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Quantum fluctuations of solitons in two-dimensional anisotropic σ models

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We study quantum fluctuations about static soliton solutions for the two-dimensional anisotropic σ models. Particularly, for the XY-like anisotropy, we find that quantum corrections induce an internal degree of freedom and lower the soliton's classical energy. For Ising-like anisotropy, quantum corrections raise the classical energy of the soliton. In both cases, the corrections are proportional to the anisotropy parameter and depend on the size of the soliton.

I. INTRODUCTION

It is well known that the nonlinear σ model in two dimensions is, in many respects, similar to a four-dimensional non-Abelian gauge theory. In particular, both theories are scale invariant and asymptotically free. Both contain pseudoparticle solutions of the field equations. Hence, this model seems to be an ideal testing ground for speculations about the effects of pseudoparticles in four-dimensional gauge theories.[1]

Besides, it should be pointed out that the isotropic σ model is also a useful model in solid-state physics. It is of interest to condensed-matter theorists, not only as a model of a classical two-dimensional ferromagnet,[2] but also as the large-s limit of a spin-s antiferromagnetic quantum chain.[3] The topological nature of this model was studied in reference to classical Heisenberg magnets by Belavin and Poliakov,[4] who obtained nontrivial metastable states producing local energy minima. Thus, considering that these metastable states have finite energy, a finite density of them will be excited at any temperature, however small. This is the reason why finite-energy static solutions become particularly relevant in the statistical mechanics of condensed-matter systems. Even though there is a small density of them at low temperature, each pseudoparticle can have an arbitrarily large size, thanks to the scale invariance of the model. Consequently, they can occupy all of space and each pseudoparticle has the spin pointing in all different directions. Thus, Belavin and Poliakov argue, long-range order is destroyed at any temperature, however small, for this system.

Our purpose in this paper is to study quantum fluctuations of solitons in two-dimensional anisotropic σ models. As was shown by Watanabe and Otsu,[5] anisotropic σ models also contain static and topologically nontrivial classical minima. This theory in one time and two space dimensions is defined by the Lagrangian

$$L=\tfrac{1}{2}J\int[(\partial_0 \mathbf{S})^2-(\partial_\mu \mathbf{S})^2-\lambda(\partial_\mu S_3)^2]d^2x \tag{1}$$

and the nonlinear constraint

$$\sum_{a=1}^{3} S_a^2=1\ , \tag{2}$$

where $\mu=1,2$; $\partial_0=(1/c)\partial/\partial t$; $\mathbf{S}$ denotes the spin field since (1) could be considered as the Lagrangian for the continuum limit of an antiferromagnetic system coupled by anisotropic nearest-neighbor exchange interactions without mass term. In this case, J is the exchange coupling constant, λ an anisotropy, and c is the velocity of the long-wavelength spin waves. Depending on the anisotropy parameter, the Lagrangian (1) reproduces the following models: for $\lambda=0$ we have the isotropic magnet; $-1\le\lambda<0$ leads to XY-like behavior and $\lambda>0$ leads to Ising-like behavior. Recently, the isotropic case ($\lambda=0$) has been successfully used to explain the low-temperature behavior of the correlation length in the antiferromagnet[6] La_2CuO_4. Topological excitations in two-dimensional (2D) antiferromagnets and their relation to high-temperature superconductivity have also recently been discussed in Refs. 7 and 8.

The plan of this paper is as follows. In Sec. II we present the pseudoparticle solutions for the anisotropic σ models. In Sec. III we compute the first quantum correction due to these configurations. A discussion of the results is given in Sec. IV.

II. PSEUDOPARTICLES IN TWO-DIMENSIONAL ANISOTROPIC σ MODELS

In this section we shall consider soliton solutions to the equations of motion. To this end, first, we will consider the static part of the Lagrangian density. Using constraint (2), we can rewrite this Lagrangian density as

$$\mathcal{L}=\frac{1}{2}J\left\{\sum_{i=1}^{2}(\partial_\mu S_i)(\partial_\mu S_i)+\frac{(1+\lambda)\sum_{i=1}^{2}\sum_{j=1}^{2}S_iS_j(\partial_\mu S_i)(\partial_\mu S_j)}{\left[1-\sum_{i=1}^{2}S_iS_i\right]}\right\}. \tag{3}$$

From Eq. (3), it is clear that soliton solutions, i.e., those with nonzero but finite energy, must satisfy

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$$\lim_{r\to\infty} S(\mathbf{r})\to\text{const} . \tag{4}$$

Then, it is convenient to introduce the new field variable[4,9]

$$W=\frac{S_1+iS_2}{1+S_3} , \tag{5}$$

obtained from the field (S_1,S_2,S_3), taking values on the unit sphere S^2 by stereographic projection. We can write $W=W_1+iW_2$, where W_1 and W_2 describe the plane in "internal space" on which S^2 has been projected stereographically.

In terms of these variables, the Lagrangian density (3) reads

$$\mathcal{L}=\frac{1}{2}J\sum_{i,j} g_{ij}(W)\partial_\mu W_i \partial_\mu W_j \quad (i,j=1,2) , \tag{6}$$

where the metric $g_{ij}(W)$ is given by

$$\begin{aligned} g_{11}&=4(1+|W|^2)^{-2}[1+4\lambda W_1^2(1+|W|^2)^{-2}] ,\\ g_{22}&=4(1+|W|^2)^{-2}[1+4\lambda W_2^2(1+|W|^2)^{-2}] ,\\ g_{12}&=g_{21}=16\lambda W_1 W_2(1+|W|^2)^{-4} . \end{aligned} \tag{7}$$

We see that the anisotropy λ deforms the usual metric on a sphere (isotropic case[10])

$$g_{ij}(W)=4\delta_{ij}(1+|W|^2)^{-2} \quad (\lambda=0) . \tag{8}$$

Considering that $W=W_1+iW_2$, we can rewrite (3) as

$$\mathcal{L}=\frac{1}{2}J\left\{\frac{4|\partial_\mu W|^2}{(1+|W|^2)^2}+\frac{4\lambda[W(\partial_\mu \overline{W})+\overline{W}(\partial_\mu W)]^2}{(1+|W|^2)^4}\right\} , \tag{9}$$

and Eq. (1) can be written in the form

$$\begin{aligned} L&=2J\int\frac{|\partial_0 W|^2}{(1+|W|^2)^2}d^2x\\ &\quad-8J\int\frac{|\partial_{\bar z} W|^2}{(1+|W|^2)^2}\left[1+\frac{2\lambda|W|^2}{(1+|W|^2)^2}\right]d^2x\\ &\quad-8\lambda J\int\frac{[W^2(\partial_z\overline{W})(\partial_{\bar z}\overline{W})+(\overline{W})^2(\partial_z W)(\partial_{\bar z} W)]}{(1+|W|^2)^4}d^2x\\ &\quad-QAJ , \end{aligned} \tag{10}$$

where we have used $\partial_z=1/2(\partial_x-i\partial_y)$, $\partial_{\bar z}=1/2(\partial_x+i\partial_y)$; likewise, $z=x+iy$, $\bar z=x-iy$. In Eq. (10), Q is the topological number of the configuration given by

$$Q=\frac{4}{A}\int\frac{[(\partial_z W)(\partial_{\bar z}\overline{W})-(\partial_{\bar z} W)(\partial_z\overline{W})]}{(1+|W|^2)^2}\times\left[1+\frac{2\lambda|W|^2}{(1+|W|^2)^2}\right]d^2x \tag{11}$$

and measures the number of times the "internal" spheroid of area A is traversed in the mapping. This area is

$$A=\frac{4\pi}{3}(3+\lambda) . \tag{12}$$

We are interested, first of all, in finding static, finite-energy solutions (solitons) to the resulting field equations. From Eq. (10), we see therefore that the minimal value of the energy of the fields having topological number $Q\ge 0$ is equal to

$$E_c=QAJ . \tag{13}$$

This value is achieved for fields satisfying $\partial_{\bar z}W=0$. Hence the field

$$W_c(z)=P_0(z)/P_1(z) , \tag{14}$$

where $P_0(z)$, $P_1(z)$ are polynomials, is a soliton; the topological number Q of the soliton (14) assumes only integer values and is equal to the maximal degree of $P_0(z)$, $P_1(z)$. It is convenient to write the general Q-soliton solution (i.e., soliton having topological number Q) in the form

$$W_c(z)=\prod_i\left[\frac{z-z_i}{\delta}\right]^{m_i}\prod_j\left[\frac{\delta}{z-z_j}\right]^{n_j} , \tag{15}$$

where δ, z_i, and z_j are complex parameters.

From Eq. (13), we see that the classical energy of these static soliton configurations, within a given λ, depend only upon the total topological number Q. Thus, at the classical level, the pseudoparticles do not interact. This is a consequence of the fact that the multiple soliton configuration is an exact solution of the equations of motion and $E_N=NE_1=NAJ$.

Stereographic coordinates allow one to generalize the static solutions: in other coordinate systems, such a generalization would be a difficult task. Notice that, although the classical soliton energy depends on the anisotropy parameter, its configuration (14) in stereographic coordinates does not depend on λ. In other coordinate systems, these solutions are not so simple. In general, pseudoparticles have the spin pointing in all different directions as $\mathbf{r}$ varies, and this configuration depends on λ. The spin field of these solutions has been found in Ref. 5 by Watanabe and Otsu for both XY-like anisotropy and Ising-like ones. It was found by specifying how the conformal mapping function W depends on $\theta(\mathbf{r})$ and $\phi(\mathbf{r})$, the two scalar fields in which S can be parametrized: $S(\mathbf{r})=(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. Qualitatively, for the cases of XY-like magnets, each metastable state classified by the topological number Q carries Q vortices and Q antivortices. The Qth inhomogeneous metastable for Ising-like magnets, contains Q locally ordered regions in which Q core spins are antiparallel to the direction of those on the boundary.

Using the maximal degree $\sum m_i>\sum n_j=Q$, we consider now the simplest nontrivial case of topological number one, that is, $m_1=1$, $m_i=0$, $i>1$, $n_j=0$. We have

$$W_c(z)=\frac{z-z_1}{\delta} . \tag{16}$$

The complex parameters δ and z_1 refer to the size and location of the soliton solution. The magnitude of δ gives

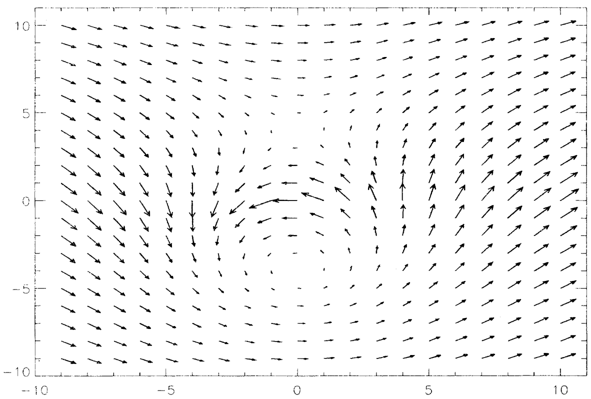

FIG. 1. Spin configuration representing a soliton ($Q=1$) for the XY model ($\lambda=-1$). The vortex is located at $(0,-4)$ and the antivortex at (0,4). The lengths of the arrows are proportional to the spin projection into the XY plane: then, small arrows mean a large out-of-plane spin component [see Eq. (17)].

the extension of the soliton, while the phase of δ gives the rotational orientation of the soliton configuration. As an example, $\lambda=-1$ (XY-like), the spin field with unit topological number is given by[5]

$$\begin{aligned}\phi(\mathbf{r})&=\tan^{-1}\left[\frac{y+|\delta|}{x}\right]-\tan^{-1}\left[\frac{y-|\delta|}{x}\right],\\ \theta(\mathbf{r})&=\cos^{-1}\frac{2y|\delta|^{1/2}}{\{|y|[x^2+(|y|+|\delta|)^2]\}^{1/2}},\end{aligned}\tag{17}$$

where we have chosen $z_1=0$ for the position of the soliton and $\pi/2$ for its phase. The soliton configuration (17) looks like a vortex-antivortex pair with the vortex at $(0,-|\delta|)$ and antivortex at $(0,|\delta|)$ (see Fig. 1). The distance between vortex-antivortex centers is $R=2|\delta|$, but the energy of this configuration is simply

$$E_c=\tfrac{8}{3}\pi J\ ,\tag{18}$$

so classically the vortex and antivortex do not interact in this model. The fact that the solution exists for arbitrary δ and z_1, and the fact that neither Q nor E_c depend on these constants are a consequence of scale and translational invariance of Lagrangian (1). Then, each pseudoparticle can have arbitrarily large size, due to the scale invariance.

The multiple soliton (and antisolitons, obtained by interchanging $z\leftrightarrow\bar{z}$) configurations discussed above are exact finite-energy solutions of the equations of motion and our attention will be focused on them. We have seen that solitons in systems with $-1\le\lambda\le 0$ can be thought as being made up of a meron-antimeron pair (vortex-antivortex). Isolated merons in related anisotropic models have been discussed elsewhere.[11–13] In contrast to the soliton (17), these merons have long-range Coulomb interaction.

III. SMALL OSCILLATIONS ABOUT SOLITON CONFIGURATION

We will examine in this section the time-dependent equation for small disturbances, $\xi(\mathbf{r},t)$, which propagate on the classical background W_c, with topological number $Q=1$. To this end, we write

$$W=W_c+\xi\ ,\tag{19}$$

where the deviation from the classical minimum, ξ, represents the spin-wave mode.

By minimizing the action corresponding to Lagrangian (10) to second order in ξ, we find

$$\nabla^2\xi-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\xi=V_1\xi+V_2\bar{\xi}\ ,\tag{20}$$

where

$$\begin{aligned}V_1=8\partial_z\ln(1+|W_c|^2)&\left[\partial_{\bar z}-\frac{\lambda}{(1+|W_c|^2)}\partial_{\bar z}+\frac{4\lambda|W_c|^2}{(1+|W_c|^2)^2}\partial_{\bar z}+\frac{4\lambda W_c(\partial_{\bar z}\bar{W}_c)}{(1+|W_c|^2)^2}\right]\\ &+\frac{8\lambda\bar{W}_c(\partial_z W_c)}{(1+|W_c|^2)^2}\partial_{\bar z}-\frac{8\lambda(\partial_z W_c)(\partial_{\bar z}\bar{W}_c)}{(1+|W_c|^2)^2}-\frac{8\lambda W_c(\partial_{\bar z}\bar{W}_c)}{(1+|W_c|^2)^2}\partial_z-\frac{2\lambda|W_c|^2}{(1+|W_c|^2)^2}\nabla^2\end{aligned}\tag{21}$$

and

$$V_2=\frac{16\lambda W_c^2}{(1+|W_c|^2)^2}[\partial_z\ln(1+|W_c|^2)\partial_{\bar z}+\partial_{\bar z}\ln(1+|W_c|^2)\partial_z]-\frac{8\lambda W_c(\partial_z W_c)}{(1+|W_c|^2)^2}\partial_{\bar z}-\frac{2\lambda W_c^2}{(1+|W_c|^2)^2}\nabla^2\ .\tag{22}$$

Equation (20) is valid for all possible values of the anisotropy, which appear as a parameter in the potential terms. The two potential operators have a finite range. This implies that the usual continuum states, with energy given by the frequency ω of small oscillations, exist. Writing $\xi(\mathbf{r},t)=\psi_{qn}(\mathbf{r})e^{i\omega t}$, we obtain, in the limit $r\to\infty$, two spin-wave solutions to Eq. (20) with frequencies $\omega(q)=cq$. Thus, the solution of Eq. (20) in the limit $r\to\infty$ results in a superposition of an incoming cylindrical wave and an outgoing phase-shifted cylindrical wave

$$\begin{aligned}\psi_{qn}(\mathbf{r})_{r\to\infty}=\frac{1}{2}\Bigg\{&H^{(1)}_{|n|}(qr)\exp(in\varphi)\\ &+\sum_m\exp[-2i\Delta_{nm}(q)]\\ &\times H^{(2)}_{|n|}(qr)\exp(im\varphi)\Bigg\}\ ,\end{aligned}\tag{23}$$

where we have used cylindrical coordinates (r,φ). Here, $\Delta_{nm}(q)$ is the phase-shift matrix which couples the different angular momentum channels n,m.

It is our purpose to calculate the quantum correction to the classical soliton energy, given by the zero-point energy of the small fluctuations measured with respect to the vacuum. This quantum correction energy depends only upon the diagonal elements[8] of $\Delta_{nm}(q)$ and is obtained by generalizing the arguments used in the semiclassical quantization of solitons in $1+1$ dimensions.[14,15] It is given by[8,16]

$$E=E_c-E_0 \ , \tag{24}$$

where

$$E_0=\frac{\hbar}{\pi}\int_0^{1/a}\left[\frac{\partial\omega}{\partial q}\right]\mathrm{tr}\Delta_{nm}(q)dq \ , \tag{25}$$

and we have introduced a cutoff for the integration, assuming a Debye model for the small oscillations excitations. The value $1/a$ was introduced considering that solid-state systems have a natural length, the lattice constant a, which provides a natural cutoff rendering the theory finite.[17]

Now, consider one-pseudoparticle classical solution $(Q=1)$ given by Eq. (16). Due to the fact that a translation of the origin in Eq. (23) simply transforms the phase shift by unitary transformation $\Delta(q)\rightarrow U^t\Delta(q)U$, under which $\mathrm{tr}\Delta(q)$ is invariant, the quantum corrections do not depend on z_1 and then it is sufficient to consider the problem corresponding to the simpler configuration $W_c(z)=z/\delta$. Substituting this soliton configuration into Eqs. (21) and (22), we note that the potential operator V_1 has cylindrical symmetry while the other potential operator, V_2, does not.

In order to obtain E_0 we have to calculate the sum of the diagonal elements of the phase shift. The corresponding phase shifts depend in a highly nontrivial way on the potential. Due to the complicated form of interaction (20), we are not able to solve exactly this problem. We can, however, find the phase shift using the Born approximation. The first-order Born terms for the diagonal and nondiagonal elements are given, respectively, in this case, by

$$\Delta_n(q)=-\frac{\pi}{2}\int_0^\infty\langle J_{|n|}(qr)\exp(-in\varphi)V_1\exp(in\varphi)J_{|n|}(qr)\rangle_\varphi r\,dr \ , \tag{26}$$

$$\Delta_{nm}(q)=-\frac{\pi}{2}\int_0^\infty i^{(m-n)}\langle J_m(qr)\exp(im\varphi)V_2\exp(-in\varphi)J_n(qr)\rangle_\varphi r\,dr \ , \tag{27}$$

where $\langle\cdots\rangle_\varphi$ denotes an angular average.

First, we will consider $-1\le\lambda<0$ (XY-like anisotropy). In this case, we will use $R=2|\delta|$ representing the separation of a vortex pair. In order to calculate $\mathrm{tr}\Delta_{nm}$, we use $W_c=z/\delta$ to obtain V_1. Thus,

$$\mathrm{tr}\Delta_{nm}(q)=-\frac{\pi}{2}\sum_{n=-\infty}^{\infty}\int_0^\infty\langle J_{|n|}(qr)\exp(-in\varphi)V_1\exp(in\varphi)J_{|n|}(qr)\rangle_\varphi r\,dr \ . \tag{28}$$

After some straightforward but lengthy work, we obtain

$$\mathrm{tr}\Delta_{nm}(q)=-2\pi\lambda\left\{1+\sum_{n=1}^{\infty}\frac{qR}{2}n^2\left[K_{n-1}\left[\frac{qR}{2}\right]I_n\left[\frac{qR}{2}\right]-K_n\left[\frac{qR}{2}\right]I_{n+1}\left[\frac{qR}{2}\right]\right]\right\} . \tag{29}$$

The nondiagonal terms given by Eq. (27) correspond to transitions between the $n,n\pm2$ angular momentum channels for V_2 given by Eq. (22) and $W_c=z/\delta$. All other transitions are not possible.

In Sec. II, we have seen that the energy of the vortex-antivortex pair configuration (soliton in the XY-like magnet) does not depend on the separation R between them and the vortices classically do not interact.

However, at the quantum level the phase shifts of small oscillations about this configuration are dependent on the distance R between vortices, and in fact by solving Eq. (25) in the limiting case $R\rightarrow0$ (small separation) we shall explicitly demonstrate how interaction between a vortex and an antivortex arises due to quantum effects in the nonlinear σ model with XY-like anisotropy.

In the limit $R\rightarrow0$, we can approximate (29) by

$$\mathrm{tr}\Delta_{nm}(q)\simeq-2\pi\lambda\left[1-\left[\frac{qR}{2}\right]^2\ln\left[\frac{qR}{2}\right]\right] . \tag{30}$$

Inserting Eq. (30) into Eq. (24), we obtain

$$E_{R\rightarrow0}\simeq\frac{4\pi}{3}(3+\lambda)J-\frac{2\hbar c}{a}\lambda\left[\frac{1}{3}\left[\frac{R}{2a}\right]^2\ln\left[\frac{R}{2a}\right]-1\right] \quad (-1\le\lambda<0) . \tag{31}$$

We see that, since $-1\le\lambda<0$, semiclassical quantum corrections lower the classical soliton energy in the XY-like anisotropy and induce an effective interaction potential between vortices in a pair. Then, small fluctuations about the static soliton configuration of the 2D nonlinear σ model with XY-like anisotropy induce an internal degree of freedom for each soliton. Quantization of these solitons breaks the static scale invariance and gives a preferred soliton size, that is, a preferred distance R_0 between the vortices in a soliton. It is obtained using $dE/dR=0$, which leads to $R_0\sim a$. Hence, this effective interaction potential is repulsive at very short range $(R<R_0)$ and attractive for $R>R_0$.

Equation (31) is also valid for the Ising-like anisotropy. In this case, we must substitute $R/2$ by $|\delta|$, where $|\delta|$ gives the soliton's size. Thus, for the Ising-like anisotropy, the quantum correction to the classical soliton energy is

$$E_{|\delta|\to 0} \simeq \frac{4\pi}{3}(3+\lambda)J - \frac{2\hbar c}{a}\lambda\left[\frac{1}{3}\left[\frac{|\delta|}{a}\right]^2 \ln\left[\frac{|\delta|}{a}\right] - 1\right] \quad (\lambda > 0) . \qquad (32)$$

Then, semiclassical quantum corrections raise the classical soliton energy in the Ising-like anisotropy. We remark that Eq. (32) has the same functional dependence with $|\delta|$ as the soliton energy calculated by Kosevich[18] in easy-axis two-dimensional ferromagnets, i.e., the energy increases, as soliton size increases, as $|\delta|^2 \ln|\delta|$.

IV. DISCUSSION

We have shown how linearized small oscillations about the static soliton in the 2D anisotropic σ model modify the energy of these configurations. Two types of anisotropy have been studied: Ising-like and XY-like. Particularly, in XY-like magnets, such contributions lower the classical energy of the pseudoparticles and induce an effective interaction between the vortices in a soliton, whose form, in the limit of small separations, is given by

$$V(R)_{R\to 0} \sim -\frac{2}{3}\lambda\frac{\hbar c}{a}\left[\frac{R}{2a}\right]^2 \ln\left[\frac{R}{2a}\right] \quad (-1 \le \lambda < 0) . \qquad (33)$$

This potential is attractive for $R > R_0$ and the dependence in R reflects the structure of the soliton (see Fig. 1), since $\ln R$ is due to in-plane spin components[19] and R^2 is the modification induced in the logarithmic interaction due to out-of-plane spin components. Solitons of size $R_0 \simeq a$ are energetically favorable, and two vortices in a soliton separated by R_0 represent a configuration of stable equilibrium. Considering the motion of the vortices separated by R, with $R \simeq R_0 + r$ where $|r| \ll R_0$, we can approximate the force between the vortices in a pair as

$$F \sim -\frac{2}{3}|\lambda|\frac{\hbar c}{a^2}\left[\frac{3}{2} - \ln 2\right] r \quad (-1 \le \lambda < 0) , \qquad (34)$$

leading to a simple harmonic motion.

Since a Q-soliton configuration can be thought of as being made up of Q vortices and Q antivortices, we can expect from results obtained here that quantum fluctuations also induce effective interactions between solitons. For the isotropic case, some authors[8,20] have extracted the interactions between pseudoparticles due to quantum fluctuations.

For Ising-like magnets, we find that quantum corrections raise the classical energy of the soliton. In particular, the energy increases as soliton size increases.

Of course, our discussion of the quantization of solitons in the anisotropic σ models has been far from rigorous. We have used the Born approximation, valid at long wavelengths for small solitons. In this regime, multiple scattering effects are not so important, but these effects must be investigated in the large-soliton regime in order to see if our calculations remain valid. Nevertheless our discussion has been at least "suggestive" and we hope that the calculation presented in this paper may be useful not only for the quantization procedures investigated here but also for use in perturbation theories[21] involving soliton response to external perturbations or forces, as well as statistical mechanics. As has been shown by Currie *et al.*,[15] the phase-shift interaction between small oscillations and solitons provides the sharing mechanism of energy and degrees of freedom among the nonlinear excitations of the system and therefore is important in the study of the statistical mechanics of the model.

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