Brillouin-zone mapping of the existence conditions for interface bilayer spin waves

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We consider the model of a ferromagnetic bilayer film, described by a standard Heisenberg Hamiltonian involving interactions between nearest neighbors. Our theory holds for arbitrary (with respect to the film normal) configuration of the film magnetization, arbitrary (ferromagnetic or antiferromagnetic) interface exchange coupling, and arbitrary (easy-axis/easy-plane) uniaxial interface anisotropy. Exact conditions for the existence of interface spin waves (ISW) are established in the limit of a very thick bilayer film. The existence of ISW is found to be related to the propagation direction of the spin wave in the plane of the film. We propose a Brillouin-zone mapping (BZM) of the regions of existence of ISW's on the two-dimensional Brillouin zone for the three interface orientations sc (100), fcc (100), and bcc (100). These regions are studied as to their size *versus* the respective interface parameters as well as the film magnetization configuration. We show that the emergence of ISW occurs much more easily on the edges of the BZ than at its center and moreover that antiferromagnetic interface coupling considerably broadens the regions of interface spin waves existence (towards the BZ center).

I. INTRODUCTION

Recent experimental data suggest that giant magnetoresistance (an effect observed in magnetic multilayers) may originate in spin-dependent scattering from magnetic states predominantly *localized* at magnetic/nonmagnetic interfaces.[1] Such findings stimulate a renewal of theoretical interest in the existence conditions for interface spin waves (ISW) in multilayers (see, e.g., Ref. 2). Historically, the subject of interface spin waves in bimagnetic systems was addressed by several authors;[3-6] different theoretical aspects concerning ISW existence have been touched upon in these papers.

Thus, Djafari-Rouhani and Dobrzynski[3] have considered the sc (100) and sc (110) interfaces formed between two semi-infinite ferromagnets. The effect of interface (first- and second-nearest neighbors) coupling exerted on the existence conditions of the respective interface spin waves as well as the influence of couplings on their energy dispersion along the high-symmetry directions of the two-dimensional Brillouin zone (2D BZ) were studied within the framework of the Green's-function approach. In particular, it has been shown that a magnetic *superstructure* may exist at one of the interfaces considered, namely the sc (110) interface, as a consequence of the softening of the interface mode energy associated with this interface. Hong and Yang[4] have considered theoretically (within the Heisenberg model) the spin-wave spectrum of a bimagnetic system composed of two different *ferromagnetic* materials, the one having the form of a thin film deposited on the (semi-infinite) surface of the other. They have shown that the properties of the energy spectrum of the magnetic overlayer were dependent on the magnitude of the interface coupling. In particular, they note that some energy states originating in the overlayer film protrude beyond the band of the semi-infinite substrate and are of the nature of states localized on the interface. They restricted their considerations to simple cubic structures with sc (100) oriented interface. They neglected the external magnetic field as well as surface and interface intrinsic anisotropies.

Yaniv[5] has considered a biferromagnetic interface within the spin-wave approximation using the Green's-function technique. The bilayer system considered was composed of two Heisenberg ferromagnets coupled *via* nearest-neighbor interface exchange coupling. The film had a sc (100) structure and the external field was absent; it was assumed that the corresponding interface exchange coupling integral was always positive. Also, no bulk, surface, or interface anisotropies were included into his considerations. It has been found that under certain circumstances either 0, 1, or 2 interface magnon branches may exist; however, none of them was located below the bulk subbands. We shall show that, in fact, the last property is a consequence of the assumption that only positive values of the interface exchange integral were admitted in Yaniv's theory. From the nowadays available data it is evident, however, that the interface coupling can be negative as well, and the ferromagnetic ground state will not be destroyed because of that, if simultaneously an external static field (sufficiently strong) is applied to the system. Therefore, in our present theory we allow the interface coupling to be antiferromagnetic, and we show that because of that a new interface magnon branch appears at the bottom of the energy spectrum of the bilayer system considered.

Xu, Mostoller, and Rajagopal[6] have extended their considerations beyond Yaniv's simple sc (100) model of a bicrystal; namely, they took into account other possible interface cuts of cubic crystals (for which first-nearest-neighbors belong to a given lattice plane and the two adjacent planes only). Within the framework of the Heisenberg model (only isotropic exchange terms were retained) the local densities of magnon states at interface planes were numerically computed for different sets of interface exchange coupling parameters involved in their model. The structure of the density spectra obtained pointed to the existence of interface states split off below the bulk bands over the whole 2D BZ. This result complemented Yaniv's findings, who established conditions for the existence of interface states with energies lying above the bulk (one to two) bands in a sc (100) biferromagnetic crystal, only.

In Ref. 2 we have established the existence conditions for ISW in bilayer sc (111) film especially including effects due to changes in orientation of the film magnetization with respect to the film surface; we moreover have shown how the regions of existence of ISW on the 2D BZ vary with the respective interface parameters and film magnetization orientation. Similarly Ref. 7 reports results for sc (110) bilayer film, showing moreover how the ISW energy evolves with varying in-plane wave vector $\mathbf{k}_\parallel$ along the high-symmetry directions of the 2D BZ. References 2 and 7 show that in most cases the existence of ISW requires (beside appropriate values of the interface parameters) well specified directions of in-plane propagation $\mathbf{k}_\parallel$. Reference 8 notes that antiferromagnetic interface coupling favors the existence of ISW (the problem has subsequently been analyzed in full detail in a number of papers, cf. the references in Ref. 2), whereas Ref. 9 gives a first indication of the possible coexistence of surface and interface spin waves (see also Ref. 10). In this paper we shall perform a detailed analysis of the ISW existence conditions for all three cubic bilayer films with (100) interface orientations. Logistically, our analysis will resemble that performed in Ref. 11 for surface spin waves. We shall take into account the influence of the orientation of the external magnetic field as well as the interface coupling and intrinsic interface anisotropy on the ISW existence conditions. We find that the existence of an ISW (at fixed interface conditions) requires in most cases an appropriate propagation direction of the spin wave in the plane of the film. This dependence will be visualized by plotting a Brillouin-zone mapping (BZM) of the existence regions of ISW in the 2D BZ. These regions are then studied as to their form and size *versus* the respective interface parameters.

II. THE MODEL

We study a bilayer film composed of two ferromagnetic layers (sublayers A and B) of the same magnetic material, coupled by exchange interactions at their interface through a nonmagnetic spacer. We thus include the effects of modified exchange interaction and *intrinsic* anisotropy at the interface. We assume that the externally applied static magnetic field $\mathbf{H}$ is oriented at some angle to the film normal and can be rotated in the plane perpendicular to the film surface. The bulk effective field acting on a given spin is defined as the sum of the external static field and the demagnetization as well as the bulk anisotropy fields. We may consider the bilayer system as made up of a number of lattice planes perpendicular to the z axis, and any magnetic ion will be specified by a set of indices $l\mathbf{j}$, where l is an integer labeling the plane containing the ion and $\mathbf{j}$ is a two-dimensional lattice vector in the xy plane. The integer l takes values from 0 to $L-1$; we assume that both sublayers are of equal thickness, i.e., $L\equiv 2N$. We write the Hamiltonian of our bilayer film in the form:

$$\hat{\mathscr{H}}=-\sum_{l\mathbf{j}\neq l'\mathbf{j}'} J_{ll'}\hat{\mathbf{S}}_{l\mathbf{j}}\cdot\hat{\mathbf{S}}_{l'\mathbf{j}'}-g\mu_B\sum_{l\mathbf{j}}\mathbf{H}_l\cdot\hat{\mathbf{S}}_{l\mathbf{j}}-\sum_{l\mathbf{j}}D_l(\hat{S}^z_{l\mathbf{j}})^2 \quad (1)$$

with terms accounting successively for the isotropic exchange interactions, the Zeeman energy of the spins, and the uniaxial (single-ion) interface anisotropy energy. We write for the NN exchange terms $J_{ll'}=J$ if both spins belong to the same sublayer and $J_{ll'}=J^{AB}$ if the spins are coupled across the interface between the sublayers A and B; the ratio of the two integrals is $J_{\text{int}}\equiv J^{AB}/J$. The third (anisotropic) term of (1) comprises the interface anisotropy effect, specified as: $D_l=0$ for surface and bulk spins, and $D_l=D'$ for interface spins.

We denote the wave-vector component of the spin waves parallel to the surface by $\mathbf{k}_\parallel=[k_x,k_y]$. The spin waves will additionally be characterized by a third wave-vector component $k_z\equiv k$ in the z direction; one should note that the quantity we are going to use is in fact a *reduced* wave vector, i.e., its components will be expressed in lattice units. It is also convenient at this stage to define the following structural functions dependent on the crystallographical structure considered:

$$\Gamma_\parallel^{\mathbf{k}_\parallel}=\sum_{\boldsymbol{\delta}_\parallel}\exp i(\mathbf{k}_\parallel\cdot\boldsymbol{\delta}_\parallel) \quad\text{and}\quad \Gamma_\perp^{\mathbf{k}_\parallel}=\sum_{\boldsymbol{\delta}_\perp}\exp i(\mathbf{k}_\parallel\cdot\boldsymbol{\delta}_\perp)\ , \quad (2)$$

where $\boldsymbol{\delta}_\parallel$ is a vector connecting a site in layer l with its nearest neighbors in the same layer, and $\boldsymbol{\delta}_\perp$ is the projection onto the xy plane of a vector connecting a site in layer l with its nearest neighbors in layer $l+1$ (also $z_\parallel$ and $z_\perp$ will denote the number of nearest neighbors in either case). We obtain as in Ref. 2 the following expression for the energy:

$$E(\mathbf{k}_\parallel,k)=2SJ\left[z_\parallel+2z_\perp-\Gamma_\parallel^{\mathbf{k}_\parallel}\right]+g\mu_B(H^{\text{eff}}\cdot\gamma)-4SJ|\Gamma_\perp^{\mathbf{k}_\parallel}|\cos k\ . \quad (3)$$

This expression has been obtained from Eq. (1) on transformation of the spin operators to boson operators and conserving only the quadratic terms of the Hamiltonian; however, it is essential to mention that the rejected terms of higher orders have first been brought to normal order. When dealing with interface boundary conditions the only approximation to be made by us in the present paper consists in assuming circular spin precession throughout the whole bilayer sample. This implies that the elliptical deformation of the spin precession cones (caused by interface anisotropy of the *uniaxial* type) in the vicinity of the interface has been neglected; this is, of course, justified only for small values of the interface anisotropy. Thus, we can apply here the final results of Refs. 12 and 13. We thus obtain the following two characteristic equations:

$$\frac{\sin(N+1)k - a\sin Nk}{\sin Nk - a\sin(N-1)k} = b \pm J_{\text{int}} \tag{4}$$

corresponding, respectively, to symmetric and antisymmetric mode solutions; above, we have used the notations:

$$a = \frac{z_\perp}{|\Gamma_\perp^{\mathbf{k}_\|}|}; \quad b = \frac{z_\perp}{|\Gamma_\perp^{\mathbf{k}_\|}|}\left[1 - J_{\text{int}} - D_{\text{int}}(3\cos^2\vartheta - 1)\right];$$

$$D_{\text{int}} = \frac{D'(S-1/2)}{2SJz_\perp}, \tag{5}$$

where ϑ is the angle between the direction of magnetization and that of the normal to the surface of the film. The mode number k can, in general, be complex, and it should be emphasized that throughout the present work we focus our attention entirely on the *acoustical* (i.e., $k = it$) localized solutions only.

We note that the boundary parameters occurring in the characteristic equation (4) are functions of the in-plane spin-wave propagation vector $\mathbf{k}_\|$ by way of the structural factors (2), so that the existence conditions for localized spin-wave solutions are functions of $\mathbf{k}_\|$, too. In what follows we illustrate the dependence of the existence conditions for interface spin waves on $\mathbf{k}_\|$ by determining the appropriate regions of ISW existence on the 2D BZ. We are interested in how propagation affects the existence conditions for waves localized on the bilayer interface. It will turn out that for fixed *static* interface conditions, interface spin waves exist for certain strictly determined directions of propagation $\mathbf{k}_\|$ only. We shall present here the results obtained for three cubic crystals with (100) interface film cut. The structural factors take the respective forms:

$$z_\perp = 1, \quad |\Gamma_\perp^{\mathbf{k}_\|}| = 1; \quad z_\| = 4, \quad \Gamma_\|^{\mathbf{k}_\|} = 2(\cos k_1 + \cos k_2)$$

for sc (100) interface cut, (6a)

$$z_\perp = 4, \quad |\Gamma_\perp^{\mathbf{k}_\|}| = 4\cos(\tfrac{1}{2}k_1)\cos(\tfrac{1}{2}k_2); \quad z_\| = 4,$$

$$\Gamma_\|^{\mathbf{k}_\|} = 2(\cos k_1 + \cos k_2)$$

for fcc (100) interface cut, (6b)

$$z_\perp = 4, \quad |\Gamma_\perp^{\mathbf{k}_\|}| = 4\cos(\tfrac{1}{2}k_1)\cos(\tfrac{1}{2}k_1); \quad z_\| = 0, \quad \Gamma_\|^{\mathbf{k}_\|} = 0$$

for bcc (100) interface cut, (6c)

where k_1, k_2 are the in-plane wave-vector components in directions determined against the background of the two-dimensional Brillouin zone, as shown in Fig. 1.

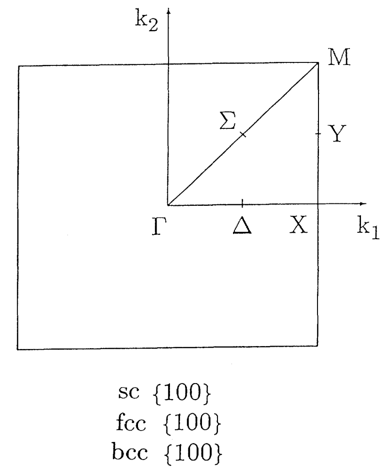

FIG. 1. Two-dimensional (in-plane) Brillouin zone for (100) interface cut in cubic structures.

III. THE EXISTENCE CONDITIONS FOR INTERFACE SPIN WAVES

By applying the procedure described in Ref. 2 we find the conditions for the existence of acoustic interface roots (i.e., $k = it$) of the characteristic equation (4); they require that the following inequalities shall be fulfilled:

$$b \pm J_{\text{int}} > \frac{N+1-aN}{N+a-aN} \tag{7}$$

for symmetric as well as antisymmetric solutions, respectively. In general, the analysis of the conditions (7) is rather complicated because the *surface* parameter a intervening therein (as well as the interface parameters) varies dynamically with varying $\mathbf{k}_\|$. However, if the thickness of the film is made to grow infinitely, $N \Longrightarrow \infty$, which is equivalent to applying the approximation of very thick bilayer films, the right-hand term of (7) tends to unity and the condition under consideration becomes dependent on interface-related quantities only: $b > 1 \mp J_{\text{int}}$, or explicitly

$$\frac{z_\perp}{|\Gamma_\perp^{\mathbf{k}_\|}|}\left[1 - D_{\text{int}}(3\cos^2\vartheta - 1) - J_{\text{int}}\right] > 1 \mp J_{\text{int}}. \tag{8}$$

If we now take into account expressions (6), Eq. (8) becomes, respectively,

$$1 - D_{\text{int}}(3\cos^2\vartheta - 1) - J_{\text{int}} > 1 \mp J_{\text{int}}$$

for sc (100) interface cut, (9a)

$$1 - D_{\text{int}}(3\cos^2\vartheta - 1) - J_{\text{int}} > (1 \mp J_{\text{int}})\cos(\tfrac{1}{2}k_1)\cos(\tfrac{1}{2}k_2)$$

for fcc (100) and bcc (100) interface cuts. (9b)

One notes that the existence conditions for fcc (100) and bcc (100) interface orientations are identical; this is of course related to their identical expressions for $|\Gamma_\perp^{\mathbf{k}_\|}|$. Figures 2 and 3 show, respectively, for sc (100) and fcc (100)/bcc (100) orientations, the Brillouin-zone mapping of the existence regions of ISW determined from the conditions (9) for three different orientations of the magnetization with respect to the film surface and six different combinations of ferromagnetic/antiferromagnetic interface coupling and easy-axis/easy-plane interface anisotropy. One notes that antiferromagnetic interface coupling considerably broadens the regions of existence of ISW (specifically, towards the center of the BZ). The detailed discussion of the results obtained will be facilitated if one first analyses the respective expressions for the ISW energy.

IV. THE INTERFACE SPIN-WAVE ENERGY

A confirmation of the influence of the in-plane wave vector $\mathbf{k}_{\parallel}$ on the generation of exchange interface spin waves is given in Figs. 4–6, which illustrate the changes in the reduced energy spectrum versus the variation of the vector $\mathbf{k}_{\parallel}$ along the high-symmetry path shown in Fig. 1. The *reduced* energy of ISW is an energy-related quantity, defined as below [expression (10) follows from Eq. (3) by inserting therein $k = it$]:

$$E_{\text{red}} = (2SJ)^{-1}\left[E(\mathbf{k}_{\parallel},t) - g\mu_B(H^{\text{eff}}\cdot\gamma)\right] = (z_{\parallel} + 2z_{\perp}) - \Gamma_{\parallel}^{\mathbf{k}_{\parallel}} - 2|\Gamma_{\perp}^{\mathbf{k}_{\parallel}}|\cosh t \ . \tag{10}$$

By solving the characteristic Eq. (4) in the limit of $N \Longrightarrow \infty$ (in this limit one is able to get *exact* solutions) we find the following two roots:

$$2\cosh t = (b \pm J_{\text{int}}) + (b \pm J_{\text{int}})^{-1} \ , \tag{11}$$

which allow us to express the reduced energy directly in terms of the interface parameters. Hence, using Eqs. (5) and (6), we finally get

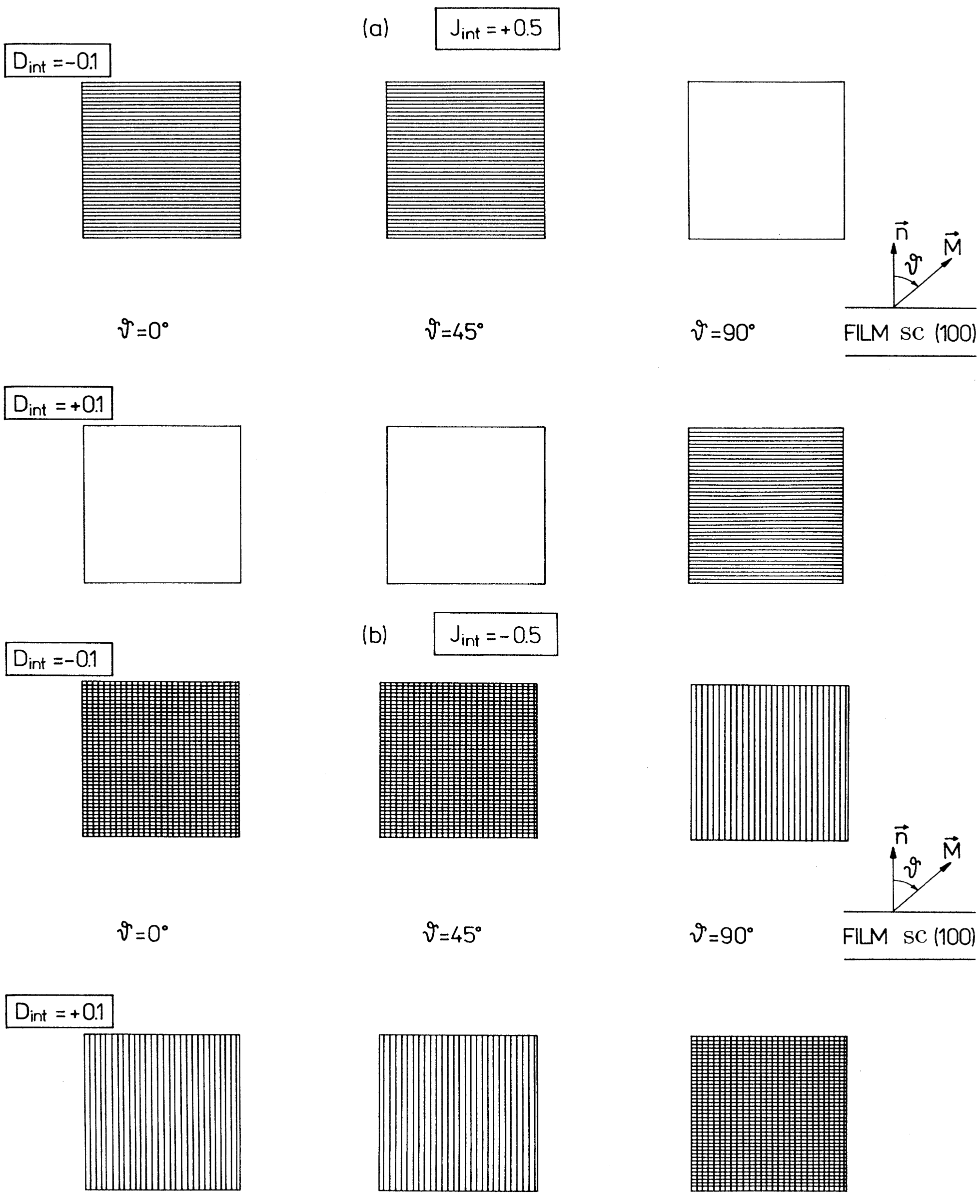

FIG. 2. Brillouin-zone mapping of the regions of existence of acoustical bilayer interface spin waves for sc (100) film. Two interface waves exist in the more densely shaded regions, one —in the less densely shaded regions, and none—in the unshaded area. Notations: J_{int} is the interface exchange coupling parameter, D_{int} is the uniaxial interface anisotropy constant, and ϑ is the angle of film magnetization orientation. Note that for the case (a) of ferromagnetic J_{int} only ISW *induced* by the intrinsic anisotropy D_{int} exists, while for the case (b) of antiferromagnetic interface coupling—the second ISW due to J_{int} is brought into existence in the whole Brillouin zone.

$$E_{\text{red}}[\text{sc }(100)]=6-2(\cos k_1+\cos k_2)-(B+B^{-1}) , \quad (12a)$$

$$E_{\text{red}}[\text{fcc }(100)]=12-2(\cos k_1+\cos k_2) -[C+(4\cos\tfrac{1}{2}k_2\cos\tfrac{1}{2}k_2)^2C^{-1}] , \quad (12b)$$

and

$$E_{\text{red}}[\text{bcc }(100)]=8-[C+(4\cos\tfrac{1}{2}k_2\cos\tfrac{1}{2}k_2)^2C^{-1}] , \quad (12c)$$

where we have made use of the following notations:

$$B=1-J_{\text{int}}-D_{\text{int}}(3\cos^2\vartheta-1)\pm J_{\text{int}} , \quad (13a)$$

$$C=4[1-J_{\text{int}}(1\pm\cos\tfrac{1}{2}k_1\cos\tfrac{1}{2}k_2)-D_{\text{int}}(3\cos^2\vartheta-1)] . \quad (13b)$$

V. DISCUSSION OF THE RESULTS

To start with, let us consider the case of sc (100). By Eq. (9a), we get the following two conditions for the existence of ISW:

$$D_{\text{int}}(3\cos^2\vartheta-1)<0 \quad (14a)$$

and

$$D_{\text{int}}(3\cos^2\vartheta-1)<-2J_{\text{int}} . \quad (14b)$$

Obviously, the fulfillment of either (14a) or (14b) corresponds to the existence of *one* ISW. Thus we can have 0, 1, or at the most 2 branches of ISW's; this result is complementary to that of Yaniv.[5] Noteworthy are the following properties of the above conditions: (i) the condi-

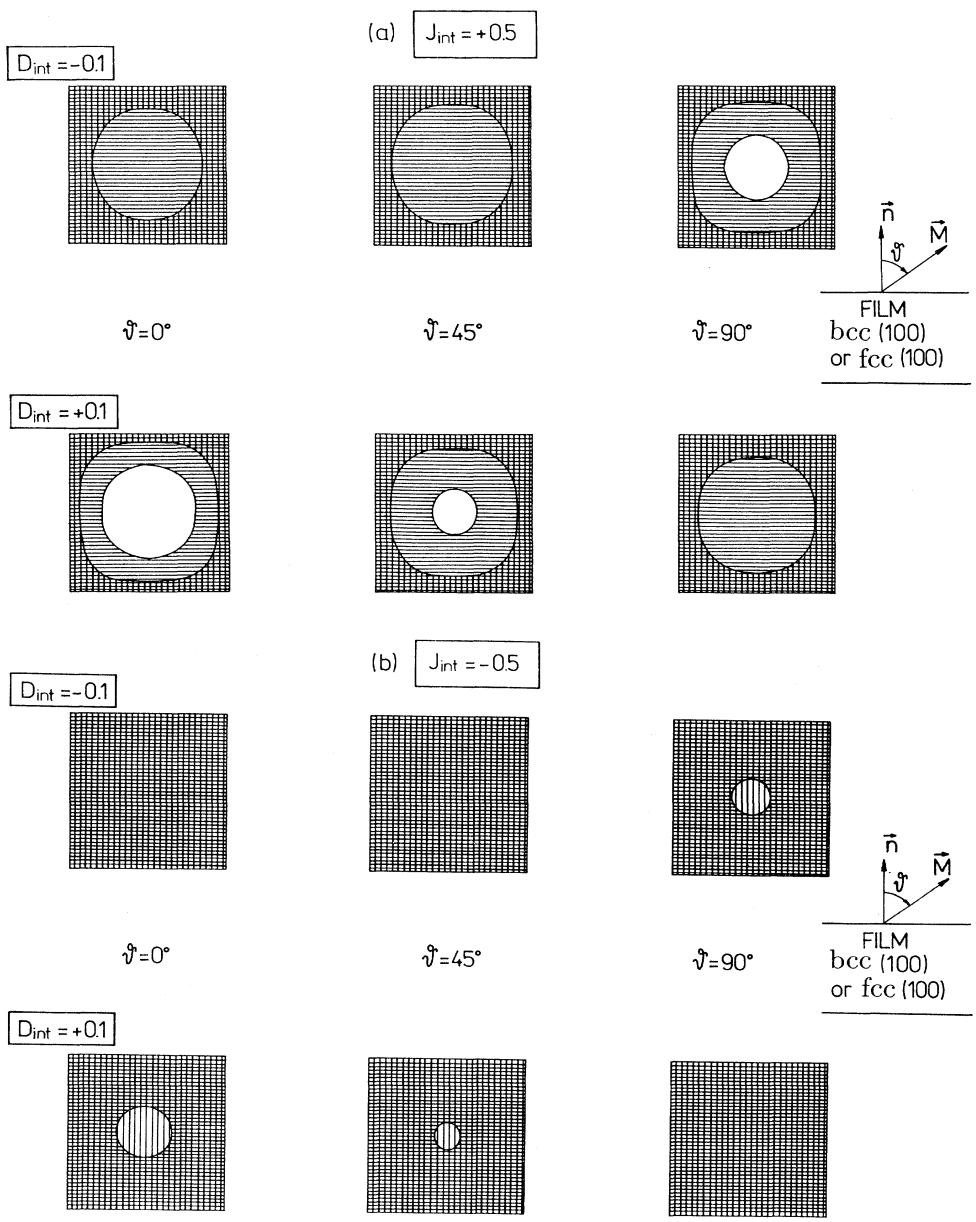

FIG. 3. The same as in Fig. 2 for fcc (100)/bcc (100) interface cuts. From the ISW existence conditions (9b) it follows that now even for ferromagnetic J_{int} two ISW exist in some regions of the BZ.

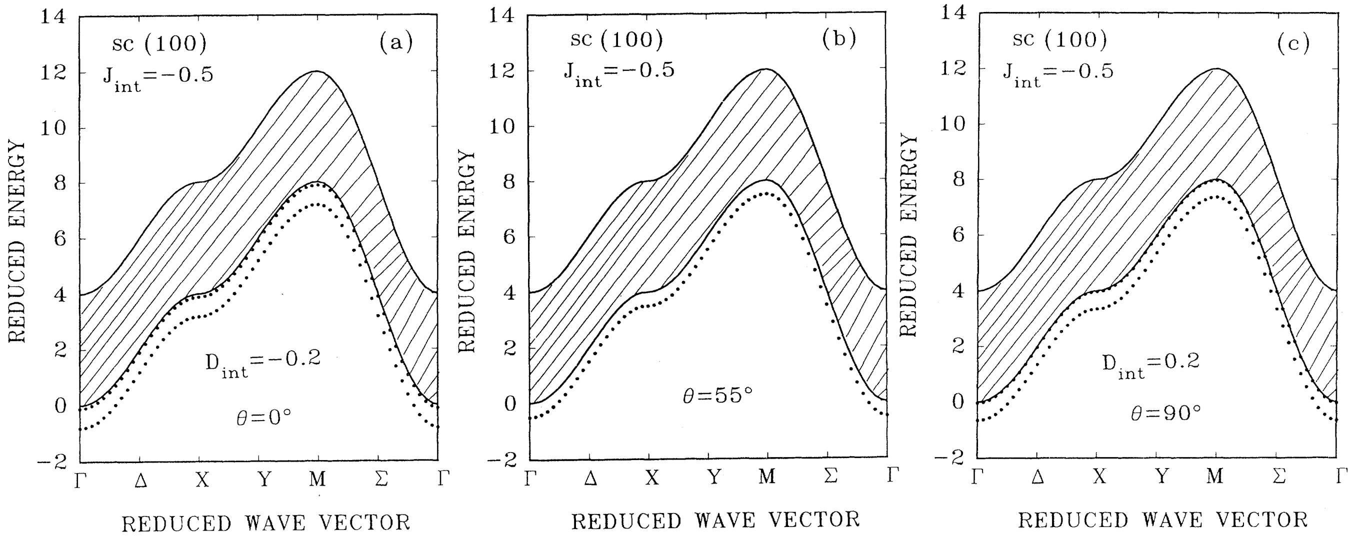

FIG. 4. Changes in the *reduced* energy of interface spin waves (dotted lines) for sc (100) bilayer structure (in the limit of very thick sublayers) *versus* the in-plane vector $\mathbf{k}_{\parallel}$ (varying along the high-symmetry path of Fig. 1) for the case of antiferromagnetic interface coupling ($J_{int}=-0.5$) and for three configurations of the film magnetization: (a) perpendicular to the film ($\vartheta=0°$), (b) $\vartheta=55°$, and (c) parallel ($\vartheta=90°$). Explanation is given in the text. Note that the values assumed for D_{int} have opposite signs for the cases (a) and (c).

tion (14a) is completely independent of J_{int}; (ii) both conditions are independent of $\mathbf{k}_{\parallel}$, signifying that the two are simultaneously fulfilled or unfulfilled in all the points of the BZ; and (iii) the configuration $\vartheta=55°$ is specifically distinguished in that it causes the expression $3\cos^2\vartheta-1$ to vanish identically, $3\cos^2\vartheta-1\equiv 0$, thus eliminating the parameter D_{int} from both conditions.

For the configuration $\vartheta=55°$ the condition (14a) is never fulfilled, whereas the condition (14b) is fulfilled if $J_{int}<0$, leading to the presence of *one* ISW branch [see Fig. 4(b)], which still exists in the other configurations, particularly in the two extremal [perpendicular and parallel, see Figs. 4(a) and 4(c)] configurations. These two extremal configurations would admit of the emergence of the other ISW branch on the assumption of appropriate values for D_{int} so as to ensure the fulfillment of the condition (14a). The fulfillment of (14a) is easily achieved if we keep in mind the following rule: the existence of ISW at parallel configuration [$\vartheta=90°$, Fig. 4(c)] is favored by interface anisotropy of the easy-axis type (i.e., $D_{int}>0$), whereas the same effect at perpendicular configuration [$\vartheta=0°$, Fig. 4(a)] requires the interface anisotropy to be

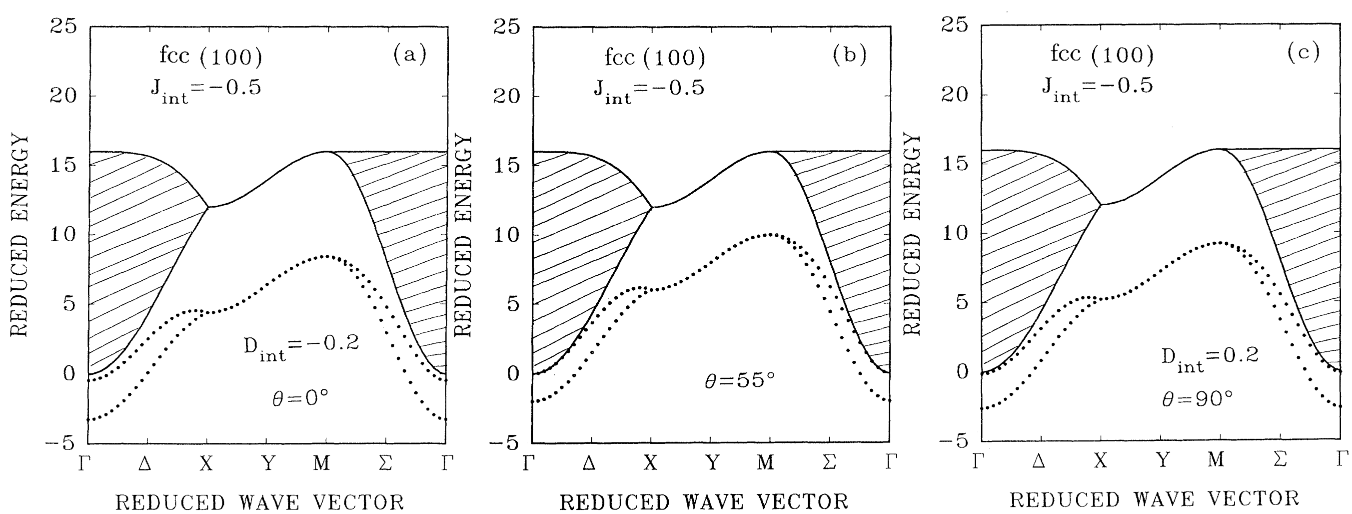

FIG. 5. The same as in Fig. 4 for fcc (100) interface cut.

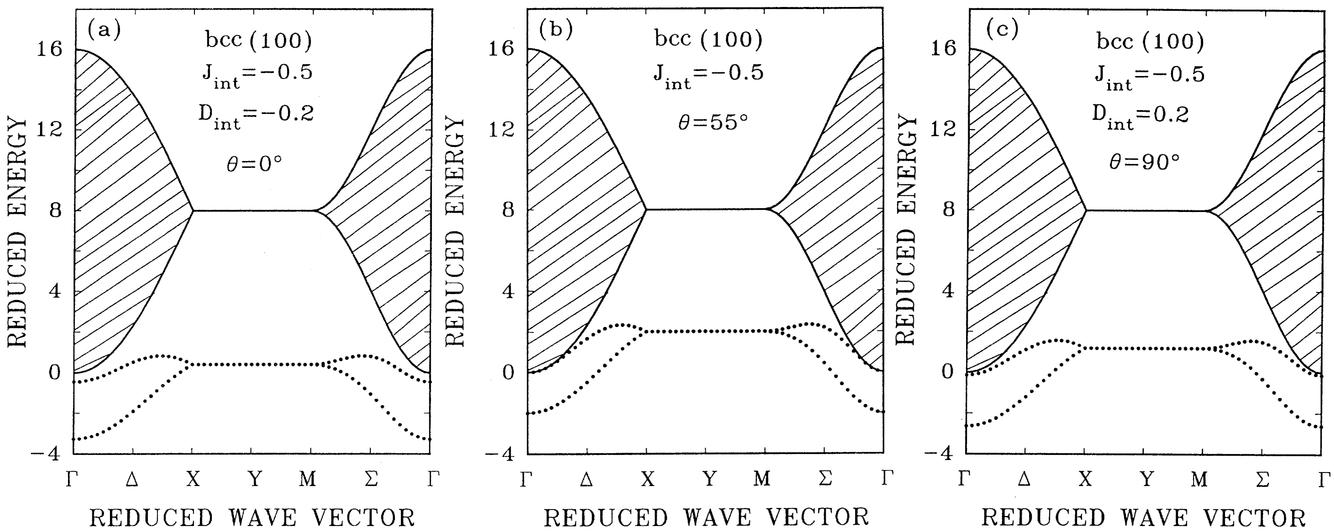

FIG. 6. The same as in Fig. 4 for bcc (100) interface cut.

easy-plane ($D_{\rm int}<0$); this rule is physically quite obvious since ISW's exist only if $D_{\rm int}$ endows the interface spins with additional pinning freedom.

Figure 2 shows the 2D BZ with the regions of existence of the two ISW branches, as brought into existence by $J_{\rm int}$ and by $D_{\rm int}$, and taking into account all the four possible combinations of ferromagnetic/antiferromagnetic interface coupling and easy-axis/easy-plane intrinsic interface anisotropy. Let us draw attention to the case of ferromagnetic $J_{\rm int}$ [Fig. 2(a)]: here, no "$J_{\rm int}$-induced" ISW occurs and only one "$D_{\rm int}$-induced" state can appear for certain configurations. Whereas in Fig. 2(b) (antiferromagnetic $J_{\rm int}$) the same configurations exhibit *two* ISW branches—the additional second branch being due specifically to the negative $J_{\rm int}$ assumed. Let us now concentrate on this latter branch, assuming for simplicity $D_{\rm int}\equiv 0$. By Eqs. (12a) and (13a) the reduced ISW energy is

$$E_{\rm red}[{\rm sc}\ (100)]=6-2(\cos k_1+\cos k_2)-(1-2J_{\rm int})-(1-2J_{\rm int})^{-1}\ , \tag{15}$$

exactly agreeing (in units of $2JS$) with the formula (4.8) of Yaniv.[5] However, there are some differences in the applicability of Yaniv's and our formula: his formula concerned ISW lying *above* the bulk band, whereas ours (15) is derived for ISW lying *below* the bulk band. Thus, what we have proved is this: on expressing the ISW energy in terms of the interface exchange coupling integral the two formulas are found to be complementary.

In the other limiting case of $J_{\rm int}\equiv 1$ we arrive at the following expression:

$$E_{\rm red}[{\rm sc}\ (100)]=6-2(\cos k_1+\cos k_2)+\left[D_{\rm int}(3\cos^2\vartheta-1)\pm 1\right]+\left[D_{\rm int}(3\cos^2\vartheta-1)\pm 1\right]^{-1}\ , \tag{16}$$

which gives the ISW energy (in units of $2JS$) if the condition (14a) or

$$D_{\rm int}(3\cos^2\vartheta-1)+1<-1 \tag{17}$$

is fulfilled, or both. By Eq. (16), the interface anisotropy can "induce" two ISW branches, while [by Eq. (15)] interface coupling can give rise to but one ISW branch. (Obviously, the latter statement results from the case now under consideration, namely that of an interface between two magnetically *identical* sublayers.)

The situation is different as we proceed to the cases of fcc (100) and bcc (100). For the specific configuration $\vartheta=55°$ the condition (9b) becomes

$$1-J_{\rm int}>(1\mp J_{\rm int})\cos\tfrac{1}{2}k_1\cos\tfrac{1}{2}k_2 \tag{18}$$

and, obviously, $J_{\rm int}$ can now "induce" two ISW's [see Figs. 5(b) and 6(b), which show the two ISW branches extending over the whole BZ]. For other configurations we additionally have to deal with an effect of $D_{\rm int}$ that shifts the two ISW branches towards lower energies [see Figs. 5(a), 5(c), 6(a), and 6(c)]. Note that the convergence of the bulk band to a single energy level for the points between X and M is a consequence of the vanishing of $|\Gamma_\perp^{\mathbf{k}_\parallel}|$. In this region only *two* separated energy levels exist: an $(L-2)$-fold degenerate *bulk* level and, lying below it, a two-fold *interface* level. This highly interesting behavior is specified for and restricted to the BZ boundary only. Now since the condition (9b) is dependent on $\mathbf{k}_\parallel$ it can happen that it will be fulfilled in certain regions only (see the Brillouin-zone mapping, Fig. 3). Thus, depending on the values assumed for the quantities $J_{\rm int}$, $D_{\rm int}$ and ϑ there can occur regions with 0, 1, or 2 ISW branches. These BZM regions always possess fourfold symmetry and (cf. Fig. 3) are disposed more and more densely as we move away from the center of the BZ. As an example of how to glean the information inherent in Fig. 3 let us consider the case of $J_{\rm int}=+0.5$, $D_{\rm int}=+0.1$, and $\vartheta=90°$ [Fig. 3(a)]. For $\mathbf{k}_\parallel$ from the center of the zone (Γ point) there is one ISW, and with $\mathbf{k}_\parallel$ approaching some point between Δ and X the second ISW emerges. At the point X both ISW's reach the farthest energetical distance from the band edge and thus become the most strongly localized.

Finally, let us make a brief assessment of how the values of $J_{\rm int}$ and $D_{\rm int}$ used in the preceding analysis suit those presented by real magnetic materials. This is best done in terms of the quantity used by the majority of experimenters—the interface pinning energy *per* surface unit of the interface, $E_{\rm int}$. This quantity,[14] in erg/cm^2, is expressed as follows in terms of $J_{\rm int}$ and $D_{\rm int}$:

$$E_{\rm int}=-A_{\rm ex}a_0^{-1}\left[J_{\rm int}+D_{\rm int}(3\cos^2\vartheta-1)\right], \qquad (19)$$

with a_0 the lattice constant and $A_{\rm ex}$ the exchange stiffness constant of the magnetic film. The existence of ISW requires that $E_{\rm int}$ shall be positive. We shall make an estimate for a material with magnetic properties close to those of permalloy $Ni_{80}Fe_{20}$; this amounts to the assumption of $a_0=3$ Å, $A_{\rm ex}=1.0\times10^{-6}$ erg/cm and $E_{\rm int}=1.0$ erg/cm^2.[15] For parallel configuration ($\vartheta=90°$) we find from Eq. (19) the difference as equal to $J_{\rm int}-D_{\rm int}=-0.03$. This value is more than one order smaller than that (-0.7) used in our present work. Obviously, our choice of so greatly overestimated values for the interface parameters was dictated by the only necessity that the effects depicted by us graphically should be conveniently readable.

Let us also estimate the expected shift of the interface mode energy away from the bottom bulk band for the center of the Brillouin zone (Γ point: $\mathbf{k}_\parallel=0$). For parallel configuration ($\vartheta=90°$) we find[14] that the resonance magnetic-field separation between the interface mode line and the position of the uniform mode ($k=0$) is expressed as follows in terms of $J_{\rm int}$ and $D_{\rm int}$:

$$\Delta H\equiv H_{\rm IM}-H_{\rm UM}=2A_{\rm ex}M^{-1}a_0^{-2}(A+A^{-1}-2)\,;$$
$$A\equiv1-(J_{\rm int}-D_{\rm int})\,. \qquad (20)$$

On taking parameter values as estimated above for permalloy (with its $4\Pi M=8.5$ kOe) we find that the resulting shift ΔH lies in the range of 2 kOe, which is large enough for the experimental detection of the IM line.

To summarize: In this paper we were interested in how the in-plane propagation $\mathbf{k}_\parallel$ affects the existence conditions for spin waves localized on the three bilayer (100) cubic interfaces. We find that for fixed *static* interface conditions interface spin waves can exist for certain strictly determined regions of the two-dimensional Brillouin zone, only. Work is under way for the other four interface cubic cuts: sc (110), sc (111), bcc (110), and fcc (111).

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