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Disorder-tuned transition between a quantum Hall liquid and Hall insulator

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We report measurements of the disorder-driven phase transition from a quantum Hall liquid to Hall insulator at $\nu=\frac{1}{3}$ and $\nu=2$ observed in a two-dimensional electron gas. The data show good scaling behavior over a wide range of densities and temperatures with an exponent $1/zx\sim0.43$ for both $\nu=\frac{1}{3}$ and 2, where z is the dynamical exponent and x is the correlation length exponent. The critical conductance at the transition is found to be $\sigma_{xx}=0.09\pm0.01e^2/h$ for the $\nu=\frac{1}{3}$ transition and $\sigma_{xx}=0.9\pm0.15e^2/h$ for the $\nu=2$ case.

In the limit of $T\rightarrow0$ there are two stable phases of the two-dimensional electron gas (2DEG) in the presence of disorder and magnetic field B: the quantum Hall liquid (QHL) characterized by $\rho_{xx}\rightarrow0$ and a quantized $\rho_{xy}=(h/e^2)(1/\nu)$, where ν is a rational number; and the Hall insulator (HI) characterized by $\rho_{xx}\rightarrow\infty$ and $\rho_{xy}\sim B/nec$ where n is the areal density of electrons.[1] By changing the applied magnetic field and/or disorder, it is possible to drive a transition between two QHL's, or from a QHL to HI, and thereby study the quantum critical phenomena associated with the transitions.

Magnetic-field-induced phase transition between different QHL's both in the integer quantum Hall effect (IQHE) and the fractional quantum Hall effect (FQHE) have been studied experimentally. Wei *et al.*[2] found that in a $In_xGa_{1-x}As/InP$ heterostructure, as the temperature is lowered, the slope in ρ_{xy} in the transition region from one plateau to the next becomes sharper as $T^{-\kappa}$ or $(d\rho_{xy}/dB)^{\max}\sim T^{-\kappa}$. Further, the exponent κ was found to be universal with a value of 0.42 ± 0.04 independent of the filling factor of the QHL. On the theoretical side, Jain, Kivelson, and Trivedi[3] obtained a microscopic law of corresponding states which related the behavior of the system in the fractional Hall regime to that in the integer Hall regime. They argued that the transition between fractional QHL's should also show the same exponents as that between the corresponding integer QHL's; for example, the exponents for the transition from $\nu=\frac{1}{3}$ to $\frac{2}{5}$ should be the same as those for the transition from $\nu=1$ to 2. This was indeed confirmed by Engel *et al.*,[4] where they obtained $\kappa=0.43\pm0.02$ for the transition between $\nu=\frac{1}{3}$ and $\frac{2}{5}$, thereby extending the universality of exponents to the fractional case as well. On the other hand, both the experiment by Koch *et al.* in a $GaAs/Al_xGa_{1-x}As$ heterostructure[5] and the experiment by Wakabayshi *et al.*[6] in a Si inversion layer have shown that the exponent κ is not universal. However, the correlation length exponent x was found to be universal by Koch *et al.*

More recently, it has become possible to study the transition from a QHL to a HI in a strongly disordered 2DEG. By changing the magnetic field a reentrant transition around $\nu=2$ has been discovered first by Jiang *et al.*,[7] and later observed by Wang *et al.*[8] and by Hughes *et al.*[9] In these experiments, two critical magnetic fields, from HI to QHL (at B_{c1}) and from QHL back to HI (at B_{c2}), were found. These observations are consistent with the global phase diagram of quantum Hall effect proposed by Kivelson, Lee, and Zhang[1] and the delocalization model given by Khmelnitskii and by Laughlin.[10] Furthermore, Hughes *et al.* found that the data for the QHL-HI transition near B_{c2} can be scaled via a single parameter $(B-B_{c2})T^{-\kappa}$ with $\kappa=0.43\pm0.05$. It was found by Wang *et al.* that the HI-QHL transition around B_{c1} has an exponent $\kappa=0.21\pm0.02$, the reduction of the exponent by roughly a factor of 2 was attributed to the fact that the transition from $\nu=2$ HI to QHL passes across two unresolved spin levels.

In the experiments mentioned above, the phase transition is induced by changing the magnetic field while disorder in the 2DEG system is "fixed." Here we address another type of phase transition that occurs between the QHL and HI by tuning effectively the degree of disorder in the system. While for a given sample there is unique impurity potential caused by the random distribution of the ionized donors, it is possible to manipulate the screened random potential experienced by the electrons by varying their density. In this paper, we will present our studies of the disorder-tuned transition in both the IQHE and FQHE regimes.

The two types of samples used in the experiments were modulation-doped $GaAs/Al_xGa_{1-x}As$ heterostructures fabricated by molecular-beam epitaxy. Type-*A* samples were cut from a wafer consisting of a 80-nm-thick undoped $Al_{0.3}Ga_{0.7}As$ spacer with high mobility ($\mu\sim1\times10^6$ $cm^2/V\,s$ at a density of $n\sim1\times10^{11}/cm^2$). For type-*B* samples, the 2DEG was formed without the undoped spacer[7] so that the 2DEG experienced large impurity potential from the random positions of the donors; thus providing a low mobility sample ($\mu\sim4\times10^4$ $cm^2/V\,s$ at a density of $n\sim4.6\times10^{11}/cm^2$). On both types of samples, Hall bar patterns of 1×3 mm^2 were etched out by standard lithographic

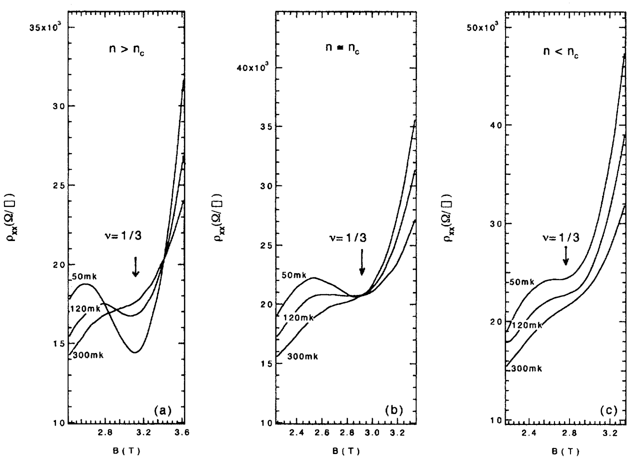

FIG. 1. The magnetoresistance curves $\rho_{xx}(T)$ of type-*A* sample at different temperatures when the 2DEG is (a) in the $\nu=\frac{1}{3}$ QHL phase; (b) on the phase boundary between QHL and HI; and (c) in the HI phase.

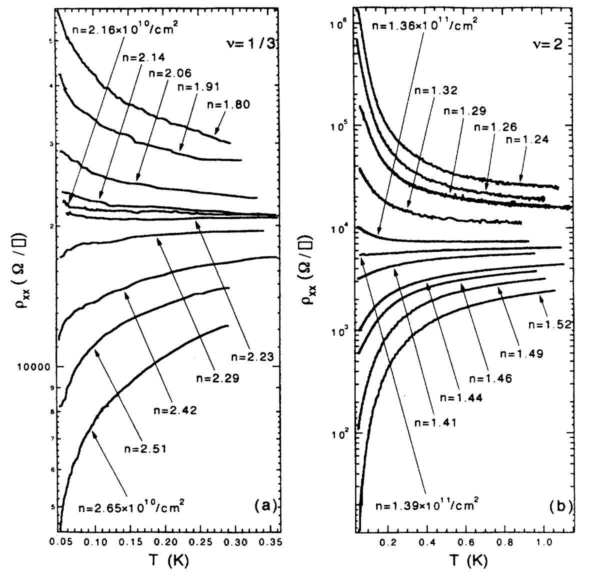

FIG. 2. (a) The magnetoresistance curves $\rho_{xx}(T)$ of a type-*A* sample at $\nu=\frac{1}{3}$ with different densities (units of $10^{10}/\mathrm{cm}^2$); (b) $\rho_{xx}(T)$ of a type-*B* sample at $\nu=2$ with different densities (units of $10^{11}/\mathrm{cm}^2$).

techniques and NiCr gates were evaporated onto their surfaces. The magnetotransport measurements were carried out by standard low-frequency lock-in techniques in either a ^{3}He cryostat or a $^3\mathrm{He}+{}^4\mathrm{He}$ dilution refrigerator.

The high-mobility and low-density type-*A* samples were used to study the phase transition in the fractional quantum-Hall-effect regime. When we swept the magnetic field with zero gate voltage on the sample at 50 mK, a sequence of FQHE states (with $\nu=\frac{5}{3},\frac{4}{3},\frac{2}{3},\frac{3}{5},\frac{3}{7},\frac{2}{5},\frac{1}{3}$) were clearly seen. As we increased the disorder of the sample by applying an increasingly negative gate voltage, at all filling factors except $\frac{1}{3}$, the minima in ρ_{xx} *vanished* and the quantized plateaus in ρ_{xy} were smeared out and approached the classical limit $\sim B/nec$. At $\nu=\frac{1}{3}$, however, the data exhibited a direct transition between the QHL and HI. Figure 1 shows the characteristic behavior of ρ_{xx} around $\nu=\frac{1}{3}$ at three different densities. In the QHL phase [Fig. 1(a)], there is a well-defined minimum in ρ_{xx} which drops in resistance as the temperature is lowered. In addition, there is a plateau in ρ_{xy} with a value of $3h/e^2$ for the same range of magnetic fields. A well-defined crossing point, independent of temperature, is seen at $B=3.41$ T, which makes the transition from QHL to HI induced by the magnetic field. On the low-field side, unlike the $\nu=2$ case, there is, however, no reentrant HI phase. Figure 1(b) shows the traces of ρ_{xx} at a slightly higher disorder (near the phase boundary). The minima of ρ_{xx} at three different temperatures intercept at $B=2.91$ T ($\nu=\frac{1}{3}$). As disorder is increased further, the dip at $\nu=\frac{1}{3}$ starts to lift up with the lowering of the temperature (which is a signature of a HI) as shown in Fig. 1(c). A series of measurements of ρ_{xx} at $\nu=\frac{1}{3}$ as a function of temperature have been performed in the proximity of the phase boundary. Figure 2(a) shows traces of ρ_{xx} as a function of temperature with different densities in a semilogarithmic plot. There appears a temperature-independent ρ_{xx} curve at a critical density n_c dividing the $\rho_{xx}(T)$ curves into two groups: one group with $n<n_c$ appears to be insulating ($\rho_{xx}\rightarrow\infty$ as $T\rightarrow 0$) and the other group of curves with $n>n_c$ appears to be dissipationless ($\rho_{xx}\rightarrow 0$ as $T\rightarrow 0$). It is theoretically predicted that the value of this critical conductivity is universal with a value of $\sigma_{xx}^c=\frac{1}{10}e^2/h$ for the $\nu=\frac{1}{3}$ QHL to HI transition.[1] The value of σ_{xx} deduced from the graph is $0.09e^2/h$ (different samples show a variation of around $\pm 10\%$). In a recent experimental study of Shahar *et al.*,[11] a universal resistivity of h/e^2 was found in the field-induced QHL to HI transitions (both around $\nu=1$ and $\nu=\frac{1}{3}$), which after matrix conversion to σ_{xx} shows good agreement with our results.

Similar measurements on type-*B* samples were performed and showed a transition from QHL to HI in the IQHE regime similar to the FQHE. As we increased disorder by decreasing the density of the sample, QHL's at different filling factors disappeared except at $\nu=2$ (no feature of $\nu=1$ at its corresponding magnetic field was seen as the spin splitting is expected to be unresolved in the presence of strong disorder). Figure 2(b) shows the curves of $\rho_{xx}(n,T)$. Again, a temperature-independent curve with a critical n_c separates the curves into a group with $n<n_c$ corresponding to the HI phase, and another group with $n>n_c$ corresponding to the QHL phase. In contrast to the case at $\nu=\frac{1}{3}$, the phase transition at $\nu=2$ is more drastic in the sense that a change of 10% in n from the critical density changes ρ_{xx} by three orders of magnitude at low temperatures, while for $\nu=\frac{1}{3}$ ρ_{xx} only changes by about an order of magnitude. This difference arises because of a large energy gap for $\nu=2$ compared to $\nu=\frac{1}{3}$. Studies of different samples have shown that the critical conductance is $0.9\pm 0.15e^2/h$, which is close to the theoretically calculated value of $\sigma_{xx}^c=0.5e^2/h$ per spin.[1,12]

The fact that there is only a direct phase transition between QHL-HI at $\nu=\frac{1}{3}$ and 2 (no intermediate metallic phase) is consistent with the global phase diagram.[1,10] In the phase diagram, transitions between the HI state and the QHL states at $\nu=\frac{1}{3}$ and 2 (spin splitting unresolved) are permitted, but all other QHL states are separated from the insulating state by a regime of at least one other QHL state.

The transition from QHL to HI at $T=0$ is an example of a continuous quantum phase transition. The transport coefficients are therefore expected to show scaling behavior. As the quantum critical point at $T=0$ and $n=n_c$ between the QHL and HI is approached by decreasing the density n, the correlation length diverges $\xi \sim \delta^{-x}$ where $\delta=|n-n_c|/n_c$ and x is the correlation length exponent.[13] In addition, since we are looking at quantum critical phenomena the statics and dynamics are linked by the Hamiltonian. So along with the divergence of the correlation length the correlation time also diverges as $\xi_\tau \sim \xi^z \sim \delta^{-zx}$ (critical slowing down) where z is the dynamical exponent. However, the physical size of the sample cuts off the divergence of the correlation length and thermal effects cut off the divergence of the correlation time. Near the transition we can make the following ansatz for the scaling function:

$$\rho_{\alpha\beta}(n,T)=\rho^c_{\alpha\beta}g(L/\xi,L_\tau/\xi_\tau).$$

The size dependence of ρ_{xx} was used previously to extract the exponent x.[5] However, the analysis is complicated by the fact that the second argument in the scaling function is not constant. Usually, in experiments one is dealing with a macroscopic sample, so we have $\rho_{\alpha\beta}(n,T)=\rho^c_{\alpha\beta}g(\infty,L_\tau/\xi_\tau)$. Upon using the definitions given above we have

$$\rho_{\alpha\beta}=\rho^c_{\alpha\beta}f(c_0|n-n_c|/T^{1/zx})$$

with c_0 a nonuniversal constant and $f(y)=g(\infty,y)$. Such a scaling function has been used successfully in studies of the field-induced QHL-HI transitions,[8,9] and has been used also to describe the superconductor-insulator transition[14] in disordered films. As can be seen from the above manipulations, the exponent κ extracted in the experiments is $1/zx$. Previously it was assumed in the literature that, at a finite temperature, ξ is cut off by the inelastic scattering length $L_{\rm in}(T)\sim T^{-p/2}$ and form the relation $\xi_{\rm max}\sim|B-B_c|^{-x}\sim L_{\rm in}$ we see that width of the transition $|B-B_c|\sim T^{p/2x}$. This suggests that $\kappa=p/2x$ may mot be universal, since it depends on the precise inelastic scattering mechanism. However, as we discussed above, since $\kappa=1/zx$ and is purely determined by exponents characterizing the $T=0$ quantum phase transition, it is universal. In fact, it has been argued theoretically that the finite temperature scaling observed by Wei *et al.* reflects the critical dynamics and the finite width of the system in the imaginary time direction (i.e., the thermal fluctuation), rather than the effective finite spatial size of the system as determined by the inelastic scattering length.[1]

In Fig. 3, $\log_{10}\rho_{xx}(n,T)$ is plotted against the scaling variable $(|n-n_c|/T^{1/zx})$ for $\nu=\frac{1}{3}$ in Fig. 3(a) and for $\nu=2$ in Fig. 3(b) with $1/zx=0.43$. While we cannot determine z and x separately, our results are consistent with $z=1$ (Ref. 15) and $x=\frac{7}{3}$. Our value of x is consistent with the prediction of the quantum-percolation model[16] as well as with numerical studies of noninteracting electrons in a random potential.[17] In fact, a recent experiment by Shashkin *et al.* has shown experimentally the QHL-HI transition is a percolation transition.[18] The curves with $n<n_c$, corresponding to the HI phase, collapse onto one branch while curves with $n>n_c$, corresponding to the QHL phase, collapse onto the other branch. While the collapse is not perfect, reasonable scaling behavior is exhibited for $1/zx=0.45\pm0.05$ for $\nu=\frac{1}{3}$ and $1/zx=0.44\pm0.05$ for $\nu=2$. We would like to note here that, to increase the disorder of our samples, a negative voltage is applied on them to reduce the electron density. In effect, this shifts the minima of $\nu=\frac{1}{3}$ and 2 to a magnetic field that is smaller by about 10%. We believe that the shift in B can consequently change the amplitude of the scaling curves, but not the resultant exponents.

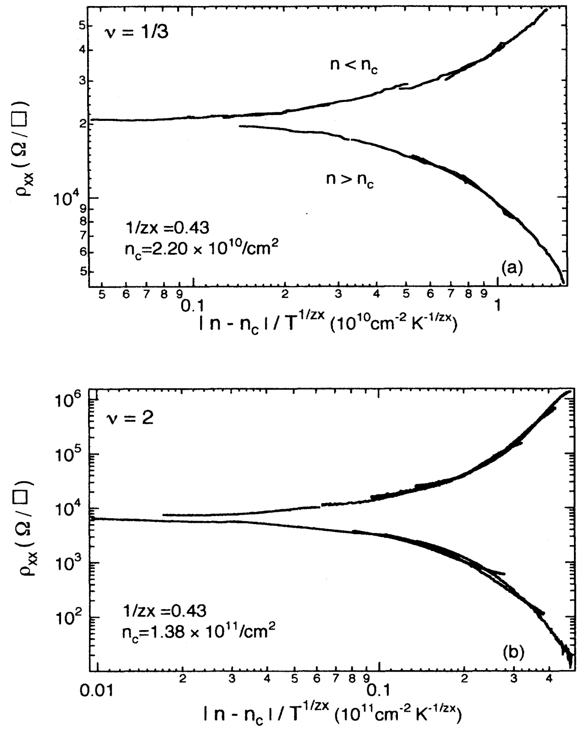

FIG. 3. Scaling analysis of ρ_{xx} vs $(|n-n_c|/T^{1/zx})$ with $1/zx=0.43$ at (a) $\nu=\frac{1}{3}$; and (b) $\nu=2$.

As mentioned earlier, it is claimed on theoretical grounds that phase transitions between different QHL's, and between QHL and HI for both fractional and integer cases all belong to the same universality class, in the sense that they are characterized by the same critical exponents.[1,3] Our results, given our error bars, are consistent with this claim. We would also like to comment here that our results at $\nu=2$ (a spin-degenerate case) clearly show no doubling[8] of the correlation length exponent x as $1/zx\approx0.2$ is clearly outside of our error bars.

The QHL-HI transition show strong resemblance to the 2D superconductor-insulator transition observed in thin films by tuning the degree of disorder.[19] This apparent similarity might not be coincidental. In a Landau-Ginzburg theory for the quantum Hall effect, an electron is mapped onto a composite of a charged boson and a flux tube with odd number of fundamental flux quanta $\phi_0=hc/e$ attached to it (Chern-Simons gauge fields).[20] At special filling factors $\nu=1/(2k+1)$, where k is an integer, the Chern-Simons field, which is determined by the particle density, exactly cancels the external magnetic field, on average, and one obtains a problem of bosons. Thus the transition from a QHL to HI at special fillings can be viewed as a superfluid-insulator transition in the Bose system. Recently, quantum Monte

Carlo simulations have been used to study the disorder-driven superfluid-insulator transition for a model of interacting bosons in a random potential.[21,22]

In summary, we have studied the disorder-tuned QHL-HI transitions. Direct phase transitions between QHL and HI are found only for the $\nu=\frac{1}{3}$ and 2 states. Scaling behavior is well exhibited in the vicinity of $1/zx=0.43$. The critical conductance at the transition is found to be $\sigma_{xx}=0.09\pm0.01e^2/h$ for the $\nu=\frac{1}{3}$ transition and $\sigma_{xx}=0.9\pm0.15e^2/h$ for the $\nu=2$ case.

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