Correlated few-electron states in vertical double-quantum-dot systems

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(Received 7 September 1994)

The electronic properties of semiconductor, vertical, double-quantum-dot systems with few electrons are investigated by means of analytic, configuration-interaction, and mean-field methods. The combined effect of a high magnetic field, electrostatic confinement, and interdot coupling induces a class of few-electron ground states absent in single quantum dots. In particular, the role played by the isospin (or quantum-dot index) in determining the appearance of ground states is analyzed and compared with the role played by the standard spin.

I. INTRODUCTION

The behavior of a small number of electrons confined into a single quasi-two-dimensional quantum dot (QD) in the presence of a magnetic field has been studied over the past few years.[1–8] Most of the work has focused on the high magnetic field regime of fully spin-polarized electrons where incompressible electron states analogous to the Laughlin states of the fractional quantum Hall effect exist. Only very recently the problem of spin and its implications has been addressed.[6–8] The ground and excited states of a spin-unpolarized QD turn out to be much more complex than those of the polarized QD. The role of spin and spin-induced interactions can be effectively simulated by the *isospin* (layer index) in double layer systems where the isospin-up isospin-down states correspond to electrons on layer 1 or layer 2. Recent experimental[9–12] and theoretical[13–24] studies of a double layer two-dimensional electron system (DL2DES) in a high magnetic field have shown a rich variety of incompressible states related to the quantum Hall effect[25] (QHE). For instance, in a recent experiment (Ref. 11) Murphy *et al.* have reported the existence of an incompressible QHE state at a filling factor of individual layers $\nu = 1/2$ (Ref. 26) in the absence of interlayer tunneling. This coupling-induced QHE state has no analog in single layer systems. The distance between layers, i.e., the strength of the Coulomb coupling, determines whether the incompressible state appears or not. This experiment seems to confirm the predictions made in Refs. 15–17, and 21 that the QHE states are supported by the interlayer Coulomb interactions and by the single-particle symmetric-antisymmetric gap in the presence of arbitrary tunneling. The breakdown of these states occurs when the Coulomb coupling is switched off and the intralayer correlations become dominant.[18–22]

In this work we examine the role of confinement, magnetic field, and interdot coupling in the generation of few-electron states in a vertical double-quantum-dot system (DQDS). We concentrate on the high magnetic field limit where electrons in the ground and low-lying states are spin polarized due to Zeeman energy, and the effect of the isospin degree of freedom can be isolated. The appearance of several ground states (GS) with an unexpected isospin is the most striking feature of such systems in comparison with isolated polarized QD's.

The paper is organized as follows: Sec. I is devoted to the description of the model; in Sec. II, we present the analytical results for the simplest possible case of two electrons in a DQDS; in Sec. III, we present numerical results for up to six electrons, and Sec. IV contains conclusions of the work.

II. THE MODEL

We consider a pair of identical vertical coupled QD's with parabolic in-plane confinement, containing N electrons, and separated by a distance D as shown in Fig. 1. The normalized single-particle states $|m,n;\sigma\rangle$ of each individual QD (in the presence of a magnetic field B oriented normal to the plane of the QD) are simply harmonic oscillator states,

$$|m,n;\sigma\rangle = \sqrt{\frac{1}{m!n!}}(a^+)^m(b^+)^n|0,0;\sigma\rangle, \tag{1}$$

with σ labeling the QD index or z component of the isospin quantum number. In analogy with the standard spin, σ takes on the values $+1/2, -1/2$ (for brevity, $+,-$, from now on) for electrons in the "upper" and "lower" QD, respectively. The single-particle energies are those of a pair of harmonic oscillators (the Zeeman energy is omitted and $\hbar = 1$ for the rest of this work),

$$\epsilon_{mn\sigma} = \Omega_+(n+\tfrac{1}{2}) + \Omega_-(m+\tfrac{1}{2}), \tag{2}$$

with $\Omega_\pm = [\sqrt{\omega_c^2 + 4\omega_0^2} \pm \omega_c]/2$. ω_c is the cyclotron frequency and ω_0 is the frequency characterizing the parabolic confinement of both QD's. We shall restrict

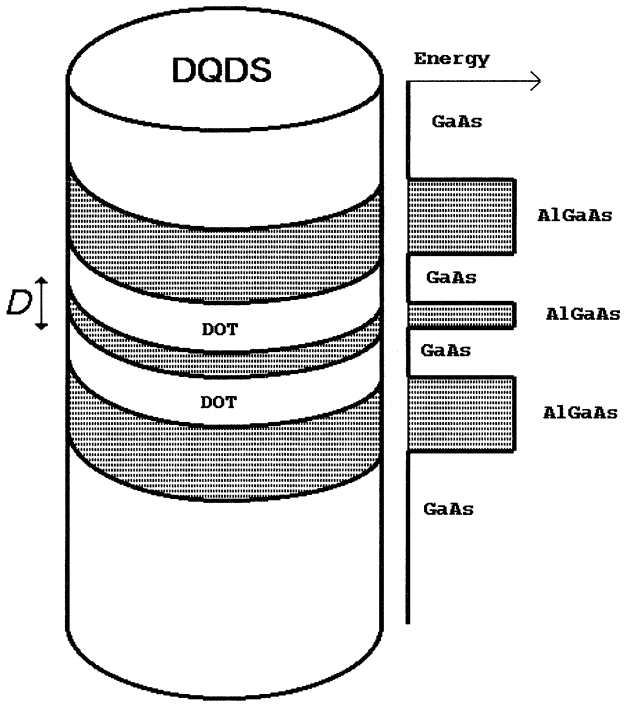

FIG. 1. A schematic picture of a DQDS. Such a type of semiconductor structures can be now routinely fabricated by means of combined growing and etching techniques.

this discussion to the case of high magnetic fields ($B > 2-3$ T) where electrons are spin polarized, and the condition $\omega_c > \omega_0$ is satisfied. In this regime, one can restrict the basis set to $n = 0$ and the problem becomes one dimensional. We shall omit index n in what follows.

Allowing for interdot tunneling, the full-interacting Hamiltonian of the DQDS can be expressed in second quantization as

$$H = \sum_{m\sigma\sigma'} [\delta_{\sigma,\sigma'}\epsilon_{m\sigma}c^{\dagger}_{m\sigma}c_{m\sigma'} + (1-\delta_{\sigma,\sigma'})tc^{\dagger}_{m\sigma}c_{m\sigma'}] + \frac{1}{2}\sum_{m_1m_2m_3m_4\sigma\sigma'} V^{m_1m_2m_3m_4}_{\sigma\sigma'}c^{\dagger}_{m_1\sigma}c^{\dagger}_{m_2\sigma'}c_{m_3\sigma'}c_{m_4\sigma}, \tag{3}$$

where $c^{\dagger}$, c are the single-particle creation and annihilation operators, $V^{m_1m_2m_3m_4}_{\sigma\sigma'}$ the Coulomb interaction terms, and t is the interdot hopping matrix element. We have included here interdot direct and exchange terms but neglected some off-diagonal scattering elements, negligible in the weak hopping ($t \to 0$) limit.

Alternatively, we can use an isospin 1/2 operator representation to define isospin operators in terms of previous creation and annihilation single-particle operators as

$$\rho_{m_1m_2} = c^{\dagger}_{m_1+}c_{m_2+} + c^{\dagger}_{m_1-}c_{m_2-},$$
$$\zeta^z_{m_1m_2} = c^{\dagger}_{m_1+}c_{m_2+} - c^{\dagger}_{m_1-}c_{m_2-},$$
$$\zeta^x_{m_1m_2} = c^{\dagger}_{m_1+}c_{m_2-} + c^{\dagger}_{m_1-}c_{m_2+}.$$

Omitting linear terms in $\rho_{m_1m_2}$ and $\zeta^z_{m_2m_3}$ allows us to emphasize the appearance of isospin-isospin interactions in the DLQD Hamiltonian:

$$H = \sum_m(\epsilon_m\rho_{mm} + t\zeta^x_{mm}) + \frac{1}{2}\sum_{m_1m_2m_3m_4} V^{m_1m_2m_3m_4}_D\rho_{m_1m_4}\rho_{m_2m_3} + \frac{1}{2}\sum_{m_1m_2m_3m_4} V^{m_1m_2m_3m_4}_E\zeta^z_{m_1m_4}\zeta^z_{m_2m_3}. \tag{4}$$

The Hamiltonian (4) contains two SU(2) symmetry-breaking fields: The first one along the x direction and proportional to the hopping t, and the second one along the z direction, the isospin-isospin interaction, proportional to V_E (in addition to the usual charge-charge interaction). The Coulomb matrix elements are given by

$$V^{m_1m_2m_3m_4}_D = \frac{1}{2}(V^{m_1m_2m_3m_4}_{++} + V^{m_1m_2m_3m_4}_{+-}),$$
$$V^{m_1m_2m_3m_4}_E = \frac{1}{2}(V^{m_1m_2m_3m_4}_{++} - V^{m_1m_2m_3m_4}_{+-}),$$

where V_{++} (identical to V_{--}) are intradot Coulomb matrix elements, and V_{+-} are interdot Coulomb matrix elements.

As far as the symmetry-breaking term t is concerned, two physical situations can be distinguished: the "incoherent" and the "coherent" one. The incoherent case ($t = 0$) describes isolated QD's coupled only by Coulomb interactions. Electrons cannot transfer (exchange) between QD's, and are localized on individual QD's. This distinguishability manifests itself in the anticommutation relation $\{c^{\dagger}_{m_1\sigma}, c_{m_2\sigma'}\} = \delta_{\sigma,\sigma'}\delta_{m_1,m_2}$. This is the well-known "layered electron gas" model.[27]

The second case is the coherent behavior of QD's, where electrons cannot be in a specific QD but occupy the symmetric (s) or antisymmetric (as) orbitals of a pair of QD's. The transformation from orbitals localized on individual dots to the symmetric-antisymmetric orbitals is equivalent to the rotation of the isospin. Let us define a rotated isospin representation, $\{\alpha\}$, which diagonalizes the hopping part of the Hamiltonian (4) for arbitrary hopping matrix element t:

$$c^{\dagger}_{m+} = \frac{1}{\sqrt{2}}(\alpha^{\dagger}_{m,s} + \alpha^{\dagger}_{m,\mathrm{as}}),$$
$$c^{\dagger}_{m-} = \frac{1}{\sqrt{2}}(\alpha^{\dagger}_{m,s} - \alpha^{\dagger}_{m,\mathrm{as}}). \tag{5}$$

We wish to emphasize that this transformation does not depend on the strength of the hopping matrix element. We can define isospin operators ρ, ζ^z, ζ^x in the space of coherent operators $\{\alpha\}$ to write the coherent Hamiltonian as

$$H = \sum_m(\epsilon_m\rho_{mm} + t\zeta^z_{mm}) + \frac{1}{2}\sum_{m_1m_2m_3m_4} V^{m_1m_2m_3m_4}_D\rho_{m_1m_4}\rho_{m_2m_3} + \frac{1}{2}\sum_{m_1m_2m_3m_4} V^{m_1m_2m_3m_4}_E\zeta^x_{m_1m_4}\zeta^x_{m_2m_3}. \tag{6}$$

In this rotated isospin space the hopping matrix element t is simply equivalent to an external field. The coherent Hamiltonian (6) is similar to the previous one (4) in the sense that it also presents two SU(2) symmetry-breaking terms: that proportional to the hopping t and the isospin-isospin interactions proportional to V_E (again, in additon to the normal SU(2) invariant charge-charge interactions). This isospin-isospin interaction V_E, present in both Hamiltonians, is, ultimately, responsible for the physics in a DQDS. Since this physics is dominated by Coulomb interaction, we shall concentrate on the $t \to 0$ limit.

III. ANALYTICAL RESULTS FOR THE TWO-ELECTRON DQDS

The case of $N = 2$ is the simplest case that deserves to be studied in detail and can be solved analytically.[28]

We can write the total wave function $\Psi(\vec{r_1}, z_1\ \vec{r_2}, z_2)$ as a product of the center-of-mass ($\vec{R}$) wave function, relative motion ($\vec{r}$) wave function, and the rotated isospin wave functions (symmetric or antisymmetric)

$$\Psi(r_1, r_2) = \psi^{M^{\text{c.m.}}}(\vec{R})\phi^{M^r}(\vec{r})|i,j\rangle \equiv |M^{\text{c.m.}}, M^r; i, j\rangle$$

$$i, j = s, \text{as}. \quad (7)$$

The center-of-mass motion (characterized by an angular momentum M^{cm}) separates from the Hamiltonian, and isospin states can be characterized by four orthogonal rotated isospin states $\{|s,s\rangle, |\text{as},\text{as}\rangle, |s,\text{as}\rangle, |\text{as},s\rangle\}$. These states can be written in a more familiar language of the usual isospin states, e.g., $\{|+,+\rangle = |I=1, I_z=+1\rangle\}$, based on orbitals localized on individual QD's:

$$\begin{aligned}
|s,s\rangle &= \frac{1}{\sqrt{2}}\left\{\frac{|I=1,I_z=+1\rangle+|I=1,I_z=-1\rangle}{\sqrt{2}} + |I=1,I_z=0\rangle\right\},\\
|\text{as},\text{as}\rangle &= \frac{1}{\sqrt{2}}\left\{\frac{|I=1,I_z=+1\rangle+|I=1,I_z=-1\rangle}{\sqrt{2}} - |I=1,I_z=0\rangle\right\},\\
|s,\text{as}\rangle &= \frac{1}{\sqrt{2}}\{|I=1,I_z=+1\rangle - |I=1,I_z=-1\rangle\},\\
|\text{as},s\rangle &= |I=0,I_z=0\rangle.
\end{aligned} \quad (8)$$

The first three states correspond to a well-defined isospin $I = 1$ but an undefined z component of I. The expectation value of I_z for the three coherent states $I = 1$ is zero, but quantum fluctuations are present in the $I = 1$ states, in contrast with the incoherent $I = 0$ state. This means that the two $I = 1$ states $\frac{1}{\sqrt{2}}\{|s,s\rangle + |\text{as},\text{as}\rangle\}$ and $|s,\text{as}\rangle$ are not eigenstates of I_z, and one cannot determine on which QD the electrons are localized. They correspond to having both electrons in one or the other QD of the DQDS. On the other hand, the $|I=1,I_z=0\rangle$ and $|I=0,I_z=0\rangle$ states ($\frac{1}{\sqrt{2}}\{|s,s\rangle - |\text{as},\text{as}\rangle\}$ and $|\text{as},s\rangle$, respectively) correspond to electrons on opposite QD's and their energy is determined only by interdot interaction $V_{+-} = V_D - V_E$.

We can now write the relative particle Hamiltonian in the rotated isospin space as a 4×4 matrix. It is easy to see that the first three states correspond to total isospin $I = 1$ and hence the in-plane relative particle wave function must be antisymmetric. This corresponds to *odd* relative angular momentum, M^r, of the relative particle. The Hamiltonian H_1 for the $I = 1$ states can be written as

$$\begin{pmatrix}
t + \Omega_- M^r + \langle M^r|V_D|M^r\rangle & \langle M^r|V_E|M^r\rangle & 0\\
\langle M^r|V_E|M^r\rangle & -t + \Omega_- M^r + \langle M^r|V_D|M^r\rangle & 0\\
0 & 0 & \Omega_- M^r + \langle M^r|V_D + V_E|M^r\rangle
\end{pmatrix}, \quad (9)$$

and for $I = 0$, we simply have

$$\Omega_- M^r + \langle M^r|V_D - V_E|M^r\rangle = H_0, \quad (10)$$

with relative angular momentum M^r being *even*.

From the rotated isospin Hamiltonian (9), it is clear that (a) only the symmetric-antisymmetric states are coupled, (b) coupling is due to the symmetry-breaking exchange interaction V_E, (c) the coupling between symmetric-antisymmetric states is present even in the absence of tunneling ($t = 0$), and (d) only these coupled symmetric-antisymmetric states are affected by interdot hopping (t).

The two-electron Hamiltonian can be easily diagonalized and simple analytical expressions for energies and wave functions obtained (not shown here for brevity).

Figure 2 shows the evolution of the total energy spectrum as a function of the distance between QD's for a given value of the total angular momentum $M = M^r + M^{\text{cm}} = 6$ and zero hopping matrix element t. Standard values of dielectric constant and effective mass for GaAs have been taken throughout the calculations and the confining energy of the QD's in the DQDS is taken to be 5 meV. From now on, it is also convinient to define an interdot coupling constant: $\alpha = V^{0000}_{+,-}/V^{0000}_{+,+}$. One can see in Fig. 2 the energy splitting of the $I = 1$ states (when M^r odd), which are degenerate when $\alpha = 1$, as the distance increases (i.e., as the coupling constant α lowers its value). Those with $I_z = 0$ go down in energy while those with $I_z = \pm 1$ remain degenerate and constant (notice that this would not be the case any more if $t \neq 0$). It must be pointed out that the total isospin I is a good

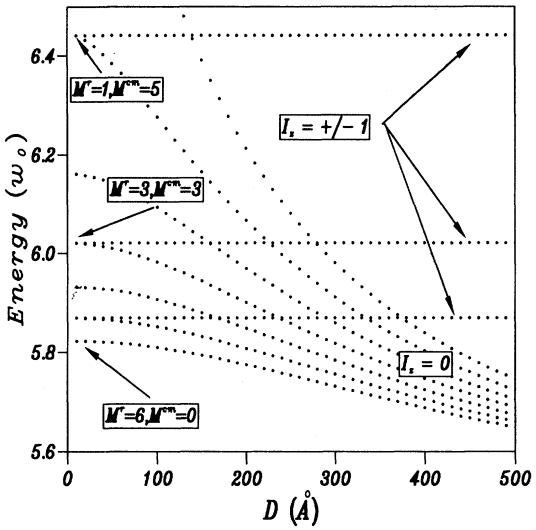

FIG. 2. Evolution of the energy spectrum as a function of the distance between QD's of a two-electron DQDS with total angular momentum $M = 6$. Notice the splitting of the $I = 1$ set of states into $I_z = 0$ and ± 1 states as the distance between QD's increases. The corresponding center of mass and relative angular momenta have been explicitly stated in the figure.

quantum number for any value of the coupling constant in the two-electron DQDS. This is no longer the case for a higher number of electrons as will be shown below. In Fig. 3, we show the phase diagram (isospin I and angular momentum M) of the two-electron DQDS GS as a function of the magnetic field B and coupling constant α. The magnetic field changes the ratio of kinetic (Ω_-) to Coulomb energy and induces changes in the isospin and angular momentum of the DQDS GS. Similar transitions of the isospin and angular momentum of the GS can be induced by changing the coupling constant as shown in Fig. 3.

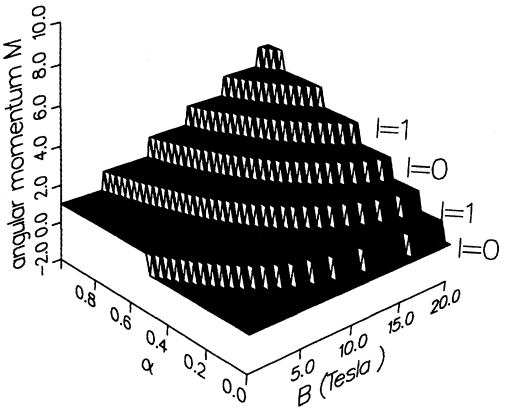

FIG. 3. The phase diagram (angular momentum and isospin) of the two-electron DQDS ground states.

IV. MORE THAN TWO ELECTRONS IN THE DQDS

A. Zero distance limit: No SU(2) symmetry-breaking interactions

The zero distance limit (ZDL) ($D = 0$, $\alpha = 1$), i.e., the limit of having the QD's forming the DQDS superimposed in real space, although unrealistic, is interesting and deserves to be studied in detail. In this particular case (always with $t \to 0$), the SU(2) isospin symmetry is not broken by the isospin-isospin interactions since $V_E = 0$. In the ZDL the DQDS of identical QD's under high magnetic fields is completely equivalent to a single QD with Landé factor $g \to 0$, i.e., in the zero Zeeman limit. The role of spin in the DQDS (frozen out by the magnetic field) is now played by the isospin. In the case of nonidentical QD's, the presence of an "isospin Zeeman energy" (for instance, a difference between the confinement energies of each QD) would make the DQDS equivalent to the spin-polarized single QD . The feasibility of fabricating identical QD's with zero "isospin Zeeman energy" is one of the most appealing possibilities presented by such systems. As will be shown below, it is the fundamental origin of the electronic properties that appear in a DQDS at high magnetic fields, compared to those appearing in the same regime of fields in a single QD (i.e., in a spin-polarized QD).

We now extend our study to a larger number of electrons by employing exact diagonalization techniques. The intradot and interdot electron-electron correlations have been taken fully into account by expanding the

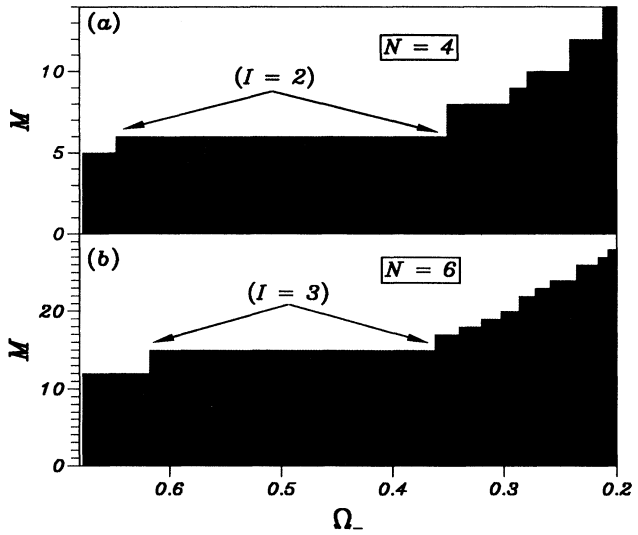

FIG. 4. (a) Evolution of the absolute GS angular momentum M as a function of the single-particle kinetic energy (magnetic field and confinement) for four electrons in the ZDL. (b) The same for six electrons. Along with M, the values of the total isospin I of the stable states discussed in the text are also shown.

many-body wave function in terms of Slater determinants (or configurations) and diagonalizing the Hamiltonian (3) for four and six electrons. The Hilbert space has been restricted to the states of the lowest Landau level of each QD ($n = 0$). Due to the circular symmetry, the diagonalization can be done in separate subspaces of configurations with the same z component of the total angular momentum, M. In the absence of hopping any subspace of given M can be split, in turn, into orthogonal subspaces of given I_z. In this way, the size of the matrices to diagonalize becomes smaller and computationally more accessible. The GS's and lowest-lying eigenvalues and eigenfunctions of all the subspaces of different M were obtained using standard diagonalization routines.

Figures 4(a) and 4(b) show the evolution of M for the DQDS absolute GS as a function of Ω_- (i.e., B) for four and six electrons. The absolute GS angular momentum M goes through a series of increasing values as Ω_- lowers its value (i.e., the magnetic field rises). The competition between kinetic energy and Coulomb repulsive energy determines the value of M: the kinetic energy (due to the confinement and magnetic field) favors electrons in the center of the QD, the Coulomb repulsion tends to spread the charge.

The results in Figs. 4(a) and 4(b) show a remarkable stability of the GS's with $M = 6, I = 2$ for four electrons, and $M = 15, I = 3$ for six electrons, against changes in Ω_-. These stable GS's are the only ones appearing with maximum total isospin I, and are fivefold degenerate ($I_z = 2, 1, 0, -1, -2$) and sevenfold degenerate ($I_z = 3, 2, 1, 0, -1, -2, -3$), for four and six electrons, respectively. In particular, the $I_z = 0$ states are described *exactly* in this ZDL by the Jastrow-type correlated wave function [111],[13]

$$\Phi(z_1, ..., z_{N/2}, w_1, ..., w_{N/2}) = A\Bigg[\prod_{1\le i<j\le N/2}(z_i - z_j)\prod_{1\le i<j\le N/2}(w_i - w_j)\prod_{1\le i,j\le N/2}(z_i - w_j) \times \exp\Bigg(-\sum_{i=1}^{N/2}|z_i|^2/4l^2 - \sum_{i=1}^{N/2}|w_i|^2/4l^2\Bigg)|+\rangle_1...|+\rangle_{N/2}|-\rangle_1...|-\rangle_{N/2}\Bigg], \quad (11)$$

proposed by Halperin in the context of fractional QHE wave functions with spin degrees of freedom, and used later in the context of a DL2DES.[16] The electron coordinates of the upper dot are given by $z_i = x_i - iy_i$ and those of the lower dot by $w_i = x_i' - iy_i'$, and + and − denote the values of the QD indices (see previous section). The symbol A represents the antisymmetrization operator. Alternatively, these states can be also expressed in terms of the single-particle occupation numbers of each QD, ν^+ and ν^-. For instance, for six electrons $\nu_m^{+,-} = 0.5$ for $m = 0, 1, 2, 3, 4$, and 5, and 0 for the following m's. If we define a filling factor for the DQDS as $\nu^{\mathrm{DQDS}} = \langle \nu_m^+ + \nu_m^- \rangle_m$ where the brackets denote average over the lowest occupied m's, the GS's $M = 6$ ($N = 4$) and $M = 15$ ($N = 6$) clearly correspond to $\nu^{\mathrm{DQDS}} = 1$. These $\nu^{\mathrm{DQDS}} = 1$ states can be considered, in turn, as precursors of the incompressible state at total $\nu = 1$ observed in a DL2DES.[11]

In Fig. 5, we show the six-electron excitation spectrum of a DQDS with a $\nu^{\mathrm{DQDS}} = 1$ degenerate GS of angular momentum $M = 15$ (marked with a star in the plot). In particular, the fully isospin-polarized state within the degenerate subspace has been considered as the absolute GS in what follows. The excitation spectrum consists of two branches: the branch with $M < 15$ and that with $M > 15$. Let us first concentrate on the branch of the spectrum with angular momentum $M < 15$. By examining the wave function, we have been able to identify those excitations corresponding to *isospin-flip* excitations (solid dots in the plot). Those with M close to $M = 15$ can be associated with *isospin-flip spin-wave-like edge* excitations.[29] Those with M farther below $M = 15$ correspond to *isospin-flip quasiparticle* excitations (magnetoexcitons[29]) and consist in flipping the electron's isospin and moving it from the edge of the $\nu^{\mathrm{DQDS}} = 1$ droplet to a reversed isospin single-particle state m closer to the center of the DQDS. The value of the total isospin of such excitations is the maximum possible value according to the spin flip and to the subspace in which it is found, i.e., the value of M of the excitation [all those shown with solid dots in Fig. 5, on the left branch, correspond to $I = (N-1)/2 = 5/2$]. We can see that isospin-flip excitations are *not*, in general, the lowest energy states in subspaces of given M, but there

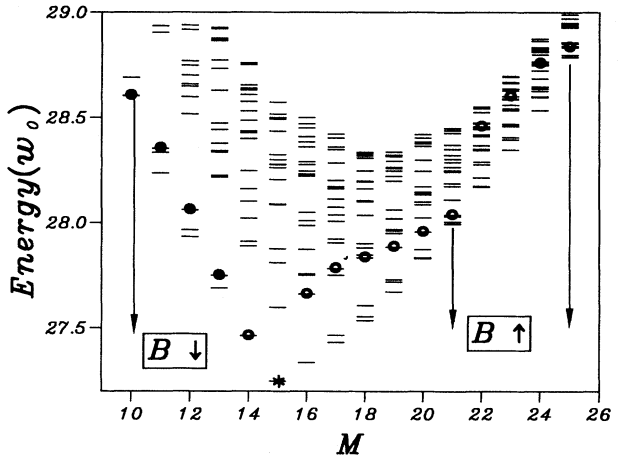

FIG. 5. The excitation spectrum of six electrons in a DQDS in the ZDL. The GS is marked with a star. Isospin-reversed quasiparticle excitations are distinguished by solid dots and quasihole excitations by open dots. The arrows show those subspaces whose GS's will become the absolute GS in the presence of isospin (spin) Zeeman energy for increasing B (to the right of $M = 15$) and for decreasing B (to the left of $M = 15$).

appear a few other states of lower energy and minimum value of I within each subspace.[30]

We have also identified isospin-conserving *quasihole-quasielectron pair* excitations (open dots in the plot) consisting of a quasihole in a single-particle state of $m < 5$ and an electron added to the edge of the droplet. Similarly to the spin-flip excitations, those with M far from $M = 15$ present the better-defined character of a single, localized hole in the droplet. These excitations present the maximum possible value of I ($N/2 = 3$). In contrast with the isospin-flip excitations, there appear *many* excitations of lower energy and minimum isospin within each subspace of given M.

From the excitation spectrum, we can understand the evolution of the absolute GS angular momentum with Ω_-. The stability of the $\nu^{\mathrm{DQDS}} = 1$ GS's is nothing but the direct consequence of the cusplike structure or gap exhibited by the excitation spectrum (see Fig. 5), and such state can be referred to as an *incompressible* GS. Up to the values of Ω_- studied, no other stable GS's seem to appear in our calculations. In the case of identical QD's almost all the lowest states of different subspaces of M will become the absolute GS of the DQDS at a certain value of B, which is in clear contrast with the case of a single, isolated polarized QD. In this ZDL the GS takes on almost all possible values of M for the range of variation of Ω_-, but such values change very quickly with Ω_- (except the $\nu^{\mathrm{DQDS}} = 1$), presenting no stability. The relevance of these kinds of absolute GS's in a single QD at low magnetic fields (when the Zeeman energy cannot be considered infinite) has been stressed in Ref. 8. Their importance lies in suppressing, through the spectral function of the system, the single-electron tunneling rates,[34] and therewith, in strongly modifying the transport properties of the system. One would expect that the presence of such states in the DQDS would give rise to similar effects on transport properties of these systems as long as they do not disappear for a realistic situation, i.e., for a certain distance between QD's. Taking into account realistic values of the distance is the topic of the following sections.

As mentioned previously, a significant difference between the confining energies of the QD's forming the DQDS (or the presence of the g factor for a single QD at high B) tends to favor fully isospin-polarized (spin-polarized for a single QD) absolute GS's. Quantum dots with fully polarized electrons have been extensively studied over the past few years. It is known that, for instance, for four and six electrons, when only the spin-polarized states are relevant to the GS properties, the value of M for the absolute GS is restricted to a series of specific numbers: 6, 10, 14, ..., for four electrons[31,2] and 15, 21, 25, ..., for six electrons.[4] These numbers are known in the literature as "magic" numbers.[2,5] This can be seen easily from Fig. 5: The lowest states in each subspace of M will correspond now (in the presence of isospin Zeeman energy) to isospin-conserving quasihole ($M > 15$) or isospin-flip quasiparticle ($M < 15$) excitations. Only those with quasiholes (quasiparticle) close to the center of the droplet will become the GS of the DQDS as B changes (those with downward-pointing arrows in Fig. 5). A simple description of the magic absolute GS's (for $M > 15$) in terms of "bosonic" operators acting upon the electronic $\nu = 1$ droplet has been presented elsewhere.[32]

B. Short distances: Weak SU(2) symmetry-breaking interactions

We now discuss how the minimum-isospin GS's of a ZDL DQDS evolve for realistic distances between QD's. Figures 6(a) and 6(b) show, for $D = 100$ Å and 50 Å, the evolution with Ω_- of the M value of the absolute GS (from now on we will restrict to the six electrons case). As was clearly shown in Fig. 2, as the distance between QD's increases, or the symmetry-breaking interaction term V_E is stronger, the isospin multiplet degeneracy is removed and the GS will always have $I_z = 0$, i.e., equal number of electrons in each dot. The total isospin, I, is no longer a good quantum number for $D \neq 0$, but one can still trace it back to its original value at $D = 0$, and use it to label the states as long as the distance is not too large. As can be seen in Figs. 6(a) and 6(b), for short distances (D =50 Å, $\alpha \approx 0.8$) many of the absolute minimum-isospin GS's in the ZDL survive. The $M = 15$ GS also remains stable. On increasing the distance ($D = 100$ Å, $\alpha \approx 0.6$) the situation changes dramatically: The symmetry-breaking interactions have made all the minimum-isospin GS's of the ZDL disappear. Instead, many stable states appear ($M = 6, 9, 12, 15, 18, ...,$), and the only stable GS in the ZDL, $M = 15$, has become less relevant. At the same time, the overlap of such GS (or the Jastrow-type wave function shown above) with those of the same M at distances different from zero decreases with D. Instead of showing this overlap we have chosen to show in Fig. 7 the

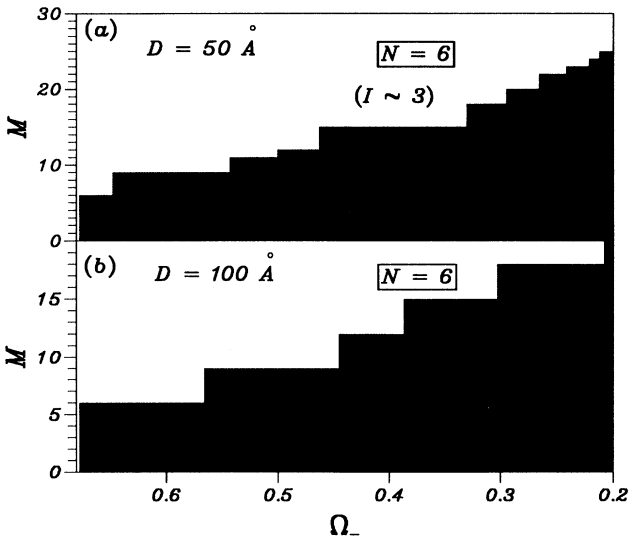

FIG. 6. (a) Evolution of the absolute GS angular momentum M as a function of the single-particle kinetic energy (magnetic field and confinement) for six electrons for $D = 50$ Å. (b) The same for $D = 100$ Å.

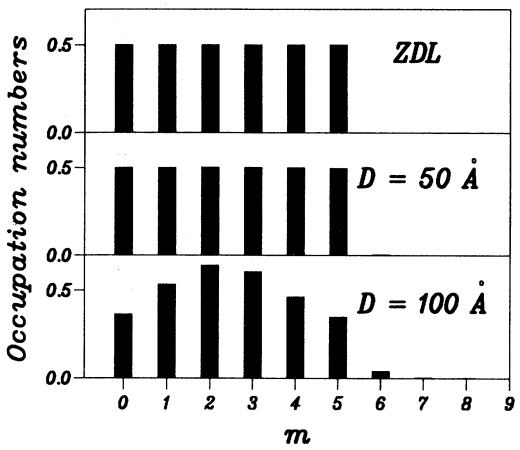

FIG. 7. Single-particle occupation numbers ν_m^+ (ν_m^-) of the $\nu^{\mathrm{DQDS}} = 1$ state as a function of the distance.

single-particle occupation numbers ν^+ (ν^-) in the ZDL, for $D = 50$ Å, and for $D = 100$ Å. One can see that the regular occupations forming the $\nu^{\mathrm{DQDS}} = 1$ state in the ZDL melt as the distance between the QD's increases, and the value of $M = 15$ becomes less relevant for the GS. These stable states have their origin in the "superposition" of the stable ones for each QD with three electrons ($M = 3, 6, 9, ...,$).[2,5] The next section will clarify what we mean by such superposition.

C. Large distance limit: Strong SU(2) symmetry-breaking interactions

We have seen in the previous section that, as the distance D between dots increases the situation seems to change noticeably. In order for this large-D limit to be understood, we have carried out a self-consistent Hartree-Fock (HF) treatment of the interdot coupling, but conserving the intradot correlation. The procedure is the following. A GS solution of total angular momentum, M^+, by means of an exact diagonalization of, for instance, the full-interacting Hamiltonian of the upper QD, H^+, is found as described in the previous section. Then, the single-particle energies of the lower dot are modified by

$$\epsilon_m'^- = \epsilon_m^- + \Sigma_m^+, \tag{12}$$

where in the calculation of the self-energy Σ_m^+, Hartree-Fock-like diagrams have been used. If the interdot hopping is forbidden, then only a Hartree-type diagram is allowed and corresponds to taking into account the single-particle occupation numbers of the upper GS, ν_m^+, together with the coupling term $V_{+,-}^{m_1 m_2 m_2 m_1}$. Now, an exact diagonalization of the lower full-interacting Hamiltonian, H^-, with the corrected single-particle energies is done, a lower GS of M^- is found, and the occupations of this GS are used to modify, in turn, the single-particle energies of the upper dot. The process continues until convergence is achieved.

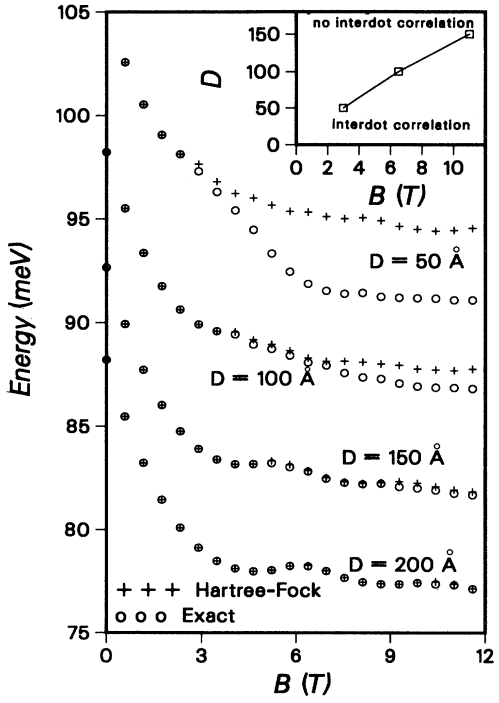

FIG. 8. Evolution of the renormalized ($-6\varepsilon_0$) GS energy as a function of B. Crosses show the HF approximation results discussed in the text and dots those from the exact diagonalization calculations. The inset shows a phase diagram for the interdot correlation as a function of B and D.

Figure 8 shows the evolution of the renormalized GS energy of our DQDS as a function of the magnetic field (or Ω_-) for the case of three electrons in each dot ($I_z = 0$) and for four different distances. The GS's for $D = 200$ Å ($\alpha \approx 0.3$) correspond to a superposition of $M^+ = 3$, $M^- = 3$ (which gives us $M = M^+ + M^- = 6$), $M^+ = 6$, $M^- = 6$ ($M = 12$), and $M^+ = 9$, $M^- = 9$ ($M = 18$). The possible values for M^+ and M^- (3,6,9,...,) are the corresponding magic angular momenta M mentioned above for the case of three electrons in a single QD.[2,5] As we bring the dots together, these states shift to lower fields, so one can think of the interdot coupling in terms of an "additional" magnetic field which adds to the bare one to give a stronger effective value of B.

Exact energies are also shown in Fig. 8. At large distances ($D = 200$ Å) both procedures give the same energy for the GS. As the dots are brought together, the exact GS energy becomes smaller than the HF energy. This fact is more noticeable for the shortest distances where this deviation takes place at $B \approx 7$ T and $B \approx 4$ T for $D = 100$ Å and $D = 50$ Å, respectively. This deviation points out the fact that interdot correlations have grown to play their role, a role that cannot be described in terms of a mean-field theory. A phase diagram is shown in the inset: The interdot correlation grows to be relevant with the inverse of the distance (as one would expect), and with the strength of the magnetic field. Now one can understand the appearance of the $\nu^{\mathrm{DQDS}} = 1$ from a dif-

ferent point of view: The incompressible state at short distances ($M = 15$) has its origin in the spatial interdot correlations, and cannot be obtained as a simple superposition of two equal QD stable electronic configurations. The origin of such particular occupation numbers can be understood. One can form a GS with $M = 15$ out of two single QD configurations with $M^+ = 3$ and $M^- = 12$. These two configurations match perfectly with each other in the sense that the three magnetic flux quanta of the $M^- = 12$, i.e., the three quasiholes in the center of the lower QD "recombine" with the three quasiparticles of the $M^+ = 3$ configuration of the upper QD. Of course, one cannot label the electrons of different dots due to the inherent particle indistinguishability so one must think in terms of a linear combination of the above total configuration with the reversed one: quasiholes in the upper dot and quasiparticles in the lower one. Thus, the origin of the $\nu^{\mathrm{DQDS}} = 1$ from the spatial interdot correlations becomes clear.

V. CONCLUSIONS

In this work we have analyzed the correlated electronic states appearing in double-quantum-dot systems. The simplest case of two electrons was solved analytically. By means of exact diagonalization of the full-interacting Hamiltonian for up to six electrons, minimum-isospin ground states were found to appear for short distances between identical quantum dots, and to disappear as the distance increased. Mean-field calculations revealed the critical distances at which the interdot correlations were significant, signaling the appearance of the minimum-isospin ground states.

ACKNOWLEDGMENTS

One of the authors (J.J.P.) wishes to acknowledge the hospitality and support of the Institute for Microstructural Sciences, NRC Canada, where this work was conceived. We are also indebted to L. Brey, A. H. MacDonald, and C. Tejedor for fruitful discussions, and to G. Aers, and, again to L. Brey, for the critical reading of the manuscript. This work has been supported in part by the Comisión Interministerial de Ciencia y Tecnología of Spain under Contract No. MAT 91-0201 and by the Commission of the European Communities under Contract No. SSC-CT-90-0020.

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[1] G. W. Bryant, Phys. Rev. Lett. **59**, 1140 (1987).
[2] P. A. Maksym and T. Chakraborty, Phys. Rev. Lett. **65**, 108 (1990); Phys. Rev. B **45**, 1947 (1992).
[3] U. Merkt, J. Huser, and M. Wagner, Phys. Rev. B **43**, 7320 (1991); M. Wagner, U. Merkt, and A. V. Chaplik, **45**, 1951 (1992); D. Pfannkuche, V. Gudmundsson, and P. A. Maksym, *ibid.* **47**, 2244 (1993); B. L. Johnson and G. Kirczenow, *ibid.* **47**, 10 563 (1993).
[4] A. H. MacDonald and M. D. Johnson, Phys. Rev. Lett. **70**, 3107 (1993).
[5] P. Hawrylak and D. Pfannkuche, Phys. Rev. Lett. **70**, 485 (1993); P. Hawrylak, *ibid.* **71**, 3347 (1993).
[6] S. R. Eric Yang, A. H. MacDonald, and M. D. Johnson, Phys. Rev. Lett. **71**, 3194 (1993).
[7] J. J. Palacios, L. Martín-Moreno, and C. Tejedor, Europhys. Lett. **23**, 495 (1993); Surf. Sci. **305**, 541 (1994).
[8] J. J. Palacios, L. Martín-Moreno, G. Chiappe, E. Louis, and C. Tejedor, Phys. Rev. B **50**, 5760 (1994).
[9] Y. W. Suen, L. W. Engel, M. B. Santos, M. Shayegan, and D. C. Tsui, Phys. Rev. Lett. **68**, 1379 (1992).
[10] J. P. Eisenstein, G. S. Boebinger, L. N. Pfeiffer, K. W. West, and S. He, Phys. Rev. Lett. **68**, 1383 (1992).
[11] S. Q. Murphy, J. P. Eisenstein, G. S. Boebinger, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **72**, 728 (1994).
[12] Y. W. Suen, H. C. Manoharan, X. Ying, M. B. Santos, and M. Shayegan, Phys. Rev. Lett. **72**, 3405 (1994).
[13] B. I. Halperin, Helv. Phys. Acta **56**, 75 (1983); Surf. Sci. **305**, 1 (1994).
[14] E. H. Rezayi and F. M. D. Haldane, Bull. Am. Phys. Soc. **32**, 892 (1987).
[15] T. Chakraborty and P. Pietilainen, Phys. Rev. Lett. **59**, 2784 (1987).
[16] D. Yoshioka, A. H. MacDonald, and S. M. Girvin, Phys. Rev. B **39**, 1932 (1989).
[17] H. A. Fertig, Phys. Rev. B **40**, 1087 (1989).
[18] A. H. MacDonald, P. M. Platzman, and G. S. Boebinger, Phys. Rev. Lett. **65**, 775 (1990).
[19] A. H. MacDonald, Surf. Sci. **229**, 1 (1990).
[20] L. Brey, Phys. Rev. Lett. **65**, 903 (1990).
[21] S. He, X. C. Xie, S. Das Sarma, and F. C. Zhang, Phys. Rev. B **43**, 9339 (1991); S. He, S. Das Sarma, and X. C. Xie, *ibid.* **47**, 4394 (1993).
[22] R. Côté, L. Brey, and A. H. MacDonald, Phys. Rev. B **46**, 10 239 (1992).
[23] K. Yang, K. Moon, L. Zheng, A. H. MacDonald, S. M. Girvin, D. Yoshioka, and S. C. Zhang, Phys. Rev. Lett. **72**, 732 (1994); K. Moon, H. Mori, K. Yang, S. M. Girvin, A. H. MacDonald, L. Zheng, D. Yoshioka, and S. C. Zhang (unpublished).
[24] C. M. Varma, A. I. Larkin, and E. Abrahams, Phys. Rev. B **49**, 13 999 (1994).
[25] See, e.g., *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1987).
[26] One must bear in mind that the definition of filling factor for a DL2DES is twice the usual definition for a single layer.
[27] P. Hawrylak, Phys. Rev. Lett. **59**, 485 (1987).
[28] An analytical treatment of more electrons under certain conditions has been recently presented in N. F. Johnson and S. Benjamin (unpublished).
[29] C. Kallin and B. I. Halperin, Phys. Rev. B **30**, 5655 (1984).

[30] Similar lowest energy, minimum-spin states appear in the numerical calculations of "spin texture" excitations (*skyrmions*) present in a two-dimensional electron gas at a filling factor $\nu = 1$ and vanishing Zeeman energy. See, e.g., S. L. Shondi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B **47**, 16 419 (1993); H. A. Fertig, L. Brey, R. Côté, and A. H. MacDonald, *ibid.* **50**, 11 018 (1994).

[31] S. A. Trugman and S. A. Kivelson, Phys. Rev. B **31**, 5280 (1985).

[32] J. J. Palacios, L. Martín-Moreno, J. H. Oaknin, and C. Tejedor (unpublished); J. H. Oaknin, L. Martín-Moreno, J. J. Palacios, and C. Tejedor (unpublished).