PHYSICAL REVIEW B VOLUME 51, NUMBER 24 15 JUNE 1995-II

Possible bistability of the persistent current of two interacting electrons in a quantum ring

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(Received 20 December 1994)

We calculate the energy band structure and the persistent current of two interacting electrons in a quantum ring in the presence of a magnetic flux for a strongly correlated situation: a rotating Wigner molecule with internal vibrations. We discuss a possible bistability and hysteresis in the persistent current in dependence on the magnetic flux.

Three very interesting experiments measuring persistent currents in normal-metal[1,2] and semiconductor rings[3] threaded by a magnetic flux Φ have been performed recently. These studies indicate that in the diffusive regime[1,2] the magnitudes of the persistent currents are much larger than those predicted for simple models of noninteracting electrons,[4–8] while in the ballistic regime[3] this simple theory seems to agree with experiment. This led to two important issues: (i) the role of the choice of the statistical ensemble to calculate average values[4–10] and (ii) the role of the electron-electron interaction (EEI) in the presence of impurities.[11–20,23,24] The disorder-averaged persistent current has been found[10] to be vanishingly small when the grand-canonical ensemble is invoked, while it is of finite magnitude within the framework of the canonical ensemble.[4,6–9] At the moment, the role of the EEI in disordered systems is still unclear and is an open subject for research. Much work has been done to explore persistent currents in the diffusive regime,[7–9,11–20] but comparatively little is known about the ballistic regime.[4,21–27] The ballistic regime of nanostructures is important because it allows the studying of quantum systems, which are classically chaotic.[25] Further, in recent papers[26,27] it was shown that in the ballistic regime a rigorous quantum-mechanical theory of persistent currents can be developed, with the result that the coupling between the different channels contributing to the persistent current causes the occurrence of higher harmonics of the flux quantum $\Phi_0 = h/e$ in the persistent current.[26] On the basis of this theory it was shown[23,24,28] for the case of few interacting electrons that this electron system, quantum confined in a narrow-width ring, forms a rotating Wigner crystal with relative angular motions of the electrons in the form of the harmonic oscillations and radial motions depending on the shape of the confining potential.

The aim of the present paper is to investigate the electronic ground state of a two-electron quantum ring (QR) (ideal or clean ring) of finite width in the presence of a magnetic flux Φ threading the opening of the ring, i.e., for an Aharonov-Bohm (AB) geometry.

We consider two electrons ($i = 1,2$) with the effective conduction-band-edge mass m_e in the plane $z = 0$, confined in a QR by the potential $V_C(\mathbf{x}_i)$ in the x-y plane. This potential is taken as zero in the region $R - W/2 < r_i < R + W/2$ and infinite otherwise. Assuming an AB geometry with magnetic field $\mathbf{B} = (0,0,B)$ and vector potential $\mathbf{A}(\mathbf{x}) = \frac{1}{2}\mathbf{B} \times \mathbf{x}$ and introducing polar coordinates in the x-y plane $\mathbf{x}_{\parallel i} = (r_i, \varphi_i)$, the relative angular coordinate $\gamma = \varphi_1 - \varphi_2$, and the angular coordinate $\Theta = (\varphi_1 + \varphi_2)/2$ describing the motion of the two-electron system as a whole, the Hamiltonian reads

$$
\begin{aligned}
H = \sum_{i=1}^{2} \Bigg\{ \left[-\frac{\hbar^2}{2m_e}\left(\frac{\partial^2}{\partial r_i^2} + \frac{1}{r_i}\frac{\partial}{\partial r_i}\right) + V_C(r_i) \right] \Bigg\} \\
-\frac{\hbar^2}{2m_e}\Bigg\{ \left(\frac{1}{r_1^2} + \frac{1}{r_2^2}\right)\left[\frac{\partial^2}{\partial \gamma^2} + \frac{1}{4}\left(\frac{\partial}{\partial \Theta} - 2i\frac{\Phi}{\Phi_0}\right)^2\right] \\
+ \left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)\frac{\partial}{\partial \gamma}\left(\frac{\partial}{\partial \Theta} - 2i\frac{\Phi}{\Phi_0}\right)\Bigg\} \\
+ \frac{e^2}{4\pi\varepsilon_0\varepsilon_s\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\gamma}},
\end{aligned}
\tag{1}
$$

where ε_0 is the permittivity in vacuum and ε_s is the static dielectric constant of the host semiconductor. For a narrow-width ring, i.e., if $W \ll R$, the radial motion is much faster than the angular motions. Hence the radial motion is *adiabatically decoupled* from the angular motions with the result[23]

$$
\Psi(r_1, r_2; \gamma, \Theta) = \sum_{P,K_1,K_2} \Xi^{P}_{K_1,K_2}(r_1, r_2)\, \psi_{P,K_1,K_2}(\gamma, \Theta)
\tag{2}
$$

for the orbital part of the *two-electron wave function.* The radial part $\Xi^{P}_{K_1,K_2}(r_2, r_1)$ is a symmetrized ($K_1 = K_2, P = 0$ and $K_1 \neq K_2, P = 2$) or an antisymmetrized ($K_1 \neq K_2, P = 1$) product of the corresponding single-particle wave functions $\chi_{K_i}(r_i)$, which have eigenenergies $\mathcal{E}_{K_i} = \hbar^2\pi^2K_i^2/(2m_eW^2)$, with quantum number $K_i = 1, 2, \ldots$ and P describes the symmetry according to particle permutation. Because the single-particle wave functions are orthonormalized, the set $\{\Xi^{P}_{K_1,K_2}(r_2, r_1)\}$ forms a closure set of orthonormalized functions. The *single-valuedness boundary conditions* $\psi_{P,K_1,K_2}(\varphi_1, \varphi_2 + 2\pi) = \psi_{P,K_1,K_2}(\varphi_1 + 2\pi, \varphi_2) = \psi_{P,K_1,K_2}(\varphi_1 + 2\pi, \varphi_2 + 2\pi) = \psi_{P,K_1,K_2}(\varphi_1, \varphi_2)$ mean, in terms of the new variables

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γ and Θ, $\psi_{P,K_1,K_2}(\gamma+2\pi,\Theta+\pi) = \psi_{P,K_1,K_2}(\gamma,\Theta+2\pi) = \psi_{P,K_1,K_2}(\gamma,\Theta)$. Removing the magnetic flux from the Hamiltonian by a gauge transformation, these conditions change to the *twisted boundary conditions* $\psi_{P,K_1,K_2}(\gamma+2\pi,\Theta+\pi) = \exp[i2\pi\Phi/\Phi_0]\psi_{P,K_1,K_2}(\gamma,\Theta)$ and $\psi_{P,K_1,K_2}(\gamma,\Theta+2\pi) = \exp[i4\pi\Phi/\Phi_0]\psi_{P,K_1,K_2}(\gamma,\Theta)$.

Now we substitute Eq. (2) into the Schrödinger equation $H\Psi = E\Psi$ with the transformed Hamiltonian of Eq. (1), multiply both parts by $\Xi^{P}_{K_1,K_2}(r_1,r_2)$, and integrate over r_1,r_2. As long as the above-stated criterion of the adiabatic approximation is satisfied, the excited states of the radial motion [the nearest ones to the ground state are those with $P = 1$ and $2, K_1 = 1, K_2 = 2$, and the energy $5\hbar^2\pi^2/(2m_eW^2)$] lie high above the ground state, which has the energy $\hbar^2\pi^2/(m_eW^2)$. As a consequence, we can restrict the consideration to the ground state of the radial motion ($P' = P = 0, K_1' = K_1 = 1, K_2' = K_2 = 1$). Hence the variables γ and Θ become separated:

$$\left\{-\frac{\hbar^2}{m_e}\left\langle\frac{1}{r_1^2}\right\rangle\left(\frac{\partial^2}{\partial\gamma^2}+\frac{1}{4}\frac{\partial^2}{\partial\Theta^2}\right) + \left\langle\frac{e^2}{4\pi\varepsilon_0\varepsilon_s\sqrt{r_1^2+r_2^2-2r_1r_2\cos\gamma}}\right\rangle + 2\mathcal{E}_1 - E\right\}\psi_{0,1,1}(\gamma,\Theta) = 0, \tag{3}$$

where $\langle\,\rangle$ denotes the average with the radial wave functions

$$\langle\,\rangle = \int dr_1\, r_1 \int dr_2\, r_2\, \Xi^0_{1,1}(r_1,r_2)\cdots\Xi^0_{1,1}(r_1,r_2). \tag{4}$$

Thus the relative angular motion is separated from the angular motion of the two-electron system as a whole. Therefore the angular wave function can be represented in the form

$$\psi_{0,1,1}(\gamma,\Theta) = \sum_{p}\sum_{j=0}^{\infty}\Phi_j^p(\gamma)\,Q_{\nu_\Upsilon}(\Theta), \tag{5}$$

where the wave function of the relative angular motion is a solution of the equation

$$\left\{-\frac{\hbar^2}{R^2m_e}\frac{\partial^2}{\partial\gamma^2} + \frac{e^2}{4\pi\varepsilon_0\varepsilon_s R\sqrt{2(1-\cos\gamma)}} - \mathcal{E}_j^{\text{r.a.},p}\right\}\Phi_j^p(\gamma) = 0. \tag{6}$$

Here p describes the possible symmetry types of $\Phi_j^p(\gamma)$ and Υ stands for $\{P = 0, K_1 = 1, K_2 = 1, p, j\}$. In Eq. (6), we have used

$$\left\langle\frac{1}{r_1^2}\right\rangle \approx \frac{1}{R^2}$$

and

$$\left\langle\frac{e^2}{4\pi\varepsilon_0\varepsilon_s\sqrt{r_1^2+r_2^2-2r_1r_2\cos\gamma}}\right\rangle \approx \frac{e^2}{4\pi\varepsilon_0\varepsilon_s R\sqrt{2(1-\cos\gamma)}}.$$

In view of the 2π periodicity of the mean Coulomb potential as a function of γ, the wave function of the relative angular motion has the form $\Phi_j^p(\gamma) = \exp(ip\gamma)u_j^p(\gamma)$, where $u_j^p(\gamma+2\pi) = u_j^p(\gamma)$ and $-\frac{1}{2} < p \leq \frac{1}{2}$. If the typical Coulomb interaction energy between the two electrons $E_{\text{Coul}} = e^2/(8\pi\varepsilon_0\varepsilon_s R)$ is much larger than the oscillation energy $\mathcal{E}_j^{\text{osc}} = \mathcal{E}_j^{\text{r.a.},p} - E_{\text{Coul}}$, the relative angular motion can be considered in the tight-binding approximation

$$\Phi_j^p(\gamma) = \sum_{Q=-\infty}^{\infty}\exp(ip\gamma_Q)\phi_j(\gamma-\gamma_Q), \tag{7}$$

where $\phi_j(\gamma-\gamma_Q)$ is the wave function of a single well with minimum at $\gamma_Q = (2Q+1)\pi$, $Q = 0, \pm1, \pm2, \ldots$. This wave function is the eigenfunction of

$$\left\{-\frac{\hbar^2}{R^2m_e}\frac{d^2}{d\gamma^2} + \frac{m_e\Omega^2r^2}{4}(\gamma-\gamma_Q)^2 - \mathcal{E}_j^{\text{osc}}\right\}\phi_j(\gamma-\gamma_Q) = 0, \tag{8}$$

where $\Omega^2 = e^2/(16\pi\varepsilon_0\varepsilon_s m_e R^3)$. In Eq. (8), we have expanded the mean Coulomb potential in a power series of $(\gamma-\gamma_Q)$, taking into account the harmonic approximation, valid if $\hbar\Omega \ll e^2/(8\pi\varepsilon_0\varepsilon_s R)$. Because tunneling between the wells is neglected, the energy becomes independent of p: $\mathcal{E}_j^{\text{r.a.},p} \equiv \mathcal{E}_j^{\text{r.a.}}$. In the following, the wave functions $\phi_j(\gamma-\gamma_Q)$ are harmonic-oscillator eigenfunctions with eigenenergies $\mathcal{E}_j^{\text{osc}} = \hbar\Omega(j+\frac{1}{2})$, $j = 0, 1, 2, \ldots$. Now it becomes obvious that the adiabatic approximation used above is valid if $\hbar\Omega \ll \hbar^2\pi^2/(m_eW^2)$. It is noticeable that the adiabatic approximation, as well as the harmonic approximation, is valid for typical semiconductor rings used in experiments.

The solution of the Schrödinger equation for the free rotation of the two-electron system as a whole

$$\left(\frac{d^2}{d\Theta^2} + \nu_\Upsilon^2\right)Q_{\nu_\Upsilon}(\Theta) = 0, \tag{9}$$

characterized by a rotational momentum

$$\nu_\Upsilon = \nu_\Upsilon(E) = \left\{\frac{4m_eR^2}{\hbar^2}\left[E - 2\mathcal{E}_1 - \frac{e^2}{8\pi\varepsilon_0\varepsilon_s R} - \mathcal{E}_j^{\text{osc}}\right]\right\}^{1/2},$$

is

$$Q_{\nu_\Upsilon}(\Theta) = \frac{1}{\sqrt{2\pi}}\exp(i\nu_\Upsilon\Theta). \tag{10}$$

The curves $\nu_\Upsilon(E)$ define the *channels*[26,27] for the angular motion of the two-electron system as a whole. On the other hand, using the wave function of Eq. (5) in the Schrödinger equation (3) with a fixed rotational momentum, we obtain the eigenenergies of two electrons in a narrow-width ring as a function of ν_Υ:

$$E_{\Upsilon,\nu_\Upsilon} = 2\mathcal{E}_1 + \frac{e^2}{8\pi\varepsilon_0\varepsilon_s R} + \mathcal{E}_j^{\text{osc}} + \frac{\hbar^2\nu_\Upsilon^2}{4m_eR^2}. \tag{11}$$

According to the Pauli principle, the total two-electron wave function (including spin) must be antisymmetric

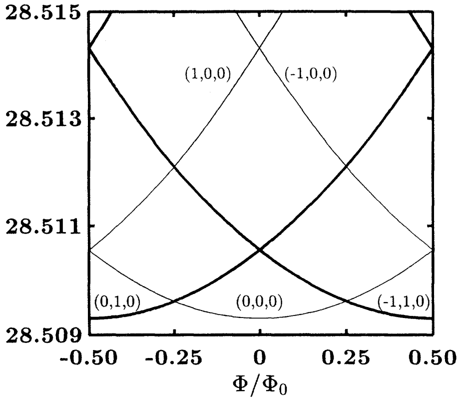

FIG. 1. Energy bands $E_{\Upsilon,\nu_\Upsilon}(\Phi,2)$ given by Eq. (10) of the lowest para ($S=0$, thin solid lines) and ortho ($S=1$, heavy solid lines) states for a two-electron quantum ring with $R = 480$ nm and $W = 20$ nm. The bands are denoted by (m,S,j). For the calculation the parameters of GaAs are used: $\varepsilon_s = 12.87$ and $m_e = 0.06624 m_0$, with m_0 the bare electron mass.

under particle permutation and hence, the orbital part fulfills $\Psi(r_2,r_1,-\gamma,\Theta) = \eta\Psi(r_1,r_2,\gamma,\Theta)$. For total spin $S=0$ it follows $\eta = 1$, the *para state* (or singlet state), where the spin function $|S,M_S\rangle$ is an antisymmetric eigenfunction of the total spin projection operator with eigenvalue $S_z = \hbar M_S$, $M_S = 0$. For $S=1$ it follows that $\eta = -1$, the *ortho state* (or triplet state), accompanied by three symmetric spin functions, which are eigenfunctions of the total spin projection operator with quantum numbers $M_S = -1, 0$, and $+1$. This imposes selection rules for the possible values of the rotational momentum ν_Υ:[24] for $S=0$ (the para state),

$$\nu_\Upsilon = 2m + 2\frac{\Phi}{\Phi_0}| \text{ if } j = 2k$$

$$\text{or } \nu_\Upsilon = 2m + 1 + 2\frac{\Phi}{\Phi_0} \text{ if } j = 2k+1$$

and for $S=1$ (the ortho state),

$$\nu_\Upsilon = 2m + 1 + 2\frac{\Phi}{\Phi_0} \text{ if } j = 2k$$

$$\text{or } \nu_\Upsilon = 2m + 2\frac{\Phi}{\Phi_0} \text{ if } j = 2k+1, \qquad (12)$$

where $m = 0, \pm1, \pm2\ldots$, and $k = 0,1,2,\ldots$. The energy bands of two interacting electrons in a narrow-width QR are plotted in Fig. 1. It is seen that for $\Phi/\Phi_0 = 0$ and 0.5 the energy bands of a certain symmetry become degenerate. In the case of a mesoscopic ring with impurities this degeneracy becomes lifted and gaps between the energy bands open.[23] Further, for a QR the bands of para and ortho states become degenerate at $\Phi/\Phi_0 = 0.25$. This degeneracy still remains for non-magnetic impurities and is lifted by the scattering of the electrons on magnetic impurities or due to the spin-orbit coupling.

Hence, from the energy bands determined by Eq. (11), the lowest energy band of the para state in the region $-\frac{1}{2} < \frac{\Phi}{\Phi_0} \le \frac{1}{2}$ is

$$E^{\text{para}}(\Phi,2) = \frac{\hbar^2\pi^2}{m_e W^2} + \frac{e^2}{8\pi\varepsilon_0\varepsilon_s R} + \frac{\hbar\Omega}{2} + \frac{\hbar^2}{m_e R^2}\left(\frac{\Phi}{\Phi_0}\right)^2, \qquad (13)$$

which gives the *persistent (azimuthal single-band) current*, defined by $I_A^{(\Upsilon,\nu_\Upsilon)} = -\partial E_{\Upsilon,\nu_\Upsilon}/\partial\Phi$, in the form

$$I_A^{\text{para}}(\Phi,2) = -\frac{2\hbar^2}{\Phi_0 m_e R^2}\frac{\Phi}{\Phi_0}. \qquad (14)$$

On the other hand, the lowest energy band in the region $-\frac{1}{2} < \frac{\Phi}{\Phi_0} \le \frac{1}{2}$ of the ortho state is

$$E^{\text{ortho}}(\Phi,2) = \frac{\hbar^2\pi^2}{m_e W^2} + \frac{e^2}{8\pi\varepsilon_0\varepsilon_s R} + \frac{\hbar\Omega}{2} + \begin{cases} \frac{\hbar^2}{m_e R^2}\left(\frac{\Phi}{\Phi_0} + \frac{1}{2}\right)^2 & \text{for } -\frac{1}{2} < \frac{\Phi}{\Phi_0} < 0 \\ \frac{\hbar^2}{m_e R^2}\left(\frac{\Phi}{\Phi_0} - \frac{1}{2}\right)^2 & \text{for } \ 0 < \frac{\Phi}{\Phi_0} \le \frac{1}{2} \end{cases} \qquad (15)$$

and the persistent current reads

$$I_A^{\text{ortho}}(\Phi,2) = -\begin{cases} \frac{2\hbar^2}{m_e R^2 \Phi_0}\left(\frac{\Phi}{\Phi_0} + \frac{1}{2}\right) & \text{for } -\frac{1}{2} < \frac{\Phi}{\Phi_0} < 0 \\ \frac{2\hbar^2}{m_e R^2 \Phi_0}\left(\frac{\Phi}{\Phi_0} - \frac{1}{2}\right) & \text{for } \ 0 < \frac{\Phi}{\Phi_0} \le \frac{1}{2}. \end{cases} \qquad (16)$$

As follows from Eq. (12), at $\Phi = 0$, the para state should have a wave function of the angular motion of the two-electron system as a whole, which is *symmetric* with respect to a rotation $\Theta \to \Theta + \pi$, $Q_{\nu_\Upsilon}(\Theta + \pi) = Q_{\nu_\Upsilon}(\Theta)$; the lowest energy of such states belongs to $\nu_\Upsilon = 0$. At the same time, the ortho state should have a wave function of the angular motion of the two-electron system as a whole, which is *antisymmetric* with respect to this rotation $\Theta \to \Theta + \pi$, $Q_{\nu_\Upsilon}(\Theta + \pi) = -Q_{\nu_\Upsilon}(\Theta)$, and hence the lowest energy of such states belongs to $\nu_\Upsilon = \pm1$ and is higher than that of the lowest energy of para states. Therefore, the ground state at $\Phi = 0$ is a para state.

The *minimal energy* in $-\frac{1}{2} < \frac{\Phi}{\Phi_0} \le \frac{1}{2}$ belongs to different states:

$$\min\{E_{\Upsilon,\nu_\Upsilon}(\Phi,2)\} = \frac{\hbar^2\pi^2}{m_e W^2} + \frac{e^2}{8\pi\varepsilon_0\varepsilon_s R} + \frac{\hbar\Omega}{2} + \begin{cases} \frac{\hbar^2}{m_e R^2}\left(\frac{\Phi}{\Phi_0} + \frac{1}{2}\right)^2 & \text{for } -\frac{1}{2} < \frac{\Phi}{\Phi_0} \le -\frac{1}{4} \\ \frac{\hbar^2}{m_e R^2}\left(\frac{\Phi}{\Phi_0}\right)^2 & \text{for } -\frac{1}{4} \le \frac{\Phi}{\Phi_0} \le \frac{1}{4} \\ \frac{\hbar^2}{m_e R^2}\left(\frac{\Phi}{\Phi_0} - \frac{1}{2}\right)^2 & \text{for } \ \frac{1}{4} \le \frac{\Phi}{\Phi_0} \le \frac{1}{2}. \end{cases} \qquad (17)$$

In correspondence to this minimal energy, the persistent current suffers *discontinuities* at $\frac{\Phi}{\Phi_0} = \pm\frac{1}{4}$:

$$I_A(\Phi,2) = \begin{cases} -\frac{2\hbar^2}{\Phi_0 m_e R^2}\left(\frac{\Phi}{\Phi_0}+\frac{1}{2}\right) & \text{for} \quad -\frac{1}{2} < \frac{\Phi}{\Phi_0} \leq -\frac{1}{4} \\ -\frac{2\hbar^2}{\Phi_0 m_e R^2}\frac{\Phi}{\Phi_0} & \text{for} \quad -\frac{1}{4} \leq \frac{\Phi}{\Phi_0} \leq \frac{1}{4} \\ -\frac{2\hbar^2}{\Phi_0 m_e R^2}\left(\frac{\Phi}{\Phi_0}-\frac{1}{2}\right) & \text{for} \quad \frac{1}{4} \leq \frac{\Phi}{\Phi_0} \leq \frac{1}{2}. \end{cases} \tag{18}$$

The persistent currents of a two-electron QR are plotted in Fig. 2(a) for the case where the system is in the lowest para state (thin solid line) and in the lowest ortho state (heavy solid line) and in Fig. 2(b) for the case where the minimal energy of Eq. (16) is realized. Whereas the first case [Fig. 2(a)] demands total spin conservation of the two-electron system, the second picture [Fig. 2(b)], however, can be realized only under a *fast relaxation* between the states $S = 0$ and $S = 1$ due to spin-flip processes that change the symmetry of the state. If such a relaxation is slow, which is a very realistic case, then the para state (or the ortho state) can still exist beyond the above limits of Φ, where they are the lowest states, becoming *metastable* states, and hence the persistent currents carried by these states can be observed. If, nevertheless, the symmetry breaking spin-flip occurs, we just obtain a hysteresis of the persistent current, schematically represented in Fig. 2(c).

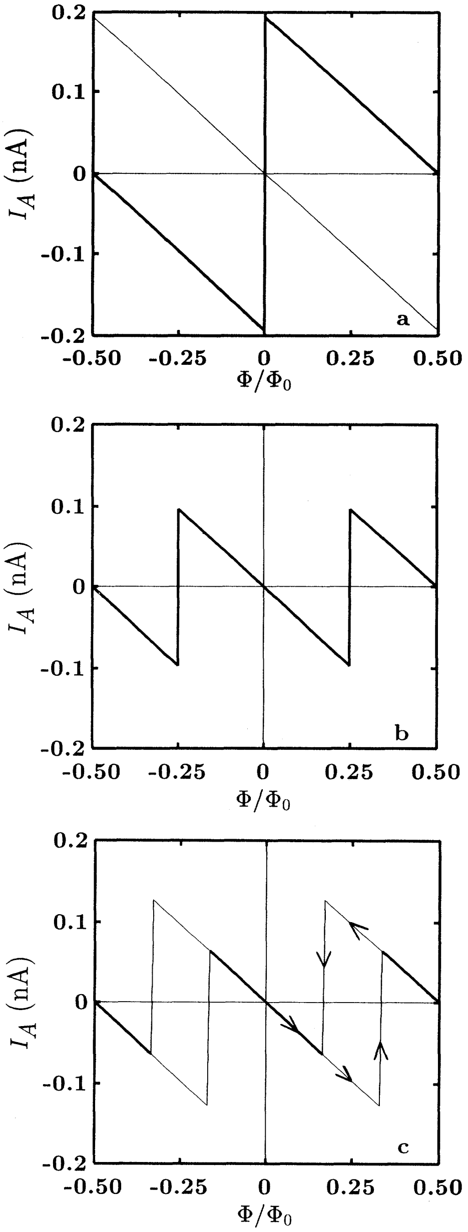

FIG. 2. Persistent currents $I_A(\Phi,2)$ of a two-electron quantum ring (parameters are the same as in Fig. 1) (a) carried by the lowest para (thin solid lines) and ortho (heavy solid lines) states calculated from Eqs. (13) and (15), (b) carried by the minimal-energy band, and (c) possible bistability and hysteresis.

The question that now arises is, What process is responsible for a spin flip from $S = 0$ (or 1) to $S = 1$ (or 0) with a change of the symmetry of the two-electron state in a QR? The probability per unit time that a quantum system changes its state from $|i\rangle$ with the energy $\mathcal{E}_i$ to $|f\rangle$ with the energy $\mathcal{E}_f$ is given in first order by $W_{i\to f} = \frac{2\pi}{\hbar}|\langle f|H_{\text{int}}|i\rangle|^2\delta(\mathcal{E}_f - \mathcal{E}_i)$, where H_{int} is a perturbation. Let us consider as a perturbation at first magnetic impurities. In this case we have to add to the Hamiltonian the interaction

$$H_{\text{imp}} = \sum_{s=1}^{N_S}\sum_{i=1}^{2} V_s(r_i,\varphi_i) \tag{19}$$

with the impurity potential

$$V_s(r_i,\varphi_i) = -J_s\delta(\mathbf{x}_i - \mathbf{x}_s)\mathbf{S}_i\cdot\mathbf{S}_s^{\text{imp}}, \tag{20}$$

where J_s is the exchange integral, $\mathbf{S}_i = \frac{\hbar}{2}\boldsymbol{\sigma}_i$ is the spin operator of the ith electron, $\boldsymbol{\sigma}_i = (\sigma_{xi},\sigma_{yi},\sigma_{zi})$ stands for the Pauli spin vector operator with $\sigma_{\alpha i}$ the Pauli matrices, and $\mathbf{S}_s^{\text{imp}} = \frac{\hbar}{2}\boldsymbol{\sigma}_s$ is the spin operator of the magnetic impurity. It is well known that such an interaction results in the *Kondo effect* of the resistivity of metals. Considering for simplicity only one impurity ($N_S = 1$), a definite state of the two-electron system plus one impurity is described by the product $\Psi(\mathbf{x}_1,\mathbf{x}_2)|1{:}m_s,2{:}m_s;I{:}M_{\text{imp}}\rangle$ of the orbital part $[\Psi(\mathbf{x}_1,\mathbf{x}_2) \equiv \langle\mathbf{x}_1,\mathbf{x}_2|\Psi\rangle]$ and the spin part. The ket $|1{:}m_{s1},2{:}m_{s2};I{:}M_{\text{imp}}\rangle$ acting in the spin state space means: electron 1 in the spin state $|m_{s1}\rangle$ with a spin quantum number $m_{s1} = \pm\frac{1}{2}$, electron 2 in the spin state $|m_{s2}\rangle$ with spin quantum number $m_{s2} = \pm\frac{1}{2}$, i.e., $M_S = m_{s1} + m_{s2}$, and the magnetic impurity in the spin state $|M_{\text{imp}}\rangle$. According to the symmetry of the two-electron system, we have (i) for the para state ($S = 0$, $M_S = 0$)

$$\Psi_{\text{para}}(\mathbf{x}_1,\mathbf{x}_2)\frac{1}{\sqrt{2}}[|1{:}\tfrac{1}{2},2{:}-\tfrac{1}{2};I{:}M_{\text{imp}}\rangle - |1{:}-\tfrac{1}{2},2{:}\tfrac{1}{2};I{:}M_{\text{imp}}\rangle] \tag{21}$$

with a symmetric orbital part $\Psi_{\text{para}}(\mathbf{x}_2,\mathbf{x}_1) = \Psi_{\text{para}}(\mathbf{x}_1,\mathbf{x}_2)$ and (ii) for the ortho state ($S = 1$, $M_S = -1,0,+1$)

$$\Psi_{\text{ortho}}(\mathbf{x}_1,\mathbf{x}_2)\times\begin{cases}|1:\tfrac{1}{2},2:\tfrac{1}{2};I:M_{\text{imp}}\rangle,\\ |1:-\tfrac{1}{2},2:-\tfrac{1}{2};I:M_{\text{imp}}\rangle,\\ \tfrac{1}{\sqrt{2}}[|1:\tfrac{1}{2},2:-\tfrac{1}{2};I:M_{\text{imp}}\rangle+|1:-\tfrac{1}{2},2:\tfrac{1}{2};I:M_{\text{imp}}\rangle]\end{cases} \tag{22}$$

with an antisymmetric orbital part $\Psi_{\text{ortho}}(\mathbf{x}_2,\mathbf{x}_1)=-\Psi_{\text{ortho}}(\mathbf{x}_1,\mathbf{x}_2)$.

Then it follows for the ortho state under the action of H_{imp} that

$$\begin{aligned}&H_{\text{imp}}\Psi_{\text{ortho}}(\mathbf{x}_1,\mathbf{x}_2)|1:\tfrac{1}{2},2:\tfrac{1}{2};I:-\tfrac{1}{2}\rangle\\ &\quad=-J_s\left(\frac{\hbar}{2}\right)^2\Big\{(2|1:-\tfrac{1}{2},2:\tfrac{1}{2}\rangle\otimes|I:\tfrac{1}{2}\rangle-|1:\tfrac{1}{2},2:\tfrac{1}{2}\rangle\otimes|I:-\tfrac{1}{2}\rangle)\delta(\mathbf{x}_1-\mathbf{x}_s)\Psi_{\text{ortho}}(\mathbf{x}_s,\mathbf{x}_2)\\ &\qquad+(2|1:\tfrac{1}{2},2:-\tfrac{1}{2}\rangle\otimes|I:\tfrac{1}{2}\rangle-|1:\tfrac{1}{2},2:\tfrac{1}{2}\rangle\otimes|I:-\tfrac{1}{2}\rangle)\delta(\mathbf{x}_2-\mathbf{x}_s)\Psi_{\text{ortho}}(\mathbf{x}_1,\mathbf{x}_s)\Big\}.\end{aligned} \tag{23}$$

The first term in each set of parentheses leads to a spin-flip process. These two terms can be rearranged as

$$\begin{aligned}H_{\text{imp}}\Psi_{\text{ortho}}(\mathbf{x}_1,\mathbf{x}_2)|1:\tfrac{1}{2},2:\tfrac{1}{2},I:-\tfrac{1}{2}\rangle|_{\text{spin-flip}}\to-J_s\left(\frac{\hbar}{2}\right)^2&\{(|1:-\tfrac{1}{2},2:\tfrac{1}{2}\rangle+|1:\tfrac{1}{2},2:-\tfrac{1}{2}\rangle)\otimes|I:\tfrac{1}{2}\rangle\\ &\times[\delta(\mathbf{x}_1-\mathbf{x}_s)\Psi_{\text{ortho}}(\mathbf{x}_s,\mathbf{x}_2)+\delta(\mathbf{x}_2-\mathbf{x}_s)\Psi_{\text{ortho}}(\mathbf{x}_1,\mathbf{x}_s)]\\ &+(|1:-\tfrac{1}{2},2:\tfrac{1}{2}\rangle-|1:\tfrac{1}{2},2:-\tfrac{1}{2}\rangle)\otimes|I:\tfrac{1}{2}\rangle\\ &\times[\delta(\mathbf{x}_1-\mathbf{x}_s)\Psi_{\text{ortho}}(\mathbf{x}_s,\mathbf{x}_2)-\delta(\mathbf{x}_2-\mathbf{x}_s)\Psi_{\text{ortho}}(\mathbf{x}_1,\mathbf{x}_s)]\}.\end{aligned} \tag{24}$$

The first term in the curly brackets is a result of a spin flip, but does not change the symmetry of the orbital wave function of the two-electron system. Multiplying Eq. (24) from the left with the conjugate wave function (21) of the para state, we find the matrix element

$$\begin{aligned}&\langle f=P|H_{\text{imp}}|i=O\rangle\\ &\quad=\frac{1}{\sqrt{2}}\int d^2x_1\int d^2x_2[\langle 1:\tfrac{1}{2},\,2:-\tfrac{1}{2}|+\langle 1:-\tfrac{1}{2},2:\tfrac{1}{2}|]\otimes\langle I:\tfrac{1}{2}|\Psi^*_{\text{para}}(\mathbf{x}_1,\mathbf{x}_2)H_{\text{imp}}|1:\tfrac{1}{2},2:\tfrac{1}{2},I:-\tfrac{1}{2}\rangle\Psi_{\text{ortho}}(\mathbf{x}_1,\mathbf{x}_2)\\ &\quad=\frac{\hbar^2}{2\sqrt{2}}J_s\int d^2x_1\int d^2x_2\Psi^*_{\text{para}}(\mathbf{x}_1,\mathbf{x}_2)[\delta(\mathbf{x}_1-\mathbf{x}_s)\Psi_{\text{ortho}}(\mathbf{x}_s,\mathbf{x}_2)-\delta(\mathbf{x}_2-\mathbf{x}_s)\Psi_{\text{ortho}}(\mathbf{x}_1,\mathbf{x}_s)]\\ &\quad=\frac{\hbar^2}{\sqrt{2}}J_s\int d^2x\Psi^*_{\text{para}}(\mathbf{x}_s,\mathbf{x})\Psi_{\text{ortho}}(\mathbf{x}_s,\mathbf{x}).\end{aligned} \tag{25}$$

As long as $\langle f=P|H_{\text{imp}}|i=O\rangle\neq 0$ is valid, we come to the conclusion that the spin flip process of a Kondo-like effect can result in a possible bistability of the persistent current. It is important to note that a homogeneous distribution of magnetic impurities would not result in a spin flip with symmetry change because of the orthogonality of the wave functions $\Psi_{\text{ortho}}(\mathbf{x}_1,\mathbf{x}_2)$ and $\Psi_{\text{para}}(\mathbf{x}_1,\mathbf{x}_2)$.

A second possible mechanism is the spin-orbit coupling, described by the Hamiltonian

$$H_{\text{so}}=\sum_{i=1}^{2}\frac{\hbar^2}{4m_0^2c^2}[\boldsymbol{\sigma}_i\times\boldsymbol{\nabla}V(\mathbf{x}_i)]\cdot\mathbf{p}_i, \tag{26}$$

where m_0 is the bare electron mass, $\mathbf{p}_i$ is the momentum operator of the ith electron, and $V(\mathbf{x}_i)$ is the total potential (crystal potential and confining potential). It is well known that in GaAs the spin-orbit coupling results in a lifting of the degeneracy of the valence bands. Further, the spin-orbit coupling results in the inelastic light scattering on GaAs in the possibility to create spin-polarized states. Hence $\langle f=P|H_{\text{so}}|i=O\rangle\neq 0$ results and the spin-orbit coupling is the mechanism responsible for a possible bistability in the persistent current of a two-electron ring.

In conclusion, for a two-electron ring synthesized from a GaAs-Ga$_{1-x}$Al$_x$As heterostructure, we expect a bistability and a hysteresis in the persistent current as plotted in Fig. 2(c). This effect should be observable in semiconductor rings because for quantum dots it is possible to tune the number of electrons up to the limit of single-electron charging.[29]

We thank A. V. Chaplik for stimulating discussions. V.M.F. acknowledges with gratitude the support of the Alexander von Humboldt Foundation and of the Martin-Luther-University Halle.

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