

## Low-magnetic-field critical behavior in strongly type-II superconductors

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A description is proposed for the low-field critical behavior of type-II superconductors. The starting point is the Ginzburg-Landau functional in presence of an external magnetic field  $\mathbf{H}$ . A set of fictitious vortex variables and a singular gauge transformation are used to rewrite this finite  $\mathbf{H}$  Ginzburg-Landau functional in terms of a complex scalar field of zero average vorticity. The continuum limit of the transformed problem takes the form of an  $\mathbf{H}=\mathbf{0}$  Ginzburg-Landau functional for a charged field coupled to a fictitious "gauge" potential, which arises from long-wavelength fluctuations in the background liquid of field-induced vortices. A possibility of a phase transition involving zero-vorticity degrees of freedom and formation of a uniform condensate is suggested. A similarity with the superconducting (Higgs) electrodynamics and the nematic-smectic- $A$  transition in liquid crystals is noted. The experimental situation is discussed.

There are two basic theoretical approaches to the fluctuation behavior of high-temperature superconductors (HTS) and other strongly type-II systems in a magnetic field: The Ginzburg-Landau (GL) theory confined to the lowest-Landau level (LLL) for Cooper pairs (the GL-LLL theory)<sup>1</sup> and the XY-model approach, in which one focuses on the field-induced London vortices and suppresses other superconducting fluctuations.<sup>2</sup> Generally, it is expected that the GL-LLL theory describes the high-field regime, close to  $H_{c2}(T)$ , behavior, while the London vortex theory should be appropriate at low fields, far from  $H_{c2}(T)$ . It is often presumed that there is a smooth crossover between the two regimes.

It is argued here that there is an important physical difference between the critical behaviors at high and low fields, reflected in the nature of low-energy fluctuations. At high fields, all such fluctuations are represented by the motion of  $N_\phi$  field-induced vortices. This is so because the states within the LLL can be expressed as different configurations of zeros of the holomorphic order parameter.<sup>3</sup> Higher LL's have a finite gap and contribute only by renormalizing various terms in the GL-LLL theory. In contrast, at low fields, even if we imagine that the field-induced vortices are fixed in their positions, there are low-energy fluctuations of many degrees of freedom which are not associated with field-induced vortices: nonsingular phase fluctuations, vortex-antivortex pairs, vortex loops, etc. It is precisely these zero vorticity degrees of freedom that produce the zero-field superconducting transition. At finite but low fields, it is reasonable to expect that these degrees of freedom still account for most of the entropy change in the critical regime and dominate various thermodynamic functions.

In this paper a description is derived of the low-field critical behavior associated with these zero vorticity fluctuations. The key step is to rewrite the original partition function of the GL theory in terms of a complex scalar field  $\Phi$  of zero average vorticity, instead of the usual superconducting order parameter field  $\Psi$  whose average vorticity is  $N_\phi$ , as fixed by the magnetic field. This vorticity shift by  $N_\phi$  is functional space of the fluctuating or-

der parameter is accomplished by first introducing a set of  $N_\phi$  auxiliary vortex variables, which we may call "shadow" vortices (or  $s$  vortices), and then performing a "singular" gauge transformation  $\Psi \rightarrow \Phi$ . The continuum hydrodynamic limit of the transformed problem is a field theory involving a complex scalar field  $\Phi$  in an effective average magnetic field equal to zero coupled to a fictitious gauge potential  $\mathbf{S}$  produced by local fluctuations of  $s$  vorticity around its average value. In intuitive terms,  $\Phi$  represents those degrees of freedom of the original superconducting order parameter which cannot be reduced to motion of field-induced vortices, while  $\mathbf{S}$  arises from the long-wavelength density and current fluctuations in the background system of these field-induced London vortices. The physical insight gained by this transformation is that we have now uncovered in the original GL problem those hidden off-diagonal correlations, represented by  $\Phi$ , whose range extends far beyond the average separation between field-induced vortices (set by the magnetic length). At the same time, the range of original superconducting correlator involving field  $\Psi$  remains limited by the magnetic length. It is these novel off-diagonal correlations with range longer than magnetic length that govern the low-field critical behavior. Furthermore, a possibility is pointed out of a finite-field (FF) phase transition involving divergence of a certain susceptibility related to  $\Phi$  and  $\mathbf{S}$ . As field tends to zero the line of the FF phase transitions terminates in the familiar zero-field (ZF) critical point at  $T=T_{c0}$ . Next, a connection is made between the FF critical fluctuations and the critical behavior of liquid crystals and the ordinary superconducting (Higgs) electrodynamics at zero field. An intuitive picture is proposed for this FF transition in the context of the three-dimensional (3D) XY model where the  $\Phi$ -ordered phase is identified as the incompressible London vortex liquid state, while the  $\Phi$ -disordered state corresponds to the high-temperature compressible fluid of unbound vortex loops. Finally, the experimental situation is discussed.

The anisotropic GL partition function is the appropriate starting point for HTS and other layered super-

conductors:  $Z = \int \mathcal{D}[\Psi(\mathbf{r}, \xi)] \exp\{-F_{\text{GL}}[\Psi(\mathbf{r}, \xi)]/T\}$ ,  
where

$$F_{\text{GL}} = \int d^2r d\xi \left\{ \alpha |\Psi|^2 + \frac{\beta}{2} |\Psi|^4 + \gamma_{\perp} \left| \left[ \nabla_{\perp} + \frac{2ei}{c} \mathbf{A} \right] \Psi \right|^2 + \gamma_{\parallel} |\partial_{\xi} \Psi|^2 \right\}, \quad (1)$$

$\mathbf{r}=(x,y)$  and  $\xi$  are the coordinates perpendicular and along  $\mathbf{H}$ ,  $\nabla \equiv [\nabla_{\perp}, \partial_{\xi}]$ ,  $\alpha = \alpha_0[(T/T_{c0}) - 1]$ ,  $\beta, \gamma_{\perp}, \gamma_{\parallel}$  are the GL coefficients, and  $\nabla \times \mathbf{A} = \mathbf{H}$ , with the magnetic field  $\mathbf{H}$  perpendicular to the layers. Fluctuations of the electromagnetic field are neglected throughout the paper ( $\kappa \gg 1$ ).

An important feature of  $F_{\text{GL}}$  is the formation of Landau levels (LL's) for Cooper pairs. The LL structure arises from the quadratic part of  $F_{\text{GL}}$ . The quartic interaction term, however, mixes different LL's and acts to suppress the LL structure in the fluctuation spectrum. It is instructive to divide the effect of the quartic term into the intra-LL and inter-LL correlations. At high fields,  $H \gg H_b$ , where the cyclotron gap between LL's is much larger than the interaction term, only the intra-LL correlations are important and the LL structure will be reflected in the theory. In this regime the GL-LLL description captures essential features of the physics. In the opposite limit of low fields,  $H \ll H_b$ , the inter-LL correlations become dominant and the LL structure is suppressed at long wavelengths. The crossover field  $H_b$ , which separates the high-field LL regime from the low-field "semiclassical" one, is given by  $H_b \sim (\theta/16)(T/T_{c0})H_{c2}(0)$ , where  $\theta$  is the Ginzburg fluctuation number. In terms of the parameters of the GL theory  $\theta \approx 2\beta H'_{c2} T_{c0}^2 / \phi_0 \alpha_0^{3/2} \gamma_{\parallel}^{1/2}$ , where  $H'_{c2} = [dH_{c2}/dT]$  at  $T = T_{c0}$ . This expression for  $H_b$  was derived in Ref. 4 by comparing the strength of quartic correlations in (1) with the cyclotron gap between LL's. In HTS  $\theta \sim 0.01 - 0.05$  and  $H_b \sim 0.1 - 1$  T.<sup>4</sup>

In the low-field limit,  $H \ll H_b$ , a semiclassical description becomes possible. The idea proposed here is to shift the overall vorticity of the fluctuation spectrum by  $N_{\phi}$ , where  $N_{\phi} = \Omega/2\pi l^2$ ,  $\Omega$  and  $l = \sqrt{c/2eH} \equiv \sqrt{\phi_0/2\pi H}$

being the area of a layer and the magnetic length, respectively. This shift is accomplished by introducing a set of  $N_{\phi}$  auxiliary variables,  $\{z_i(\xi)\}$ , which one may call shadow vortices ( $s$  vortices). We now change variables in the functional integral:  $\Psi(\mathbf{r}, \xi) \rightarrow \Phi(\mathbf{r}, \xi) = \Psi(\mathbf{r}, \xi) \exp[-i\Theta(\mathbf{r}, \xi)]$ , where  $\Theta(\mathbf{r}, \xi) = \sum_i \tan^{-1}\{[y - y_i(\xi)]/[x - x_i(\xi)]\}$ ,  $z_i(\xi) = x_i(\xi), y_i(\xi)$ , and rewrite the GL functional (1) as

$$\int d^2r d\xi \left\{ \alpha |\Phi|^2 + \frac{\beta}{2} |\Phi|^4 + \gamma_{\perp} \left| \left[ \nabla_{\perp} + i\nabla_{\perp}\Theta + \frac{2ei}{c} \mathbf{A} \right] \Phi \right|^2 + \gamma_{\parallel} |\partial_{\xi} + i\partial_{\xi}\Theta|^2 \right\}, \quad (2)$$

while simultaneously introducing the function integral over  $s$  vortex variables,  $\int \prod_{i,\xi} dz_i(\xi) \mathcal{T}[\Phi, \{z_i(\xi)\}]/N!$ , in the measure.  $\mathcal{T}[\Phi, \{z_i(\xi)\}]$  is the Jacobian of this [singular] gauge transformation and it ensures that in the low-temperature limit, where  $\Phi(\mathbf{r}, \xi)$  takes the form which minimizes  $F_{\text{GL}}$  for a given configuration of  $\{z_i(\xi)\}$ , the above formulation becomes equivalent to the standard London description. Note, however, that  $s$  vortices are fictitious objects and do not directly correspond to the physical vortex excitations of  $\Psi(\mathbf{r}, \xi)$ . The whole transformation is an identity if the short-wavelength behavior is properly regularized, for example, by having  $\Psi(\mathbf{r}, \xi)$  and  $s$  vortices defined on dual sublattices.

The integration over  $s$  vortex variables would now lead to a new, and complicated, representation of the original problem (1). Such an exact integration is beyond reach. What has been gained, however, is that we now have a reformulation of the problem in terms of field  $\Phi$  whose average vorticity equals zero. Therefore, certain aspects of the physics become more visible. In particular, the long-wavelength functional for  $\Phi$  can be extracted by the following "smoothing out" procedure: New variables are introduced,  $\rho(\mathbf{r}, \xi) \equiv \sum_i \delta[\mathbf{r} - \mathbf{z}_i(\xi)]$ ,  $\mathbf{j}(\mathbf{r}, \xi) \equiv \sum_i [dz_i/d\xi] \delta[\mathbf{r} - \mathbf{z}_i(\xi)]$ , which represent microscopic  $s$  vortex density and "current," respectively. In terms of  $[\rho, \mathbf{j}]$  we have  $[\nabla_{\perp}\Theta, \partial_{\xi}\Theta] \equiv \int d\mathbf{r}' (\mathbf{r} - \mathbf{r}') \times [\mathbf{e}_z \rho(\mathbf{r}', \xi), -\mathbf{j}(\mathbf{r}', \xi)] / |\mathbf{r} - \mathbf{r}'|^2$ , where  $\mathbf{e}_z$  is a unit vector along  $\mathbf{H}$ . The GL partition function is transformed into

$$Z = \int \mathcal{D}[\Phi] \mathcal{D}[\rho] \mathcal{D}[\mathbf{j}] \exp\{-F'_{\text{GL}}[\Phi, \rho, \mathbf{j}]/T + W[\rho, \mathbf{j}]\}, \quad (3)$$

$$\exp\{W[\rho, \mathbf{j}]\} \equiv \int \prod_{i,\xi} \frac{dz_i(\xi)}{N!} \mathcal{T} \prod_{\mathbf{r}} \delta \left[ \rho(\mathbf{r}, \xi) - \sum_i \delta[\mathbf{r} - \mathbf{z}_i(\xi)] \right] \delta \left[ \mathbf{j}(\mathbf{r}, \xi) - \sum_i \frac{dz_i}{d\xi} \delta[\mathbf{r} - \mathbf{z}_i(\xi)] \right],$$

where  $F'_{\text{GL}}$  is given by Eq. (2) with  $[\nabla_{\perp}\Theta, \partial_{\xi}\Theta]$  expressed via  $[\rho, \mathbf{j}]$ .

The above expression is formally exact but useless, since  $\rho$  and  $\mathbf{j}$  are wildly varying functions. For  $H \ll H_b$ , however, it is expected that replacing  $\rho(\mathbf{r}, \xi) \rightarrow \bar{\rho}(\xi) + \delta\rho(\mathbf{r}, \xi)$  in (3), where  $\bar{\rho}(\xi) = (2\pi l^2)^{-1}$  is the average  $s$  vortex density and  $\delta\rho(\mathbf{r}, \xi)$  is a smooth function describing variations around the average, should be adequate at wavelengths long compared to  $l$ . Similar smoothing out is performed in  $\mathbf{j}(\mathbf{r}, \xi)$ . This leads to the hydrodynamic limit of the GL functional:

$$Z = \int \mathcal{D}[\Phi(\mathbf{r}, \xi)] \mathcal{D}[\delta\rho(\mathbf{r}, \xi)] \mathcal{D}[\mathbf{j}(\mathbf{r}, \xi)] \exp\{-F''_{\text{GL}}[\Phi(\mathbf{r}, \xi), \delta\rho(\mathbf{r}, \xi), \mathbf{j}(\mathbf{r}, \xi)]/T + W[\delta\rho, \mathbf{j}]\}, \quad (4)$$

$$F''_{\text{GL}} = \int d^2r d\xi \left\{ \alpha |\Phi|^2 + \frac{\beta}{2} |\Phi|^4 + \gamma_{\perp} \left| \left[ \nabla_{\perp} + i\mathbf{s}(\mathbf{r}, \xi) \right] \Phi \right|^2 + \gamma_{\parallel} \left| \partial_{\xi} + i\mathbf{s}_{\xi}(\mathbf{r}, \xi) \right|^2 \right\},$$

with the form of  $W[\delta\rho, \mathbf{j}]$  apparent from (3) and a new vector field  $\mathcal{S} \equiv [\mathbf{s}, s_\xi] = \int d\mathbf{r}'(\mathbf{r}-\mathbf{r}') \times [\mathbf{e}_z \delta\rho(\mathbf{r}', \xi), -\mathbf{j}(\mathbf{r}', \xi)]/|\mathbf{r}-\mathbf{r}'|^2$ .

The main feature of  $F''_{\text{GL}}$  is that  $\mathbf{H}$  has now disappeared from the problem. The vector potential  $\mathbf{A}$  has been canceled by the average  $s$  vortex density,  $\bar{\rho}$ , which appears in  $\langle \nabla_\perp \Theta(\mathbf{r}, \xi) \rangle$  ( $\langle \cdots \rangle$  denotes thermal average). The average magnetic field felt by  $\Phi$  is zero. We have thus arrived at the following simple description: The critical behavior of the GL partition function (1) can be represented by a complex scalar field  $\Phi$  of zero average vorticity coupled to a fictitious “ $s$  gauge” potential  $\mathcal{S}$  produced by local fluctuations of vorticity around its average value  $N_\phi$  set by the external field.

While the above picture appears intuitive it is by no means obviously justified. The above derivation relies on the assumption of separation of long-wavelength fluctuations in  $\Phi$  from the rapidly changing short-wavelength variations of the microscopic, i.e., not smoothed out,  $s$  vortex density. To demonstrate that such separation indeed takes place in (1) is no trivial task.<sup>5</sup> In fact, the high-field, LL regime is entirely dominated by such core effects. In the low-field limit of the XY model, where the microscopic core size  $a$  satisfies  $a \ll l$ , it is hoped that these core effects eventually become irrelevant for the long-wavelength behavior. For the rest of the paper it is assumed that the magnetic field is sufficiently low so that core effects in (1), even if relevant, affect critical behavior only at distances much too long to be of practical interest. The range of the validity of this semiclassical description can then be established empirically.

These clarifications noted, I now demonstrate the utility of this description of low-field critical behavior. The free energy of Eq. (4) can be evaluated in the mean-field approximation  $\Phi(\mathbf{r}, \xi) = \Phi_0$ :

$$F_{\text{mf}}[\Phi_0] = \alpha |\Phi_0|^2 + \frac{\bar{\beta}}{2} |\Phi_0|^4 + \frac{\gamma_\perp}{2\pi l^2} |\Phi_0|^2 f_C(\Gamma), \quad (5)$$

where  $\bar{\beta} = \beta = 2C\gamma_\perp^2 \phi_0 / a_\parallel^4 T_{c0} H$ ,  $C$  is of order unity and  $a_\parallel = \max(\xi_\parallel, d)$ , with  $\xi_\parallel$  and  $d$  being the coherence length along  $\mathbf{H}$  and the interlayer separation, respectively. In deriving (5), I assumed  $2C\gamma_\perp^2 \phi_0 / a_\parallel^4 \beta T_{c0} H \ll 1$ , which is appropriate for Bi-Sr-Ca-Cu-O HTS.  $Q^2 f_C(\Gamma)$ , with  $\Gamma = Q^2 / T \equiv |\Phi_0|^2 \gamma_\perp \xi_\parallel / T$ , is the free energy of the 2D one-component Coulomb plasma (OCP),  $f_C = (1/2) \ln(\sqrt{2}l/a) + g(\Gamma)$ .<sup>6</sup> An approximate form, reliable for  $1 \gtrsim \Gamma$ , is

$$g(\Gamma) = -\frac{1}{4} \left\{ 2E + \ln \left[ \frac{\Gamma}{\Gamma+2} \left[ \frac{2}{\Gamma+2} \right]^{2/\Gamma} \right] \right\},$$

where  $E = 0.5772 \dots$  is Euler's constant.  $\Phi_0$  is determined by minimizing  $F_{\text{mf}}$ . Below some  $T_\Phi(H)$ , which has to be determined numerically, the minimum of  $F_{\text{mf}}$  shifts from  $\Phi_0 = 0$  to finite  $\Phi_0$ . A simple approximate formula is

$$T_\Phi(H) \cong T_{c0} - c_1 \frac{4\pi\gamma_\perp T_{c0}}{\alpha_0 \phi_0} H \times \left\{ \frac{1-E}{2} + \frac{1}{4} \ln \left[ \frac{2c_2 \bar{\beta} T_{c0} \phi_0^2}{\pi a^2 a_\parallel (4\pi\gamma_\perp)^2 H^2} \right] \right\}, \quad (6)$$

where  $c_{1,2}$  are constants of order unity. The transition is weakly first order due to long-range interactions in OCP, with the jump  $\Delta\Phi_0 \cong \sqrt{4\pi\gamma_\perp H / 2\bar{\beta}\phi_0}$ .

$T_\Phi(H)$  [or  $H_\Phi(T)$ , as in Fig. 1] could be interpreted as the “fluctuation renormalized”  $H_{c2}(T)$ . This  $\Phi$  transition is driven by the growth of a novel off-diagonal order associated with  $\Phi$ , and not the original superconducting field  $\Psi$ . Above the mean-field  $H_{c2}(T)$ , the correlation length of  $\chi_\Phi(\mathbf{r}, \xi) \equiv \langle \Phi(\mathbf{r}, \xi) \Phi^*(0, 0) \rangle$  in the  $xy$  plane  $\xi_\Phi$  is much shorter than  $l$  and is approximately equal to  $\xi_\Psi$ , the latter being the usual superconducting correlation length, associated with  $\chi_\Psi(\mathbf{r}, \xi) \equiv \langle \Psi(\mathbf{r}, \xi) \Psi^*(0, 0) \rangle$  (the XY region in the inset of Fig. 1). In the critical region below  $H_{c2}(T)$ ,  $\xi_\Psi$  saturates at  $\sim l$ , but  $\xi_\Phi$  grows rapidly and becomes  $\gg l$  (the  $\Phi$  region in the inset of Fig. 1). This illustrates the main physical idea of this paper: While the original pairing correlations in the critical region remain limited by the motion of field-induced vortices, which form a liquid both above and below  $H_\Phi(T)$ , there are other off-diagonal correlations in (1) whose range greatly exceeds  $l$ .

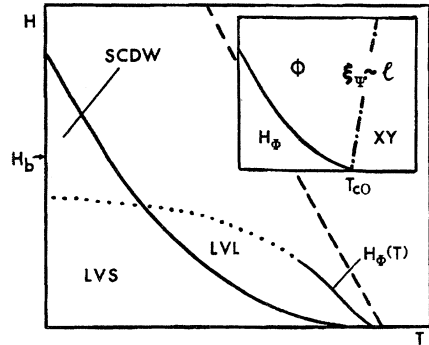


FIG. 1. A schematic representation of the critical region. LVL—London ( $s$ ) vortex liquid; LVS—London ( $s$ ) vortex solid; SCDW—charge-density-wave of Cooper pairs (see Tešanović in Ref. 1). The dotted line indicates that  $H$  is too high for the semiclassical approximation to be reliable in determining  $H_\Phi(T)$ . The dashed line is the mean-field  $H_{c2}(T)$ . The full line represents the London ( $s$ ) vortex solid-liquid melting transition in the low-field regime while it separates the normal state from the density-wave of Cooper pairs at high fields. The low-field melting line is well-separated from  $H_\Phi(T)$  because the melting transition takes place only for a rather large  $\Gamma$  ( $\Gamma_{\text{melt}} \sim 140$ ) and is well below the nominal critical region [see Eq. (6) and the text above it]. The arrow on the  $H$  axis indicates the crossover from the high-field to the low-field regime of critical behavior,  $H_b \sim (\theta/16)(T/T_{c0})H_{c2}(0) \sim 1$  T in HTS. The inset shows regions of XY and  $\Phi$  critical behavior as described in text. The dashed-dotted XY- $\Phi$  crossover line is set by  $\xi_\Phi(T, H=0) \sim l$ .

$H_\Phi(T)$  has no analog in the high-field regime. There  $H_{c2}(T)$  is only a smooth crossover.

The mean-field theory predicts that these  $\Phi$  correlations become long ranged below  $H_\Phi(T)$ . To examine this prediction we consider fluctuations in  $\Phi$ . The smoothed out variables  $[\delta\rho, \mathbf{j}]$  are well-defined only on length scales

$$\mathcal{F}_{\text{GL}} = \int d^2r d\xi \left\{ \alpha |\Phi|^2 + \frac{\beta}{2} |\Phi|^4 + \gamma_\perp |[\nabla_\perp + i\mathbf{s}(\mathbf{r}, \xi)]\Phi|^2 + \gamma_\parallel |[\partial_\xi + i s_\xi(\mathbf{r}, \xi)]\Phi|^2 + K_\parallel^0 (\nabla \times \mathcal{S})_\parallel^2 + K_\perp^0 (\nabla \times \mathcal{S})_\perp^2 \right\}, \quad (7)$$

where  $K_{\parallel, \perp}^0(T, H)$  is the bare ‘‘stiffness’’ of the  $s$  gauge field  $\mathcal{S} \equiv [\mathbf{s}, s_\xi]$ , produced by the short-wavelength ( $< \Lambda$ ) fluctuations in  $\Phi(\mathbf{r}, \xi)$ , which also renormalize the  $\frac{\text{GL}}{\text{GL}}$  coefficients. At  $T \sim T_{c0}$ ,  $K_{\parallel, \perp}^0 \sim [\sqrt{\gamma_\perp/\gamma_\parallel}, \sqrt{\gamma_\parallel/\gamma_\perp}] \times T_{c0}\Lambda$ .<sup>7</sup> Higher-order terms in  $\mathcal{S}$  which also are generated by the integration of short-wavelength ( $< \Lambda$ ) modes are presumed irrelevant. This procedure is internally consistent in the hydrodynamic limit. Similarly,  $\mathcal{W}[\mathbf{s}, s_\xi]$ , which is simply  $\mathcal{W}$  of Eq. (3) reexpressed in terms of  $\mathcal{S}$ , is assumed to have an expansion in  $\mathcal{S}(\mathbf{r}, \xi)$ . In addition, the above integration procedure with a specified cutoff may produce quadratic terms in  $\mathcal{S}$  which violate the fictitious  $s$  gauge invariance, an example being the mass term  $m_s^2 \mathcal{S}^2$ . All such terms will be absorbed into the redefinition of  $\mathcal{W}[\mathbf{s}, s_\xi]$ . Conversely, the  $s$  gauge invariant part of  $\mathcal{W}$  will be absorbed into  $K_\parallel^0, K_\perp^0$ . The remaining fluctuations in  $\Phi$  and those of  $\mathcal{S}$  come from wavelengths  $> \Lambda$ .  $\mathcal{F}_{\text{GL}}$  in (7) defines the bare level functional which serves as the starting point for study of the finite-field critical behavior. Note that, for  $H \rightarrow 0$  along the  $T = T_\Phi(H)$  line,  $K_{\parallel, \perp}^0 \propto \Lambda \propto 1/\sqrt{H} \rightarrow \infty$ , the  $\mathcal{S}$  fluctuations in  $\mathcal{F}_{\text{GL}}$  are suppressed, and the  $\Phi$  transition goes into the zero-field superconducting transition.

$\mathcal{F}_{\text{GL}}$  has a form reminiscent of the standard superconductor (Higgs) electrodynamics (SHE) at zero field (the anisotropy is easily rescaled out<sup>8</sup>). The finite structure constant is unity rather than  $1/137$ , the GL parameter is  $\kappa_{s(\parallel, \perp)}^2 = \alpha_0 K_{\parallel, \perp}^0(T, H)/\gamma_{\parallel, \perp}^2$ , and the gauge is fixed in an unusual way.<sup>9</sup> This  $s$  vortex gauge has physical origin in the underlying connection between  $[\mathbf{s}, s_\xi]$  and  $[\delta\rho, \mathbf{j}]$ . It is, however, awkward to work with since the long-wavelength nonsingular phase fluctuations of  $\Phi$ , with  $|\Phi| = \Phi_0$  fixed, still couple to  $\mathcal{S}$ . The presence of this coupling allows only power-law correlations in  $\langle \Phi(\mathbf{r}, \xi)\Phi^*(0, 0) \rangle$  at low temperatures. It is expedient to introduce new variables:  $\Phi \rightarrow \bar{\Phi} = \Phi \exp[i\Pi(\mathbf{r}, \xi)]$ ,  $\mathcal{S} \rightarrow \mathbf{S} = \mathcal{S} - \nabla\Pi$ , where  $\Pi = \nabla^{-2} \partial_\xi s_\xi$ . This is a simple gauge transformation as far as  $\mathcal{F}_{\text{GL}}$  is concerned. This allows us to extract the leading two-point correlations in (7), which are  $\langle \bar{\Phi}(\mathbf{r}, \xi)\bar{\Phi}^*(0, 0) \rangle = \langle \Phi(\mathbf{r}, \xi) \times \exp[i\Pi(\mathbf{r}, \xi)]/\Phi^*(0, 0)\exp[-i\Pi(0, 0)] \rangle$ , and not  $\langle \Phi(\mathbf{r}, \xi)\Phi^*(0, 0) \rangle$ . These  $\bar{\Phi}$  correlations involve combined phase fluctuations in  $\Phi$  and the  $s$  vortex transverse current fluctuations which enter through  $\Pi$  via  $s_\xi$ ,  $\bar{\Phi}$  order is expected to be long ranged below  $T_\Phi(H)$ .

After this gauge transformation,  $\mathcal{F}_{\text{GL}}$  in (7) becomes

longer than inter- $s$  vortex separation ( $\sim \sqrt{2\pi}l$ ). Let us define a cutoff,  $\Lambda(T, H) \gtrsim \sqrt{2\pi}l$ . The fluctuations in  $\Phi$  at wavelengths shorter than  $\Lambda$  are integrated out. Finally, the functional integral over  $[\delta\rho, \mathbf{j}]$  is replaced by the one over  $\mathcal{S} \equiv [\mathbf{s}, s_\xi]$ . The result is  $Z \rightarrow \int \mathcal{D}[\Phi]\mathcal{D}[\mathcal{S}]\exp\{-\mathcal{F}_{\text{GL}}/T + \mathcal{W}\}$ ,

equivalent to SHE in the Coulomb gauge,  $\nabla \cdot \mathbf{S} = 0$  (apart from the anisotropy).  $\mathcal{W}[\mathcal{S}]$  in contrast, does not possess this fictitious gauge invariance of  $\mathcal{F}_{\text{GL}}$ . Its form reflects only the spatial symmetries of the original problem (1). This form changes when  $\mathcal{S} \rightarrow \mathbf{S}$ . A similar situation arises in studies of the nematic-smectic- $A$  transition in liquid crystals.<sup>10</sup> There one also has the part which is equivalent to SHE and additional terms which reflect spatial symmetries of that problem. Here I adopt the renormalization-group (RG) analysis of the nematic-smectic transition by Halperin, Lubensky, and collaborators.<sup>11</sup> An important result is that the pure isotropic SHE is one of the fixed points of (7). At this fixed point  $K_\parallel, K_\perp \rightarrow K^*$  and  $\mathcal{W}/K^* \rightarrow 0$ . Furthermore, this fixed point is stable to finite  $\mathcal{W}$  perturbations to all orders in the  $\epsilon$  expansion. Consequently, if the bare  $K_\parallel^0, K_\perp^0$  are large enough, the RG flows should be attracted to the SHE fixed point. It is difficult, however, to evaluate  $K_\parallel^0(T, H)$  and  $K_\perp^0(T, H)$  from first principles with a precision greater than what is given below Eq. (7). At this point further approximations become necessary. An estimate based on the self-consistent integration of fluctuations in  $\Phi$  at wavelengths short compared to  $\Lambda$  suggests  $K_\parallel^0, K_\perp^0$  that are growing exponentially for  $H \rightarrow 0$ , at  $T < T_{c0}$ , but this is probably too crude. A more efficient alternative approach might be to treat  $K_\parallel^0, K_\perp^0$  given below Eq. (7) as a phenomenological input to the theory and proceed to compute various consequences of  $\mathcal{F}_{\text{GL}}$ . The comparison to experiments and numerical simulations can then be used to establish more precise values of  $K_\parallel^0, K_\perp^0$ . At any rate, for low enough fields,  $K_\parallel^0, K_\perp^0$  become large in the critical region and the plausible scenario within the  $\epsilon$  expansion is that the fluctuation behavior would show crossover from mean-field to anisotropic SHE to isotropic SHE and ultimately to a very weak first-order transition at renormalized  $H_\Phi(T)$ .<sup>12</sup> This SHE scenario is valid if the mass terms for  $\mathcal{S}$  are either absent or small at the bare level, i.e., if  $K_{\parallel, \perp}^0/m_s^2 \gg \Lambda^2$ , where  $m_s$  is the ‘‘bare’’ mass of  $\mathcal{S}$ .<sup>13</sup>

The SHE phase transition has been studied also by Dasgupta and Halperin<sup>14</sup> using the lattice superconductor model (LSM). They have concluded that, within LSM, the transition appears continuous and is in the inverted  $XY$  universality class, defined by the interacting vortex loops. In our model this implies a proliferation of unbound large vortex loops in  $\bar{\Phi}(\mathbf{r}, \xi)$  at  $H_\Phi(T)$ . They also find that the transition moves to lower temperature

as the stiffness of the gauge field is reduced and completely disappears below certain critical value of the stiffness. This should be the case here for higher fields, where  $K_{\parallel}^0, K_{\perp}^0$  are limited by core effects and may become large only at comparatively low temperatures. Ultimately, it is expected that  $T_{\Phi}(H) \rightarrow 0$  as  $H$  increases toward  $H_b$ . At these higher fields, where  $K_{\parallel}^0, K_{\perp}^0$  are getting smaller, the  $s$  gauge field fluctuations are enhanced and transition may become discontinuous as observed in numerical simulations.<sup>15</sup> This transition to the high-field limit, represented by the dotted line in Fig. 1, is beyond the semiclassical approximation of this paper. An interesting question in this context is the interference between the  $\Phi$  transition and the London ( $s$ ) vortex liquid-solid (LVL-LVS) transition (the crossing between the dotted  $H_{\Phi}(T)$  line and the solid LVL-LVS line in Fig. 1) which is left for future study.

The above connection to the critical behavior of SHE is theoretically appealing and deserves additional discussion (it is now assumed that the bare mass  $m_s$  is negligible). The problem here is that the critical behavior of SHE itself is still not fully understood, as illustrated above. The  $\epsilon$  expansion predicts the first-order transition while various numerical studies indicate a continuous transition, at least for strongly type-II systems. Furthermore, the  $1/N$  expansion also favors continuous transition.<sup>16</sup> Interestingly, the problem of the finite-field (FF) critical behavior in type-II superconductors provides additional motivation for the study of SHE since the fictitious  $s$  gauge Higgs electrodynamics introduced here has a much larger intrinsic fine-structure constant than the real SHE (unity versus  $1/137$ ). Consequently, the domain of FF critical fluctuations could be considerably wider than that of the real SHE at the zero-field (ZF) normal-superconducting transition. While the above uncertainties concerning SHE remain to be resolved, it is still possible to exploit this connection to make some general remarks on type-II superconductors in a low magnetic field, assuming their fluctuation behavior is faithfully represented by the 3D  $XY$  model. The low-temperature, Meissner phase of SHE is related to the London ( $s$ ) vortex liquid (LVL) state of a type-II superconductor (see Fig. 1). This is an incompressible  $\bar{\Phi}$  ordered liquid phase with long-range interactions between ( $s$ ) vortices and with the overall vorticity locked at  $N_{\Phi}$ . Large thermally excited vortex loops are suppressed. The strength of this long-range London interaction is given by the “photon” mass of the fictitious  $s$  gauge electrodynamics which is directly related to the “helicity modulus” of the condensed  $\bar{\Phi}$  field. Thus, as we approach  $T_{\Phi}(H)$  from below, the long-range component of the ( $s$ ) vortex interaction is renormalized by fluctuations in the same fashion as the London magnetic penetration depth in the real SHE. Above  $T_{\Phi}(H)$  large thermally excited vortex loops proliferate across the sample and the system behaves like a compressible ( $s$ ) vortex “gas,” with the strongly fluctuating overall vorticity. The fictitious  $s$  gauge photon is now massless, the  $\bar{\Phi}$  order is absent and ( $s$ ) vortex density-density interactions and  $\Phi$  correlations are both short ranged. This phase can be identified with the normal state.

I now state potential sources of concern with the above description. The  $\mathcal{W}[\mathcal{S}]$  term could produce subtle non-perturbative effects on the critical behavior. Furthermore, the core effects, which have been ignored on the basis of  $a/l \ll 1$ , might have a nontrivial effect at long wavelengths which could modify critical behavior and suppress  $\bar{\Phi}$  order, even at low fields. Still, I expect the description proposed here to retain its usefulness at low fields near  $T_{c0}$  and at the length scales typically encountered in experiments. This is so because the present theory does perform at least one important task: it extracts from the original GL functional those off-diagonal correlations whose range extends well beyond the magnetic length,  $l$ . This is illustrated in the inset of Fig. 1. Above the dashed-dotted line the  $\bar{\Phi}$  correlator has basically the same range as the standard superconducting  $\Psi$  correlator. In this region, labeled as  $XY$ , we can exploit the proximity of the zero-field (ZF) critical point to describe the physics and construct various thermodynamic quantities. Below the dashed-dotted line, in the region labeled as  $\Phi$ , the range of the superconducting  $\Psi$  correlator saturates at  $l$ , while  $\bar{\Phi}$  correlations continue to grow and ultimately diverge, at least within the present description based on the effective functional  $\mathcal{F}_{GL}$  (7). To be sure,  $\mathcal{F}_{GL}$  itself results from a particular approximate way of taking the continuum limit and thus one must allow for the possibility that additional relevant terms, not included in the present description, might modify the ultimate long-wavelength behavior of the problem. Such subtle issues notwithstanding, the clear message of the present approach, and the one likely to remain in place, is that the fluctuation behavior in the  $\Phi$  region (Fig. 1) must be governed by the proximity to some new finite-field (FF) critical point and not to the zero-field transition. It appears likely that  $\mathcal{F}_{GL}$  captures at least basic features of the physics associated with this new critical behavior.

There is empirical support for the picture presented in this paper.  $H_{c2}(T)$  determined from magnetization measurements of Ref. 17 shows expected linear behavior at high fields but deviates from linearity at low fields. This deviation is consistent with  $T_{\Phi}(H)$  of Eq. (6) if we relate the crossover in magnetization to the point where  $\langle |\Phi|^2 \rangle$  starts growing. A detailed analysis of the specific-heat data in 1-2-3 HTS (Ref. 18) in terms of the  $XY$ -model critical scaling leads to poor agreement unless one allows for the (unexplained) strong field dependence of the coefficients.<sup>19</sup> This strong field dependence can be interpreted as the crossover from the  $XY$  to the  $\Phi$ -critical behavior described by (7). This crossover takes place as one moves from the high-temperature regime,  $\xi_{\Phi} < l$ , to the true critical regime,  $\xi_{\Phi} \gg l$ . Finally, Li and Teitel<sup>20</sup> have reported Monte Carlo simulations of the  $XY$  model which show a suppression of large vortex loops followed by a sharp onset of the helicity modulus for field-induced vortices. This effect should arise as a consequence of the  $\bar{\Phi}$  ordering, with the disappearance of unbound vortex loops leading to a sharp increase in the line tension of field-induced vortices. It is clear, however, that additional experimental and computational effort will be needed before a complete picture of the low-field critical behavior is in place. It is hoped that the present work

will stimulate such developments.

In summary, the main advances reported in this paper can be viewed as twofold: First, on a conceptual front, a description is derived for the low-magnetic field critical behavior of the GL theory. In contrast to the GL theory in high fields, where the only fluctuating degrees of freedom are positions of field-induced vortices, the low-field critical behavior is dominated by thermally induced zero-vorticity excitations, like vortex-antivortex pairs, vortex loops, etc. By shifting the vorticity in the original GL partition function and taking the hydrodynamic limit, the low-field critical behavior is related to the field theory describing a complex scalar field in a zero average magnetic field coupled to a fictitious gauge field produced by the long-wavelength fluctuations in the background system of field-induced vortices. The conceptual advance here is that this derivation uncovers the hidden off-diagonal correlations of the GL theory whose range is far longer than that of the original superconducting correlator. Second, when it comes to the theory of real type-II superconductivity, the main utility of this novel formulation is in the fact that it describes the finite-field (FF) crit-

ical behavior in contrast to the familiar zero-field (ZF) critical point at  $T = T_{c0}$ . The strong fluctuation regime of type-II superconductors for  $T < T_{c0}$  and low fields will be controlled by the proximity to such a FF transition rather than the ZF one, contrary to what is often assumed in the literature.<sup>21</sup> Even if the predicted FF transition itself turns out to be hard to access in a real experiment,<sup>12</sup> the description introduced here should still be valuable in providing a systematic approach to physical problems which up to now were beyond analytical reach: The construction of thermodynamic functions describing the ZF-FF crossover, the renormalization of London vortex interactions by critical fluctuations, the issue of the high-field versus the low-field thermodynamic scaling, etc.

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<sup>12</sup>As shown in Ref. 11 the standard SHE also exhibits a first-order transition, both within the mean-field [compare with Eq. (6)] and the RG analysis. Note that, in our case, at very large distances and/or very low fields, the real electromagnetic field fluctuations may have to be included as well since  $\kappa^2$ , while large, is still finite. This will clearly modify the FF critical behavior and may lead to the  $\bar{\Phi}$  transition becoming only a sharp crossover or a weak first-order transition involving no symmetry breaking. The effect of these real electromagnetic field fluctuations, while clearly of theoretical interest, is also of practical significance in HTS and other extremely type-II systems. In these materials  $\kappa^2$  is as large as  $10^3 - 10^4$  and the magnetic-field penetration depth in the critical region will typically be longer than the length scale over which the system can be considered homogeneous. This is reminiscent to the issue of smearing of the Kosterlitz-Thouless-Berezinskii (KTB) transition in superconducting films by the electromagnetic screening. In practice, the effective magnetic-field penetration depth is often larger than the sample size and one can observe the original  $\kappa \rightarrow \infty$  KTB fluctuation behavior.

<sup>13</sup>Since our theory does not have the local gauge symmetry of the SHE, such terms are generally permitted. If  $K_{||,1}^0/m_s^2 \sim \Lambda^2$ ,  $s$  gauge-field fluctuations will be reduced in the critical region. For discussion of related issues in the pure SHE, see M. Kiometzis, H. Kleinert, and A. M. J. Schakel, *Phys. Rev. Lett.* **73**, 1975 (1994).

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- <sup>20</sup>Y. H. Li and S. Teitel, *Phys. Rev. B* **49**, 4136 (1994); S. Teitel (private communication). More precisely, these authors observe that, at some well-defined temperature  $T_{cz}$ , an interconnected tangle of wandering field-induced vortices and thermally induced vortex rings percolates through the system in the plane perpendicular to the applied field. At the same temperature, the helicity modulus along the field direction, which was found to be finite below  $T_{cz}$ , goes continuously to zero. This is just what one expects at the  $\bar{\Phi}$  transition of our model: As one approaches  $T_{cz}$  from below, large thermally induced vortex loops in  $\bar{\Phi}$  proliferate across the system. Such vortex excitations are confined within a flux tube of the fictitious gauge field. This flux tube represents the screening cloud of field- and thermally induced vorticity and corresponds to the above “interconnected tangle” observed in the Monte Carlo simulations of Li and Teitel. The cross-

sectional area of such a tangle is given by the product of pertinent penetration depths of our fictitious electrodynamics. The helicity modulus along the field directly measures the formation of a *uniform*  $\bar{\Phi}$  condensate. Thus, their  $T_{cz}$  can be identified with  $T_{\Phi}(H)$ . The resistivity anomalies along the field observed by H. Safar *et al.*, *Phys. Rev. Lett.* **72**, 1272 (1994), are presumably also related to  $T_{\Phi}(H)$ . In this context, it should be noted that the label London vortex liquid (LVL) below  $T_{\Phi}(H)$  (Fig. 1) indicates a translationally invariant phase in which the long-wavelength density-density interaction of the fluctuating vorticity has a characteristic London form, the strength of which is given by the superfluid density of the  $\bar{\Phi}$  condensate. This phase has strong thermal fluctuations throughout the critical region which makes it impossible to locally separate field-induced from thermally induced vortex excitations. Thus, the LVL phase (Fig. 1) should be clearly distinguished from the often used London line liquid description (Ref. 1) in which only field-induced vortices are considered and all other fluctuations are ignored. Such description is applicable only at temperatures well below  $T_{\Phi}(H)$ .

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