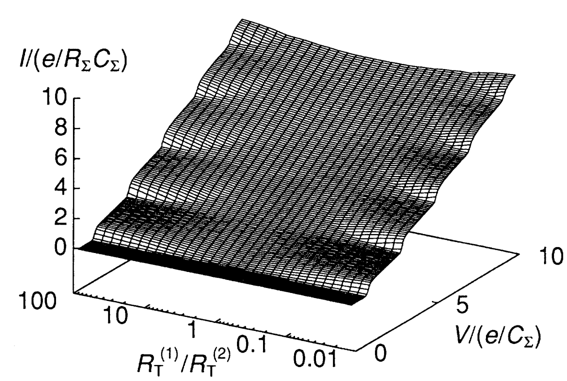

FIG. 8. Current-voltage characteristics of double junctions with $C_1 = C_2$ and various tunnel resistances at temperature $U/kT = 25$ under the low impedance environment ($\hbar\omega_L/E_c = 1$). Coulomb staircases are seen only when $R_T^{(1)}C_1 \neq R_T^{(2)}C_2$.

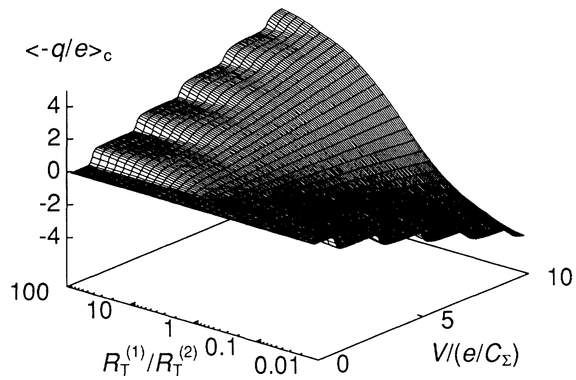

FIG. 9. Number of charges on the island of double junctions. Junction parameters, environmental impedance, and temperature are the same employed in Fig. 8. The crossover of charged states of the island from positive through neutral to negative is clearly seen depending on the sign of $R_T^{(1)}C_1 - R_T^{(2)}C_2$.

of the charging can be clearly seen in Fig. 9. Since an electron tunnels through from the third electrode to the first electrode for $V > 0$, the net change in the number of electrons tunneling into the island increases ($n > 0$) for $R_T^{(1)}/R_T^{(2)} > 1$, and decreases ($n < 0$) for $R_T^{(1)}/R_T^{(2)} < 1$.

V. CONCLUSIONS AND DISCUSSIONS

In this paper we have proposed the theory on the Coulomb blockade in ultrasmall double junctions with external circuits which self-consistently describes tunneling currents and charged states of the island without resorting to the stochastic technique. First we derived the island Hamiltonian, which comes from charging energy on the island and work done by external bias and contains the chemical potential (charged state) of the island to be determined self-consistently through the current continuity conditions.

According to the previous work,[21] the general expression for the tunneling current was obtained together with self-consistency equations for the charged state of the island. Current-voltage characteristics were discussed analytically at zero temperature in the limits of high and low impedance environments. Coulomb gaps were obtained analytically in both limits. The difference in current-voltage characteristics between symmetric and asymmetric double junctions (Coulomb staircases) were also analytically discussed in connection with the concept of neutral and charged islands. At finite temperatures current-voltage characteristics and the number of charges on the island under equilibrium configuration were calculated numerically for various junction parameters, environmental impedances, and temperatures. The theory reasonably describes the fundamental features of the ultrasmall double junctions with external circuits.

Throughout the paper we have only considered inductance $Z(\omega) = i\omega L$ as the external environment. This is just for simplicity. As already noted,[8,11,21] we can easily extend the discussion to the general case only by replacing Eq. (3.9) by

$$J(\tau) = \int_0^\infty \frac{d\omega}{\omega} \frac{\mathrm{Re} Z_t(\omega)}{R_q} \times \left\{ \coth \frac{\hbar\beta\omega}{2} (\cosh \omega\tau - 1) - \sinh \omega|\tau| \right\} \quad (5.1)$$

with

$$Z_t(\omega) = \frac{1}{i\omega C + Z^{-1}(\omega)} \quad (5.2)$$

for the general external impedance.

In this paper we have not considered the effect of the external charge modulation which can be realized by a gate electrode facing the island. The gate electrode is a fundamental element of the single electron tunneling devices. In this case, an extra charge Q_3, which is not C_gV_g (C_g is the gate capacitance, V_g is the gate voltage), appears on the gate electrode as a *dynamical variable.* In principle, we need the same treatment for Q_3 as we did for Q_1 and Q_2. More specifically, it is not trivial to identify a phase φ_3 which is canonically conjugate to Q_3. Therefore, the starting point is the Lagrangian formalism with a constraint,[22,23] by which we can successfully show what the φ_3 is, the e periodic variation of I-V_g characteristics, and the new aspect of the environmental effect in somewhat complicated external circuits. This will be reported elsewhere. Furthermore, it is straightforward to generalize the present theory to the cases where junction systems contain superconducting electrodes. This will be also reported elsewhere.

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