Enhanced inelastic backscattering of electrons from disordered media

E. Kanzieper
The Jack and Pearl Resnick Institute of Advanced Technology, Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel
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The backscattering of electrons undergoing single inelastic collision in disordered medium is considered. It is shown that apart from the usual weak-localization correction to the backscattering angular spectrum, an additional backscattering enhancement should exist. In contrast to the weak-localization correction, the angular width of the new interference correction is found to be about tens of degrees.

Inelastic quantum transport of the fast (in comparison with conduction electrons) charged particles in the disordered media has recently attracted attention.[1–4] In these works the angular spectra of electrons of intermediate energies (from hundreds to thousands of eV) reflected by a disordered sample with fixed energy loss $\hbar\omega$ owing to a *single inelastic collision* were considered. It has been established that, in distinction to an elastic scattering channel, two types of interference corrections arise to the backscattering angular spectra in inelastic channel.

First, the *weak localization correction*[1] was shown to remain under the condition of strong interference $\omega \ll \gamma$ (here γ is the frequency of electron collisions) and be almost as pronounced as in the pure elastic backscattering. This kind of the coherent angular feature is connected with the diffusive nature of electron motion in a medium and can be interpreted as a result of the interference of particle wave fields associated with different realizations of scattering processes in an inelastic channel. Namely, for arbitrary particle trajectory T that involves at least double elastic scattering along with a single inelastic collision, one can always find such a complementary trajectory T^* (or set of trajectories) which will interfere constructively with a given trajectory T in the narrow vicinity of backward direction: $T \Longleftrightarrow T^*$. Thus, if we consider for simplicity the scattering processes which start or end with single inelastic collision i and include double elastic scattering on the scatterers e_1 and e_2, then four possibilities exist to represent the propagation of electron in the medium: $i-e_1-e_2$, $i-e_2-e_1$, e_1-e_2-i, and e_2-e_1-i. In this approach the pairs of trajectories $(i-e_1-e_2) \Longleftrightarrow (e_2-e_1-i)$ and $(i-e_2-e_1) \Longleftrightarrow (e_1-e_2-i)$ with reversed consequences of scattering events will interfere near the backscattering situation, giving rise to the weak localization correction. This assertion is true for any relationship between the characteristic wave vector q_c of inelastic excitation and the electron mean free path l. It can be shown[1] that the interference of trajectories $(i-e_1-e_2) \Longleftrightarrow (i-e_2-e_1)$ and $(e_1-e_2-i) \Longleftrightarrow (e_2-e_1-i)$ contributes to the weak localization correction only when $q_c l \ll 1$.

Second, the *wide-angle correction* to the backscattering angular spectra was discovered in Ref. 2 for the first time.[5] It was shown that wide-angle correction arises at some scattering angles which are determined by the nature of the inelastic collision. The angular width of this correction is much larger than the width of the correction connected with usual weak localization. Moreover, unlike the weak-localization correction, for which the role of the multiple elastic scattering is crucial, the wide-angle correction was already found to be manifested in the approach that only involves single elastic and inelastic collisions and interference between them: $(i-e) \Longleftrightarrow (e-i)$. This mean that a relevant correction can even exist in the absence of diffusionlike motion of electrons in the medium. Recently an attempt was made to analyze the influence of multiple elastic scattering on the wide-angle correction.[3]

Hitherto weak localization[1,4] and wide-angle[2,3] quantum corrections were treated separately in the different approximations, whereas they can comprehensively be studied in the framework of common formalism. Below we shall consider both corrections jointly in the limit of strong interference, $\omega \ll \gamma$, when they are emphasized most strongly.

In order to do this we shall use the formalism developed in Ref. 1 in the approximation of isotropic elastic scattering. If the characteristic wave vector q_c transferred in inelastic collision is much larger than the inverse electron mean free path l^{-1}, $q_c l \gg 1$, and much smaller than electron wave vector $k, q_c \ll k$, three terms contributing to the angular spectrum of electrons reflected with energy loss $\hbar\omega$ from the disordered sample, which occupies half-space $z>0$, can be extracted from Eq. (38) of Ref. 1. The first term

$$J_{\mathrm{inc}}(\mu_0 \to |\mu|,\omega) = \frac{\pi}{v l_{\mathrm{el}}} \left[\frac{\hbar^2}{m}\right]^2 \int \frac{d\mathbf{q}_\parallel dq_z}{(2\pi)^3} W_{\mathrm{inel}}(\mathbf{q}_\parallel, q_z, \omega) \int\int dz dz' [\delta(z-z') + \Gamma^{\mathrm{el}}(0,z,z')] |\Psi_{\mathbf{k}_\omega^{(-)}}(z)|^2 |\Psi_{\mathbf{k}}(z')|^2 \times [F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v},z',z') + F_{\omega\mathbf{q}}(\mathbf{v}',\mathbf{v}',z,z)] \tag{1}$$

contains no coherent effects and arises from single inelastic and multiple elastic collisions, which are strictly successive relative to each other (ladder approximation). The second term

$$J_{\rm wide}(\mu_0\to|\mu|,\omega)=-\frac{2\pi}{vl_{\rm el}}\left[\frac{\hbar^2}{m}\right]^2{\rm Re}\int\frac{d\mathbf{q}_\parallel dq_z}{(2\pi)^3}W_{\rm inel}(\mathbf{q}_\parallel,q_z,\omega)\int dz\,|\Psi_{\mathbf{k}_\omega^{(-)}}(z)|^2|\Psi_{\mathbf{k}}(z)|^2F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}',z,z) \tag{2}$$

corresponds to the wide-angle correction and results from the interference of single elastic and inelastic collisions only. This circumstance emphasizes the nondiffusive origin of $J_{\rm wide}$ in the present case. The third term

$$J_{\rm weak}(\mu_0\to|\mu|,\omega)=-\frac{2\pi}{vl_{\rm el}}\left[\frac{\hbar^2}{m}\right]^2{\rm Re}\int\frac{d\mathbf{q}_\parallel dq_z}{(2\pi)^3}W_{\rm inel}(\mathbf{q}_\parallel,q_z,\omega)\int\int dzdz'\Gamma^{\rm inel}(\mathbf{k}_\parallel+\mathbf{k}'_\parallel,z,z')\Psi^*_{\mathbf{k}_\omega^{(-)}}(z)\Psi_{\mathbf{k}_\omega^{(-)}}(z')$$
$$\times\Psi^*_{\mathbf{k}}(z)\Psi_{\mathbf{k}}(z')F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}',z',z') \tag{3}$$

has weak localization nature, $kl\gg1$. In Eqs. (1)–(3) we have use notations from Ref. 1. Here $\mathbf{v}$ and $\mathbf{v}'$ [$\mathbf{k}=(\mathbf{k}_\parallel,k_z)$ and $\mathbf{k}_\omega^{(-)}=(\mathbf{k}'_\parallel,-|k'_z|)$] are the velocities (wave vectors) of incident and inelastically reflected electrons, respectively, μ_0 and μ are cosines of incidence and emission angles, $l_{\rm el}$ is the electron elastic mean free path, $W_{\rm inel}(\mathbf{q}_\parallel,q_z,\omega)$ is the probability of inelastic scattering per unit time with wave-vector transfer $\mathbf{q}=(\mathbf{q}_\parallel,q_z)$ and energy loss $\hbar\omega$, $\Psi_{\mathbf{k}}(z)=\exp(ik_zz)\exp(-zk/2|k_z|l)$, and

$$F_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}',z,z')=\frac{\exp[-iq_z(z-z')]}{\hbar^2(\omega+\mathbf{vq})(\omega+\mathbf{v}'\mathbf{q})}\times\left[1-\exp\left[iz\frac{\omega+\mathbf{v}'\mathbf{q}}{v'_z}\right]\right]\times\left[1-\exp\left[-iz'\frac{\omega+\mathbf{vq}}{v_z}\right]\right]. \tag{4}$$

$\Gamma^{\rm inel}$ and $\Gamma^{\rm el}=\Gamma^{\rm inel}|_{\omega=0}$ are the so-called diffusion propagators for half-space $z,z'>0$, which for $\omega\ll\gamma=v/l$ can be written as

$$\Gamma^{\rm inel}(\mathbf{Q}_\parallel,z,z')=\frac{3}{2ll_{\rm el}}\frac{1}{\sqrt{\mathbf{Q}_\parallel^2+\beta^2}}\times[\exp(-\sqrt{\mathbf{Q}_\parallel^2+\beta^2}|z-z'|)-\exp(-\sqrt{\mathbf{Q}_\parallel^2+\beta^2}|z+z'|)]. \tag{5}$$

Here

$$\beta^2=\frac{3}{l^2}\left[\frac{l}{l_{\rm inel}}+i\frac{\omega}{\gamma}\right], \tag{6}$$

$l_{\rm inel}$ is the electron inelastic mean free path, and ${\rm Re}\sqrt{\mathbf{Q}_\parallel^2+\beta^2}>0$. Equations (1)–(6) take into account the surface effect on electron motion in a disordered sample and describe both wide-angle and weak-localization corrections.

As regards the *weak localization correction* it has already been shown[1] to manifest itself in the case of *strong interference*, $\omega\ll\gamma$, and be displaced from an "exactly backward" direction at the oblique incidence of electron on a disordered sample surface in accordance with the equation $|\mathbf{k}_\parallel+\mathbf{k}'_\parallel|=0$. As in the case of pure elastic scattering the backscattering enhancement is more pronounced in the weakly absorbing samples.

To explore the properties of the *wide-angle correction* in the limit of *strong interference,* let us compare $J_{\rm wide}$ with the contribution of that part $J_{\rm sing}$ of Eq. (1), which contains $\delta(z-z')$, and therefore is associated with single elastic collision. Integration over z in Eqs. (1) and (2) and elementary algebraic transformations yield

$$J_{\rm sing}(\mu_0\to|\mu|,\omega)=J_0{\rm Re}\int\frac{d\mathbf{q}}{(2\pi)^3}W_{\rm inel}(q,\omega)[M_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v})+M_{\omega\mathbf{q}}(\mathbf{v}',\mathbf{v}')], \tag{7}$$

$$J_{\rm wide}(\mu_0\to|\mu|,\omega)=-J_0{\rm Re}\int\frac{d\mathbf{q}}{(2\pi)^3}W_{\rm inel}(q,\omega)[M_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}')+M_{\omega\mathbf{q}}(\mathbf{v}',\mathbf{v})]. \tag{8}$$

Here

$$M_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}')=\frac{1}{v_zv'_z}\left[\left[\frac{\omega+\mathbf{vq}}{v_z}\right]-\frac{i}{\bar\mu l}\right]^{-1}\times\left[\left[\frac{\omega+\mathbf{v}'\mathbf{q}}{v'_z}\right]-\left[\frac{\omega+\mathbf{vq}}{v_z}\right]+\frac{i}{\bar\mu l}\right]^{-1}, \tag{9}$$

$J_0=2\pi\bar\mu\hbar^2l/vm^2l_{\rm el}$, and $\bar\mu=|\mu|\mu_0/(|\mu|+\mu_0)$.

The angular distribution $J_{\rm sing}$ contains angular dependence caused by surface influence only, whereas $J_{\rm wide}$, because of $M_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}')$, yields to the backscattering spectrum an additional dependence on scattering angle $\chi=\widehat{\mathbf{v},\mathbf{v}'}$ due to the interference of single elastic and inelastic collisions if the inequality $q_cl\gg1$ is fulfilled. This dependence on scattering angle χ remains even in the infinite disordered medium and is defined by the nature of single inelastic excitation.[2] In the opposite limit, $q_cl\ll1$, this nontrivial angular dependence on χ disappears and only dependence remains, which is associated with surface influence. [Really, dependence on χ results from the second brackets in Eq. (9) because of the term $(\mathbf{v}'\mathbf{q})/v'_z-(\mathbf{vq})/v_z$. As $\omega l/v\ll1$ one can disregard the terms ω/v_z and ω/v'_z as compared to the imaginary quantity $i/\bar\mu l$. The remaining terms $(\mathbf{vq})/v_z$ and $(\mathbf{v'q})/v_z'$ can be neglected as well due to the inequality $q_c l \ll 1$.] Therefore in the limit $q_c l \ll 1$ there is no sense in distinguishing $J_{\rm wide}$ from the total spectrum, and the concept of wide-angle correction loses its meaning.

Equations (7)–(9) allow us to estimate relative contributions of $J_{\rm wide}$ and $J_{\rm sing}$ in the backward direction. Since from Eq. (9) it follows that

$$M_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}'=-\mathbf{v}) \simeq -M_{\omega\mathbf{q}}(\mathbf{v},\mathbf{v}) \left[1+i\frac{\omega}{\gamma}\right]^{-1} \tag{10}$$

and

$$M_{\omega\mathbf{q}}(\mathbf{v}'=-\mathbf{v},\mathbf{v}) \simeq -M_{\omega\mathbf{q}}(-\mathbf{v},-\mathbf{v}) \left[1-i\frac{\omega}{\gamma}\right]^{-1}, \tag{11}$$

one immediately obtains from Eqs. (7) and (8) that in the strong interference limit in the backward direction $(\chi=\pi)$

$$J_{\rm wide}(\chi=\pi,\omega) \simeq J_{\rm sing}(\chi=\pi,\omega) \ . \tag{12}$$

Thus, under the condition of strong interference $J_{\rm wide}$ contributes in the backward direction as large as $J_{\rm sing}$, reaching its maximal value. This means that in the strong interference limit the maximum of wide-angle correction lies in the *backward* direction.

Of course, to be observable, the wide-angle correction, associated in the scattering regime $q_c l \gg 1$ with single elastic collision, must be comparable with the contribution $(J_{\rm inc}-J_{\rm sing})$ of multiply scattered electrons. It is well revealed that absorption decreases the contribution of scattering processes of high multiplicity into backscattering intensity (see, for example, Ref. 6). Therefore appreciable absorption is needed for observation of the wide-angle correction. On the contrary, in weakly absorbing samples the multiple elastic scattering will diminish the relative contribution of $J_{\rm wide}$ to the full angular spectrum of inelastically reflected electrons.

In order to illustrate the conclusions obtained we turn to the results of numerical calculations based on the integration of Eqs. (1)–(6) under the realistic assumption $W_{\rm inel}(q,\omega) \sim q^{-2}\Theta(q_m-q)$ (here q_m is a maximal wave vector of inelastic excitation). We have computed the enhancement factor $\eta_{\rm sing}(\mu_0 \to |\mu|,\omega) = 1 + J_{\rm wide}/J_{\rm sing}$ in the simplest approximation of single elastic collision and the enhancement factor $\eta_{\rm mult}(\mu_0 \to |\mu|,\omega) = 1 + (J_{\rm weak}+J_{\rm wide})/J_{\rm inc}$ taking into account multiple elastic scattering. This allows us to trace the effect of multiple scattering on the interference corrections.

For the case of *weakly absorbing samples*, $l_{\rm inel} \gg l_{\rm el}$, the results are plotted in Fig. 1 for different angles of electron incidence on the surface of disordered sample. In this case the behavior of both corrections is defined by the decisive role of multiple elastic scattering in the formation of the angular distribution of electrons backscattered by weakly absorbing sample. Thus, the weak-localization correction is strongly pronounced. Comparison of curves for $\eta_{\rm sing}$ with those for $\eta_{\rm mult}$ shows that in this scattering regime the wide-angle correction is practically suppressed by the multiply scattered electrons.

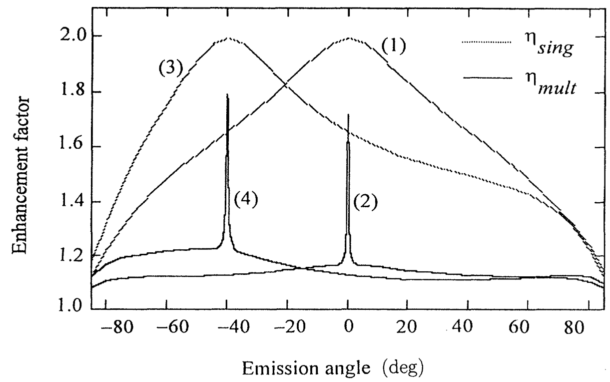

FIG. 1. Enhancement factor as a function of emission angle for normal ($\theta_0=0°$, curves 1 and 2) and oblique ($\theta_0=40°$, curves 3 and 4) incidence. Parameters: $kl=200$, $q_m l=9$, $\omega/\gamma=0.1$, $a=(1+l_{\rm el}/l_{\rm inel})^{-1}=0.95$.

The numerical results for *strongly absorbing samples*, $l_{\rm inel} \le l_{\rm el}$, are presented in Fig. 2. Unlike in the previous case, the wide-angle correction with broad maximum in the backward direction is not suppressed by multiple scattering since in the strong absorption regime the angular distribution of backscattered particles is mainly defined by the scattering processes of low multiplicity. Despite this the weak-localization correction is pronounced as well, but its relative height, being renormalized to the altitude of its pedestal (as it has been done in Ref. 1 and in the theories of backscattering enhancement in an elastic scattering channel), is smaller than in the case of weak absorption.

Let us stress that in the case of strong absorption the relative magnitude of the new coherent effect is appreciable (~ 2), and this result is of most interest. Indeed, the fact that under the above-noted conditions the wide-angle correction $J_{\rm wide}$ has a maximum in the *backward* direction (see Fig. 2) tells it essentially from the incoherent part $J_{\rm inc}$ of angular spectrum, which always has a maximum in the direction *normal* to the surface of the sam-

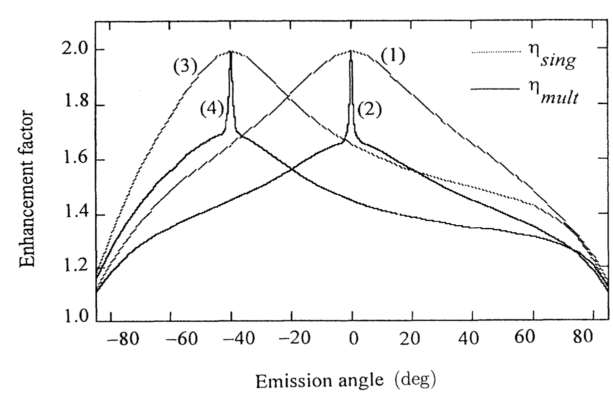

FIG. 2. Enhancement factor vs emission angle, $\omega/\gamma=0.1$, $a=0.40$. Other parameters coincide with those in Fig. 1.

ple. This attests that the phenomenon of backscattering enhancement in the inelastic channel can even take place in the absence of multiple elastic scattering, although the width of the corresponding angular features is about tens of degrees.

Experimentally, such specific backscattering enhancement might be found by means of reflection of electrons of intermediate energies from strongly absorbing amorphous or polycrystalline samples, $l_{\rm inel} \ll l_{\rm el}$. Two angular spectra must be recorded under the condition of oblique incidence of electrons on the surface of the sample: the angular spectrum $J_{\rm el}$ of electrons reflected elastically and the angular spectrum $J_{\rm inel}$ of electrons reflected with small energy loss, $\omega \ll \gamma$, associated with single inelastic collision. Since in the very strongly absorbing samples the multiple elastic collision practically do not give rise to angular distribution of backscattered electrons, the ratio $J_{\rm inel}/J_{\rm el}$ being independent of real elastic electron-atom cross section,[2] must manifest the same angular behavior as does the enhancement factor $\eta_{\rm sing}$ calculated above.

In summary, we have shown that two types of backscattering enhancement exist in the inelastic scattering channel under the condition of strong interference. First, the enhancement, associated with multiple elastic scattering, manifests itself in the backscattering of electrons from weakly absorbing samples (weak localization correction). Second, the enhancement, connected to the interference of single elastic and inelastic collisions, is found to be pronounced in the opposite case of electron reflection from the strongly absorbing samples (wide-angle correction). Both corrections have appreciable magnitude and depend on the nature of the single inelastic collision. In contrast to the weak-localization correction the wide-angle one not only modifies the angular distribution of inelastically backscattered electrons, but also can significantly change the integral cross section of backscattering in the inelastic scattering channel. This circumstance must play an important role in the quantitative analysis of experiments on inelastic backscattering of particles by disordered samples.

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