Coulomb gap in a two-dimensional electron gas with a close metallic electrode

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The tunneling density of states of a classical two-dimensional electron gas with large external disorder and strong electron-electron interaction is studied by computer modeling and by solving a self-consistent equation. The Coulomb interaction is supposed to be strongly suppressed at large distances by a parallel metallic plane. At low temperatures the Coulomb gap is found to exist even if the metallic plane is at a smaller distance from the electron gas than the electron-electron separation. These results are explained in terms of a self-consistent equation. The tunneling density of states as obtained from computer modeling is compared to recent experimental data by Ashoori *et al.* For a strong magnetic field our results are in a good agreement with the experiment.

I. INTRODUCTION

Tunneling into a two-dimensional electron liquid (2DEL) in a strong magnetic field has been the subject of intensive research in the past few years. Attention has been directed to this area after the discovery of the gap in tunneling conductivity between a metal gate and a 2DEL (Ref. 1) and between two layers of a 2DEL.[2] The experiments have shown that in a strong magnetic field the tunneling conductivity is significantly suppressed at small bias and low temperature. Several theoretical groups have offered both classical[3] and quantum-mechanical[4,5] explanations or this phenomenon. It was shown that the observed tunneling gap arises from a gap in the single-particle density of states (DOS) of a 2DEL at the Fermi level. However, all these theories mostly assume no external disorder. This approach is applicable to the cleanest heterostructures with wide spacer layers and low impurity concentration, like the samples used in Ref. 2. The samples of Ref. 1 are quite different: they have highly doped material pretty close to the 2DEL layer, and, therefore, the disorder potential in these samples may be large. In addition, the proximity of the metal contact leads to a strong screening of the electron-electron interaction.

In this paper, we investigate the DOS of a classical 2DEL, subjected to a random potential, and study the conductivity for tunneling into 2DEL. We have shown before[3] that the classical description gives good results for the tunneling between two layers of a 2DEL in a strong magnetic field. We were able to get the voltages, corresponding to maxima of the tunneling current in a good agreement with the experimental data of Ref. 2 without any fitting parameters. No disorder was assumed.

We think that the classical description of the gap in the tunneling DOS becomes more reliable in the presence of a disorder. This is because the Coulomb gap in a disordered system is connected with the long-range part of the interaction. The localization of electrons is the only condition of applicability of this approach. In the system with the Coulomb interaction the characteristic length R_ϵ responsible for the DOS at the energy ϵ is $R_\epsilon \approx e^2/\kappa|\epsilon|$. Here, κ is the dielectric constant of the surrounding medium and ϵ is the energy with the reference point at the chemical potential, where the DOS has minimum.[6] The Coulomb gap appears if R_ϵ becomes larger than the localization length of electrons. The length R_ϵ increases with the decreasing $|\epsilon|$. That is why we think that to get the DOS at small $|\epsilon|$ the system with the Coulomb interaction can be approximated as interacting classical point charges. Strong magnetic field suppresses the quantum overlap and increase the classical region. The experimental data of Ref. 1 support this point. The tunneling gap appears only in a strong magnetic field and it becomes more pronounced with in-

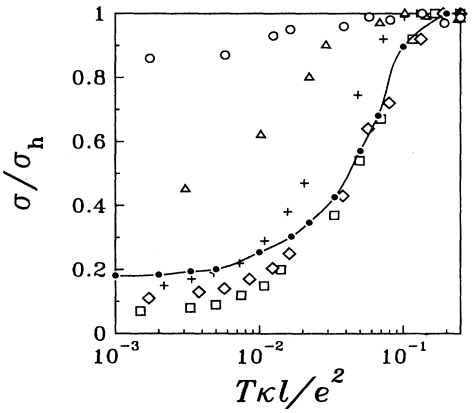

FIG. 1. Comparison of our simulation results with experimental data from Ref. 1: ratio of the conductivity to its high-temperature value σ_h vs temperature in dimensionless units. Solid line shows the TDOS $G_t(u)$ normalized by its high-temperature value for $\nu = 0.3$, $A = 0.4e^2/\kappa l$, and $d = 0.4l$. The markers show the data from Fig. 7 of Ref. 1 for different magnetic fields: $B = 1.0$ T—circles; $B = 2.0$ T—triangles; $B = 4.0$ T—pluses; $B = 6.5$ T—diamonds; $B = 8.5$ T—squares.

creasing field (see Fig. 1). One can expect that near the peaks of σ_{xx}, the gap might become less pronounced due to the increase of the localization length. This effect cannot be described in the framework of our classical theory and, as far as we know it has not been observed.

In the Hartree-Fock approximation Yang and MacDonald[7] has recently derived the classical Hamiltonian from the quantum one in the presence of disorder. Here, we consider the case when the Coulomb interaction is screened by a metallic gate and the characteristic length is smaller than R_ϵ. In this case, the DOS at the chemical potential is nonzero even at $T = 0$.

We show by a computer modeling that the gap is suppressed by the screening, but it still exists. This means that the characteristic length of the interaction is still larger than the interelectron distance and the classical description should be reasonable. Using the classical approach, it is demonstrated below that the tunneling gap, observed in Ref. 1, can be described as a Coulomb gap in the DOS of a classical electron system, even despite the screening of the Coulomb interaction between the electrons.

The striking feature of the gap observed in Ref. 1 is that it is independent of the electron density at a given magnetic field, i.e., on filling factor. Such behavior is a characteristic feature of a "classical" Coulomb gap,[6] which has a universal DOS dependence on energy in the vicinity of chemical potential. This type of gap exists in systems with long-range Coulomb electron-electron interaction and with a strong external disorder. The second requirement is, probably, met in the samples used in Ref. 1, since the concentration of impurities in these samples is fairly large, and the distance between the doped layer and the 2DEL is small. However, the first requirement is violated because the distance d between the 2DEL and the metallic gate is small: it is of the order of electron-electron distance (for the sample A of Ref. 1 $d \approx 200$ Å, and for electron density $n = 10^{11}$ cm^{-2}, we have $d \approx 0.6n^{-1/2}$). This means that the interaction at larger distances is strongly screened and falls off faster than the $1/r$ Coulomb interaction. In fact, at $r \gg d$ the potential $V(R) \sim 1/r^3$, so the interaction becomes dipole.

The other type of the Coulomb gap, the gap in the electron liquid without any external disorder, was recently predicted theoretically[8] and, in our opinion, observed experimentally.[2,3] We show in Sec. III that this gap exists even for dipole electron-electron interaction, and should be understood as a kind of polaronic effect in the system of interacting electrons. However, this gap is strongly dependent on the filling factor. The question raises if there is a gap of an "intermediate" type, which would combine the features of the two above types: the gap survives at dipole interaction and is universal in filling factor. Such a gap may exist only at some range of moderate magnitudes of external disorder, because at small disorder the gap would not be universal, and at large disorder the Coulomb interaction is a necessary condition for the gap to exist.

In a search for this type of the Coulomb gap, we have performed a computer simulation of the system of classical interacting electrons in an external random potential. The model and the algorithm are described in Sec. II. Our simulation has shown that the DOS has a dip at the Fermi level, even when the screening length d is less than the mean electron-electron distance. Consequently, the tunneling conductivity at small biases is strongly suppressed. Moreover, the conductivity at zero bias becomes independent of the filling factor at moderate disorder $A > 0.4 - 0.6$, and its temperature dependence is in good agreement with the experimental data. The results of the calculations are discussed in detail in Sec. III and Sec. IV.

II. MODEL AND ALGORITHM

We study a system of classical point charges on a two-dimensional lattice in an external disorder potential. The Hamiltonian of the system has the following form:

$$H = \sum_i n_i \phi_i + \frac{1}{2} \sum_{i \neq j} (n_i - \nu)(n_j - \nu) V(r_{ij}), \qquad (1)$$

where $n_i = 0, 1$ are occupation numbers of electron on the lattice sites, ν is the filling factor of the lattice, ϕ_i is the external random potential, distributed uniformly in the interval $-A < \phi_i < A$, r_{ij} is the distance between the sites i and j, and $V(r)$ is the interaction potential. To simulate a plane of 2DEL, parallel to a metallic gate at a distance d, we have used the interaction of the following form:

$$V(r) = \frac{e^2}{\kappa} \left[\frac{1}{r} - \frac{1}{\sqrt{r^2 + 4d^2}} \right]. \qquad (2)$$

The Hamiltonian Eq. (1) with the interaction Eq. (2) takes into account the Coulomb interaction of the electrons of 2DEL with other electrons and their images in the metal plane. The interaction of an electron with its own image charge is not included, because it would lead only to a change in a total energy of the system by some constant value. We take the lattice constant to be $l = 1/\sqrt{n_0}$, where $n_0 = eB/hc$ is the electron density at $\nu = 1$. Thus, ν is also the filling factor of the lowest Landau level. Our model contains two-dimensionless parameters: filling factor ν and the ratio d/l. By changing ν at a fixed d/l one simulates an experiment, where an electron density is changed at a constant magnetic field.

In order to find the conductivity for the tunneling between the metallic gate and the 2DEL, we must know the DOS of single-particle energies $G(\epsilon)$ of the 2DEL at a given temperature T. The single-particle energy ϵ_i of the site i is

$$\epsilon_i = \phi_i + \sum_j (n_j - \nu) V(r_{ij}). \qquad (3)$$

We assume that the tunneling probability itself depends only on the parameters of the sample (such as the thickness and height of the tunneling barrier) and not on the electron density, temperature, or magnetic field. Then the tunneling conductivity from the 2DEL to a metal at

a given bias voltage u is proportional to the tunneling density of states (TDOS) defined as follows:

$$G_t(u) = \int d\epsilon G(\epsilon) \left(-\frac{\partial f}{\partial \epsilon}\right). \quad (4)$$

Here,

$$f(\epsilon) = \frac{1}{\exp\beta(\epsilon - \mu_g) + 1}, \quad (5)$$

$\beta = 1/T$ is a reciprocal temperature and μ_g is the chemical potential of the metal. The latter can be expressed through the chemical potential of the 2DEL μ and the bias u between the gate and the 2DEL,

$$\mu_g = \mu + u. \quad (6)$$

We calculate here the tunneling density of states of the 2DEL $G_t(u)$ as a function of filling factor ν, distance from 2DEL to the gate d, and temperature T. We have used the Metropolis Monte Carlo procedure[9] to average the TDOS over many-electron configurations. We have also employed quasiperiodic boundary conditions[10,11] to reduce the dependence of the results on the finite size of the system. Doing finite temperature calculations, we did not average the DOS over different realizations of the external disorder ϕ_i. Instead, we have used systems of large enough size to achieve a self-averaging. We have found that when the distance between the 2DEL and the gate d is of the order of lattice constant l, i.e., the screening of the electron-electron interaction by the gate is strong, a square sample with 10 000 sites is sufficiently large, so that the results do not depend on the realization of the disorder. A typical calculation for $d \approx l$, $\beta = (10 - 30)(e^2/\kappa l)^{-1}$, and $L = 100l$, which means that we have $L \times L = 10\,000$ lattice sites, takes about 30 min on Cray-YMP.

We have also calculated the DOS of the 2DEL at zero temperature. To do this we have found the pseudoground state of the system by minimization of the Hamiltonian Eq. (1), with respect to all one-electron hops. The minimization algorithm of Ref. 12 has been used. An averaging over different realizations of disorder was then performed. It has been shown by Mobius *et al.*[13] that many-electron hops do not change substantially the DOS as obtained by this algorithm.

III. RESULTS OF THE SIMULATION

The first question is if the Coulomb gap exists at all when the long-range part of the Coulomb interaction is strongly suppressed by a gate. To answer this question, we have calculated the TDOS of the 2DEL at $d = 0.7l$, which is a typical value for the experimental conditions of Ref. 1. Some of the results are shown in Fig. 2 for different values of the filling factor ν, different magnitudes of disorder A, and two temperatures. First, one can see that the TDOS has a dip at zero bias, but does not vanish there. This is consistent with the experimental results of Ref. 1, where the tunneling conductivity first decreases as temperature decreases, then it saturates at some nonzero value.

One can also see from Fig. 2 that the value of TDOS at zero bias $G_t(0)$ becomes independent of the filling factor at large magnitude of disorder. Since we keep d/l constant, this means that $G_t(0)$ is independent of an electron density. At $\nu < 0.5$, this independence appears at $A \geq 0.4$. The situation is a little different in the vicinity of $\nu = 0.5$, where at low temperatures a larger disorder is necessary to overcome a tendency to form a crystalline state and achieve the independence of the filling factor. For example, for $T = 1/30$ and $\nu = 0.5$ the conductivity becomes independent of the filling factor for $A \geq 0.6$. We understand that the tendency to crystallization at $\nu = 0.5$ is an artifact of our model which is completely classical. At other filling factors our system remains in a liquid state at all temperatures under the study even at $A = 0$.

Figure 3 displays the temperature dependence of the TDOS. Zero-temperature results are also given there.

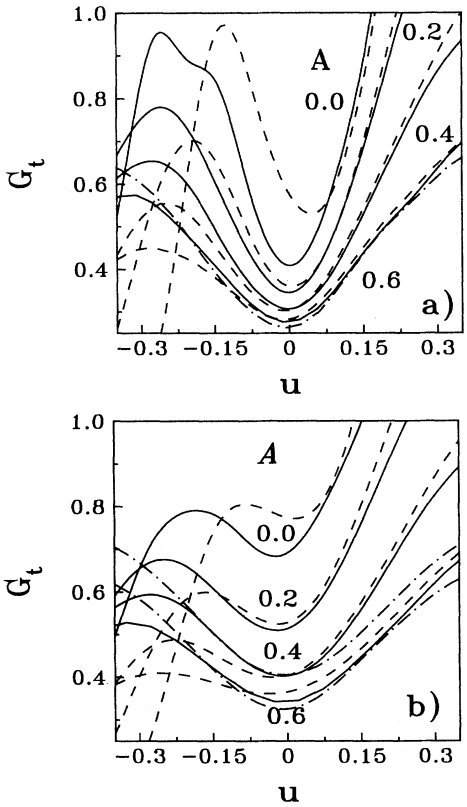

FIG. 2. Tunneling density of states G_t in units of κ/e^2l vs bias u between the 2DEL and the gate (in units of $e/\kappa l$). The temperature is $T = 1/30$ (a) and $T = 1/20$ for (b) in units of $e/\kappa l$, for different values of disorder A. Solid curves are for the filling factor $\nu = 0.3$, dashed curves are for $\nu = 0.2$, and dotted-dashed curves are for $\nu = 0.5$. The distance between the 2DEL and the gate is $d = 0.7l$. The magnitude of disorder A is shown in the figure for each set of curves, in units of $e^2/\kappa l$. Note that the conductivity at $u = 0$ becomes filling factor independent at large enough A.

One can see that the finite temperature affects the TDOS only in the vicinity of zero bias, and the width of the region where the TDOS is changed by the temperature is of the order of the temperature itself.

Thus, we can conclude that the Coulomb gap in the TDOS survives the strong screening. Moreover, within some interval of A the value $G_t(0)$ is independent of the electron density. This is just a behavior, observed in Ref. 1.

Now we discuss the dependence of $G_t(0)$ on other parameters, such as temperature T, disorder A, and screening length d. In this section, we study the behavior of the conductivity at zero bias $G_t(0)$ in the region of disorder, temperatures, and filling factors, where it is independent of the filling factor (the limitations on temperatures and filling factors are chosen to avoid the vicinity of $\nu = 0.5$ at low temperatures and small magnitudes of disorder).

We have found that there is an unexpected universality in these dependencies. This universality is illustrated in Fig. 4, where the values of $G_t(0)/T$ are plotted versus ATd^2 for different values of individual parameters. Figure 4(a) gives the temperature dependence, while Figs. 4(b) and (c) give dependence of A and d, receptively. All data clearly collapse in this scale onto some universal curve, which suggests that within a certain range of the parameters the TDOS at zero bias is given by the universal formula,

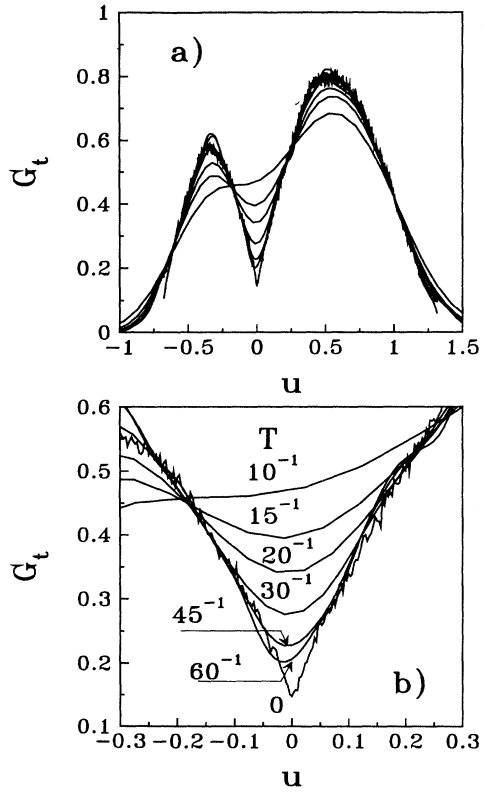

FIG. 3. The TDOS G_t in units of κ/e^2l vs bias u in units of $e/\kappa l$. The magnitude of disorder is $A = 0.6\ e^2/\kappa l$, the filling factor $\nu = 0.3$, and the distance between the 2DEL and the gate is $d = 0.7l$. (b) shows the conductivity at small biases in more details. The temperature is shown in the (b) for each of the curves, in units of $e^2/\kappa l$.

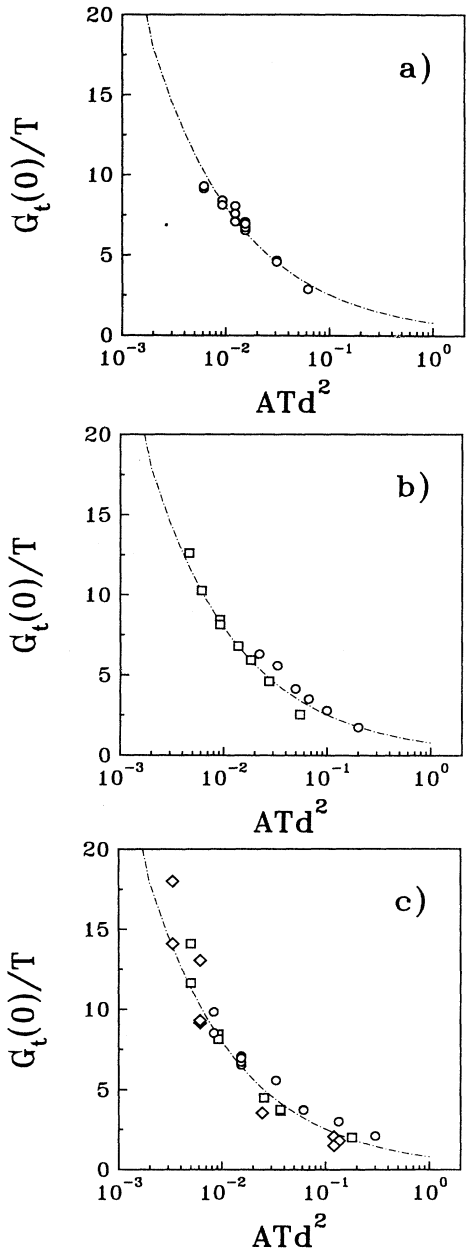

FIG. 4. Universal behavior of the conductivity at zero bias: $G_t(0)/T$ in units of κ^2/e^4 vs the parameter ATd^2 (in units of e^4/κ^2 for different values of A, T, and d). On the plot (a) the dependence of $G_t(0)/T$ on A at fixed $d = 0.7l$ and $T = 1/30\ (e^2/\kappa l)$ is presented. Plot (b) shows the dependence $G_t(0)/T$ on T for $A = 0.6\ (e^2/\kappa l)$, $d = 0.7l$ (squares), and for $A = e^2/\kappa l$, $d = l$ (circles). Finally, the dependence of $G_t(0)/T$ on d at constant A and T is shown on plot (c) for $A = 0.4\ (e^2/\kappa l)$, $T = 1/30\ (e^2/\kappa l)$ (diamonds), $A = 0.6\ (e^2/\kappa l)$, $T = 1/30\ (e^2/\kappa l)$ (squares), and $A = e^2/\kappa l$, $T = 1/30\ (e^2/\kappa l)$ (circles). The dotted-dashed curve is a fit, given by Eq. (8).

$$G_t(0) = Tf\left(ATd^2\right). \tag{7}$$

The best fit for the function gives the expression

$$f(x) \approx \frac{0.8}{\sqrt{x}}. \tag{8}$$

This function is shown in Fig. 4 by the dashed-dotted line.

The universality equations (7,8) exists only within a certain range of the parameters A, T, and d. First, the magnitude of disorder A has to be sufficiently large for $G_t(0)$ to be independent of the filling factor. Then, the distance d has to be small: $d < 1.5l$, where l is the lattice constant. Finally, we have found that the universality always breaks at very low temperatures. For example, for $d = 0.7l$, the equations (7,8) hold for $T > 1/60$ in units of $e^2/\kappa l$. As temperature decreases or d increases, the TDOS $G_t(0)$ eventually becomes *lower* than the value predicted by Eqs. (7,8), but we have never observed the deviation in the opposite direction. In other words, if we start from such a low T or large d that the universality is broken, and then increase T or decrease d, the TDOS $G_t(0)$ increases until it reaches the solid curve in Fig. 4, and then it follows the universal law. Also, the universal dependence manifests itself as an upper limit for $G_t(0)$: a low temperature of large d can cause the conductivity to be less than the universal value, but the larger values have never been realized in our calculations. If we take, for example, $d = 1.4$, then at $T < 1/20$ the conductivity is lower than the universal one. At $T = 1/20$ it reaches the value, given by Eqs. (7,8) and remains on the universal curve when the temperature continues to increase. One might suppose that for each d there is a temperature above which the universality begins, but at high temperatures the gap in conductivity disappears. This determines the maximum value of d at which the universality can still be observed, which value we have found to be about $1.5l$.

The parameter ATd^2 has a dimension of κ/e^2, therefore is it does not depend on the unit of length, the lattice constant. Thus, we may expect that this universality is not the result of a lattice effect. To test this directly, we have repeated some calculations for a triangle lattice and found no deviation in $G_t(0)$ within the computational accuracy.

IV. DISCUSSION OF SQUARE ROOT TEMPERATURE DEPENDENCE. COMPARISON WITH SELF-CONSISTENT EQUATION

The very existence of the Coulomb gap at $A \approx e^2/\kappa l$ is connected with the long-range part of the Coulomb interaction. At large d we would expect that at zero temperature DOS $G(0) \approx \kappa/e^2 d$ and it is temperature independent if $T \ll e^2/\kappa d$. It follows that if $e^2/\kappa d \approx A \approx e^2/\kappa l$ the gap should not exist. However, Fig. 2 shows very pronounced gap at different A and $d = 0.7l$. We think that this can be explained as a game of small numerical coefficients.

At large A this problem was approached with the self-consistent equation (SCE). Mogilyanskii and Raikh[14] have solved SCE at nonzero temperature. Their result for large A reads:[15]

$$G(\mu) \approx 0.085\frac{\kappa}{e^2 d} + 0.86\frac{\kappa^2 T}{e^4}. \tag{9}$$

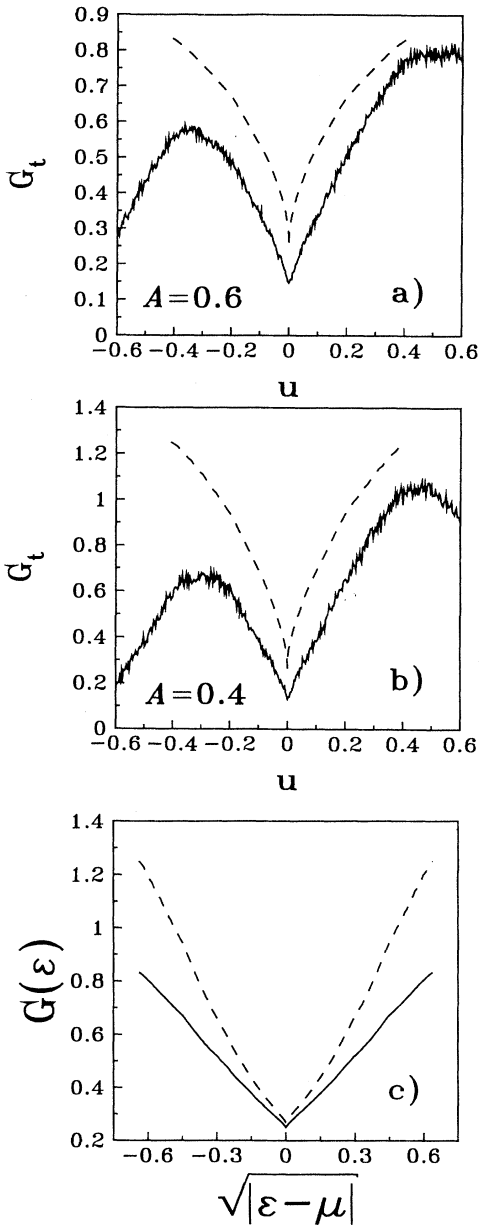

FIG. 5. Comparison of the simulation results for zero temperature with the solution of the self-consistent equation Eq. (10). Solid lines show TDOS in units of $\kappa/e^2 l$ vs bias in units of $e/\kappa l$, as obtained from the computer simulation, dashed lines show the solution of the self-consistent equation. The filling factor is $\nu = 0.3$, the distance between the 2DEL and the gate is $d = 0.7l$, and the magnitude of disorder A is 0.4 (a) and 0.6 (b) in units of $e^2/\kappa l$. (c) shows the solution of the SCE for $A = 0.4$ (dashed line) and $A = 0.6$ (solid line) as a function of $\sqrt{|\epsilon - \mu|}$; all energies are in units of $e^2\kappa l$.

This result confirms the above qualitative arguments, however, the small numerical coefficient in the first term allows us to suggest that the gap exists at much smaller d than one can expect. We have solved a zero-temperature SCE:[6]

$$G(\epsilon) = G_\infty \exp\left[-\frac{2}{\pi}\int_0^\infty d\epsilon' G(\epsilon') R^2(\epsilon+\epsilon')\right], \quad (10)$$

where $G_\infty = 1/2Al^2$ and $R(\epsilon)$ is the reciprocal function of the interaction potential:

$$V[R(\epsilon)] = \epsilon. \quad (11)$$

For the interaction potential Eq. (2) the equation (10) can be solved only numerically. At large A and d the solution in the gap can be approximated almost everywhere as

$$G(\epsilon) = 0.085\frac{\kappa}{e^2 d} + \frac{2\kappa^2}{\pi e^4}\epsilon. \quad (12)$$

This result is in a good agreement with Eq. (9) obtained at nonzero temperature. It also contains a small number in the first term, and we think that this small number is responsible for the existence of the gap at small d (see also discussion in Ref. 16).

The solution of SCE at small A and d and at zero temperature gives a square root energy dependence of a DOS rather than linear (see Fig. 5). It is interesting to compare this result with the result of computer modeling at $T = 0$. It follows from Eq. (4) that at zero temperature, when $-(\partial f/\partial\epsilon) = \delta(\epsilon - \mu_g)$, the TDOS at zero bias is just equal to the DOS at the Fermi level. We have calculated DOS of 2DEL at zero temperature, using the minimization algorithm of Ref. 12, as described in Sec. II. The TDOS can be obtained from the DOS by shifting the energy axis in such a way that the chemical potential μ becomes an origin. The resulting TDOS is shown in Fig. 5 together with a solution of the SCE.

One can conclude from Fig. 5 that the energy dependence, as obtained by SCE, is close to the result of computer modeling. On the other hand, $G(\mu)$ is different. We think that the reason is as follows. Strictly speaking, SCE is valid at large A only (see Ref. 6). However, one can see by iterating this equation that the energy dependence at small energies and at a given $G(\mu)$ is determined by the the values of the function $G(\epsilon)$ also corresponding to small energies. In this case, one can expect to get a correct self-consistent solution. On the other hand, the value $G(\mu)$ is determined by the integral of $G(\epsilon)$, which is not limited by small energies. At large energies Eq. (10) gives $g(\epsilon) = g_\infty = 1/2Al^2$. This is definitely wrong if A is not larger than the Coulomb energy. Thus, we think that at small A one can use SCE to find an energy dependence of the DOS, but this is not the way to obtain $G(\mu)$.

The square root energy dependence of $G(\epsilon)$ at $T = 0$ explains square root temperature dependence, which has been obtained by computer modeling in the moderate temperature range. However, we cannot explain by the same way the universal dependence of the TDOS on the factor Ad^2.

V. COMPARISON WITH EXPERIMENT

We now compare the results of our simulation for the conductivity at zero bias with the experimental data of Ref. 1. These data are averaged over a wide range of electron densities. For the comparison we use a large enough value of disorder, where the results of our calculations are virtually independent of the density. Since we use the classical model with electrons considered as point charges, our results may apply to the limit of large magnetic fields. In magnetic field $B = 8.5$ T the lattice constant is $l = 1/\sqrt{n_0} = \sqrt{hc/eB} \approx 220$ Å. The distance d between the 2DEL and the metal for the sample A of Ref. 1 is about $d = 160$ Å$\approx 0.7l$. In Fig. 1 we compare the experimental data from Fig. 7 of Ref. 1 with the results of our simulation with these parameters. One can see that we have achieved a good agreement, except at the lowest temperatures, where our saturation value for conductivity appears to be higher than the experimental one.

The reason for this discrepancy may be the correlation in the spatial distribution of electron in the 2DEL and in the tunneling contact. It was shown that such correlation in a system of two parallel quantum wells reduces the conductivity significantly at low temperatures.[3] The equation (4) for conductivity ignores any effects of such correlations. In fact, this expression is valid when the tunneling contact is a perfect metal. However, in the samples of Ref. 1 the tunneling is measured between the 2DEL and the strongly doped semiconductor layer, with the Debye screening length of the order of 50 Å, which is comparable with the distance d. This also means that the value of d itself is known only approximately, and the error in its determination may be of the order of the Debye length in the tunneling contact.

In conclusion, we have performed a computer simulation of a two-dimensional system of classical point electrons on a lattice with a parallel metal plane. We have calculated the density of states of the two-dimensional electron liquid and the tunneling conductivity between the metal and the electron layer. We have shown that the electron-electron interaction leads to the suppression of the conductivity at small bias. The results obtained are in a good agreement with the experimental data.

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