Anderson-Yuval approach to the multichannel Kondo problem

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We analyze the structure of the perturbation expansion of the general multichannel Kondo model with channel anisotropic exchange couplings and in the presence of an external magnetic field, generalizing to this case the Anderson-Yuval technique. For two channels, we are able to map the Kondo model onto a generalized resonant-level model. Limiting cases in which the equivalent resonant-level model is solvable are identified. The solution correctly captures the properties of the two-channel Kondo model, and also allows an analytic description of the crossover from the non-Fermi-liquid to the Fermi-liquid behavior caused by the channel anisotropy.

I. INTRODUCTION

The single-channel Kondo model has a long history as the simplest model believed to contain the relevant physics of magnetic impurities embedded in metals. A lot of effort has been devoted to study this model and presently one can safely claim that it has been completely understood from the theoretical point of view. Apart from the original perturbative scaling approach, which already gave the correct qualitative description,[1,2] there is also an exact solution of this model obtained by the Bethe-ansatz technique.[3] The physics underlying the single-channel Kondo model is the formation of a nondegenerate singlet state at low temperature. The impurity spin is screened by the conduction electrons; hence the magnetic susceptibility, obeying the Curie-Weiss law at high temperatures, undergoes a Kondo crossover, saturating to a constant upon lowering the temperature.

In an attempt to describe realistic magnetic impurities (which have orbital structure), different generalizations of the simple Kondo model have been proposed.[4] Among these generalizations, the simplest one is probably the model describing an impurity spin S coupled to N channels of conduction electrons (commonly referred to as the multichannel Kondo model). Surprisingly, it has been realized that, for $N > 2S$, this model exhibits a behavior qualitatively different from the single-channel one,[5] which is due to a nontrivial ground state with a residual degeneracy. This gives rise to divergent low-temperature susceptibility $\chi_{\rm imp}$ and specific heat coefficient $\gamma = C_{V,\rm imp}/T$ (so-called non-Fermi-liquid behavior). It should be said that experiments on dilute magnetic alloys (the systems for which the Kondo model was originally proposed) do not give any clear evidence for such a behavior but rather suggest that the ground state is always a singlet. A possible explanation would be that the channel symmetry (which is necessary for the non-Fermi-liquid behavior to occur) is broken (since typically no exact symmetry guarantees channel equivalence). If the energy scale of this symmetry breaking term is small, a slow crossover from non-Fermi-liquid to Fermi-liquid behavior is expected. However, it might be difficult to experimentally distinguish it from the usual Kondo crossover. Consequently, since early 1980s, interest in the issue declined, even though, in the meanwhile, some exact solutions for the case of equivalent channels became available.

In recent years, however, interest in the subject grew again, as other, more promising, realizations of the multichannel Kondo model have been proposed, for instance, two-level systems in metals,[6] heavy fermion compounds,[7] and high-T_c superconductors.[8] While for the two-level systems there is convincing experimental evidence[9] of the non-Fermi-liquid behavior predicted by the theory, the other proposed realizations have been until now controversial.

The general multichannel Kondo Hamiltonian is given by

$$H_K = \sum_{a=1}^{N} \sum_{\sigma} H_0(\psi_{a\sigma}, \psi^{\dagger}_{a\sigma}) + \sum_{a=1}^{N} \left\{ J_{za} S^z \sigma^z_a(\mathbf{0}) + \frac{J_{\perp a}}{2} \left[S^+ \sigma^-_a(\mathbf{0}) + S^- \sigma^+_a(\mathbf{0}) \right] \right\}, \tag{1}$$

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where

$$H_0(\psi,\psi^\dagger)=\sum_{\mathbf{k}}\epsilon_{\mathbf{k}}\psi^\dagger_{\mathbf{k}}\psi_{\mathbf{k}} \tag{2}$$

is the kinetic energy of the conduction electrons $\psi_{a\sigma}$, $a=1,...,N$ being the channel index and $\sigma=\uparrow,\downarrow$ being the spin index (we assume a spin-1/2 impurity). Notice, however, that in some realizations of this model, the channel index is the physical spin while the spin index labels an orbital quantum number (see also Sec. IV). The electron spin densities in Eq. (1) are defined by

$$\begin{aligned}\sigma^z_a(\mathbf{x}) &= \tfrac{1}{2}\left[\psi^\dagger_{a\uparrow}(\mathbf{x})\psi_{a\uparrow}(\mathbf{x})-\psi^\dagger_{a\downarrow}(\mathbf{x})\psi_{a\downarrow}(\mathbf{x})\right],\\ \sigma^+_a(\mathbf{x}) &= \psi^\dagger_{a\uparrow}(\mathbf{x})\psi_{a\downarrow}(\mathbf{x}).\end{aligned} \tag{3}$$

In the single-channel case ($N=1$) the exchange couplings J flow to infinity under scaling, if positive. The spin anisotropy $J_z\neq J_\perp$ disappears at the fixed point. This is interpreted as the formation of a singlet at the impurity site. On the contrary for the $N>1$ *channel symmetric* case, the infinite coupling fixed point is unstable as well as the weak coupling one, hence a stable intermediate coupling fixed point.

Until recently, information on the behavior of the model around this fixed point could only be extracted from the Bethe-ansatz solution[10] and conformal field theory.[11] In 1992, Emery and Kivelson[12] provided a simple solution for the symmetric two-channel case at a particular value of the longitudinal exchange coupling. The solution was obtained by means of the Abelian bosonization technique, generalizing a procedure previously applied to the single-channel Kondo model by Schlottmann.[13] The original Schlottmann approach was in turn inspired by Toulouse's mapping of the single-channel Kondo model onto a resonant-level model, which he achieved by the analysis of the partition function,[14] borrowing the perturbative treatment of Yuval and Anderson.[15] The two methods, bosonization and Yuval-Anderson method, are equivalent, in the sense that they give qualitatively similar results (it has been checked in the single-channel case). Still, the latter is formally more rigorous (e.g., it does not fully rely on band linearization) and straightforward (it simply amounts to comparing the perturbation expansions of two different models).

In this paper we generalize the Yuval-Anderson method to the multichannel Kondo model with channel anisotropic exchange couplings. For the two-channel case, we are able to show that the perturbation expansion of the Kondo model is equivalent to that of a generalized resonant-level model. In the channel isotropic case, the model is of the type found by Emery and Kivelson via the bosonization technique. The novel feature is the channel anisotropy which gives rise to interesting crossover phenomena. We demonstrate that even in this case a mapping of the Kondo model (for particular values of the longitudinal exchange couplings) onto a solvable resonant-level model does exist. For $N>2$ we are unable to find any kind of resonant-level model which would reproduce the perturbation expansion.

II. GENERALIZATION OF THE ANDERSON YUVAL APPROACH

In this section, we analyze the structure of the perturbation expansion of the Hamiltonian (1) in the transverse exchange couplings.

A. Single-channel model

Consider first the single-channel problem. We allow for an exchange anisotropy ($J_x=J_y=J_\perp$ is different from J_z). We want to calculate the impurity partition function in *time* space, using a perturbation expansion in powers of $J_\perp$. A term of order $2n$ involves $2n$ alternate impurity spin flips. Let t_i be the times of up flips, t'_i those of down flips (n of each). The philosophy is to calculate that particular term exactly, for a given history $\{t_i,t'_i\}$, and to show that it is identical to the corresponding expansion for another problem (a resonant level), with appropriately chosen parameters. The two problems are mapped on each other term by term: They are equivalent. Note that we thus bypass summation of the perturbation series in $J_\perp$. For a specific value of J_z (the "Toulouse limit") the equivalent problem happens to be trivially solvable: If we can scale through that value, we have an explicit description of the crossover to low temperature: the $(100-\epsilon)\%$ exact solution of Anderson. The error stems from the fact that universal scaling is not just a change of J_z. That error is supposed not to change the qualitative behavior, and anyway it is implicit in the equivalent bosonization technique (less powerful since it relies on a Born approximation for phase shifts).

Assume first that $J_z=0$. Each vertex flips a conduction electron spin. A t_i vertex creates a $\downarrow$ electron and an $\uparrow$ hole; a t'_i vertex does the reverse. In a typical diagram, the electron propagators go from any t_i to any t'_i for $\downarrow$ spins, and from any t'_i to any t_i for $\uparrow$ spins. Since $J_z=0$, these propagators are *free electron local* propagators

$$G_0(t)=\frac{\nu_0}{it+\xi_0^{-1}\mathrm{sgn}t},$$

where ν_0 is the density of states for one spin at the Fermi level and ξ_0 is a high-energy cutoff of the order of the conduction bandwidth. Let $D_\sigma(t_i,t'_i)$ be the contribution of n lines with spin σ that join t_i and t'_i vertices. D_σ has a pole whenever $t_i=t'_j$ (one propagator is divergent). Moreover *crossing symmetry* implies a zero whenever $t_i=t_j$ (or $t'_i=t'_j$): Exchanging the extremities of two propagators changes the sign. Hence the Cauchy determinant found by Yuval and Anderson (here and in what follows we omit time-independent prefactors containing the density of states ν_0 and to the short-time cutoff, restoring them in the final expressions),

$$D_\sigma=(-i)^n\,\frac{\prod_{i>j}(t_i-t_j)\,(t'_j-t'_i)}{\prod_{i,j}(t_i-t'_j)}\,. \tag{4}$$

Expression (4) is homogeneous with degree $(-n)$, as expected for D_σ. The proof is completed by looking at the

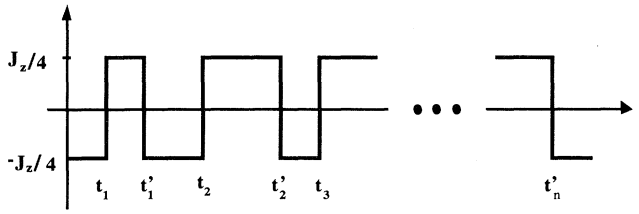

FIG. 1. Time-dependent potential seen by a spin-up conduction electron. The impurity spin flips from $\downarrow$ to $\uparrow$ at times t_i and vice versa at times t'_j.

asymptotic behavior. The corresponding contribution to the impurity propagator is $U_0 = D_\uparrow D_\downarrow$.

We now restore J_z. The potential felt by a spin σ electron changes at each impurity flip — hence an edge singularity that will modify the long-time behavior of U; see Ref. 16. In Fig. 1 we have drawn the time-dependent potential felt, e.g., by the up-spin conduction electrons at the impurity site (the down-spin one is the opposite) The phase shift is δ_+ (δ_-) when the electron and impurity have parallel (antiparallel) spins. What matters is the *discontinuity* of phase shifts when a flip occurs, $\delta = \delta_+ - \delta_-$. If we assume electron hole symmetry, then

$$\delta_+ = -\delta_- = \tan^{-1}(\pi\nu_0 J_z/4). \tag{5}$$

The effect of the flip is twofold.

(i) The *open lines* that contribute to D_σ can scatter any number of times on the impurity. That generates once again the Cauchy determinant (via a Muskhelishvili type of analysis[17]). For each spin D_σ is replaced by $D^{1-2\delta/\pi}$; thus the open line contribution acquires an extra factor $U_L = D^{-4\delta/\pi}$.

(ii) In addition the underlying Fermi sea reacts to the flip via *closed loops* that exponentiate (for a given history the potential is structureless). The resulting contribution is $U_C = D^{2\delta^2/\pi^2}$.

Altogether $U = U_0 U_L U_C$, and, after inserting back the prefactors, we obtain

$$U = \left(\frac{J_\perp \nu_0 \xi_0 \cos^2(\delta/2)}{2}\right)^{2n} \left(\frac{D}{\xi_0^n}\right)^{\eta}, \tag{6}$$

where η is an exponent that depends on J_z,

$$\eta = 2\left(1 - \frac{\delta}{\pi}\right)^2.$$

(Note that $\eta = 0$ if $\delta = \pi$, which corresponds to the strong coupling limit $\delta_+ = -\delta_- = \pi/2$.)

Let us now consider a resonant-level model for spinless electrons, characterized by the Hamiltonian

$$H = H_0(\Psi, \Psi^\dagger) + \lambda\left[\Psi^\dagger(\mathbf{0})d + d^\dagger\Psi(\mathbf{0})\right] + \frac{V}{2}\left[\Psi^\dagger(\mathbf{0})\Psi(\mathbf{0}) - \Psi(\mathbf{0})\Psi^\dagger(\mathbf{0})\right]\left(d^\dagger d - \frac{1}{2}\right), \tag{7}$$

where d is an impurity orbital at the Fermi energy located at the origin. Ψ is a free Fermi field, the kinetic energy of which is the same as in Eq. (1). The interaction potential V produces a phase shift discontinuity

$$\delta' = 2\tan^{-1}\left(\frac{\pi\nu_0 V}{2}\right) \tag{8}$$

between the empty and full d states. We expand in powers of λ which plays the role of $J_\perp$. The t_i and t'_i vertices correspond to $d^\dagger$ and d operators. The structure of the expansion is the same as for the Kondo case, except that there is no spin degeneracy. We have one set of open lines originating from λ vertices, which can scatter off the flipping δ'. We also have one set of closed loops — hence altogether

$$U' = \left[\lambda\cos(\delta'/2)\sqrt{\nu_0\xi_0}\right]^{2n}\left(\frac{D}{\xi_0^n}\right)^{\eta'}, \tag{9}$$

with

$$\eta' = \left(1 - \frac{\delta'}{\pi}\right)^2.$$

Thus the Kondo problem with coupling δ is mapped term by term onto the resonant level with coupling δ' if the two propagators (6) and (9) are identical. This implies $\eta = \eta'$ (that can always be achieved by appropriately choosing V) and

$$\lambda = \frac{J_\perp}{2}\sqrt{\nu_0\xi_0}\frac{\cos^2(\delta/2)}{\cos(\delta'/2)}.$$

The Toulouse limit corresponds to $\delta' = 0$, i.e., a phase shift $\delta = \pi(1 - 1/\sqrt{2})$. The resonant-level Hamiltonian can then be trivially diagonalized, yielding a low-temperature Fermi-liquid behavior. The resonant-level Hamiltonian of the form (7) has been previously derived by Wiegmann and Finkelstien.[18]

B. Multichannel model

We now turn to the N-channel case. In a first stage we assume flavor degeneracy: What does remain of the previous analysis? In order to answer that question we proceed in reverse.

(i) The alternation of up and down spins is unchanged. The flipping potential due to δ is the same as before (see Fig. 1), whatever the flavor involved at each vertex. Edge singularities are consequently unaffected. The scattering contribution to open lines is again $U_L = D^{-4\delta/\pi}$ (flavor is fixed by extremities). Each closed loop can have an arbitrary flavor and therefore $U_C = D^{2N\delta^2/\pi^2}$.

(ii) Paradoxically the difficulties come from the part U_0 (in the absence of J_z). U_0 still has poles whenever $t_i = t'_j$, but one *loses crossing symmetry.* If the ends of two lines are interchanged, one usually changes the number of closed loops C — hence a change in the degeneracy N^C. As a result U_0 cannot be expressed simply in terms of D.

In order to proceed, we must assign to each vertex its

flavor index a. The t_i and t'_j then break into N subclasses t_{ia} and t'_{ja} ($a = 1, ..., N$). For a given diagram the number of vertices in different subclasses need not be the same, but spin and flavor conservation implies an equal number of t_{ia} and t'_{ja} within a given subclass. Since flavor is conserved along an open line, U_0 is a product of independent factors U_{0a}. For each factor crossing symmetry holds and U_{0a} is the square of a Cauchy determinant D_a as it would be for a single channel. D_a is still given by (4), the products running only over a-type times t_{ia} and t'_{ja}. In the end we find [again omitting prefactors proportional to $J_\perp$, ν_0, ξ_0, and $\cos(\delta/2)$]

$$U = (D_1 \cdots D_N)^2 D^{-4\frac{\delta}{\pi}+2N\left[\frac{\delta}{\pi}\right]^2} = \frac{(D_1 \cdots D_N)^2}{D} D^{\beta_N}, \tag{10}$$

where

$$\beta_N = 1 - 4\frac{\delta}{\pi} + 2N\left(\frac{\delta}{\pi}\right)^2. \tag{11}$$

Note that D is *not* the product of individual D_a: We have instead $D = D_1 \cdots D_N F$, in which we have set

$$F = \frac{\prod_{ij}\prod_{a<b} |t_{ia} - t_{jb}|\,|t'_{ia} - t'_{jb}|}{\prod_{ij}\prod_{a<b} |t_{ia} - t'_{jb}|\,|t_{ia} - t'_{jb}|}.$$

[Notice that, having defined D_i as in Eq. (4), $F = D/(D_1 \cdots D_N)$ is by construction positive which justifies the use of the moduli.] The factor F couples the channels. For the case of two-level systems, an expression similar to our Eq. (10) has been derived in Ref. 6.

The Emery-Kivelson solution to the two-channel case, $N = 2$, is based on a mapping of the Kondo problem onto the spinless resonant level Hamiltonian

$$\begin{aligned} H = {} & H_0(\Psi, \Psi^\dagger) + H_0(\Psi_s, \Psi_s^\dagger) \\ & + \lambda\left(d^\dagger - d\right)\left[\Psi^\dagger(\mathbf{0}) + \Psi(\mathbf{0})\right] \\ & + \frac{V}{2}\left[\Psi_s^\dagger(\mathbf{0})\Psi_s(\mathbf{0}) - \Psi_s(\mathbf{0})\Psi_s^\dagger(\mathbf{0})\right]\left(d^\dagger d - \frac{1}{2}\right), \end{aligned} \tag{12}$$

where d is again a fictitious spinless Fermi operator. Notice that we have introduced two Fermi fields Ψ and Ψ_s, coupled to the impurity in a different way (the reason why we have not used the same field will become clear later). In order to establish the equivalence we first consider the case $V = 0$. We divide the d and $d^\dagger$ flips into two subclasses, depending on whether a fermion Ψ is emitted or absorbed. Times t_{i1} and t_{i2} correspond to $d^\dagger$ flips with a fermion emitted or absorbed, respectively; t'_{i1} and t'_{i2} are their Hermitian conjugates. A spinless fermion propagator can go as usual from t_{i1} to t'_{j1} (t'_{i2} to t_{j2}) *or* from t_{i1} to t_{j2} (t'_{i2} to t'_{j1}). The latter possibility is the new feature. The corresponding impurity propagator U' has poles whenever a propagator has a zero time range, i.e., when $t_{ia} = t'_{ja}$, $t_{i1} = t_{j2}$, or $t'_{i2} = t'_{j1}$. Due to crossing symmetry it has zeros when $t_{ia} = t_{ja}$, $t'_{ia} = t'_{ja}$, $t_{i1} = t'_{j2}$, or $t'_{i1} = t_{j2}$. Once again one thereby builds a Cauchy determinant which happens to be

$$U' = \frac{D_1 D_2}{F}. \tag{13}$$

Expression (13) has the right poles and zeros. It moreover has the right overall power of t and asymptotic behavior: It is the correct answer. Comparing (13) with the definition of F we see that

$$U' = \frac{(D_1 D_2)^2}{D}.$$

We now restore the flipping potential V. Since it involves a different Fermi field, it gives rise only to a closed loop contribution. Altogether we have

$$U' = \frac{(D_1 D_2)^2}{D} D^{\left(\frac{\delta'}{\pi}\right)^2}, \tag{14}$$

with the same δ' as in Eq. (8). Comparing (14) with (10) we see that the two problems are mapped onto each other if [see Eq. (11)]

$$\beta_2 = \left(1 - 2\frac{\delta}{\pi}\right)^2 = \left(\frac{\delta'}{\pi}\right)^2, \tag{15}$$

which is always possible since both right and left sides are positive. Notice that for a given δ the interaction potential V in Eq. (12) is, according to (8) and (15), given by

$$V = \frac{2}{\pi\nu_0}\tan\left(\frac{\delta'}{2}\right) = \frac{2}{\pi\nu_0}\tan\left(\frac{\pi}{2} - \delta\right). \tag{16}$$

The problem is directly solvable if $V = 0$, i.e., when $\delta = \pi/2$. In the electron-hole symmetric case that implies $\delta_+ = -\delta_- = \pi/4$, a typical intermediate coupling as expected for the two-channel overscreened Kondo impurity. Indeed from the expression of β_2 we see that the model is symmetric under $\delta \to \pi - \delta$. This extends the result of Ref. 5 that the two-channel Kondo model behaves similarly around $J_z = 0$ (i.e., $\delta = 0$) and $J_z = \infty$ (i.e., $\delta = \pi$). By symmetry the fixed point should be exactly at $\delta = \pi/2$, that is, at the solvable line $V = 0$.

It should be noted that for both single- and two-channel models the solvable limits are, strictly speaking, spin anisotropic, since the Yuval-Anderson approach is only valid for small $J_\perp$. However, spin anisotropy is known to be an irrelevant perturbation around both the strong coupling and the intermediate coupling fixed points. Therefore it does not influence the low-temperature behavior of the model.

The argument can be extended to a flavor-dependent exchange J. Due to anisotropy we must treat separately the channel dependence of $J_\perp$ and J_z. Different $J_{\perp 1}$ and $J_{\perp 2}$ do not affect the structure of the perturbation expansion. As we have shown above the mapping works as follows:

$$\begin{aligned}
\Psi^\dagger_{1\downarrow}(\mathbf{0},t_{i1})\Psi_{1\uparrow}(\mathbf{0},t_{i1})S^+(t_{i1}) &\longmapsto d^\dagger(t_{i1})\Psi(\mathbf{0},t_{i1}),\\
\Psi^\dagger_{2\downarrow}(\mathbf{0},t_{i2})\Psi_{2\uparrow}(\mathbf{0},t_{i2})S^+(t_{i2}) &\longmapsto d^\dagger(t_{i2})\Psi^\dagger(\mathbf{0},t_{i2}),\\
\Psi^\dagger_{1\uparrow}(\mathbf{0},t'_{i1})\Psi_{1\downarrow}(\mathbf{0},t'_{i1})S^-(t'_{i1}) &\longmapsto \Psi^\dagger(\mathbf{0},t'_{i1})d(t'_{i1}),\\
\Psi^\dagger_{2\uparrow}(\mathbf{0},t'_{i2})\Psi_{2\downarrow}(\mathbf{0},t'_{i2})S^-(t'_{i2}) &\longmapsto \Psi(\mathbf{0},t'_{i2})d(t'_{i2});
\end{aligned} \tag{17}$$

thus we need only modify accordingly the flipping matrix elements of the equivalent model, which becomes

$$\lambda_1\left[d^\dagger\Psi(\mathbf{0})+\Psi^\dagger(\mathbf{0})d\right]+\lambda_2\left[d^\dagger\Psi^\dagger(\mathbf{0})+\Psi(\mathbf{0})d\right], \tag{18}$$

where

$$\lambda_i=\frac{J_{\perp i}\cos^2(\delta/2)}{2\cos(\delta'/2)}\sqrt{\nu_0\xi_0}\,. \tag{19}$$

Notice that, if $J_{\perp 1}=J_{\perp 2}$, (18) reduces to (7) with $\lambda=J_{\perp 1}\sqrt{\nu_0\xi_0}\cos^2(\delta/2)/\cos(\delta'/2)$.

A difference between J_{z1} and J_{z2} gives rise to different phase shifts δ_1 and δ_2. Let us first consider the scattering correction to the 1 open line U_{L1}. The Muskhelishvili propagator for a channel-1 spin-up electron is

$$G_1(t,t')=-i\cos^2(\delta_1/2)\frac{\nu_0}{t-t'}\times\prod_{i,a}\left[\frac{(t-t_{ia})\,(t'-t'_{ia})}{(t-t'_{ia})\,(t'-t_{ia})}\right]^{-\frac{\delta_1}{\pi}}.$$

Its contribution to U_{L1} is obtained by putting t equal to any t_{i1}, t' to any t'_{i1}, hence a factor $\left[D_1^2F\right]^{-\delta_1/\pi}$, being $2n_1$ the number of flips which involve the channel 1. We square it in order to account for spin and we multiply by the corresponding term for channel 2. The closed line contribution is straightforward since flavor is conserved along a loop,

$$U_C=D^{2\left(\delta_1^2+\delta_2^2\right)/\pi^2}.$$

Altogether the impurity propagator is

$$U=D_1^{2-4\delta_1/\pi}D_2^{2-4\delta_2/\pi}F^{-2(\delta_1+\delta_2)/\pi}D^{2\left(\delta_1^2+\delta_2^2\right)/\pi^2}. \tag{20}$$

(Remember that $F=D/D_1D_2$.) If $\delta_1=\delta_2=\delta$, we recover the previous result (10).

In general let us write

$$\delta_1=\delta+\varepsilon\,,\quad \delta_2=\delta-\varepsilon\,.$$

Then (20) reduces to

$$U=\frac{(D_1D_2)^2}{D}D^{\beta_2}D^{4\left(\frac{\varepsilon}{\pi}\right)^2}\left(\frac{D_2}{D_1}\right)^{4\frac{\varepsilon}{\pi}}. \tag{21}$$

The additional factors with respect to (14) can be reproduced with an extra potential

$$\frac{W}{2}\left[\Psi^\dagger(\mathbf{0})\Psi(\mathbf{0})-\Psi(\mathbf{0})\Psi^\dagger(\mathbf{0})\right]\left(d^\dagger d-\frac{1}{2}\right).$$

With the choice

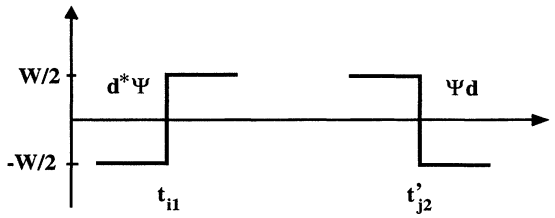

FIG. 2. Time-dependent local potential felt by an electron in the effective resonant-level model. While the same annihilation operator Ψ is involved in the two types of flips, the potentials are opposite. The flips at times t'_{i1} and t_{j2} are the Hermitian conjugates.

$$W=\frac{2}{\pi\nu_0}\tan\varepsilon=\frac{2}{\pi\nu_0}\tan\left(\frac{\delta_1-\delta_2}{2}\right), \tag{22}$$

the closed loop contribution generates the factor $D^{(2\varepsilon/\pi)^2}$. The last factor of (21) comes from the fact that the phase shift discontinuity is $+\varepsilon$ on the 1 vertices, $-\varepsilon$ on the 2 vertices (see Fig. 2). Notice also that, as a consequence of $\delta_1\neq\delta_2$, the correct definition of λ_i in Eq. (18) changes to

$$\lambda_i=\frac{J_{\perp i}\cos^2(\delta_i/2)}{2\cos(\delta'/2)}\sqrt{\nu_0\xi_0}\,, \tag{23}$$

so that, even though $J_{\perp 1}=J_{\perp 2}$, $\lambda_1\neq\lambda_2$. In that way one can map any version of the two-channel Kondo impurity onto an extended Emery-Kivelson Hamiltonian.

It is interesting to examine to what extent such an analysis could be pursued if $N>2$. We return to the flavor symmetric case, for which (7) holds. If we manage to have $\beta_N=0$, then

$$U=\frac{(D_1\cdots D_N)^2}{D}=\frac{D_1\cdots D_N}{F}. \tag{24}$$

Hence two questions: (i) Can we achieve $\beta=0$? (ii) If we can, is there a solvable model that gives the same U? It is clear that no real phase shift δ will achieve $\beta=0$ if $N>2$. That may be a definitive objection since poor man's scaling scans the real δ axis. Let us ignore it, hoping that some analytic continuation argument might help. Then in order to reproduce (24) we must introduce a coupling

$$S^+\left(\Psi_1+\cdots+\Psi_N\right)+\text{H.c.},$$

in which the Ψ_a operators are such that the corresponding propagators are

$$\begin{aligned}
\langle\Psi_a(t)\Psi^\dagger_b(t')\rangle &= g(t-t')\,(1-\delta_{ab})\,,\\
\langle\Psi_a(t)\Psi_b(t')\rangle &= g(t-t')\delta_{ab}
\end{aligned} \tag{25}$$

[$g(t)\approx 1/t$ is the free electron propagator]. Then U will have poles whenever $t_{ia}=t'_{ja}$ on the one hand, $t_{ia}=t_{jb}$, $t'_{ia}=t'_{jb}$ $(a\neq b)$ on the other. It will have zeros if $t_{ia}=t_{ja}$, $t'_{ia}=t'_{ja}$, or $t_{ia}=t'_{jb}$. That just generates

the combination (24). It remains to be seen what kind of algebra could produce (25): We do not know of any.

III. MAGNETIC FIELD EFFECTS

Let us consider the effects of a uniform magnetic field $\vec{B} = (0, 0, B)$ in the framework of the Yuval-Anderson approach. The magnetic field appears in the Hamiltonian with a term

$$H_B = -\mu_B B \left[g_i S^z + g_c \int d\mathbf{x} \sum_{a=1}^{N} \sigma_{az}(\mathbf{x}) \right], \qquad (26)$$

where the electron spin density is defined in Eq. (3), g_i and g_c are the Landé factors of the impurity and the conduction electrons, respectively, and μ_B is the Bohr magneton.

As before we will treat the transverse exchange perturbatively. This implies that the reference states $|\uparrow\rangle$ and $|\downarrow\rangle$, which are used for the perturbation expansion, are the eigenstates of the Hamiltonian with fixed impurity spin direction in the presence of the magnetic field

$$H_{\uparrow/\downarrow} = \sum_{a\sigma} H_0(\psi^\dagger_{a\sigma}, \psi_{a\sigma}) \pm \sum_{a=1}^{N} \frac{J_{az}}{2} \sigma_{az}(\mathbf{0}) \mp \frac{g_i \mu_B}{2} B - g_c \mu_B B \sum_{a=1}^{N} \int d\mathbf{x} \sigma_{az}(\mathbf{x}). \qquad (27)$$

We have to understand how the magnetic field modifies the perturbation expansion in $J_\perp$. B gives rise to two effects.

(i) It shifts the chemical potential for up- and down-spin electrons (in opposite directions). This causes a small change in the spin-up and -down phase shifts if the band has a finite curvature at the Fermi energy. This effect is negligible at low temperature.

(ii) It causes a difference $\Delta E = E_\uparrow - E_\downarrow$ in the ground state energies of (27) for the two impurity spin directions, which appears in the Muskhelishvili propagators.

By standard phase shift arguments, based on Friedel's sum rule for the displaced charge, we find

$$\Delta E = -\mu_B g_i B + \frac{1}{\pi} \sum_{a=1}^{N} \int_{\epsilon_F - \frac{g_c \mu_B}{2} B}^{\epsilon_F + \frac{g_c \mu_B}{2} B} d\epsilon \, \delta_a(\epsilon), \qquad (28)$$

where ϵ_F is the Fermi energy. For small magnetic field (28) reduces to

$$\Delta E = -\mu_B g_i B + \frac{g_c \mu_B}{\pi} B \sum_{a=1}^{N} \delta_a. \qquad (29)$$

The above energy difference enters in the impurity propagator (10) via the phase factor

$$\exp\left[-i \Delta E \sum_{i=1}^{n} (t_i - t_i') \right]. \qquad (30)$$

The conduction electron part of ΔE actually represents the leading term of closed loop diagrams, that one which grows linearly with $(t_i - t_i')$ instead of logarithmically.

Which term has to be added to the resonant-level model in order to reproduce (30)? It is easy to realize that the corresponding term is simply

$$\Delta E S_z \longmapsto \Delta E \left(d^\dagger d - \frac{1}{2} \right). \qquad (31)$$

Notice that at the Emery-Kivelson line for the two-channel case

$$\sum_{a=1,2} \delta_a = \pi,$$

so that if $g_i = g_c$, then $\Delta E = 0$ (at first order in B). Consequently the impurity magnetic susceptibility vanishes, in agreement with conformal field theory[11] and bosonization approaches.[19] This in turns means that at the Emery-Kivelson line the reference states are such as to perfectly screen the impurity spin. When the departure away from the Emery-Kivelson line is treated as a perturbation,[19] both the specific heat and susceptibility acquire logarithmic singularities, leading to the universal Wilson ration $R_W = 8/3$.

As to the $N > 2$ channel symmetric case, conformal field theory[11] and Abelian bosonization approaches (which until now exist only for $N = 4$; see Ref. 20) again predict the impurity susceptibility to vanish at the fixed point. From Eq. (29), we see that $\Delta E = 0$ for $\delta = \pi/N$, and therefore the impurity susceptibility is rigorously zero. If this is the true property of the fixed point, as follows from the analysis of Ref. 11, then $\delta = \pi/N$ is the fixed point. Such a conclusion would also agree with renormalization group (RG) arguments of Ref. 6.

IV. SOLUTION OF THE TWO-CHANNEL ANISOTROPIC MODEL

In this section we discuss the two-channel Kondo model in more detail, focusing on the effects of channel anisotropy. The starting Hamiltonian is

$$H_K = \sum_{a=1}^{2} \sum_{\sigma} H_0(\psi_{a\sigma}, \psi^\dagger_{a\sigma}) + \sum_{a=1}^{2} \left\{ J_{za} S^z \sigma^z_a(\mathbf{0}) + \frac{J_{\perp a}}{2} \left[S^+ \sigma^-_a(\mathbf{0}) + S^- \sigma^+_a(\mathbf{0}) \right] \right\}. \qquad (32)$$

If the exchange couplings are channel symmetric $J_{z1} = J_{z2}$ and $J_{\perp 1} = J_{\perp 2}$, it is known[5] that the Hamiltonian (32) flows towards a nontrivial fixed point. At this fixed point the model exhibits non-Fermi-liquid behavior; namely,

the impurity susceptibility $\chi_{\rm imp}$ and the specific heat over temperature, $C_{V,\rm imp}/T$, diverge at low temperatures as $\ln(1/T)$, and the zero-temperature entropy is finite and equal to $\ln(2)/2$, as if half of the impurity spin degrees of freedom were decoupled from the conduction electrons. Physically this occurs because two (and more) channels tend to overscreen the impurity spin, so that the complete screening characteristic of the single-channel Kondo model cannot take place, thus leaving a ground state degeneracy. In the case of a finite channel anisotropy, the system will always choose the channel with the strongest exchange to screen the impurity spin, and the usual Fermi-liquid behavior of the single-channel model will finally take place at zero temperature. The corresponding RG flow diagram is sketched in Fig. 3.

As we have shown in the previous sections, the Hamiltonian (32) can be mapped onto the resonant-level Hamiltonian

$$H_{\rm RL} = H_0(\Psi,\Psi^\dagger) + H_0(\Psi_s,\Psi_s^\dagger) + \lambda_1\left[d^\dagger\Psi(\mathbf{0}) + \Psi^\dagger(\mathbf{0})d\right] + \lambda_2\left[d^\dagger\Psi^\dagger(\mathbf{0}) + \Psi(\mathbf{0})d\right] + \frac{W}{2}\left[\Psi^\dagger(\mathbf{0})\Psi(\mathbf{0}) - \Psi(\mathbf{0})\Psi^\dagger(\mathbf{0})\right]\left(d^\dagger d - \frac{1}{2}\right) + \frac{V}{2}\left[\Psi_s^\dagger(\mathbf{0})\Psi_s(\mathbf{0}) - \Psi_s(\mathbf{0})\Psi_s^\dagger(\mathbf{0})\right]\left(d^\dagger d - \frac{1}{2}\right) + \Delta E\left(d^\dagger d - \frac{1}{2}\right), \tag{33}$$

where the interaction potentials are related to the longitudinal exchange couplings via [see Eqs. (5), (16), (22), and (23)]

$$W = \frac{1}{2}\frac{J_{z1} - J_{z2}}{1 + \pi^2\nu_0^2 J_{z1}J_{z2}/16}, \tag{34}$$

$$V = \frac{8}{\pi^2\nu_0^2}\frac{1 - \pi^2\nu_0^2 J_{z1}J_{z2}/16}{J_{z1} + J_{z2}}, \tag{35}$$

$$\lambda_i = \frac{J_{\perp i}}{2}\sqrt{\nu_0\xi_0}\frac{1}{1 + (\pi\nu_0 J_{zi}/4)^2}\sqrt{\frac{1}{1 + (\pi\nu_0 V/2)^2}}, \tag{36}$$

and (assuming equal impurity and conduction electron Landé factors $g_i = g_c = g$)

$$\Delta E = -\frac{2g\mu_B}{\pi}\tan^{-1}\left(\frac{\pi\nu_0 V}{2}\right)B \equiv -g\mu_B\Lambda(V)B.$$

In the case of symmetric exchange couplings, the resonant-level model (33) reduces to the Hamiltonian (12) originally considered by Emery and Kivelson. Then the combination $d^\dagger + d$ is decoupled from the conduction electrons (hence the ground state degeneracy and the non-Fermi-liquid behavior). A finite channel anisotropic transverse exchange couples this combination to conduction electrons and moves the system away from the non-Fermi-liquid fixed point towards the Fermi-liquid single-channel fixed point.[21] The smaller the anisotropy, the lower the crossover temperature. In what follows we analytically study this crossover in the solvable limit $V = W = 0$.

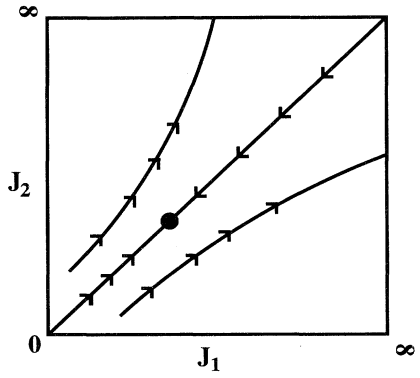

FIG. 3. Qualitative (J_1, J_2) RG flow diagram for the anisotropic two-channel Kondo model.

Since the total number of fermions is not conserved by the Hamiltonian, there are anomalous Green functions. In the Nambu representation

$$D = \begin{pmatrix} d \\ d^\dagger \end{pmatrix},$$

the impurity Green function

$$\hat{G}_d(t) = -i\langle T\left[D(t)D^\dagger(0)\right]\rangle$$

is a 2×2 matrix. Its Fourier transform can easily be evaluated. For ω much smaller than the bandwidth, we find

$$\hat{G}_d(\omega) = \frac{1}{2}\frac{\hat{\tau}_0 - \hat{\tau}_x}{\omega + i\Gamma\mathrm{sgn}\omega} + \frac{1}{2}\frac{\hat{\tau}_0 + \hat{\tau}_x}{\omega + i\gamma\mathrm{sgn}\omega}, \tag{37}$$

where the resonance widths are defined by

$$\Gamma = \pi\nu_0(\lambda_1 + \lambda_2)^2,$$
$$\gamma = \pi\nu_0(\lambda_1 - \lambda_2)^2,$$

$\hat{\tau}_i$ being the Pauli matrices and $\hat{\tau}_0$ the unit matrix. The impurity spectral function is

$$\hat{A}(\omega) = \frac{1}{2}(\hat{\tau}_0 - \hat{\tau}_x)\frac{\Gamma}{\omega^2 + \Gamma^2} + \frac{1}{2}(\hat{\tau}_0 + \hat{\tau}_x)\frac{\gamma}{\omega^2 + \gamma^2},$$

and it is therefore equally shared by two Lorentzians with different widths Γ and γ. In the channel isotropic case $\gamma \to 0$, one of the two Lorentzians tends to $\delta(\omega)$, representing the impurity degree of freedom which is decoupled from the conduction band in this particular limit.[12]

The impurity contribution to the free energy can be calculated in a standard way by integration over the coupling constant. The result is

$$F(T) = F_0(T) + \int\frac{d\omega}{2\pi}f(\omega)\left[\tan^{-1}\left(\frac{\Gamma}{\omega}\right) + \tan^{-1}\left(\frac{\gamma}{\omega}\right)\right], \tag{38}$$

where $F_0(T)$ is the free energy in the absence of coupling between the impurity and conduction electrons, $f(\omega)$ is the Fermi distribution function, and the integral should be limited to the conduction bandwidth. The entropy can be calculated by $S(T) = -\partial F(T)/\partial T$. By defining the function

$$\bar{S}(z) = \frac{1}{2\pi z}\left[\psi\left(\frac{1}{2}+\frac{1}{2\pi z}\right)-1\right] - \ln\Gamma\left(\frac{1}{2}+\frac{1}{2\pi z}\right) + \frac{1}{2}\ln\pi\,, \tag{39}$$

where $\psi(z)$ is the psi function and $\Gamma(z)$ is the gamma function, the entropy turns out to be

$$S(T) = \ln(2) + \bar{S}\left(\frac{T}{\Gamma}\right) + \bar{S}\left(\frac{T}{\gamma}\right) = \begin{cases} \dfrac{\pi T}{6}\left(\dfrac{1}{\Gamma}+\dfrac{1}{\gamma}\right)\,, & T \ll \gamma, \\ \ln\sqrt{2}\,, & \gamma \ll T \ll \Gamma, \\ \ln 2 - \dfrac{\Gamma+\gamma}{2\pi T}\,, & T \gg \Gamma, \end{cases} \tag{40}$$

the last equality being valid for $\gamma \ll \Gamma$. $S(T)$ is shown in Fig. 4. We see that $S(0) = 0$, as expected since no degeneracy is left, but there is a region of temperatures (the wider the smaller γ is) where the entropy is close to that of the symmetric two-channel model.

Another quantity of physical interest is the longitudinal impurity susceptibility. As we know from the above analysis, exactly on the Emery-Kivelson line, $\chi^{zz}_{\rm imp} = 0$, and one has to consider deviations from this line (i.e., $V \neq 0$) in order to account for a finite impurity susceptibility.[19] The resulting susceptibility is

$$\chi^{zz}_{\rm imp} = [g\mu_B\Lambda(V)]^2 \int_0^\beta d\tau \langle T\,[S^z(\tau)S^z(0)]\rangle = [g\mu_B\Lambda(V)]^2\,\frac{1}{\pi(\Gamma-\gamma)}\left[\psi\left(\frac{1}{2}+\frac{\Gamma}{2\pi T}\right)-\psi\left(\frac{1}{2}+\frac{\gamma}{2\pi T}\right)\right].$$

In the case $\gamma \ll \Gamma$ the susceptibility shows the same kind of cross-over behavior as the entropy:

$$\chi^{zz}_{\rm imp} = [g\mu_B\Lambda(V)]^2 \begin{cases} \dfrac{1}{\pi(\Gamma-\gamma)}\ln\left(\dfrac{\Gamma}{\gamma}\right)\,, & T \ll \gamma, \\ \dfrac{1}{\pi(\Gamma-\gamma)}\ln\left(\dfrac{\Gamma}{T}\right)\,, & \gamma \ll T \ll \Gamma, \\ \dfrac{1}{4T}\,, & T \gg \Gamma. \end{cases} \tag{41}$$

As expected the magnetic susceptibility saturates at low temperature, although at intermediate temperatures it shows the logarithmic behavior of the two-channel Kondo model.

It follows from (40) and (41) that the Wilson ratio R_W is *not* universal: It depends on the amount of anisotropy γ/Γ. Such a conclusion is obvious in the limit of small anisotropy, when the energy scales are well separated. Then the residual entropy $\ln\sqrt{2}$ must be quenched in a temperature range $\sim \gamma$, implying $C_{V,\rm imp} \sim T/\gamma$, while the susceptibility $\chi_{\rm imp}$ just rounds off the logarithmic singularity, $\chi_{\rm imp} \sim \ln(\Gamma/\gamma)$: The Wilson ratio is very *small*. Such a lack of universality is also apparent in the phenomenological, Fermi-liquid description of the low-temperature limit, $T \ll \gamma$. Then the impurity is quenched into a singlet, and the residual conduction electron phase shift in the channel (m,σ) may be expanded as

$$\delta_{m\sigma}(\epsilon) = \delta_{m0} + \alpha_m\epsilon + \psi_m\delta n_{m,-\sigma} + \sum_{m'\neq m}\phi^{m'\sigma'}_{m\sigma}\delta n_{m'\sigma'}\,,$$

where $\delta n_{m'\sigma'}$ is the change in the occupation measured from the ground state. Universality implies that $\delta_{m\sigma}(\epsilon)$ is invariant (i) if the chemical potential of the other channel is changed (there is no channel flip) and (ii) if ϵ and the chemical potential are changed by the same amount (the Kondo singularity is attached to the Fermi level).

Hence in our two-channel case

$$\delta_{1\sigma} = \delta_{10} + \alpha_1\left[\epsilon - \frac{\delta n_{1-\sigma}}{\nu_s}\right] + \theta_1\sigma\sigma'\delta n_{2\sigma'},$$

$$\delta_{2\sigma} = \delta_{20} + \alpha_2\left[\epsilon - \frac{\delta n_{2-\sigma}}{\nu_s}\right] + \theta_2\sigma\sigma'\delta n_{1\sigma'}$$

(the cross terms θ_1, θ_2 are equal in the electron-hole symmetric case $\delta_{10} = \delta_{20} = \pi/2$). ν_s is the one-channel density of s states at the Fermi level. It is then straightforward to extend the analysis of Ref. 5: The resulting impurity corrections are

$$\frac{C_{V,\rm imp}}{C_V} = \frac{\alpha_1+\alpha_2}{\pi\nu_s}\,, \quad \frac{\chi_{\rm imp}}{\chi} = \frac{2(\alpha_1+\alpha_2)}{\pi\nu_s} + \frac{\theta_1+\theta_2}{2\pi}\,.$$

Due to the channel interaction θ the Wilson ratio $R_W = C_V\chi_{\rm imp}/\chi C_{V,\rm imp}$ departs from the single-channel value

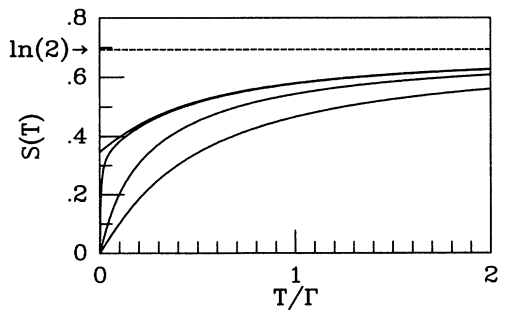

FIG. 4. Entropy $S(T)$ for various values of the anisotropy $\lambda^2 = \gamma/\Gamma$: from the top $\lambda = 0, 0.1, 0.5, 1$.

2. Put another way, one has a line of fixed points rather than a unique one. If channel 1 is the dominant screening channel, J_1 goes to infinity while J_2 may evolve towards any arbitrary value: Once the spin S is screened, J_2 no longer scales. This arbitrariness is reflected in the Wilson ratio.

To our knowledge, the two-channel Kondo model is most convincingly realized by two-level systems in metal alloys.[6] This has recently been experimentally confirmed thanks to the development of point contact spectroscopy.[9] In these systems, the role of the spin is played by some orbital degree of freedom, while the physical spin plays the role of the channel index. Thus the model is by construction channel isotropic. However, an external magnetic field breaks the channel symmetry and generates an effective channel anisotropy proportional to the curvature of the conduction electron band times the magnetic field B. In this case, the coupling to the magnetic field is described by the following term in the Hamiltonian:

$$H_B = -\frac{g\mu_B}{2} B \int d\mathbf{x} \sum_\sigma \left[\psi^\dagger_{1\sigma}(\mathbf{x})\psi_{1\sigma}(\mathbf{x}) - \psi^\dagger_{2\sigma}(\mathbf{x})\psi_{2\sigma}(\mathbf{x}) \right],$$

where σ is now the pseudospin index, and the channel indices 1 and 2 correspond to the physical spin up and down, respectively. The magnetic field shifts the Fermi level of channel 1 with respect to that of channel 2. Electron-hole symmetry is thereby broken within each channel. It follows that the magnetic field induces a phase shift anisotropy $\delta_1 - \delta_2 \propto B$, reflected into a finite $W \propto B$ and in different λ_1 and λ_2. Such a correction comes both from the correction to the Fermi level density of states,

$$\nu_1 - \nu_2 = g\mu_B \nu_0' B\,, \quad \nu_0' = \left.\frac{\partial \nu}{\partial \epsilon}\right|_{\epsilon=\epsilon_F},$$

and from the Zeeman shift of band edges. The modified pseudospin flip amplitudes in the equivalent resonant level model are

$$\lambda_i = \frac{J_\perp \cos^2(\delta_i/2)}{2\cos(\delta'/2)} \sqrt{\nu_i \xi_0} = \lambda_0 \left(1 + \frac{3\nu_0'}{4\nu_0} g\mu_B B\right).$$

Correspondingly this reflects onto a finite γ,

$$\gamma = \frac{9\pi}{16} (g\mu_B)^2 \xi_0 \frac{(\nu_0')^2}{\nu_0} J_\perp^2 B^2. \tag{42}$$

Thus, B causes the crossover to a Fermi-liquid behavior at low temperature as observed in Ref. 9. As to the physical magnetic susceptibility, it is related to the first derivative of the free energy (38) with respect to γ. The low-temperature (low-magnetic-field) behavior of the susceptibility is given by

$$\chi_{\rm imp} = \frac{(3g\mu_B \nu_0')^2}{16\nu_0} \xi_0 J_\perp^2 \ln \left[\min\left(\frac{1}{T}, \frac{1}{B^2}\right)\right].$$

(One can show that finite $W \propto B$ does not contribute to the logarithmic-divergent part of the susceptibility.)

Very recently, the channel anisotropic (but spin isotropic) Kondo model has been solved using Bethe-ansatz methods by Andrei and Jerez.[22] Their conclusions are qualitatively similar to ours.

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[1] P.W. Anderson, G. Yuval, and D.R. Hamann, Phys. Rev. B **1**, 4664 (1970).
[2] K.G. Wilson, Rev. Mod. Phys. **47**, 773 (1975); P. Nozières, J. Low Temp. Phys. **17**, 31 (1974).
[3] N. Andrei, Phys. Rev. Lett. **45**, 379 (1980); P.B. Wiegmann, JETP Lett. **31**, 364 (1980).
[4] For a general review see A.C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, England, 1993). The early discussion of the orbital degeneracy may already be found in the Appendix of the original Anderson paper [P.W. Anderson, Phys. Rev. **124**, 41 (1961)].
[5] P. Nozières and A. Blandin, J. Phys. (Paris) **41**, 193 (1980).
[6] A. Zawadowski, Phys. Rev. Lett. **45**, 211 (1980); A. Muramatsu and F. Guinea, *ibid.* **57**, 2337 (1986); G. Zaránd and A. Zawadowski, *ibid.* **72**, 542 (1994).
[7] D.L. Cox, Phys. Rev. Lett. **59**, 1240 (1987); C.L. Seaman, M.B. Maple, B.W. Lee, S. Ghamaty, M.S. Torikachvili, J.-S. Kang, L.Z. Lui, J.W. Allen, and D.L. Cox, *ibid.* **67**, 2882 (1991); B. Andraka and A.M. Tsvelik, *ibid.* **67**, 2886 (1991).
[8] D.L. Cox, M. Jarrell, C. Jayaprakash, H.R. Krishna-murty, and J. Deisz, Phys. Rev. Lett. **62**, 2188 (1989); T. Giamarchi, C.M. Varma, A.E. Ruckenstein, and Ph. Nozières, *ibid.* **70**, 3967 (1993); V.J. Emery and S. Kivelson, *ibid.* **71**, 3701 (1993).
[9] D.C. Ralph and R.A. Buhrman, Phys. Rev. Lett. **69**, 2118 (1992); D.C. Ralph, A.W.W. Ludwig, Jan von Delft, and R.A. Buhrman, *ibid.* **72**, 1064 (1994).
[10] N. Andrei and C. Destri, Phys. Rev. Lett. **52**, 364 (1984); A.M. Tsvelik and P.B. Wiegmann, Z. Phys. B **54**, 201 (1984).
[11] A.M. Tsvelik, J. Phys. C **2**, 2833 (1990); I. Affleck and A.W.W. Ludwig, Nucl. Phys. **B360**, 641 (1991).
[12] V.J. Emery and S. Kivelson, Phys. Rev. B **47**, 10 812 (1992).
[13] P. Schlottmann, J. Phys. (Paris) **6**, 1486 (1978).
[14] G. Toulouse, C. R. Acad. Sci. **268**, 1200 (1969).
[15] G. Yuval and P.W. Anderson, Phys. Rev. B **1**, 1522 (1970).
[16] P. Nozières and C. De Dominicis, Phys. Rev. **178**, 1097

(1969).
[17] N.I. Muskhelishvili, *Singular Integral Equations* (Noordhoff, Groningen, The Netherlands, 1953).
[18] P.B. Wiegmann and A.M. Finkelstein, Sov. Phys. JETP **48**, 102 (1978).
[19] D.G. Clarke, T. Giamarchi, and B.I. Shraiman, Phys. Rev. B **48**, 7070 (1993); A.M. Sengupta and A. Georges, *ibid.* **49**, 10 020 (1994).
[20] M. Fabrizio and A.O. Gogolin, Phys. Rev. B **50**, 17 732 (1994).
[21] A different kind of crossover is that induced by a local magnetic field. For the two-channel case, this has been studied by D. Sacramento and P. Schlottmann, Phys. Lett. A **142**, 245 (1989).
[22] N. Andrei and A. Jerez, Phys. Rev. Lett. (to be published).