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Generation of ultrasonic waves by ac magnetic fields in the mixed state of high-T_c superconductors

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We discuss the interaction of a vortex array in a superconductor with sound oscillations, taking into account pinning forces between vortices and crystal displacements. The generation of ultrasound by an ac electromagnetic field acting on the vortex array is considered. We find that at low temperatures and in the linear regime the amplitude of the induced sound wave is proportional to the applied dc magnetic field and independent of pinning strength. We show how the subsequent multiple echo amplitudes allow one to obtain information on the Magnus force and viscosity of the vortex array.

In a recent experiment, Haneda and Ishiguro observed generation of ultrasonic waves in the mixed state of high-T_c superconductors induced by the motion of vortices subjected to an ac magnetic field.[1] In a polycrystalline $YBa_2Cu_3O_{7-\delta}$ (YBCO) sample under a dc magnetic field, a pulse of ac magnetic field was generated with a coil attached to one end of the sample, see Fig. 1. After the supply of the ac pulse, acoustic waves were detected by a quartz transducer attached at the other end of the sample. Also the echo of the acoustic signal was detected by the coil as an ac magnetic field induced by the returning acoustic wave. These effects arise because an ac electromagnetic field excites vortex oscillations, and the motion of vortices and sound waves are coupled through pinning mechanisms.[2,3] Therefore, as pointed out by Haneda and Ishiguro, these measurements may provide new means to study the pinning and dynamics of vortices. In Ref. 1 a semiquantitative theoretical discussion of the experimental results was given, in the framework of the nonlinear regime of vortex motion. In this work we provide a theoretical treatment of these effects in the linear regime. We calculate the induced acoustic wave and its echo for a given ac magnetic pulse, as a function of pinning, dc magnetic field, and thermally activated vortex relaxation. We provide a criterion for the applicability of the linear regime using results of Ref. 4. We also show how the measurement of multiple echo signals may provide information on the propagation of sound in the mixed state, i.e., on sound velocity, attenuation, and the rotation of the polarization plane, as discussed in Ref. 3.

In the linear regime, the equation of motion for vortex displacements $\mathbf{v}(\mathbf{r},\omega)$ slowly varying on distances of the order of the London penetration depth λ_L is[3]

$$-i\omega\eta_v\mathbf{v}-i\omega\alpha_M\mathbf{v}\times\hat{\mathbf{n}}=C_{11}\frac{\partial^2\mathbf{v}}{\partial x^2}+\alpha_p(\mathbf{v}-\mathbf{u}), \qquad (1)$$

for the compression mode, where $C_{11}=B^2/4\pi$ is the compression modulus, $\hat{\mathbf{n}}=\mathbf{B}/B$, $\mathbf{B}$ is the applied dc magnetic field, and the x axis is perpendicular to the sample surface and along the direction of sound propagation, see Fig. 1. The compression mode is excited for $\mathbf{B}\perp\hat{\mathbf{x}}$ (i.e., $\hat{\mathbf{n}}=\hat{\mathbf{z}}$). For $\mathbf{B}\|\hat{\mathbf{x}}$, the transverse mode is excited by the ac magnetic field, in this case in Eq. (1) C_{11} should be replaced by C_{44}, which is also $B^2/4\pi$ in the limit of slowly varying $\mathbf{v}(\mathbf{r},\omega)$.[5] We use here $\mathbf{v}(\mathbf{r},\omega)$ which is the Fourier transform of $\mathbf{v}(\mathbf{r},t)$ to describe vortex displacements. In Eq. (1) the parameter η_v characterizes vortex dissipation and α_M is the Magnus force coefficient. We consider in (1) the dissipation term given by $i\omega\eta_v\mathbf{v}$ only. This is valid in the high-frequency regime where electrons are weakly involved in crystal displacements, and the dissipation is therefore determined by the velocity of vortices with respect to the electronic system. Note that the Magnus force mixes the components of $\mathbf{v}$ that are perpendicular to $\mathbf{B}$.

The term $\alpha_p(\mathbf{v}-\mathbf{u})$ was introduced by Pankert[2] to describe the interaction of sound waves with vortices because of pinning in the thermally activated flux flow regime (TAFF). In the absence of sound waves ($\mathbf{u}=0$), Eq. (1) with

$$\alpha_p=\alpha_L(1-i/\omega\tau_T)^{-1} \qquad (2)$$

was used by many authors, see Refs. 4, 6, and 7. Here α_L is the Labusch constant and τ_T is the relaxation rate which takes into account thermally activated hopping of vortices between different pinning centers. Using heuristic arguments, Brandt[6] obtained

$$\tau_T=(\eta_v/\alpha_L)\exp[U(B)/T], \qquad (3)$$

where $U(B)$ is the characteristic pinning potential barrier. A similar expression was obtained by Coffey and Clem for a periodic pinning potential.[7] The generalization made by Pankert is transparent: replacement of $\mathbf{v}$ by $(\mathbf{v}-\mathbf{u})$ accounts

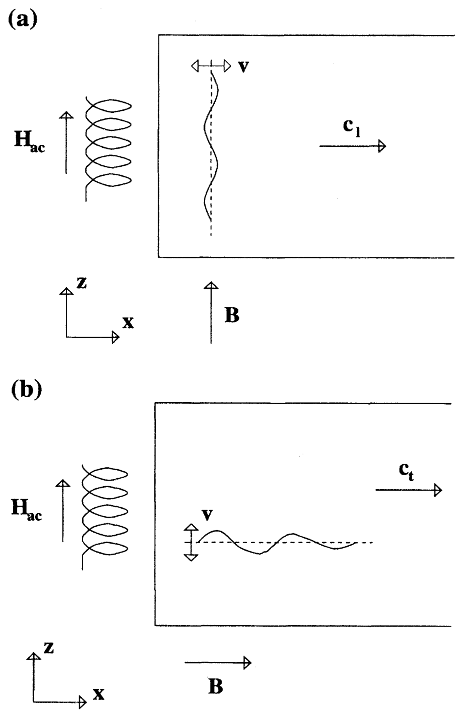

FIG. 1. Configurations of external magnetic field: (a) $\mathbf{B}\|\mathbf{H}_{ac}$, generation of longitudinal sound waves propagating with velocity $\mathbf{c}_l$; (b) $\mathbf{B}\perp\mathbf{H}_{ac}$, generation of transversal sound waves propagating with velocity $\mathbf{c}_t$. The direction of the induced displacements $\mathbf{v}$ in the vortex array are shown.

for the absence of pinning when vortices and ions move with the same velocity. We will use the same term, $\alpha_p(\mathbf{v}-\mathbf{u})$, with $\alpha_p=\alpha_L$ below the irreversibility line as well, thus neglecting jumps of vortices between pinning centers in the vortex-glass phase. The Labusch parameter may be expressed in terms of the critical current J_c as $\alpha_L=J_cB/cr_p$, where r_p is the pinning interaction range ($\approx\xi_{ab}$ in high-T_c superconductors).[4] For $Bi_2Sr_2CaCu_2O_8$ (Bi-2:2:1:2), the critical current is in the range $(5\times10^5-5\times10^6)$ A/cm^2, at helium temperature in fields of several T,[8] and in YBCO the critical current is of the same order of magnitude. Therefore we estimate $\alpha_L\Phi_0/B$ to be in the interval $(5\times10^4-5\times10^5)$ g/cm s^2. From the resistivity data[2] in this system in the TAFF regime $U\approx500$ K.

As was mentioned previously, Eq. (1) is valid at distances larger than λ_L from the sample surface at $x=0$. The vortex displacements produce an ac magnetic induction $B_{ac,z}=B\,\mathrm{div}(\mathbf{v})$ for $\mathbf{B}\|\hat{\mathbf{z}}$, and $\mathbf{B}_{ac}=B\,\partial\mathbf{v}/\partial x$ for $\mathbf{B}\|\hat{\mathbf{x}}$. At the boundary we should have $B_{ac,z}(0,t)=h_{ac}(t)$, where $h_{ac}(t)$ is the ac magnetic field produced by the coil. Thus we obtain the boundary condition for $\mathbf{v}(\mathbf{r},t)$:

$$\frac{\partial v}{\partial x}=\frac{h_{ac}(t)}{B},\quad x=0, \tag{4}$$

with $v=v_x$ for $\mathbf{B}\|\hat{\mathbf{z}}$ and $v=v_z$ for $\mathbf{B}\|\hat{\mathbf{x}}$.

In this approach we do not consider the region of length λ_L near the surface. As we see from Eq. (1), the characteristic length of variation of $\mathbf{v}$ is the ac penetration depth $\lambda_{ac}=(B^2/4\pi\alpha_p)^{1/2}$ which is larger than λ_L at fields $B\gtrsim1$ T.[4,6]

The equation for crystal displacements is

$$-\rho\omega^2\mathbf{u}-i\omega D\mathbf{u}=\rho c_s^2\frac{\partial^2\mathbf{u}}{\partial x^2}+\alpha_p(\mathbf{u}-\mathbf{v}), \tag{5}$$

where c_s is the sound velocity, $c_s=c_l$ for a longitudinal mode (excited when $\mathbf{B}\perp\hat{\mathbf{x}}$) and $c_s=c_t$ for a transversal mode (excited when $\mathbf{B}\|\hat{\mathbf{x}}$), and ρ is the crystal mass density. The term with the coefficient $D=\eta_0+\eta_q$ accounts for sound dissipation in the absence of vortices (η_0) and sound dissipation caused by quasiparticles inside the vortex core (η_q). Here, and in (1), we have neglected electromagnetic interaction forces, which are relevant above the irreversibility line, see Ref. 3. The boundary condition for the crystal displacements is

$$\frac{\partial\mathbf{u}}{\partial x}=0,\quad x=0, \tag{6}$$

for a free crystal surface, in the absence of external forces acting on the surface.

Using Eqs. (1) and (5), we will now describe the following processes.

(1) Under the magnetic ac pulse, displacements of vortices are induced. At this stage the crystal displacements u are much smaller than v, and the solution of (1) with the boundary condition (4) for a semi-infinite sample is direct; see the analysis of Brandt[6] and van der Beek *et al.*[4]

(2) Once the displacements v are found, we can solve Eq. (5) for u, with $\mathbf{v}(\mathbf{r},t)$ as a drive term and with the boundary condition (6), to obtain the amplitude of the induced sound pulse.

(3) This sound wave will propagate along the crystal, it will reach the other side of the sample where it can be detected by a quartz transducer, and where it will be reflected producing an echo propagating back to the originating side. This wave propagation was described with the modified sound equations (1) and (5) in Ref. 3.

(4) The returning sound echo induces displacements of vortices of magnitude v_e, which in turn cause an ac magnetic field that can be detected in the coil.

Let us discuss these processes in the low-temperature Campbell regime,[4] where we can take $\alpha_p=\alpha_L$. To describe the first process we can drop the Magnus and viscous force terms in (1) since $\alpha_L/\omega\gg\eta,\alpha_M$. Then, taking into account that here $u\ll v$, we have[4,6]

$$v(x,t)=\frac{Bh_{ac}(t)}{4\pi\lambda_{ac}\alpha_L}e^{-x/\lambda_{ac}}. \tag{7}$$

In a second step, we find the amplitude of the generated sound, solving Eq. (5), which reduces to the wave equation

$$\frac{\partial^2u}{\partial t^2}-c_s^2\frac{\partial^2u}{\partial x^2}=\frac{\alpha_L}{\rho}v(x,t). \tag{8}$$

The solution of (8) is given by the sum of the propagating wave $f(x-c_st)$ (solution of the homegeneous equation with $v=0$), plus the solution $g(x,t)$ of the inhomegneous wave equation. The latter is

$$g(x,t)=-\frac{B\lambda_{ac}}{4\pi\rho c_s^2}h_{ac}(t)e^{-x/\lambda_{ac}}, \quad (9)$$

where we have taken into account that $\omega \ll c_s/\lambda_{ac}$, which is fulfilled for the typical magnetic fields and frequencies in the experiments. From the boundary condition $\partial u/\partial x =\partial(f+g)/\partial x=0$ at $x=0$, we finally get for the propagating wave,

$$f(x)=\frac{B}{4\pi\rho c_s}\int_{t_0}^{-x/c_s}h_{ac}(t)dt, \quad (10)$$

with t_0 the starting time of the initial ac pulse. Now we verify that $u(x,t)\ll v(x,t)$ by noting that $u/v\approx\lambda_{ac}^2/\alpha_L\rho c_s^2=B^2/\rho c_s^2\ll 4\pi$, for reasonable magnetic fields. This allowed us to neglect u in Eq. (1) when we obtained the induced $v(x,t)$ in Eq. (7), and also in Eq. (8).

We see in Eq. (10) that the amplitude of the sound pulse reaching the other side of the sample is

$$f_0\approx\frac{Bh_0}{\rho c_s\omega}, \quad (11)$$

where h_0 is the amplitude of the ac magnetic pulse. Note that the sound amplitude is proportional to B and h_0 and it does not depend on pinning in the Campbell regime.

Now we consider the third step, propagation of sound through the sample. During propagation, the sound waves change their polarization and amplitude due to the Magnus force and the dissipation, as was discussed in Ref. 3. This effect will be evident when measuring the multiple echoes of propagating sound. For transverse sound, when the magnetic field $\mathbf{B}$ is along the $\hat{\mathbf{x}}$ direction which coincides with the c axis of the uniaxial crystal, we obtain the maximum effect for the rotation of the sound polarization (acoustic Faraday effect). In this case the amplitude of the nth echo is

$$f_n=\frac{Bh_0}{\rho c_s\omega}\left|\cos\left(\frac{\alpha_M\ell n}{\rho c_s}\right)\right|\exp\left(-\frac{2\tilde{D}\ell n}{\rho c_s}\right), \quad (12)$$

where ℓ is the length of the sample, and $\tilde{D}=D+\eta_v$. The cosine term is due to the rotation of the polarization angle. A similar acoustic Faraday effect was observed previously in ferromagnetic crystals because of the interaction between spin waves and sound waves (magnetostriction effect).[9] Here the effect will be notable at low temperatures in the superclean regime where $\eta_v \lesssim \alpha_M$ (see Ref. 10) and D is small enough. When the magnetic field $\mathbf{B}$ is along the $\hat{\mathbf{z}}$ direction, the propagating sound is longitudinal, the rotation of polarization is negligible, and the multiple echo amplitude f_n is given by (12) without the cosine term, see Ref. 3.

Finally, when the nth sound echo comes back to the originating side, it induces vortex displacements $v_e(x,t)\approx f_n(x+c_st)$. This is because in Eq. (1) α_L is the largest parameter at low temperatures, i.e., pinning centers almost completely involve vortices. The vortex displacements now induce an ac magnetic field with amplitude

$$B_{ac}^{(n)}=B\left|\frac{\partial v_e^{(n)}}{\partial x}\right|\approx\frac{h_0B^2}{\rho c_s^2}\left|\cos\left(\frac{\alpha_M\ell n}{\rho c_s}\right)\right|\exp\left(-\frac{2\tilde{D}\ell n}{\rho c_s}\right). \quad (13)$$

The signal in the coil is proportional to $B_{ac}^{(n)}$.

The condition for the linear regime is that the displacements of vortices at the surface (where they are maximum) are much smaller than the typical radius of the pinning centers. The latter is of the order of the correlation length ξ, and therefore we have the condition $|v(0,t)|\ll\xi$. We obtain for the amplitude of the electromagnetic pulse $h_0 \ll(4\pi J_c\xi B/c)^{1/2}$. For critical currents $J_c\approx 10^6$ A/cm this gives $h_0\ll 50(B/[1\ \mathrm{T}])^{1/2}$ G which was obtained also in Ref. 8.

In the measurements of electromagnetic generation of ultrasound in YBCO by Haneda and Ishiguro,[1] an ac magnetic field with amplitude $h_0=50$ G was used at a temperature $T=13$ K and dc magnetic fields up to 5 T. At $T=13$ K these conditions correspond to the linear Campbell regime. The amplitude of the induced sound wave was found to be linear in B from 1 T up to 5 T, in agreement with our result on Eq. (11).

At high temperatures, when we cross the irreversibility line, α_L should be replaced by the complex parameter α_p. The signals in the transducer and in the coil do not depend on λ_{ac}, and thus they do not depend on α_p either. Therefore as long as we stay in the linear regime, i.e., $|v(0,t)|\ll\xi$, our results still hold at high T. However, near the irreversibility line, $|\alpha_p|$ disminishes roughly by a factor $\omega\tau_T$, and therefore v increases. The region of validity of the linear regime shrinks here, as was discussed in Ref. 4.

Here we have considered standard weak pinning centers. For pinning centers produced by irradiation (columnar or splayed defects) our treatment in Eq. (1) is invalid. In this case the restoring force for a vortex pinned inside a defect made of insulating material is given as $F\mathrm{sgn}(\mathbf{v}-\mathbf{u})$ instead of $\alpha_p(\mathbf{v}-\mathbf{u})$ in Eq. (1). (Here $\mathbf{v}$ is accounted from the edge of a defect.) This force reamains nonzero when $|\mathbf{v}-\mathbf{u}|\to 0$, see Ref. 11. Then a weak ac field does not induce vortex displacements and sound waves for magnetic fields B below the matching field B_Φ, since almost all the vortices are sitting inside the strong irradiation defects. For $B>B_\Phi$ a fraction $(B-B_\Phi)/B$ of vortices is pinned by weak pinning centers, and our previous results with B replaced by $B-B_\Phi$ apply.

In conclusion, in unirradiated superconductors in the linear regime the amplitude of the generated sound wave does not depend on pinning strength. Therefore in the experiments discussed we can only obtain information about the sound propagation in the presence of vortices, which includes the vortex viscosity η_v and the Magnus force coefficient α_M. However, in the nonlinear regime the amplitude of the sound wave may depend on the pinning mechanisms. In irradiated samples the generation of sound is determined by the vortices which are not trapped by the strong pinning centers produced by irradiation.

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